

# SPIN-DEPENDENT ELECTRON–ELECTRON INTERACTION IN RASHBA MATERIALS

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**Abstract.** We review the effects of the pair spin-orbit interaction (PSOI) in Rashba materials. The PSOI is the electron–electron interaction component that depends on the spin and momentum of the electrons. Being produced by the Coulomb fields of interacting electrons, it exists already in vacuum, but becomes orders of magnitude larger in materials with the giant Rashba effect. The main nontrivial feature of the PSOI is that it is attractive for electrons in certain spin configurations tied to their momentum and competes with the Coulomb repulsion of the electrons. Under certain conditions attainable in modern low-dimensional structures the PSOI prevails. The resulting attraction between electrons leads to the formation of bound electron pairs, the binding energy of which can be controlled by electrical means. In many-electron systems the PSOI results in the instabilities of the uniform ground state with respect to the density fluctuations, which develop on different spatial scales, depending on the geometry of the electric fields that produce the PSOI. If the PSOI is not too strong the electronic system is stable, but its collective excitations reveal the highly unusual spin-charge structure and spectrum, which manifest themselves in the frequency dependence of the dynamic conductivity.

## INTRODUCTION

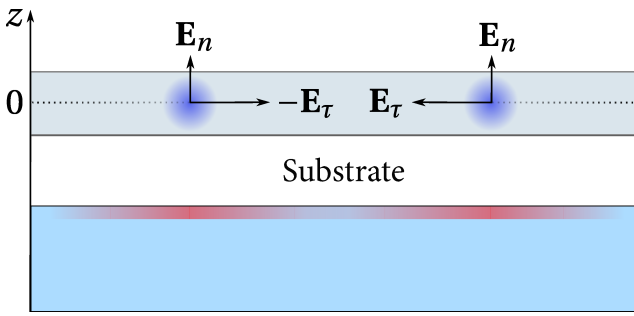
From the early days of relativistic quantum mechanics it is known that the electrostatic Coulomb potential is insufficient to describe the interaction between electrons, and in the first quasi-relativistic approximation the potential of the electron–electron ( $e$ – $e$ ) interaction in addition to the purely Coulomb component  $\mathcal{U}(\mathbf{r})$  contains a contribution that depends on the spin of the electrons and on their momenta [1],

$$\hat{H}_{PSOI} = \frac{\alpha}{\hbar} \sum_{i \neq j} [\mathcal{E}(\mathbf{r}_i - \mathbf{r}_j) \times \hat{\mathbf{p}}_i] \cdot \hat{\boldsymbol{\sigma}}_i, \quad (1)$$

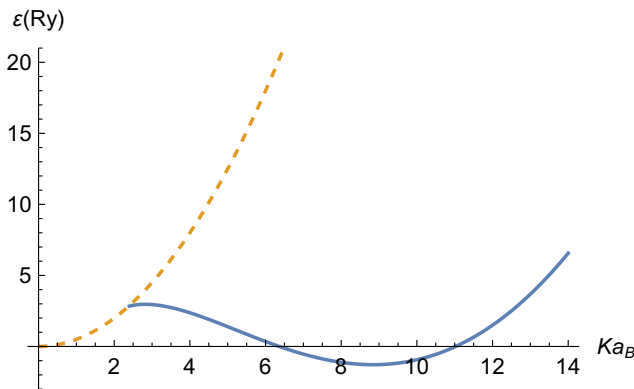
where  $\mathcal{E}(\mathbf{r}) = (1/e)\nabla_{\mathbf{r}}\mathcal{U}(\mathbf{r})$  is the Coulomb field of  $e$ – $e$  interaction,  $\hat{\mathbf{p}}_i$  is the momentum operator of the  $i$ -th electron,  $\hat{\boldsymbol{\sigma}} \equiv (\sigma_x, \sigma_y, \sigma_z)$  is the Pauli vector. This contribution is usually referred to as the pair spin-orbit interaction (PSOI). In vacuum its magnitude  $\alpha = e\lambda^2/4$  is set by the square of the Compton length  $\lambda$ , that is relativistically small.

