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DARK-ENERGY COSMOLOGICAL MODELS IN $f(G)$ GRAVITY

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We discuss dark-energy cosmological models in $f(G)$ gravity. For this purpose, a locally rotationally symmetric Bianchi type I cosmological model is considered. First, exact solutions with a well-known form of the $f(G)$ model are explored. One general solution is discussed using a power-law $f(G)$ gravity model and physical quantities are calculated. In particular, Kasner's universe is recovered and the corresponding $f(G)$ gravity models are reported. Second, the energy conditions for the model under consideration are discussed using graphical analysis. It is concluded that solutions with $f(G) = G^{5/6}$ support expansion of universe while those with $f(G) = G^{1/2}$ do not favor the current expansion.

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1. INTRODUCTION

Recent study of Planck data for models of dark energy shows that modified gravity may be a useful approach to analyze the cosmic expansion of the universe [1]. The equation-of-state (EoS) parameter $\omega = p/\rho$ can be used to describe the dark energy, where ρ and p represent the energy density and pressure of dark energy. Modified gravity has gained much popularity in the recent years; $f(R)$ and $f(R,T)$ theories of gravity have been investigated by many researchers in the last decade [2–7]. In particular, $f(R)$ gravity is proved to be equivalent to the scalar–tensor theory of gravity [8]. The cosmic acceleration can be justified just by adding the term $1/R$, which is required at small curvature. Reviews [9–11] help to get a deeper understanding of why modified theories are important in discussing expansion of the universe. The $f(T)$ theory of gravity [12–14] is another alternate theory that generalizes teleparallel gravity. The interesting feature of the theory is that cosmic acceleration can be justified without involving dark energy. Modified Gauss–Bonnet (GB) gravity is another theory that has gained popularity in the last few years [15–17]. It is also known as the $f(G)$ theory of gravity, where $f(G)$ is a generic function of the GB invariant G. The GB term plays an important role because it may allow avoiding ghost contributions and is helpful in regularizing the gravitational action [18]. It has been suggested that this theory may describe the late-time cosmic acceleration. Moreover, the theory also passes the solar system tests for some specific choices of $f(G)$ gravity models. Some interesting work has been done so far in this theory.

The investigation of exact solutions in $f(G)$ gravity is not only important but also challenging due to the involvement of higher-order derivatives in the equations of motion. Nevertheless, many interesting achievements have been reported so far. Spherical symmetry has been used to find noncommutative static wormhole solutions in modified GB gravity in [19]. Anisotropic power-law solutions in $f(G)$ gravity were explored in [20], where Bianchi type I power-law solutions were proved to be valid only for some special cases of $f(G)$ gravity models. Cylindrical symmetry in $f(G)$ gravity has been investigated and it was shown that only three choices of $f(G)$ models are compatible with the exact solutions [21]. Exact cylindrically symmetric solutions of GB-modified field equations recovered the cosmic string space–time [22]. Noether $f(G)$ symmetries for the Friedmann–Robertson–Walker (FRW) metric were recently discussed in [23]. The same authors [24] studied the role of the GB term in the late-time accelerated phases of the universe. The $f(G)$ theory of gravity is used to discuss warm inflation for the FRW universe model [25]. A further generalized version of GB gravity, the $f(R,G)$ gravity, has also been proposed recently. Spherically symmetric exact solutions in this theory were investigated in [26]. The cosmological inflation

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in $f(R,G)$ gravity has been studied in [27]. Stability criteria for Schwarzschild solutions with metric perturbations were given in [28]. In [29], finite-time singularities in modified $f(R,G)$ gravity were investigated and it was concluded that singularities could be avoided in the $f(R,G)$ theories of gravity using higher-order curvature corrections.

The energy conditions have been used considerably to discuss some important issues in cosmology. The energy conditions can be categorized as the null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC), and dominant energy condition (DEC). These are helpful in studying the validity of the second law of black-hole thermodynamics and Hawking–Penrose singularity theorems [30]. In particular, the SEC is important because its violation indicates cosmic expansion [31]. Energy conditions using metric $f(R)$ gravity were studied in [32]. The authors of [33] investigated energy conditions to find the stability and viability of cosmological models in $f(R)$ gravity. In [34], the charged black hole metric was constructed such that the WEC was satisfied. These conditions were explored in [35] to analyze the viability of some $f(G)$ gravity models. The $f(R,G)$ gravity energy conditions have been recently explored, where the WEC was used along with the recent estimated values of cosmological parameters to determine the viability of some specific choices of $f(R,G)$ gravity models [36]. Thus, it seems interesting to explore modified theories of gravity, and the $f(G)$ gravity in particular.

In this paper, we are interested in $f(G)$ gravity in an anisotropic background. We consider a locally rotationally symmetric (LRS) Bianchi type I space–time. The paper can be divided into two parts. The first part is devoted to finding the exact solutions of the LRS Bianchi type I field equations in the $f(G)$ theory of gravity. Some exact solutions with a well-known form of $f(G)$ model are explored. One general solution is discussed using a power-law $f(G)$ gravity model and physical quantities are calculated. In particular, Kasner's universe is recovered and the corresponding $f(G)$ gravity models are reported. In the second part, the energy conditions for the model under consideration are discussed using graphical analysis. Two cosmological models are discussed in detail and it is concluded that solutions with $f(G)=G^{5/6}$ support expansion of the universe while those with $f(G)=G^{1/2}$ do not favor the current expansion. The paper is organized as follows: Sec. 2 gives a brief introduction into $f(G)$ gravity and the corresponding field equations. Exact solutions of the field equations for a specific choice of the $f(G)$ model are investigated in Sec. 3. Section 4 is used to discuss the validity of solutions using energy conditions. The last section gives the summary and conclusion of the work.

