CONSTRAINTS ON WORMHOLE FORMATION FROM PHANTOM DARK ENERGY IN DESI

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Received September 9, 2025,
revised version October 7, 2025
Accepted for publication October 9, 2025

For dark energy (DE) being a dynamical field, an equation-of-state parameter $w < -1$ leads to the phantom DE state, allowing wormhole (WH) throats to be stabilized effectively. We investigated the possibility of the existence of traversable WHs, whose stability is fully ensured by phantom DE, the dominance of which was recently indicated by the DESI project. Within the framework of the simple Morris–Thorne model, we derived a phenomenological relation connecting the throat radius b with the energy density of the phantom dynamical field $\rho_{\mathsf{ph}}(z)$. This establishes a direct connection between cosmological parameters and the properties of traversable WHs, showing that phantom DE could, in principle, serve as the exotic matter required to sustain WHs with throat sizes spanning from the gravitational radii of stellar-mass BHs and SMBHs up to cosmological scales. We investigated possible WH formation channels and showed extreme suppression of two mechanisms (Euclidean instanton tunneling and thermal fluctuation nucleation). Using gravitational lensing SQLS constraints on Ellis–Bronnikov WHs, we quantified the fraction of phantom energy that can be trapped in such WHs, $f \approx 10^{-11}$, indicating that only a small fraction of the phantom DE can be trapped in WH throats. Overall, our results show both the theoretical consistency and the observational limitations of phantom-supported WHs.

DOI: 10.7868/S3034641X25120078

1. INTRODUCTION

Dark energy (DE), which is responsible, among other things, for the observed accelerated expansion of our Universe, can be described not only with the help of a fundamental physical constant (Λ-term), but also with the help of some field with a certain equation of state.

A promising concept for describing DE is to represent it as dynamic, with an equation of state

$$p = w\rho,$$

where p is the spatially homogeneous DE pressure, ρ is DE energy density, and the equation-of-state (EoS) parameter $w = w(t)$ ($w = -1$ corresponds to a cosmological constant).

In practice, the introduction of the parameter w dependence on time t into cosmological models is carried out by assuming the existence of one or more scalar fields in the Universe [1]. This dependence (or, in the equivalent form, $w = w(a)$), is one of the most promising areas in an attempt to solve the Hubble tension problem (see [2] and references therein).

Despite the fact that null energy condition (NEC) implies $w(t) \geq -1$ to avoid instabilities in DE behavior, the instability timescale can be greater than the age of the Universe [3] (it is possible to avoid the Big Rip scenario). Therefore, from a theoretical point of view one can construct realistic models of DE with $w < -1$. This so-called phantom DE, with positive energy density, negative pressure, and $w < -1$ has been investigated by many authors, [4–8].

It is very important to note that observational limits obtained in the last two decades also do not exclude phantom DE. Limits obtained from CMB data, large scale structure data, HST measurements of Type Ia supernovae, give $-1.62 < w < -0.74$, 95% c.l. [3]. According to Planck data, $w_0 = -1.006 \pm 0.045$ [9].

The result, recently obtained combining the CMB, SDSS, and DESI data [10, 11] could challenge the in-

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terpretation of the present-day accelerated expansion of the Universe within the framework of the standard ΛCDM-model, pointing to a dynamical DE component characterized by present-day quintessence-like behavior ($w > -1$) that transitioned into the phantom-like regime in the past ($w < -1$) in $z \approx 0.3$ (for detailed error bars see [11]). We should immediately note here and emphasize further that error bars in determining the dynamic parameter of the DE state create great uncertainty for predicting the effects associated with phantom energy. Thus, without denying the existence of phantom DE as such, the more subtle effects associated with it require improved accuracy in future experiments.

Due to such significant statistical indications of the presence of the DE in the form of phantom energy for redshifts approximately $z \geq 0.3$, objects whose appearance, stability, and evolution require phantom energy become particularly interesting. First of all, these are number of models of wormholes (WHs), for the stabilization and traversability of which this DE type is preferable. As this is the phantom DE to sustain traversable WH, this DE type presents us with a natural scenario for the existence of these exotic geometries.