In crystalline solids the situation fundamentally changes due to the features of the band states, which under certain conditions lead to an extremely strong spin-orbit interaction (SOI) that depends on the electric field external to the crystalline one, including the Coulomb field of interacting electrons. This typically occurs as a result of the combined effect of momentum-dependent mixing of electron and hole subbands split by intra-atomic SOI and symmetry breaking created by the electric field [2]. At present, the circle of these Rashba materials is very wide and the SOI parameter reaches gigantic values [3, 4].

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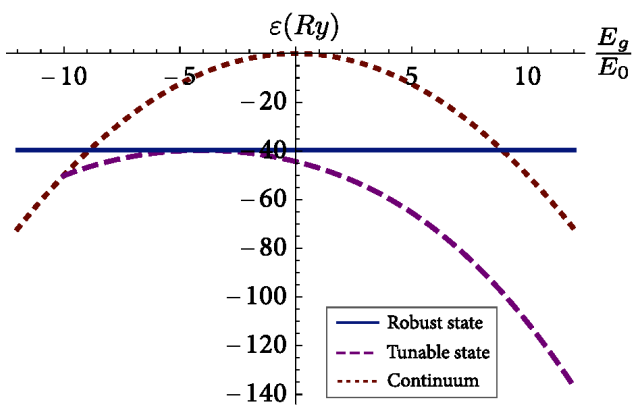


**Fig. 1.** A 2D layer with a proximate gate. Each electron experiences the electric fields from the neighboring electrons, the polarization charges, and the total charge of the gate

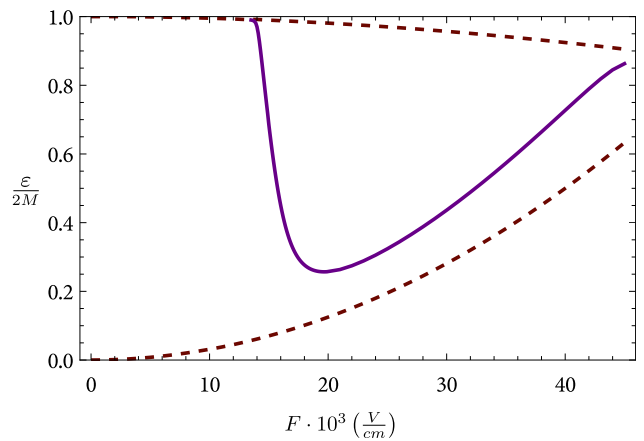


**Fig. 2.** The energy level of the convective BEP (solid line) and the kinetic energy of the center of mass (dashed line) vs  $Ka_B$  for  $\tilde{\alpha} = 1$

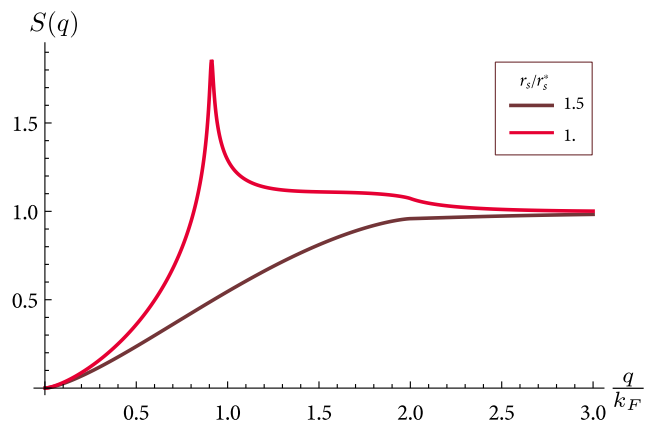
The Rashba constant  $\alpha$  of the currently known materials with giant SOI varies from  $10^2 \text{ e\AA}^2$  in InAs to  $10^3 \text{ e\AA}^2$  in such materials as  $\text{Bi}_2\text{Se}_3$  [5], the monolayers of BiSb [6], the oxide heterostructures and films [7]. In such materials, mainly two-dimensional ones, the PSOI



**Fig. 3.** The binding energy of the robust and tunable BEPs as well as the continuum boundary vs the gate field  $E_g$  normalized at  $E_0 = e/2\epsilon a_B^2$