2. FIELD EQUATIONS IN $f(G)$ GRAVITY

Modified GB gravity is described by the action

$$S=\frac{1}{2\kappa}\int d^4x\sqrt{-g}\,[R+f(G)]+S_M(g^{\mu\nu},\psi), \qquad (1)$$

where κ is the coupling constant, g is the determinant of the metric tensor $g_{\mu\nu}$, and $S_M(g^{\mu\nu},\psi)$ is the matter action, in which matter is minimally coupled to the metric tensor and ψ denotes the matter fields. This coupling of matter to the metric tensor suggests that $f(G)$ gravity is a purely metric theory of gravity. The $f(G)$ is an arbitrary function of the GB invariant G:

$$G\equiv R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}, \qquad (2)$$

where R is the Ricci scalar and $R_{\mu\nu}$ and $R_{\mu\nu\sigma\rho}$ denote the Ricci and Riemann tensors. Gravitational field equations are obtained by varying the action in Eq. (1) with respect to the metric tensor:

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+8\Big[R_{\mu\rho\nu\sigma}+R_{\rho\nu}g_{\sigma\mu}-R_{\rho\sigma}g_{\nu\mu}-{}$$
$$-R_{\mu\nu}g_{\sigma\rho}+R_{\mu\sigma}g_{\nu\rho}+\frac{1}{2}R(R_{\mu\nu}g_{\sigma\rho}-R_{\rho\sigma}g_{\nu\rho})\Big]\times$$
$$\times\nabla^\rho\nabla^\sigma F+(Gf_G-f)g_{\mu\nu}=\kappa T_{\mu\nu}, \qquad (3)$$

where ∇_μ denotes the covariant derivative and f_G represents the derivative of f with respect to G.

The line element for a spatially homogeneous, anisotropic and LRS Bianchi type I space–time is given by

$$ds^2=dt^2-L^2(t)dx^2-M^2(t)[dy^2+dz^2], \qquad (4)$$

where L and M are cosmic scale factors. The corresponding Ricci scalar and GB invariant turn out to be

$$R=-2\Big[\frac{\ddot{L}}{L}+2\frac{\ddot{M}}{M}+\frac{2\dot{L}\dot{M}}{LM}+\frac{\dot{M}^2}{M^2}\Big], \qquad (5)$$

$$G=8\Big[\frac{\ddot{L}\dot{M}^2}{LM^2}+2\frac{\dot{L}\dot{M}\ddot{M}}{LM^2}\Big], \qquad (6)$$

where the dot denotes the derivative with respect to t. Here, we assume that the universe is filled with a dark energy fluid and hence the energy–momentum tensor is taken as

$$T^\mu_\nu=\mathrm{diag}[\rho,-p_x,-p_y,-p_z], \qquad (7)$$

where ρ denotes the energy density of the fluid and p_x, p_y, and p_z are the pressures along the x, y, and z axes. The anisotropic fluid is characterized by the EoS $p = \omega\rho$, where ω is not necessarily constant. From Eq. (7), it follows that

$$T^\mu_\nu = \mathrm{diag}[\rho, -\omega_x, -\omega_y, -\omega_z]\rho, \tag{8}$$

where ω_x, ω_y, and ω_z are the directional EoS parameters along the x, y, and z axes; ω is a deviation-free EoS parameter of the fluid. We parameterize the deviation from isotropy in such a way that $\omega_x = \omega$ and introduce the skewness parameter δ as the deviation from ω along the y and z axes. In this case, the energy–momentum tensor takes the form

$$T^\mu_\nu = \mathrm{diag}[\rho, -\omega, -(\omega+\delta), -(\omega+\delta)]\rho. \tag{9}$$

The average scale factor a and the volume scale factor V are defined as

$$a = \sqrt[3]{LM^2}, \quad V = a^3 = LM^2. \tag{10}$$

The average Hubble parameter H, the expansion scalar θ, and the shear scalar σ are given in the form

$$H = \frac{1}{3}\left(\frac{\dot L}{L} + \frac{2\dot M}{M}\right), \quad \theta = \frac{\dot L}{L} + 2\frac{\dot M}{M}, \quad \sigma^2 = \frac{1}{3}\left[\frac{\dot L}{L} - \frac{\dot M}{M}\right]^2. \tag{11}$$

With Eqs. (4) and (9), field equations (3) now take the form

$$2\frac{\dot L\dot M}{LM} + \frac{\dot M^2}{M^2} - 24\frac{\dot L\dot M^2}{LM^2}\dot f_G + Gf_G - f = \kappa\rho, \tag{12}$$

$$-2\frac{\ddot M}{M} - \frac{\dot M^2}{M^2} + 16\frac{\dot M\ddot M}{M^2}\dot f_G + 8\frac{\dot M^2}{M^2}\ddot f_G - Gf_G + f = \kappa\omega\rho, \tag{13}$$

$$-\frac{\ddot L}{L} - \frac{\ddot M}{M} - \frac{\dot L\dot M}{LM} + 8\left(\frac{\dot L\ddot M}{LM} + \frac{\dot M\ddot L}{ML}\right)\dot f_G + 8\frac{\dot L\dot M}{LM}\ddot f_G - Gf_G + f = \kappa(\omega+\delta)\rho. \tag{14}$$

These are complicated and highly nonlinear differential equations. We use a physical condition that the expansion scalar θ is proportional to the shear scalar σ, which leads to

$$L = M^n, \tag{15}$$

where n is an arbitrary real number and we consider $n \neq 0, 1$ for nontrivial solutions. The physical reason for this assumption is justified because the observations of the velocity red-shift relation for extragalactic sources suggest that the Hubble expansion of the universe may achieve isotropy when σ/θ is constant [37]. Collins [38] provided the physical significance of this condition for a perfect fluid with a barotropic EoS. In the literature [39–44], many authors have proposed this condition in order to find exact solutions of the field equations. With Eq. (15), field equations (12)–(14) take the form

$$(1+2n)\frac{\dot M^2}{M^2} - 24n\frac{\dot M^3}{M^3}\dot f_G + Gf_G - f = \kappa\rho, \tag{16}$$

$$-2\frac{\ddot M}{M} - \frac{\dot M^2}{M^2} + 16\frac{\dot M\ddot M}{M^2}\dot f_G + 8\frac{\dot M^2}{M^2}\ddot f_G - Gf_G + f = \kappa\omega\rho, \tag{17}$$

$$-(n+1)\frac{\ddot M}{M} - n^2\frac{\dot M^2}{M^2} + 8\left(2n\frac{\dot M\ddot M}{M^2} + n(n-1)\frac{\dot M^3}{M^3}\right)\dot f_G + 8n\frac{\dot M^2}{M^2}\ddot f_G - Gf_G + f = \kappa(\omega+\delta)\rho. \tag{18}$$

We now investigate the solutions of these field equations.