The existence of WHs in the context of the presence of phantom energy has been considered by many authors (see [12–15] and references therein). Despite the fact that phantom energy is some kind of a homogeneously distributed fluid, it can be extended to inhomogeneous spherically symmetric spacetimes like WHs. Macroscopic WH could naturally be grown from the submicroscopic constructions that originally pervaded the quantum foam [14]. It was shown in [16] that as a (submicroscopic) WH accretes phantom energy, the radius of the respective throat will gradually increase to macroscopic dimensions. Thus, it seems that as the phantom energy dominates for $z \geq 0.3$, a natural process for the formation and growth of macroscopic traversable WH exists.

Then the WH size could increase by a factor which is proportional to the scale factor of the Universe, and still increases significantly if the cosmic expansion is driven by phantom energy [17]. Thus, supermassive black holes (SMBHs) in the center of galaxies (especially in our own Milky Way), which may turn out to be WH, are beginning to be of particular interest.

Until recently, assuming WH were formed during the period of inflation [18], it turned out to be theoretically possible, in principle, to obtain WH of any size at present time.

Literature provides several possibilities for WH nucleation, such as Euclidean instantons (EI) [19] and Coleman–De Luccia (CDL) Tunneling [20]. Yet the CDL-approach is somewhat arguable to be applied since it requires action to be bound from below, which is not true for a phantom field. The EI formalism is general, but we specialize it to a phantom background for the support of the throat. However, as will be shown in this paper, both scenarios are untenable, at least within the framework of our simple WH model under consideration.

The main purpose of our work is to investigate the possibility of the existence of traversable WH stabilized by phantom DE, the dominance of which has recently been discovered [10].

The paper is organized as follows. In Section 2 we discuss the relation between phantom DE and WH throat conditions. We found the connection of the WH throat conditions with a simple equation-of-state (EoS) of phantom DE to derive a phenomenological relation between the WH throat radius and phantom DE density, assuming it can be responsible for sustaining WHs. In Section 3 we discuss for the WH formation the Euclidean instantons (EI) formalism and Morris – Thorne formalism (instanton-like scaling and WH nucleation as thermal fluctuation). In Section 4 we estimate the fraction of phantom DE needed to support the Ellis – Bronnikov WHs using the SQLS+DESI observational data. Section 5 is discussion. Section 6 is conclusion.

2. PHANTOM DE AND WH THROAT CONDITIONS

Traversable WHs are known to require exotic matter that violates NEC for stabilizing their throats. The Morris – Thorne ansatz [21] provides a canonical example for analyzing such configurations. One of the main properties that describe the Morris – Thorne WH geometry is the shape function $b(r)$, which sets the throat radius and, all in all, determines how strongly the stress-energy tensor (SET) violates the NEC.

For the Morris – Thorne WH [21]

$$ds^2 = -e^{2\Phi(r)}dt^2 + \left(1 - \frac{b(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (1)$$

we know the relation between the SET and the WH shape function $b(r)$. We can then connect the throat conditions to a simple EoS of phantom energy $p = w\rho$ to derive a phenomenological relation between the WH throat radius and cosmological energy density, assuming it can be responsible for sustaining WHs.

Relation between SET and WH throat size b_0 in the framework of NEC-violation can be obtained as follows. The EoS and NEC-violation:

$$p_r = w\rho,$$

$$p_r + \rho = (1+w)\rho < 0.$$

The relation between the pressure p and the shape function, which follows from the field equations, is given by [21]:

$$p = -\tau = -\frac{1}{8\pi G c^{-4} r^2}\,\frac{b}{r},$$

which, after substituting the EoS and replacing r_0 with b_0 at the WH throat, immediately yields

$$b_0 = \left(\frac{1}{8\pi G c^{-4}\rho_{\rm ph}|\omega|}\right)^{1/2}. \qquad (2)$$

Expression (2) is rather a phenomenological scaling, but not an exact solution of the field equations themselves. Under the assumption that the exotic matter supporting the WH throat can be substituted by the homogeneous phantom energy of cosmological origin we conclude that the energy density at the throat ρ is defined by the background value of $\rho_{\rm ph}$. This heuristic scaling provides a bridge between cosmological observations and WH properties and allows for a broad spectrum of WH sizes, being sensitive to the $\rho_{\rm ph}$ dependence on z as well as to the CPL parameters w_a and w_0, where these DE parameters are taken in the form

$$w = w(z) = w_0 + w_a\left(1 - \frac{1}{1+z}\right). \qquad (3)$$

The parameters w_0 and w_a have been estimated and constrained with current observational data in [11].