**Fig. 4.** The BEP energy level as a function of the gate electric field  $F$ . The continuum boundaries are shown by the dashed lines

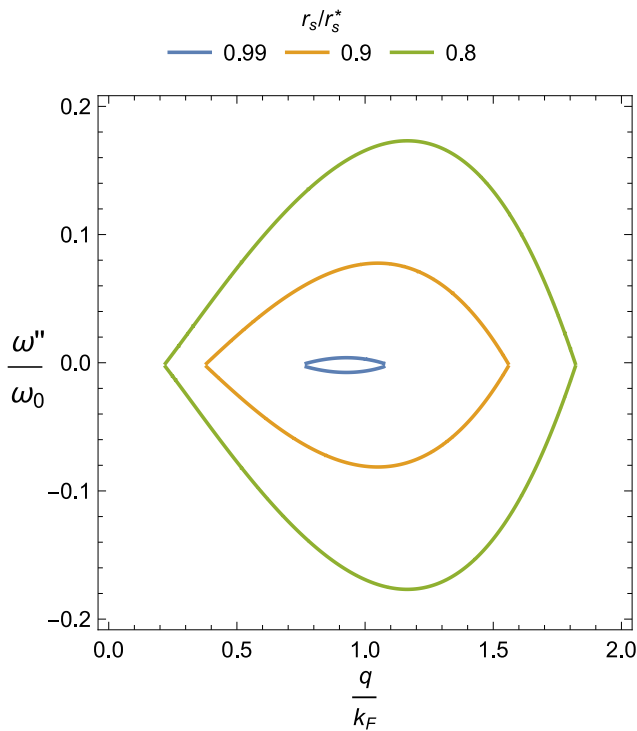


**Fig. 5.** The structure factor  $S(q)$  as a function of  $q$  for two values of the  $r_s$  parameter. The Rashba constant is  $\tilde{\alpha} = 0.1$ , which corresponds to  $r_s^* = 0.3$

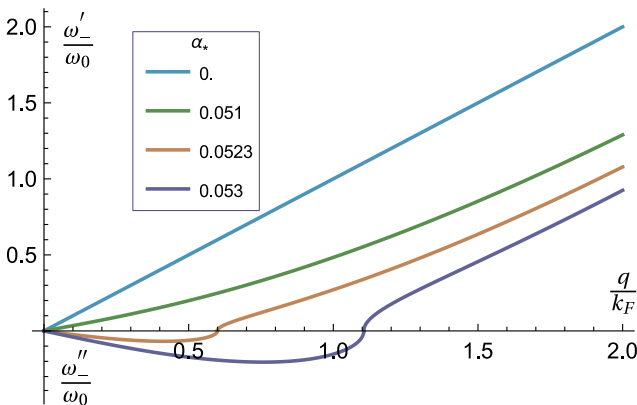
is described by the same Hamiltonian as of Eq. (1), and for a sufficiently smooth potential  $\mathcal{U}(r)$  the value of the  $\alpha$  parameter is estimated at the level of the Rashba constant in a given material.

A distinctive feature of the PSOI as compared to the Coulomb interaction is that it is determined by the electric field  $\mathcal{E}(r) \sim r^{-2}$ , which grows faster than the potential  $\mathcal{U}(\mathbf{r}) \sim r^{-1}$  when we bring two electrons closer together. Hence there appears a new characteristic scale of  $r = \sqrt{\alpha/e}$  where the PSOI prevails over the Coulomb interaction. This scale can be large enough in modern heterostructures based on  $\text{LaAlO}_3/\text{SrTiO}_3$  [8,9], in specifically crafted structures based on two-dimensional (2D) layers of van der Waals materials with heavy adatoms [10,11], etc.