3. DARK UNIVERSE WITH A POWER-LAW $f(G)$ MODEL

We consider the $f(G)$ model with

$$f(G) = \alpha G^{m+1}, \tag{19}$$

where α and m are arbitrary constants. This model has already been proposed in [16] and it is interesting because the Big Rip singularity may not appear. Also, power-law $f(G)$ models are compatible with the observational data and predict the unification of the early-time inflation with late-time acceleration [45]. The viability of this model has already been shown in cosmological contexts [46–48]. Moreover, this model belongs to the general class of models without irregular spin-2 ghosts [49]. It follows from Eq. (19) that

$$f_G(G) = \alpha(m+1)G^m. \tag{20}$$

For simplicity and without any loss of generality, we choose $\alpha = 1/(m+1)$ for the further analysis. We consider the power-law form for the metric coefficient

$$M(t) = \beta t^{1/(n+2)}, \tag{21}$$

where β is an arbitrary constant. Using this in Eqs. (16)–(18), we find the energy density and pressure components

$$\rho = \frac{t^2(m+1)(2n^2+5n+2) + 16mn(5+6m)\left(\frac{-16n}{t^4(n+2)^3}\right)^m}{t^4(m+1)(n+2)^3\kappa}, \tag{22}$$

$$p_x = \frac{1}{t^4(m+1)(n+2)^3\kappa}\Big[t^2(m+1)(2n^2+5n+2) + 16m(8m^2n+7n+14mn+16m^2+24m+8)\times \left(\frac{-16n}{t^4(n+2)^3}\right)^m\Big], \tag{23}$$

$$p_y = p_z = \frac{1}{t^4(m+1)(n+2)^3\kappa}\times \Big[t^2(m+1)(2n^2+5n+2)+16m(8m^2n^2+12mn^2 + 16m^2n+26mn+4n^2+11n)\left(\frac{-16n}{t^4(n+2)^3}\right)^m\Big]. \tag{24}$$

Thus, the solution metric takes the form [50, 51]

$$ds^2 = dt^2 - \beta^{2n}t^{2n/(n+2)}dx^2 - \beta^2 t^{2/(n+2)}(dy^2+dz^2). \tag{25}$$

The Ricci scalar and the GB invariant for this solution turn out to be

$$R = \frac{2(2n+1)}{(n+2)^2t^2}, \quad G = -\frac{16n}{(n+2)^3t^4}. \tag{26}$$

The average Hubble parameter, the average scale factor, and the volume scale factor of the universe take the form

$$H = \frac{1}{3t}, \quad a = \beta^{(n+2)/3}t^{1/3}, \quad V = \beta^{n+2}t. \tag{27}$$

The redshift for a distant source is directly related to the scale factor of the universe at the time when the photons were emitted from the source. The scale factor a and the redshift z are related as

$$a = \frac{a_0}{1+z}, \tag{28}$$

where a_0 is the present value of the scale factor. Using Eq. (27), we obtain

$$\frac{H}{H_0} = \frac{t_0}{t}, \quad \frac{a_0}{a} = 1+z = \left(\frac{t_0}{t}\right)^{1/3}, \tag{29}$$

where H_0 is the present value of Hubble's parameter. Therefore, the value of Hubble's parameter in terms of the redshift parameter turns out to be

$$H = H_0(1+z)^3. \tag{30}$$

The deceleration, jerk, and snap parameters become

$$q = 2, \quad j = 10, \quad s = -80. \tag{31}$$

The expansion scalar and the shear scalar turn out to be

$$\theta = \frac{1}{t}, \quad \sigma^2 = \frac{1}{3}\left[\frac{n-1}{(n+2)t}\right]^2. \tag{32}$$

The isotropy condition $\sigma^2/\theta \to 0$ as $t \to \infty$ is also satisfied in this case. It is also observed from Eqs. (27) and (32) that the spatial volume is zero at $t = 0$ while the expansion scalar is infinite, which suggests that the universe starts evolving with zero volume at $t = 0$ (a Big Bang scenario). It is further observed that the average scale factor is zero at the initial epoch $t = 0$ and the model becomes singular at $t = 0$ for $n < -2$. To have a better analysis of isotropy, graphical behavior of the isotropy condition is shown in Fig. 1. It is evident that $\sigma^2/\theta \to 0$ for small values of n even if t is not very large. This indicates that isotropy can be quickly recovered for some suitable values of n and t (other than ∞). With Eqs. (16)–(18), the EoS and the skewness parameters turn out to be

$$\omega = \frac{t^2(m+1)(2n^2+5n+2) + 16m(8m^2n+7n+14mn+16m^2+24m+8)\left(\frac{-16n}{t^4(n+2)^3}\right)^m}{t^2(m+1)(2n^2+5n+2) + 16mn(5+6m)\left(\frac{-16n}{t^4(n+2)^3}\right)^m}, \tag{33}$$

$$\delta = \frac{64m(m+1)(2m+1)(n^2+n-2)\left(\frac{-16n}{t^4(n+2)^3}\right)^m}{t^2(m+1)(2n^2+5n+2) + 16mn(5+6m)\left(\frac{-16n}{t^4(n+2)^3}\right)^m}. \tag{34}$$