For this general, time-varying EoS $w(z)$, the standard parametrization of phantom DE leads to the evolution law in the following form [2]:

$$\rho_{\rm ph}(z) = \Omega_{DE}\frac{3H_0^2 c^2}{8\pi G}\exp\left(-3\int\limits_{0.3}^{z}\frac{1+w(z)}{1+z}dz\right). \qquad (4)$$

Here $\Omega_{DE} = 0.685$ [9], $H_0 = 69.6 \pm 1.8$ [22]. (The method to calculate the modern value of Hubble parameter was as follows. The distances to the SNe Ia supernovae were refined by calibrating the distances to the cepheids using Hubble and Gaia data.)

The lower limit in (4) is assumed to be equal to the value of the redshift at which the phantom energy was replaced by the quintessence, $z \approx 0.3$ [11]. Formally, the DE dynamic EoS retains its form up to a current moment in time ($z = 0$), however, in the absence of phantom DE, phantom stabilized WHs do not arise at all, therefore, we can consider the process of their creation only for redshifts from $z \approx 0.3$ and more.

The relations (2) and (4) effectively specify, for a given redshift z, the maximum throat radius $b(z)$ that could in principle be sustained by the background phantom energy density $\rho_{\rm ph}(z)$. Since there already exist observational constraints from gravitational lensing [23], this scaling can also be used to estimate the redshift intervals in which WHs of a given size might have been supported.

The scaling expressed in Fig. 1 illustrates that the allowed WH throat radius spans over several orders of magnitude, depending on a chosen cosmological probe. While this approach is only a phenomenological scaling, it nevertheless shows that the cosmological phantom energy could, in principle, supply the exotic matter needed to sustain traversable WHs – not only on cosmological scales, but also on smaller ones, comparable to present-day BHs and SMBHs, including ones, discussed in our resent paper [24].

This result does not imply that WHs are necessarily formed at such epochs but rather that, if they exist, they could be maintained by phantom DE at the level of $\rho_{\rm ph}(z)$. In that sense, one could equally argue that WHs may have originated at earlier times but are stabilized with throat radii $b(z)$ consistent with the evolving density.

So, a WH that could form at $z > 20$ could not exceed a radius of about 10^{10} pc, and there is virtually no lower limit, which is due to the large uncertainty in the phantom DE parameters from (3). In later epochs, only WH of exceptionally large masses, exceeding in size 10^9 pc, could form from phantom DE. This fact indicates that we should rather expect the birth of WHs in the early Universe.

Therefore the natural next step is to apply this scaling to WH formation details. Noticeably, the intriguing interval for $b(z)$, that contains throat radii comparable to the modern Schwarzschild radii of known BHs of wide range of masses, lies in a substantially high-redshift region with $z \sim 20$ according to the obtained results. Thus the scenario of WH formation clearly lies outside the realm of stellar collapse origins. That suggests that a different – likely quantum – mechanism would be required. Possible candidates for such processes and their effectiveness will be discussed in the following sections.

Finally, we emphasize that the present discussion applies only to WHs supported by cosmological phantom energy. If other exotic forms of matter or stabiliza-

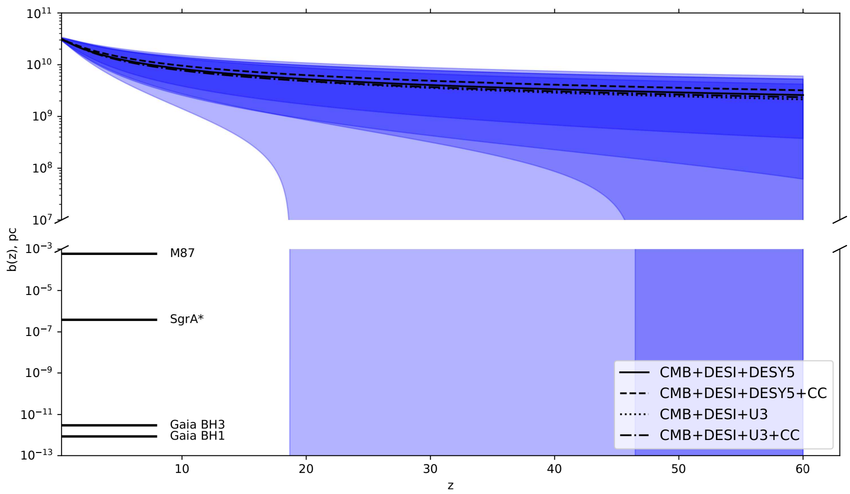

Fig. 1. Constraints on the WH throat radius $b(z)$ using parameters w_0, w_a derived from various cosmological probes. From all the datasets in [11], the ones showing the greatest preference for dynamic DE have been chosen. The lines represent the mean value and the shaded regions correspond to the 1σ uncertainty. Schwarzschild radii of selected BHs are plotted for comparison.

tion mechanisms are introduced, then the corresponding constraints on $b(z)$ may differ significantly, but the general framework presented here can still be adapted to such cases.