Most importantly, several related scales due to the competition of the PSOI and Coulomb interaction arise

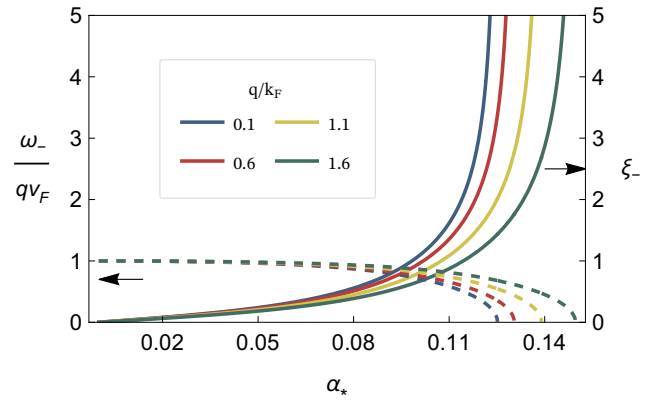


**Fig. 6.** The imaginary part of the frequency of a new solution of the dispersion equation due to the PSOI as a function of wave vector. The dispersion line is shown for three values of  $r_s$  to trace how the instability develops in the system with increasing the PSOI interaction parameter of  $\tilde{\alpha}/r_s$ . The Rashba constant equals  $\tilde{\alpha} = 0.1$ , which corresponds to  $r_s^* = 0.3$

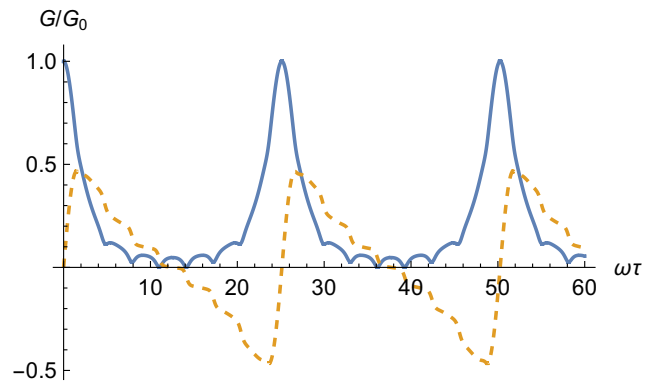


**Fig. 7.** The real ( $\omega'_-$ ) and imaginary ( $\omega''_-$ ) parts of the frequency of the collective mode as a function of the wave vector for several values of the PSOI constant. The frequency is normalized to  $\omega_0 = v_F k_F$

in many-electron systems, regulated by the electron concentration, density of states, and a particular geometry of the electric fields that generate the PSOI. The dependence of the pair interaction on the geometric configuration of the electric fields is not uncommon, for



**Fig. 8.** The spin-charge separation parameter (solid line) and normalized phase velocity (dashed line) for the  $\omega_-$  branch of collective excitations as a function of the PSOI interaction parameter for several  $q$



**Fig. 9.** The real (solid line) and imaginary (dashed line) admittance components vs frequency

example, in the physics of excitons in thin films [12,13], but here it is much more pronounced as the PSOI is generated not by a scalar potential, but by a vector field, which gives rise to more possibilities.

Qualitatively new physical effects arise on these spatial scales because the PSOI proves to be attractive for a certain spin configuration of electrons tied to their momenta. The attraction has a clear origin. The PSOI due to the electric field of a given electron lowers down the energy of another electron, provided that the latter is in a particular spin orientation relative to its momentum. This effect gets stronger as the distance between electrons decreases, which means the attraction arises between the electrons. When the PSOI becomes larger than or comparable with the Coulomb repulsion, a wide scope opens up for many non-trivial effects both at the few-particle level and collective phenomena in the systems of many particles.

Our manuscript is divided into 4 sections. After this introduction, in Sec. 2 of the full text of this paper, we review the formation of a bound electron pair (BEP) due to the attractive interaction of two electrons [14–17]. There appears an interesting problem of their binding energy and possible types of spin-charge structure. This is a highly nontrivial issue, since the interaction is attractive only for certain spin and momentum configurations.