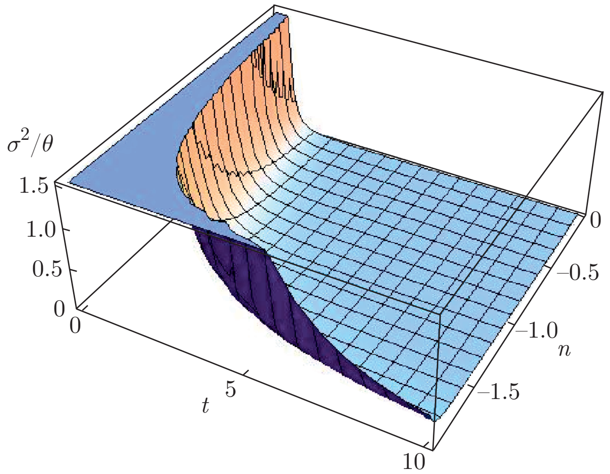

Fig. 1. Behavior of the isotropy parameter σ^2/θ

Many solutions can be reconstructed from Eq. (25). However, we here discuss only one special case to justify the physical relevance. Putting $n = -1/2$ in Eq. (25), we have

$$ds^2 = dt^2 - \beta^{-1} t^{-2/3} dx^2 - \beta^2 t^{4/3}(dy^2 + dz^2). \quad (35)$$

After redefining the parameters, it is exactly the same as the well-known Kasner metric [52]. Here, the EoS parameter takes the form

$$\omega = -\frac{24m^2 + 34m + 9}{6m + 5}. \quad (36)$$

For $m = -1/6$, we have $\omega = -1$, which describes accelerated expansion of the universe [53–55]. Moreover, when $m = -1/2$, the EoS parameter has the positive value $\omega = 1$ corresponding to a stiff-fluid universe. It has been shown that the EoS parameter can be positive due to classical and quantum mechanical contributions [56]. It is observed from Eqs. (22)–(24) that expressions for the energy density and pressure components are defined for $-\infty < n < -2$ and $-2 < n < 0$ when $m = -1/6, -1/2$. We hence choose $-2 < n < 0$ for graphical analysis. The behavior of the energy density of the universe and pressure (x-component) can be seen from Fig. 2, 3. Both models give a positive energy density for $-2 < n < 0$. Figure 4 shows the behavior of the EoS parameter ω and Fig. 5 shows the behavior of the skewness parameter δ. It can be seen that ω takes negative values as well. It is worthwhile to mention here that the phantom-like dark energy is found to be in the region where $\omega < -1$. The universe with phantom dark energy ends up with a finite-time future singularity known as the cosmic doomsday or Big Rip [57, 58].

4. EXACT SOLUTIONS AND ENERGY CONDITIONS

In modern-day cosmology, the energy conditions are considered useful to establish some important theorems about black holes. The viability of some important cosmological models is linked to the energy conditions. The energy conditions are reflected by the term $R_{\mu\nu} v^\mu v^\nu \geq 0$ in the Raychaudhuri equation for the expansion of the universe

$$\frac{d\theta}{d\tau} = -\frac{\theta^2}{2} - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu} v^\mu v^\nu, \quad (37)$$

where θ, $\sigma_{\mu\nu}$, and $\omega_{\mu\nu}$ respectively denote the expansion, shear, and rotation, while v^μ is a null vector. Because the Raychaudhuri equation is valid for any geometrical theory of gravity, this can also be used to investigate the energy conditions for modified theories, the $f(G)$ gravity in particular. The NEC, WEC, SEC, and DEC are as follows:

$$\begin{aligned}
&\text{NEC:}\ \ \rho + p_x \geq 0, \quad \rho + p_y \geq 0,\\
&\text{WEC:}\ \ \rho \geq 0, \quad \rho + p_x \geq 0, \quad \rho + p_y \geq 0,\\
&\text{SEC:}\ \ \rho + 3p_x \geq 0, \quad \rho + 3p_y \geq 0, \quad \rho + p_x \geq 0, \\
&\qquad\qquad \rho + p_y \geq 0,\\
&\text{DEC:}\ \ \rho \geq 0, \quad \rho \pm p_x \geq 0, \quad \rho \pm p_y \geq 0.
\end{aligned} \quad (38)$$

We now analyze both $f(G)$ models.

4.1. Case I. Cosmological model $f(G) = G^{5/6}$

This model corresponds to $m = -1/6$. Figure 6 shows the region where the NEC is satisfied. It can be seen that the NEC involving the x-component of pressure is satisfied at an initial epoch and is violated as time increases and n decreases from 0 to -2. In particular, the NEC is satisfied for our solutions with $n = -1/2$. Moreover, the WEC is also satisfied for this model (Figs. 2a and 6). It is observed from Fig. 7 that the SEC is satisfied at an initial epoch, but is violated as the time progresses. This is an indication of the presence of dark energy responsible for accelerated expansion of the universe. The DEC is also satisfied (Fig. 8) and hence this model is viable in the context of energy conditions, and it also justifies the physical importance of exact solutions of modified field equations.