3. TUNNELING AND WH FORMATION LIMITS

3.1. WH nucleation in Euclidean Instantons models

Let us discuss the following possibility for WH nucleation, such as Euclidean instantons (EI) [19] formalism in application to a phantom background for the support of the WH throat.

The nucleation rate Γ of WHs via instanton-mediated quantum tunneling is given by the semiclassical approximation:

$$\Gamma \sim A\, e^{-S_E},$$

where S_E is the Euclidean action of the instanton and A is a prefactor determined by quantum fluctuations. As the computing of S_E is highly complicated, we will take some asymptotical features from [19]. The numerical factor in the exponent is sometimes written as e^{-2S_E} although it does not affect the general scaling arguments below.

The prefactor A is subdominant to the exponential suppression so we omit it henceforth. In natural units ($c = \hbar = 1$) the Euclidean action S_E is dimensionless and our goal is to estimate the suppression by dimensional analysis using the characteristic features of WH throat geometry. With this setup, we proceed to the Morris – Thorne WH formation.

3.2. Morris – Thorne WH formation

3.2.1. Instanton-like scaling

In Sec. 2 we obtained the WH throat radius as

$$b(z) = \left(\frac{1}{8\pi G c^{-4}\, \rho_{\rm ph}(z)|w(z)|}\right)^{1/2},$$

where

$$\rho_{\rm ph}(z) = \Omega_{DE}\frac{3H_0^2c^2}{8\pi G}\exp\left(-3\int\limits_{0.3}^{z}\frac{1+w(z)}{1+z}dz\right),$$

$w(z)$ is defined by (3) and [11], $\Omega_{DE} = 0.685$ [9], $H_0 = 69.6 \pm 1.8$ [22].

To estimate the Euclidean action for a phantom-supported WH we use dimensional analysis to capture the basic scaling behavior. The Einstein–Hilbert action in Euclidean signature scales as

$$S_E \sim \frac{1}{G}\int d^4x\sqrt{-g}R + S_{matter} \geq \frac{1}{G}\int d^4x\sqrt{-g}R.$$

Since the dominant curvature scale for a Morris – Thorne WH is

$$R \sim \frac{1}{b^2}$$

and the characteristic 4D-volume scales as b^4, combining these gives

$$S_E(z) \sim \frac{1}{G}b^4(z)\frac{1}{b^2(z)} = \frac{b^2(z)}{G}.$$

Substituting the throat radius $b(z)$ and restoring SI units yields

$$S_E(z) \sim \frac{1}{8\pi G^2 c^{-4}\hbar\,\rho_{\mathrm{ph}(z)}|w(z)|}.$$

The corresponding nucleation rate is then

$$\Gamma \sim e^{-S_E(w)} \le \exp\left(-\frac{1}{8\pi G^2 c^{-4}\hbar\,\rho_{\mathrm{ph}}(z)|w(z)|}\right).$$

This expression represents an instanton-like scaling law that allows us to bypass obtaining of an exact Euclidean action and numerical factors in order to estimate the scaling behavior of the formation rate as a function of $\rho_{\mathrm{ph}}(z)$ and $w(z)$. It demonstrates the suppression effectively for WHs tied to the cosmological phantom energy.

3.2.2. WH nucleation as a thermal fluctuation

An alternative possibility for the WH formation is thermal nucleation – a fluctuation that assembles the phantom energy into a localized configuration that later becomes a WH throat stabilized by the energy inside. This idea, originally proposed for BH nucleation [25], can be heuristically applied to WHs as well.

We note two important caveats:

- phantom energy instability might result in the absence of thermal equilibrium;
- the WH energy E_{WH} is well-defined only for static settings.

We therefore proceed within a static order-of-magnitude approximation.

