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ALGEBRAIC TIME CRYSTALLIZATION IN A TWO-DIMENSIONAL SUPERFLUID

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In the theory of superfluidity, the phenomenon of spontaneously breaking the time-translation symmetry — time crystallization (TC), in modern terminology [1, 2] — is inseparable from the phenomenon of spontaneously breaking the global U(1) symmetry. The very fact of existence of the global superfluid phase, Φ, implies that Φ evolves in time, obeying the universal Beliaev–Josephson–Anderson relation (in the reference frame of the normal component)

$$\dot{\Phi} = -\mu, \tag{1}$$

with μ the chemical potential (in the units of $\hbar$ for bosonic systems) [3]. Any periodic function of Φ then is also a periodic function of time.

While the standard discussion of TC in a superfluid is based on the notion of the global phase and genuine condensate, for the superfluidity to take place in two dimensions an algebraic (topological) order is sufficient. We find that the absence of long-range order in a finite-temperature two-dimensional superfluid translates into algebraic time crystallization caused by the temporal phase correlations. The exponent controlling the algebraic decay is a universal function of the superfluid-stiffness-to-temperature ratio; this exponent can be also seen in the power-law singularity of the Fourier spectrum of the AC Josephson current. We elaborate on subtleties involved in defining the phenomenon of time crystallization in both classical-field and all-quantum cases and propose an experimental protocol in which the broken time translation symmetry — more precisely, temporal correlations of the relative phase, with all possible finite-size, dimensional, and quantum effects included — can be observed without permanently keeping two superfluids in a contact.

We derive the following expression for the temporal correlator of the Josephson current [4] between the superfluids A and B:

$$\langle J(0)J(t)\rangle = J_0(t_*)^2 \cos(\Delta\mu\, t) \left(\frac{t_*}{t}\right)^{\alpha}, \tag{2}$$

$$\Delta\mu = \mu_B - \mu_A, \qquad \alpha = \frac{1}{2\pi}\left(\frac{T_A}{\Lambda_s^{(A)}} + \frac{T_B}{\Lambda_s^{(B)}}\right). \tag{3}$$

Here $\mu_{A,B}$, $T_{A,B}$, and $\Lambda_s^{(A,B)}$ are (respectively) the chemical potentials, temperatures, and superfluid stiffnesses of the two superfluids. (In terms of the dependence on the ultraviolet time-cutoff parameter t_*, formula (2) implies a "conspiracy" between the amplitude of the effective Josephson coupling $J_0(t_*)$ and the factor $(t_*/t)^{\alpha}$: the net result has to be independent of t_*.)

Since Wilczek's remark in Ref. [1], it became common to argue [5] that the AC Josephson effect is fundamentally nonequilibrium/driven and as such does not qualify for the phenomenon of TC. We emphasize that AC Josephson effect is nothing but an experimental probe of the phenomenon that exists regardless of the method used to study it; there is nothing special about

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the probe being weakly dissipative. Correspondingly, TC can be also revealed either by releasing tiny fractions of atoms from each system and recording matter wave interference patterns [6] separated by some amount of time t, or by switching on the Josephson link between the two superfluids only for two infinitesimally short time intervals separated by some waiting time t. During the waiting time both superfluids — fully equilibrium on their own — evolve absolutely independently. Finally, Ramsey spectroscopy approach allows one to measure the temporal correlator of the phase with a single system [7].

Recently, Watanabe and Oshikawa presented a "no go" theorem [5] establishing the absence of TC in the ground and finite-temperature states of macroscopic systems for which energy is the only thermodynamically relevant constant of motion. Neither classical nor quantum superfluids satisfy the conditions of the theorem because of the total amount of matter conservation law. The freedom of adding to the system's Hamiltonian any function of conserved quantities formally allows one to suppress the breaking of the time-translation symmetry in a particular isolated superfluid, but this cannot be achieved for all superfluids simultaneously!

Cold atomic systems apparently provide the best platform for measuring the algebraic TC effect. Conceptually, all one needs is to establish a weak link between two superfluid states. Creating multiple well-isolated two-dimensional superfluids was already achieved for phase interference experiments some time ago [8,9], while more recent work on superfluid Fermi gases [10] has established the possibility of measuring transport through point contacts. As a natural part of the algebraic TC effect, one establishes a way to measure superfluid density using purely local in space probe.

Here, we propose an experimental protocol of measuring temporal phase correlations between two disconnected finite-size quantum superfluids that fully preserves the quantum uncertainty of the relative phase between the two samples.

• Prepare two isolated superfluid samples at close but different chemical potentials.

• At $t = 0$, connect the two samples by a local weak link for a short, $\Delta t\, \Delta\mu \ll 1$, period of time.

• Keep samples isolated for a much longer time interval $t\, \Delta\mu > 1$ and then repeat the previous step.

• Quickly, on time scales $\ll 1/\Delta\mu$, apply a deep optical lattice to localize all atoms in the system and count atom numbers N_A and N_B using single-site microscopy [11,12].

Repeating the protocol many times under identical conditions allows one to accumulate representative statistics and process the data with the help of an auxiliary experimental run that skips the next-to-last step of the above-described protocol. The outcome of the auxiliary run is the expectation value $\bar{N}_{AB} = \langle N_A - N_B \rangle$ that averages typical particle number differences taking place before the two samples are disconnected for a period of time t. The key statistical observable is then

$$K(t) \,=\, \langle\, (N_A(t) - N_B(t) - \bar{N}_{AB})^2 \,\rangle. \qquad (4)$$

This quantity is the sum of two fundamentally different dispersions of the random number $N_A(t) - N_B(t) - \bar{N}_{AB}$: one is characterizing irreproducibility of the initial state preparation, and the other is reflecting temporal current–current correlations. The latter dispersion reveals the time crystallization effect, while the former dispersion is t-independent and thus creates no problem except for that of a signal-to-noise ratio, which can be improved by collecting more statistics and optimizing setup parameters. To ensure that the time-dependent contribution to dispersion is large, one needs to have $J_0(t_*)/\Delta\mu \gg 1$.

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REFERENCES

1. F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012).

2. A. Shapere and F. Wilczek, Phys. Rev. Lett. **109**, 160402 (2012).

3. B. Svistunov, E. Babaev, and N. Prokof'ev, *Superfluid States of Matter*, Taylor & Francis (2015).

4. B. D. Josephson, Phys. Lett. **1**, 251 (1962).

5. H. Watanabe and M. Oshikawa, Phys. Rev. Lett. **114**, 251603 (2015).

6. Y. Castin and J. Dalibard, Phys. Rev. A **55**, 4330 (1997).

7. H. Kurkjian, Y. Castin, and A. Sinatra, Compt. Rend. Phys. **17**, 789 (2016).

8. Z. Hadzibabic, S. Stock, B. Battelier, V. Bretin, and J. Dalibard, Phys. Rev. Lett. **93**, 180403 (2004).

9. S. Stock, Z. Hadzibabic, B. Battelier, M. Cheneau, and J. Dalibard, Phys. Rev. Lett. **95**, 190403 (2005).

10. D. Husmann, S. Uchino, S. Krinner, M. Lebrat, T. Giamarchi, T. Esslinger, and J.-P. Brantut, Science **350**, 1498 (2015).

11. S. Trotzky, L. Pollet, F. Gerbier, U. Schnorrberger, and I. Bloch, Nature Physics **6**, 998 (2010).

12. W. S. Bakr, A. Peng, M. E. Tai, R. Ma, J. Simon, J. I. Gillen, S. Fölling, L. Pollet, and M. Greiner, Science **329**, 547 (2010).