4.2. Case II. Cosmological model $f(G) = G^{1/2}$

This model corresponds to $m = -1/2$. The NEC is satisfied for $-2 < n < 0$ as shown in Fig. 9. The WEC

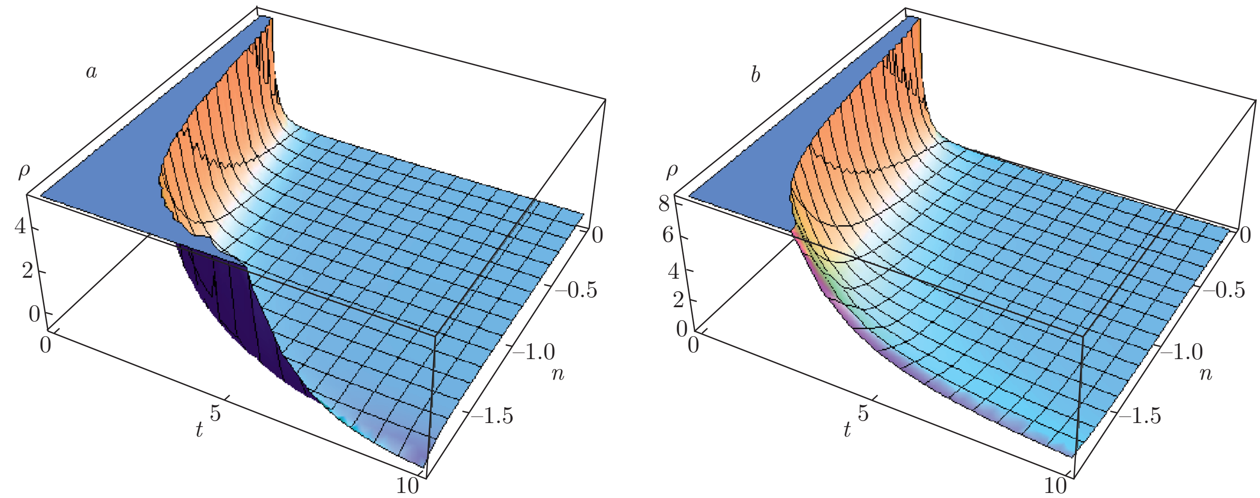

Fig. 2. Behavior of the energy density for $m = -1/6$ (a), $-1/2$ (b)

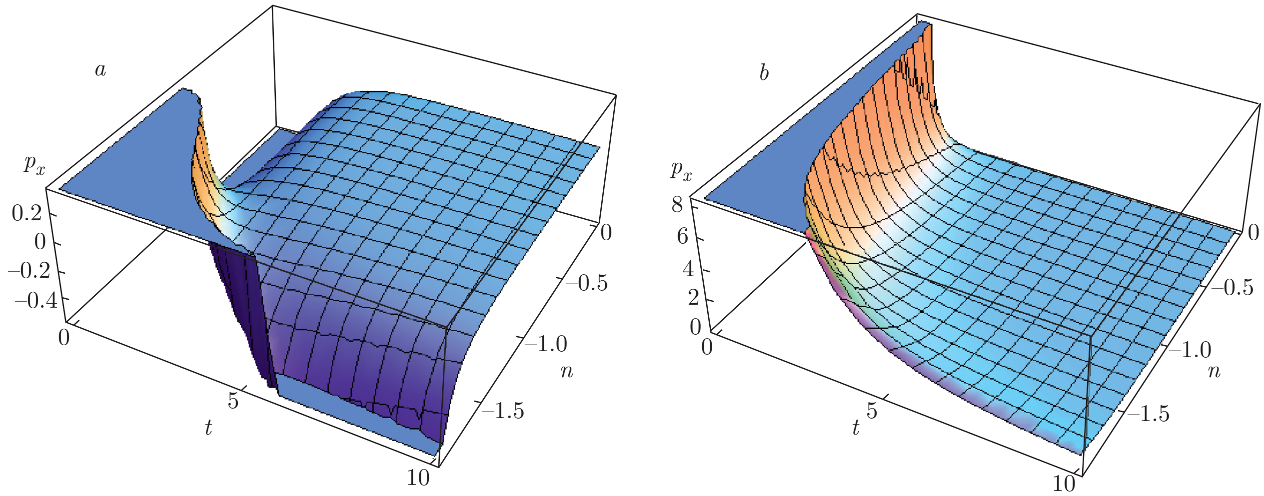

Fig. 3. Behavior of the pressure (x-component) for $m = -1/6$ (a), $-1/2$ (b)