Section 3 is devoted to the problem of a many-particle correlated state formed owing to the PSOI. The presence of the Coulomb interaction simultaneously with the PSOI greatly complicates the problem, moreover, the studies carried out to date [18, 19] show that the sufficiently strong PSOI leads to an instability of the homogeneous state of the system and therefore it is necessary to find out the adequate stabilizing mechanisms. At the present stage, studies were focused on the collective excitations under conditions where the PSOI is not too strong, but the system can approach the instability threshold. In this way, the spectrum and structure of the collective modes can be elucidated and the spectral functions of electronic fluctuations leading to the loss of stability can be found. Such studies were carried out for two specific situations of one-dimensional (1D) and two-dimensional (2D) systems.

In 1D quantum wires, collective excitations have been studied when the PSOI is produced by the image charges on a nearby gate [18]. It was found that the spectrum of collective modes contains two branches with a mixed spin-charge structure, the composition of which strongly depends on the magnitude of the PSOI. Of most interest is the behavior of one of them, which originates from the spinon branch in the absence of the PSOI. As the PSOI parameter increases, this mode strongly softens and ultimately leads to the instability of the system in the long-wavelength region of the spectrum. At the same time, its structure changes continuously from purely spin-like to purely plasmon-like.

In the direction of 2D systems, the situation of an electron gas with the in-plane mirror symmetry was studied. In this case an unconventional correlated electronic state is found to arise due to the PSOI. This state is characterized by a sharp peak in the structure factor, indicating a tendency to form a striped structure with a certain spatial scale set by the competition between the Coulomb repulsion and the PSOI-induced attraction of electrons. The system becomes unstable on this scale if the density of electrons is larger than critical [19]. Interestingly, the fluctuations that grow rapidly as the system approaches the instability thresh-

old consist mainly of the charge density fluctuations, just like in the 1D case.

The last Sec. 4 summarizes the main results discussed in our review and provides an outlook for the most prospective direction of the future research.

The results from the list of Refs. [1–30] are used or/and discussed in our work. The figures illustrating our results are presented below.

## CONCLUSION AND OUTLOOK

The pair spin-orbit interaction between the electrons, which is usually considered relativistically weak, becomes extremely strong in modern materials with a strong Rashba spin-orbit interaction. The giant PSOI in Rashba materials is of slightly different nature as compared to that known in relativistic quantum mechanics, although it certainly has a relativistic origin. It arises because of the Rashba effect — the combined effect of the momentum-dependent hybridization of the electron subband with other subbands split due to the intra-atomic spin-orbit interaction, and the inversion symmetry breaking produced by an electrical field, in this case, the Coulomb field of interacting electrons.

PSOI has two main features:

- PSOI leads to the attraction of electrons for certain configurations of the spins relative to their momenta;
- the magnitude of the PSOI is determined by the electric field of the interacting electrons and hence increases with decreasing distance between them much faster than the interaction potential.

Owing to these features, strong PSOI opens up wide prospects for the appearance of many nontrivial effects due to the formation of new strongly correlated states of electrons, which are still very poorly understood. According to the studies conducted to date, such effects are manifested for sufficiently strong Rashba SOI, which is apparently attained in the already available materials. Because of the impressive progress in the creation of new low-dimensional materials with a giant SOI, theoretical studies of such states are very important.

Nontrivial effects arise at the level of systems with few particles, as well as in many-particle systems.

PSOI leads to the formation of bound pairs of electrons with various properties, depending on the screening conditions and the presence of the SOI induced by the gate potential. An attractive feature of such states

is the possibility to achieve a high binding energy of the pairs and to control their spectrum. Prospects for further research in this direction are associated primarily with the investigation of the stability of such pairs with respect to the interaction with other electrons and to the formation of more complex electron aggregations.

In many-particle systems the PSOI, even if not very strong, dramatically affects the spectra of the collective excitations and their spin-charge structure. Of particular interest is the softening of the collective modes due to the attraction between the electrons in certain spin configurations, which is produced by the PSOI and is accompanied by a rearrangement of the structure of electron correlations. Such studies for 2D systems at the current stage reveal a lot of interesting things. But of most importance is the fact that at a sufficiently strong PSOI the uniform state of a many-electron system becomes unstable. The structure of growing fluctuations indicates a tendency towards the formation of an inhomogeneous stripe phase. The search for the stabilization mechanisms and possible structures of the stable state is an open question most promising for further research.

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