We estimate the "energy cost" to nucleate a WH as the energy in a throat-sized region:

$$E_{\mathrm{WH}} \sim \rho_{\mathrm{ph}} V \sim \rho_{\mathrm{ph}} b^3,$$

which using the scaling formula (2) transforms into

$$E_{\mathrm{WH}} \sim \frac{1}{(8\pi G c^{-4})^{\frac{3}{2}}}\frac{1}{\rho_{\mathrm{ph}}^{\frac{1}{2}}|w|^{\frac{3}{2}}}.$$

The thermal nucleation rate is then given by a Boltzmann factor

$$\Gamma \sim e^{-\frac{E_{\mathrm{WH}}}{T}},$$

where the temperature according to [26] is related to a surface gravity as

$$T = \frac{\kappa}{2\pi},$$

and the surface gravity of a spacetime as a whole is related to a Hubble parameter $H(z)$ as follows (see [27], [28]):

$$\kappa = \lim_{r_s \to R} Va = R^{-1} = H$$

at a given z.

Then the resulting expression (with the SI units restored) is as follows:

$$\Gamma(z) \sim \exp\left(-\frac{E_{\mathrm{WH}}}{k_B T(z)}\right) = \exp\left(-\frac{E_{\mathrm{WH}}}{k_B}\frac{2\pi k_B c}{\hbar c\, H(z)}\right),$$

$$\Gamma(z) \sim \exp\left(-\frac{2\pi}{\hbar}\frac{1}{(8\pi G c^{-4})^{\frac{3}{2}}\, H(z)}\frac{1}{\rho_{\mathrm{ph}}(z)^{\frac{1}{2}}|w(z)|^{\frac{3}{2}}}\right),$$

where

$$H^2(z) = H_0^2\Bigg(\Omega_m(z+1)^3 + \Omega_{rad}(z+1)^4 + \Omega_k(z+1)^2 + \\ + \Omega_{DE}\exp\left(-3\int_0^z \frac{1+w(z)}{1+z}dz\right)\Bigg).$$

Here $\Omega_m = 0.315$, $\Omega_{rad} = 0.0001$, $\Omega_k = 0$, $\Omega_{DE} = 0.685$, $H_0 = 69.6$.

This expression differs in z-dependence from the instanton result, as expected, because it originates from a thermodynamic rather than semiclassical gravity argument, still yielding negligible formation rates.

The comparison of two formation mechanisms is shown in Fig. 2. The estimates show that both tunneling and thermal nucleation mechanisms are extremely suppressed with the tunneling channel being dominant but still negligible at any redshifts. In other words, although the phantom background could in principle provide the exotic energy needed to maintain WHs, it is extremely unlikely to induce their formation via mechanisms considered.

4. PHANTOM ELLIS – BRONNIKOV WH IN SQLS+DESI DATA

Let us estimate the fraction of phantom DE needed to support the Ellis – Bronnikov WH [29, 30] using the SQLS+DESI observational data, [10, 31].

The Ellis – Bronnikov WH is known to be an exact solution of the Einstein–scalar system where the scalar field has negative kinetic energy (phantom DE). If we put in (1) $\Phi = 0$ and $b(r) = q^2/r$ then the metric takes the previously found in [29, 30] form

$$ds^2 = -dt^2 + \frac{r^2}{r^2 - q^2}dr^2 + r^2 d\Omega^2$$

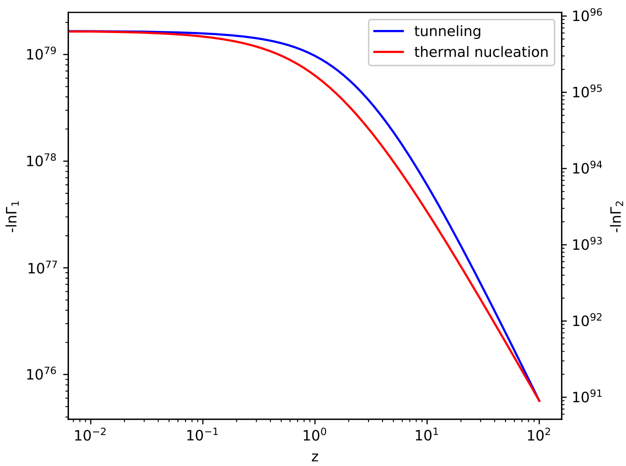

Fig. 2. Comparison of the considered formation mechanisms. Γ_1 refers to the tunneling, Γ_2 refers to the thermal nucleation channel. Only exponents are plotted since in both mechanisms the overall impact is vanishing. The tunneling channel exhibits a greater contribution, but still remains negligible at any z.

or

$$ds^2 = -dt^2 + dR^2 + (R^2 + q^2)d\Omega^2,$$

where

$$R^2 = r^2(R) - q^2.$$

Although asymptotically massless, Ellis – Bronnikov WHs curve spacetime and can deflect light producing distinctive lensing signature. Therefore their abundance could be constrained by using the Galactic microlensing surveys and using the quasar observations in the SQLS data based on SDSS II.