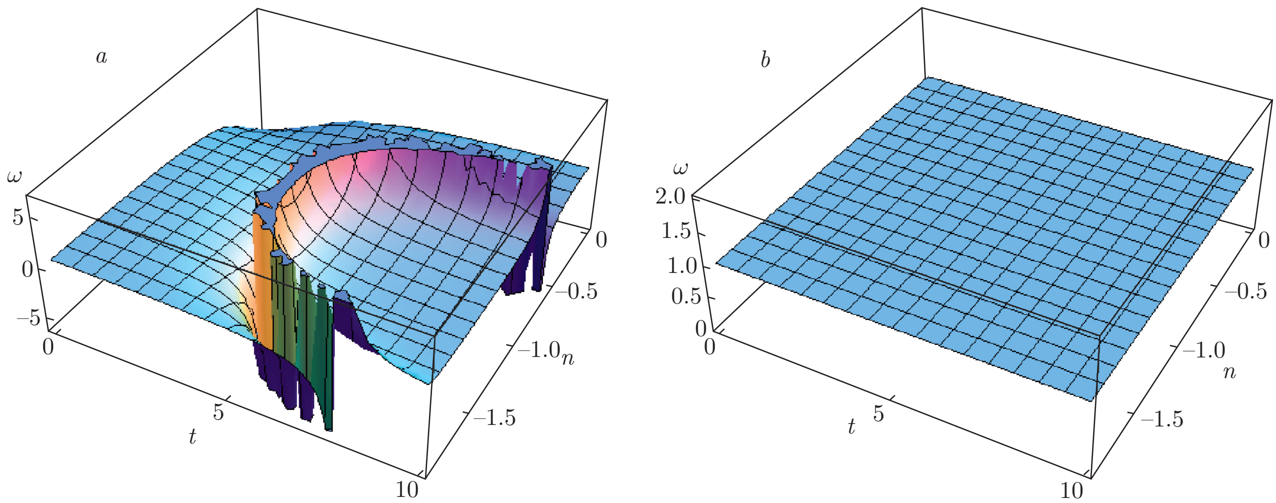

Fig. 4. Behavior of ω for $m = -1/6$ and $m = -1/2$

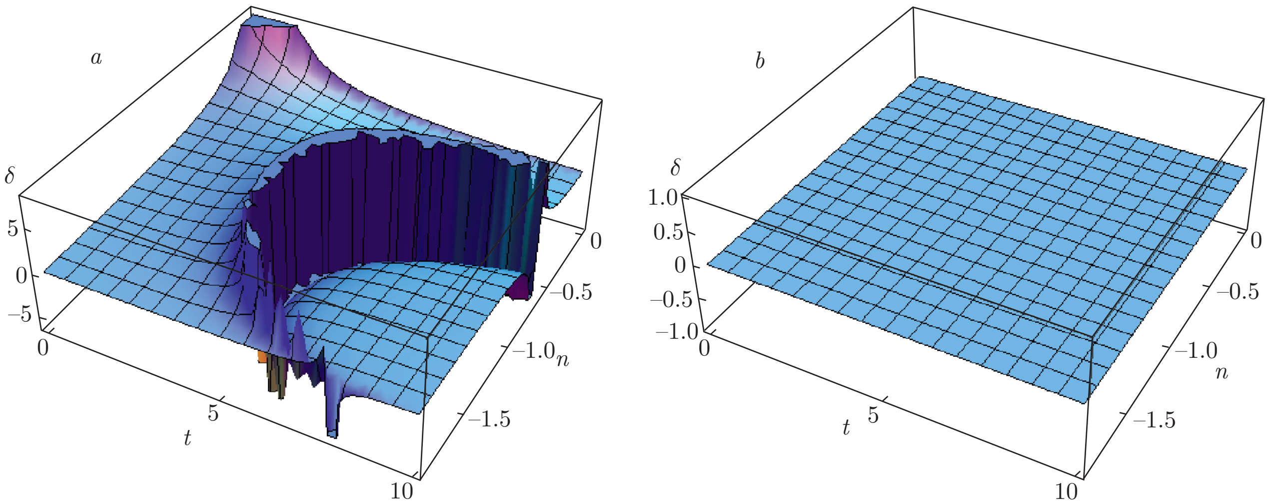

Fig. 5. Behavior of δ for $m = -1/6$ and $m = -1/2$

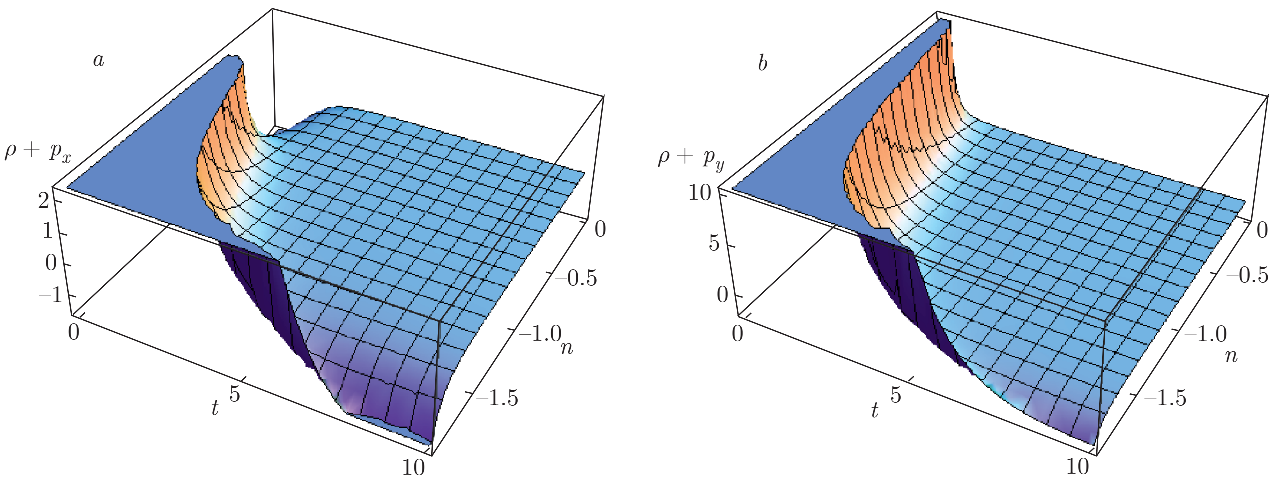

Fig. 6. Plots of the NEC for $f(G) = G^{5/6}$

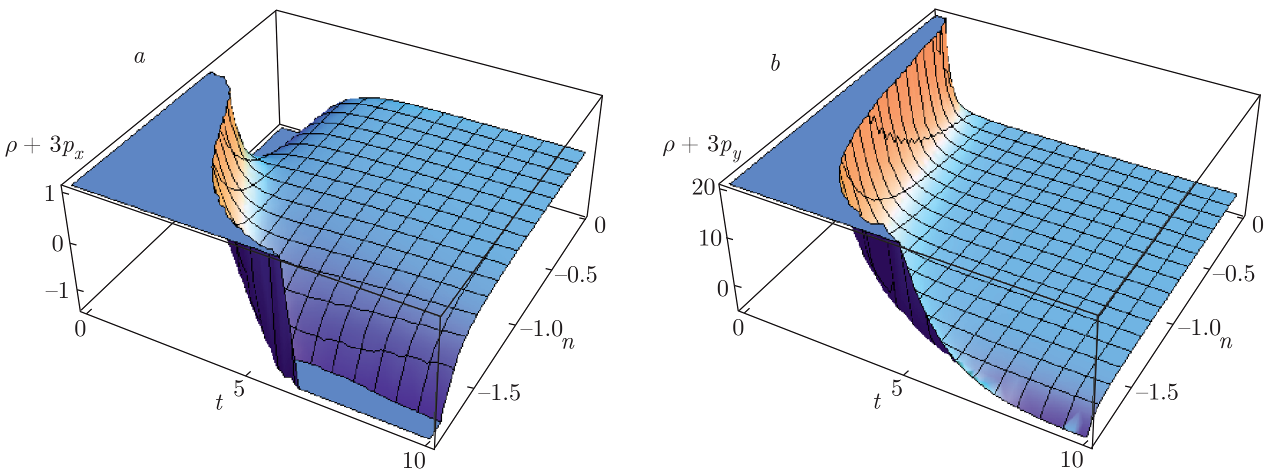

Fig. 7. Plots of the SEC for $f(G) = G^{5/6}$

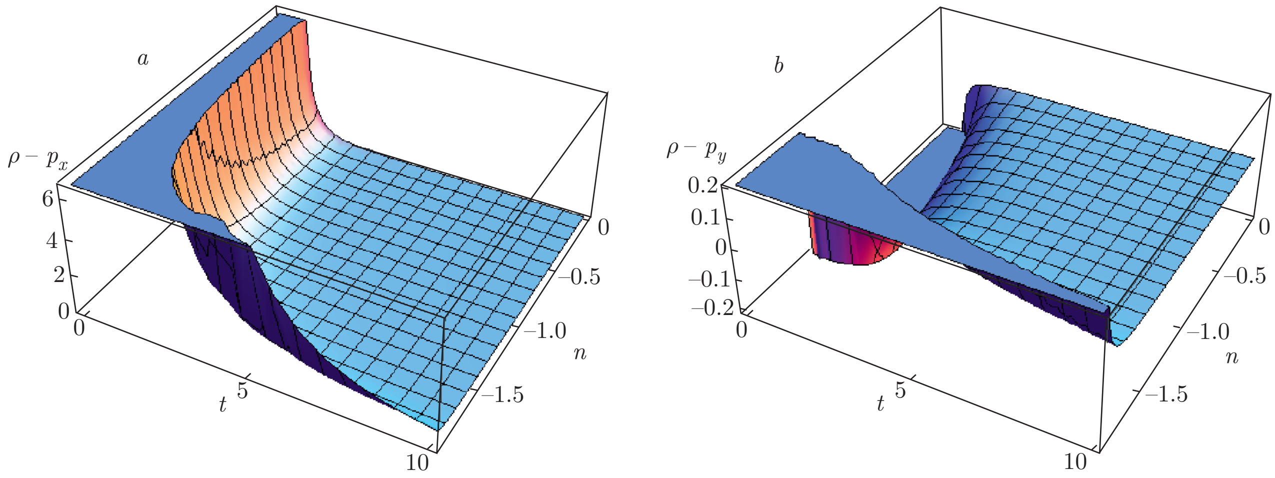

Fig. 8. Plots of the DEC for $f(G) = G^{5/6}$

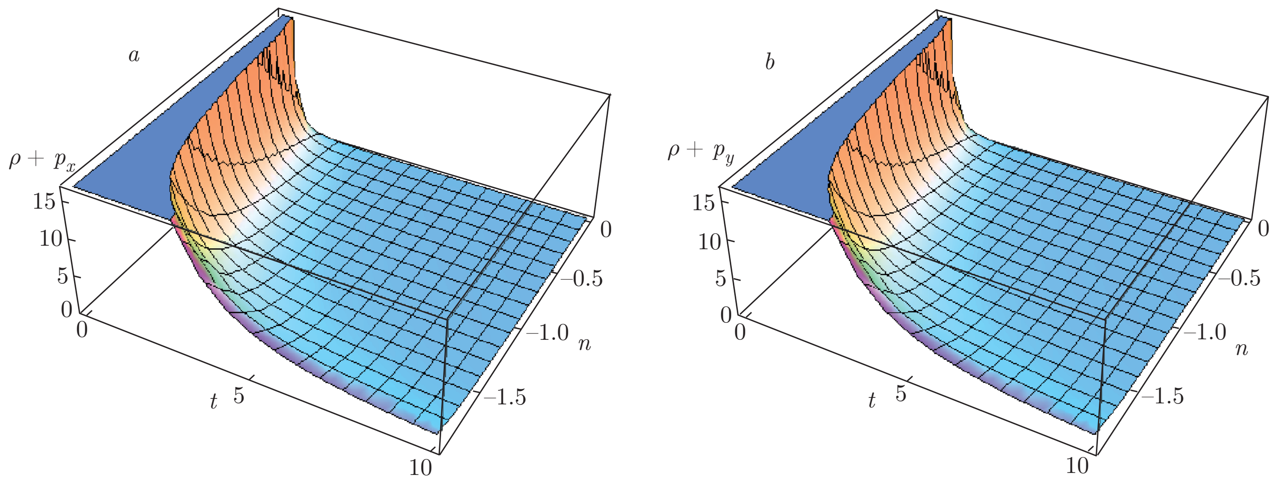

Fig. 9. Plots of the NEC for $f(G) = G^{1/2}$

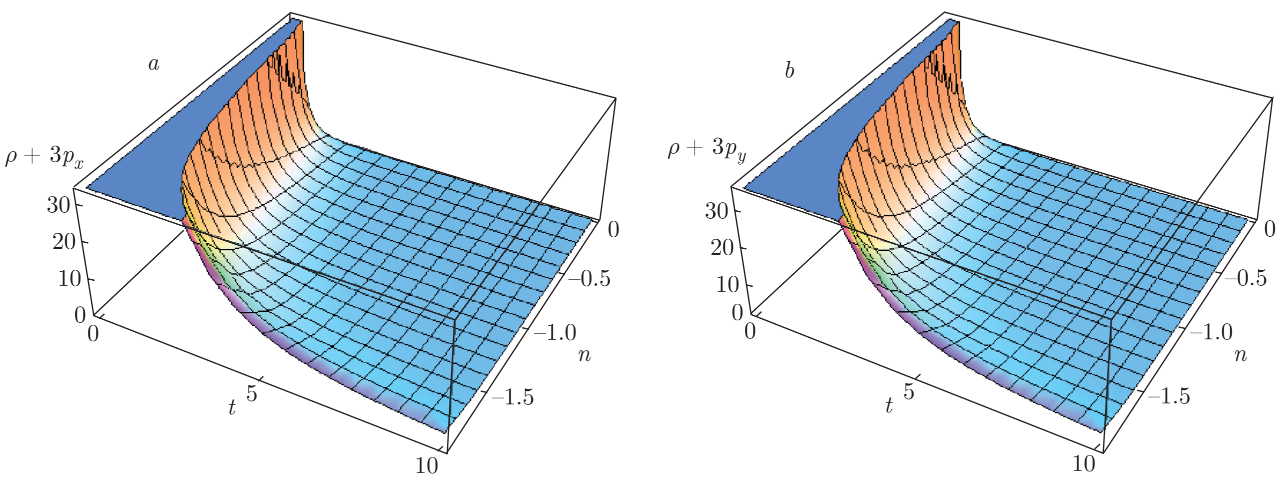

Fig. 10. Plots of the SEC for $f(G) = G^{1/2}$

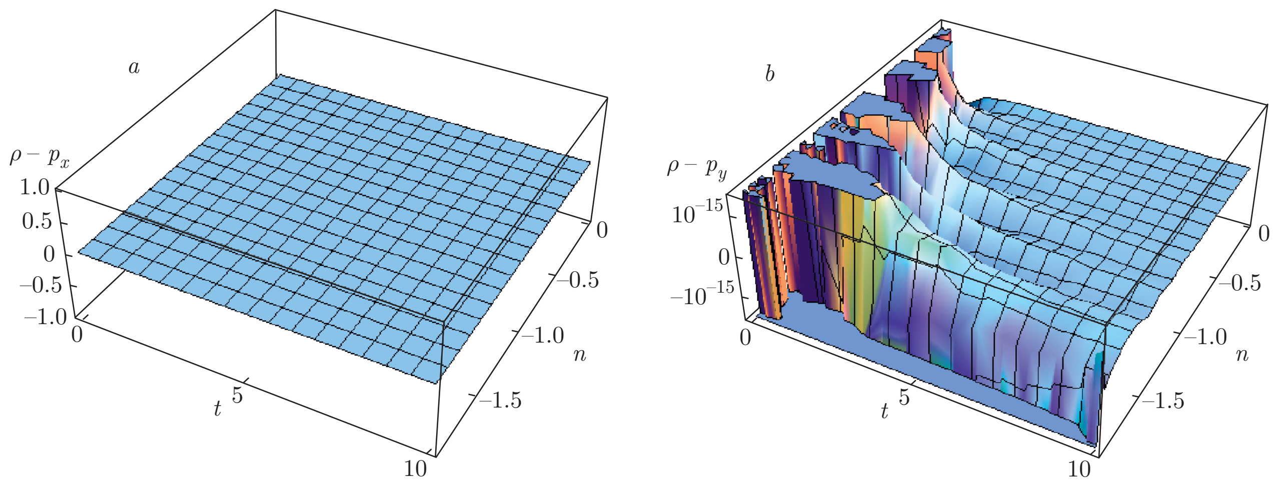

Fig. 11. Plots of the DEC for $f(G) = G^{1/2}$

is also satisfied for this model (Figs. 2*b* and 9). It is observed from Fig. 10 that the SEC is satisfied even as the time progresses. Therefore, this model does not represent accelerated expansion of the universe. Moreover, the DEC is also violated here and hence this model does not support accelerated expansion of the universe for $-2 < n < 0$ (Fig. 11).

5. CONCLUDING REMARKS

$f(G)$ gravity with an anisotropic background is discussed in this paper. The LRS Bianchi type I space–time is chosen for this purpose. We have given exact solutions for the LRS Bianchi type I space–time. With a highly nonlinear and complicated nature of the field equations, we restrict ourself to the assumption that the shear scalar σ is proportional to the expansion scalar θ. The energy conditions for the model under consideration are discussed using graphs. The parameters for graphical analysis are assumed such that the validity of the exact solutions can be checked. A brief summary and conclusion of the work is as follows.

• An important power-law $f(G)$ gravity model already available in the literature [16] is considered. This model is interesting because the chances for a Big Rip singularity to appear vanish. Moreover, it is compatible with the observational data predicting the existence of a transient phantom epoch. The viability of this model has already been shown in cosmological contexts [46–48].

• The metric coefficients involve the anisotropy parameter n for the power-law solution. In particular, two $f(G)$ gravity models are discussed in the context of the solution. The first model $f(G) = G^{5/6}$ gives $\omega = -1$, which describes accelerated expansion of the universe [53–55]. However, the second model involves a square root of the GB invariant, which leads to a viable inflation in the presence of a massive scalar field [59]. The graphical analysis indicates that the EoS parameter ω also takes negative values for $-2 < n < 0$. It is worthwhile to mention that the phantom-like dark energy is found to be in the region where $\omega < -1$.

• One general power-law solution is reported using a power-law $f(G)$ gravity model and some important physical quantities are calculated. In particular, Kasner's universe has been recovered as a special case where the anisotropy parameter is $n = -1/2$. Expression for the skewness parameter δ is calculated and graphical behavior is shown in Fig. 5. The first model shows both positive and negative values of δ. For Kasner's universe, the skewness parameter has the value $\delta = 2.5$. The second model gives $\delta = 0$ corresponding to perfect-fluid solutions without any deviation in the y or z axis.

• The energy conditions are developed for the considered $f(G)$ gravity model. Long expressions involving inequalities are not easy to analyze directly. Thus, to check the viability of the model, present-day values of cosmological parameters are assumed. The graphical analysis is given that shows that the NEC, WEC, and DEC hold, while the SEC is violated for the first model $f(G) = G^{5/6}$. This violation supports the phenomenon of expansion of the universe. However, the NEC, WEC, and SEC are satisfied for the second model $f(G) = G^{1/2}$ while the DEC is violated here and hence this model does not support accelerated expansion of the universe for $-2 < n < 0$.

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