The Ellis – Bronnikov WH could form multiple quasar images. The lens equation in the weak field approximation becomes

$$\beta = \theta - \theta_0^3 \frac{\theta}{|\theta|^3}$$

with the Einstein angular radius

$$\theta_0 = \left(\frac{\pi a^2}{4} \frac{D_{LS}}{D_L^2 D_S} \right)^{1/3},$$

where D_{LS}, D_L, D_S are distances «lens–source», «observer–lens», «observer-source», respectively.

The typical throat radius for a given θ_0 is estimated as [31]

$$q \approx 10h^{-1}\,[\mathrm{pc}] \sqrt{\left(\frac{\theta_E}{1''}\right)^{3/2} \frac{D_L^2 D_s / D_{LS}}{(1h^{-1}\,[\mathrm{Gpc}])^2}}$$

and the sensitive throat radius is $q = 10 - 100$ pc from the image separation of $1'' - 20''$.

Given [31] that the effects of lensing also depend on the ratio of distances, the radius range is wider. SQLS data yields the number density of massless compact objects of order $n_{\mathrm{WH}} \leq 10^{-4} h^3\,\mathrm{Mpc}^{-3}$ for WH throat radii $q = [10, 10^4]$ pc over 50836 quasars in the redshift range $z = 0.6 - 2.2$ under consideration [31].

Let us assume that all phantom WH are Ellis – Bronnikov WH. Then we adopt the same phantom energy density (4) and form a redshift interval specified background for SQLS data as

$$\rho_{\mathrm{ph}} = \Omega_{DE} \frac{3H_0^2 c^2}{8\pi G} \exp\left(-3 \int\limits_{0.6}^{2.2} \frac{1 + w(z)}{1 + z} dz \right).$$

As previously, $w(z)$ is defined by (3) and [11], $\Omega_{DE} = 0.685$ [9], $H_0 = 69.6 \pm 1.8$ [22].

Assuming once again that each WH is sustained by phantom energy and adopting the static order-of-magnitude estimate of its energy

$$E_{\mathrm{WH}} \sim \rho_{\mathrm{ph}} V \sim \rho_{\mathrm{ph}} b^3$$

we can set an upper limit on the WH number density n_{WH} as:

$$n_{\mathrm{WH}} \leq \frac{f \rho_{\mathrm{ph}}}{E_{\mathrm{WH}}},$$

where f denotes the effectiveness of WH formation which can be interpreted as the fraction of phantom energy utilized for sustaining the WHs. The dependence of the number density on the redshift z is as follows:

$$n_{\mathrm{WH}}(z) \leq f \frac{\rho_{\mathrm{ph}}(z)}{E_{\mathrm{WH}}},$$

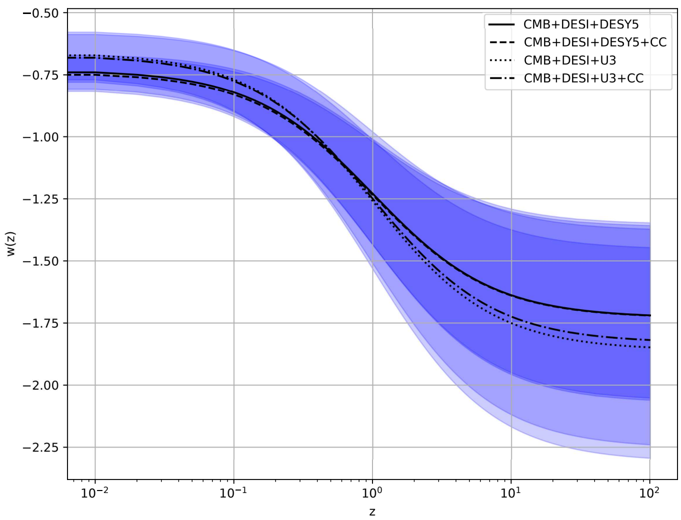

Fig. 3. Reconstruction of the results in [11] on the evolution of the CPL $w(z)$. The lines represent the mean value and the shaded regions correspond to the 1σ uncertainty.

then using throat size scaling (2) and substituting $E_{\rm WH}$ yields

$$n_{\rm WH} \leq f \left(\frac{8\pi G \rho_{\rm ph}(z)|w(z)|}{c^4} \right)^{3/2}.$$

For the formation effectiveness f, we can estimate it as

$$f \geq E_{\rm WH} n_{\rm WH} \frac{1}{\rho_{\rm ph}(z)} = b^3 n_{\rm WH}.$$

Numerically, taking the most permissive SQLS bound $n_{\rm WH} \leq 10^{-4}\ h^3\ \mathrm{Mpc}^{-3}$ $(h = 0.7)$ and the largest allowed throat radius $b \sim 10^4$ pc gives

$$f \sim (10^{-2}\mathrm{Mpc})^3 \cdot 0.7^3 \cdot 10^{-4}\mathrm{Mpc}^{-3} = 3 \cdot 10^{-11},$$

meaning that only a small fraction of phantom energy could be locked into Ellis – Bronnikov WHs without violating the observational constraints from SQLS.

This simple yet effective approach links DESI-affected cosmological parameters w_0, w_a to the constraints on $n_{\rm WH}$ directly and shows how evolving $w(z)$, if reduces the energy density $\rho_{\rm ph}(z)$, can tighten the number density of WHs. In all cases, current SQLS limits imply that only a small fraction of phantom energy can be trapped in Ellis – Bronnikov-like WH throats.

However, recently, many binary gravitational-lens systems have been discovered that do not have a visible lens object. So the resulting restrictions require revision (see, for instance, [23]) and can be exploited for estimating the efficiency of WH formation.

5. DISCUSSION

An important point to emphasize is the nature of wide error corridors obtained for the WH throat radius $b(z)$ in Fig.1. The origin of these uncertainties lies in the original estimates of the parameters w_0 and w_a, particularly in their observational uncertainties. The reconstruction of results in [11] on $w(z)$ is represented in Fig.3. When estimating $b(z)$, we applied a Monte Carlo sampling of the $w(z)$ parameter space within the asymmetric errors of w_0 and w_a. Since these errors reach up to 60% of the initial values, the resulting spread in the sampled $b(z)$ is large as well.

In fact, this feature, stemming from the looseness of the observational constraints on the DE parametrization, might play an important role not only in WH studies. In Fig. 4 we plot the DE density $\rho_{\rm ph}$, calculated via (4), for various sets of w_0 and w_a inside the uncertainties of those parameters. One can see that the resulting estimates diverge drastically with increasing redshift. In other words, any physical quantity that is sensitive to the DE EoS parameter $w(z)$ inevitably inherits this broad uncertainty, as the WH throat radius in our model does.

Another important aspect concerns the obtained formation rate. It might be instructive to compare our estimates with the scalings obtained in different spacetimes in order to capture the similarities. In AdS back-

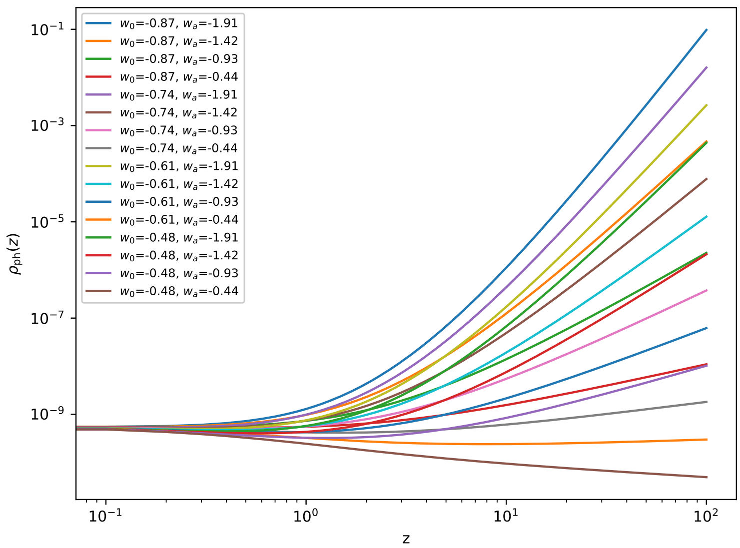

Fig. 4. DE density $\rho_{\mathrm{ph}}(z)$ for various sets of w_0 and w_a within their observational uncertainties.

grounds, for instance, the Euclidean action scales as

$$S_E \approx \pi m L \left[1 + \mathcal{O}(\epsilon \log \epsilon)\right],$$

where m is the mass of the nucleated objects and L is the AdS radius. Here again the large action leads to a negligible formation of large-scaled structures which is to some extent similar to our phenomenological estimates.

More generally, for a typical gravitational instanton one finds

$$S_E \sim \frac{M_{\mathrm{Pl}}^2}{H^2},$$

(see [32]) so the resulting nucleation rate is exponentially suppressed.

The physical interpretation is straightforward. The extreme suppression arises because

- gravitational instantons require overcoming a huge energy barrier ($\propto M_{\mathrm{Pl}}^2$);
- the Hubble scale H is many orders of magnitude smaller than M_{Pl};
- only in the extreme regime ($H \sim M_{\mathrm{Pl}}$) could nucleation be unsuppressed.

Thus, while instantons provide a mechanism for WH nucleation in principle, the rate is negligible for any astrophysical or cosmological scales. This suggests that the desired WH formation mechanism should be sought in early-Universe physics or exotic quantum gravity effects rather than semiclassical nucleation on cosmological background.

A final point concerns the possible evolution of WHs within our framework. Since we developed a simplified and static model, our constraints on the throat size $b(z)$ may be related directly only to the moment of WH formation or stabilization and the subsequent evolution therefore remains uncertain. Here we should mention a possibility of DE accretion and changes in its density inside the WH, potentially leading to the WH throat growth or lessening. Conceptually, once phantom energy is captured inside a WH throat, it should remain trapped there even after the background DE undergoes a transition into a quintessential regime. Thus, long-term evolution questions are open for further studies.

We want to emphasize that the dynamics of phantom energy in a WH spacetime are not deeply understood and definitely lie beyond the scope of this paper. Our goal here is not to speculate on a detailed scenario but to point out this intriguing possibility that cosmological phantom energy may in principle sustain WH throats and thereby stimulate possible interest in further theoretical research and observations.

6. CONCLUSION

In this work we investigated the possibility of traversable WHs stabilized by cosmological phantom

DE in the light of recent DESI results suggesting dynamical DE with a transition to the phantom regime at redshift $z \approx 0.3$. In the Morris – Thorne framework we derived a phenomenological scaling relation linking the WH throat radius b to the background phantom energy density $\rho_{\rm ph}(z)$, thereby establishing a direct bridge between cosmological parameters in CPL parametrization w_0, w_a and WH properties. This approach, although heuristic, shows that phantom energy could in principle serve as the exotic matter required to sustain WHs with throat sizes spanning from gravitational radii of stellar mass BHs and SMBHs up to cosmological scales.

According to our estimates, the birth of WHs should be expected at redshifts $z > 20$. In connection with this assessment, it is interesting to note the following [33]. It was at redshifts of about $z \approx 20$ that no galaxies were expected to be observed before the launch of the JWST telescope. However, the first data from this telescope shows the presence of galaxies and SMBHs at redshifts greater than $z \geq 17$. WHs could serve as the seeds for the formation of SMBHs, along with other mechanisms [34–36] , since it is currently not possible to distinguish between WHs and BHs.

We explored possible formation channels, including Euclidean instanton tunneling and thermal fluctuation nucleation. Both mechanisms exhibit extreme suppression, with instanton-mediated tunneling dominating over thermal nucleation yet still yielding negligible formation probabilities across all redshifts. Thus, while phantom energy may in principle stabilize WHs, it is extremely unlikely to serve as a natural channel for their formation in the present days.

We employed SQLS-driven constraints on Ellis – Bronnikov WHs and combined them with DESI-driven DE parametrization to quantify the fraction of phantom energy that could be trapped in such objects. The resulting bound on the WH formation efficiency is $f \sim 10^{-11}$, which indicates that only a small fraction of the phantom energy can be locked into WH throats without violating observational constraints. We also established a formation efficiency inspired constraint on WH number density $n_{\rm WH}$ in this framework, confirming and expanding existing estimates from gravitational lensing experiments.

Overall, our results show both the theoretical consistency and the observational limitations of phantom-supported WHs. The concept of traversable WHs and their formation remains intriguing as a phenomenological connection between cosmology and exotic geometries, but current data disfavor efficient phantom WH formation under standard semiclassical or thermodynamic scenarios.

Acknowledgements. The research was supported by the Russian Science Foundation, project № 25-22-20026.

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