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# MODELING THE EVOLUTION OF GALACTIC MAGNETIC FIELDS

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An analytic model for evolution of galactic magnetic fields in hierarchical galaxy formation frameworks is introduced. Its major innovative components include explicit and detailed treatment of the physics of merger events, mass gains and losses, gravitational energy sources and delays associated with formation of large-scale magnetic fields. This paper describes the model, its implementation, and core results obtained by its means.

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## 1. MODEL FOR GALACTIC MAGNETIC FIELDS

### 1.1. Initial assumptions

By creating this model, we aim to develop a semi-analytic approach for modeling galactic magnetic fields, which solves several known problems, including:

• the problem of overprediction of magnetic field strengths for high-mass galaxies, as pointed out in [1];

• problems arising as a result of treating evolution of magnetic fields completely independently of mass evolution;

• computational efficiency problems.

We proceed first to formulate assumptions that define the galaxy formation and evolution framework.

• Each galaxy consists of the central supermassive black hole; the bulge (or spheroid); the disk; the halo of hot gas; the dark matter halo. The spheroid and the disk contain cold gas and stars.

• All components can grow through mergers and accretion. However, mergers can also trigger mass transfer from the disk to the spheroid through the disk instability mechanism, leading to a decrease in the disk mass.

• Both star formation and supernovae reduce the amount of cold gas. Star formation, however, returns some gas into the system, while supernovae input some energy.

We next consider assumptions on the structure and behavior of galactic magnetic fields.

• Magnetic fields in a disk of a galaxy are represented by ordered magnetic fields, which exhibit large-scale structure, and chaotic (also called random or turbulent) fields, which have no explicit structure on the galactic scales, but can show ordered behavior on smaller scales.

• The gas in a galactic disk can be treated as a magnetohydrodynamic fluid, which results in equipartition of the total energy $\mathcal{E}_\Sigma$ of the system between the energy of the turbulent motions $\mathcal{E}_t$ of the disk gas and its magnetic fields $\mathcal{E}_m$ [1]

$$\dot{\mathcal{E}}_t = \dot{\mathcal{E}}_m. \quad (1)$$

• In the assumed approximation, magnetic fields are tied to the components where they formed and their evolution in each component proceeds independently.

• We further assume that the ordered magnetic fields form merely in galactic disks; we therefore consider disks only. Nevertheless, the same reasoning with minor corrections can be used to derive various properties of magnetic fields for other components of a galaxy if needed.

We next consider the assumptions that are new to this model and, to the best of our knowledge, have not been implemented in other semi-analytic models.

• All energy components of a system are tied to a cold gas, and therefore any decrease in the mass in a gas container results in the corresponding loss of energy

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$$\dot{\mathcal{E}} = \ldots - \mathcal{E}\frac{\dot{\mathcal{M}}_-}{\mathcal{M}}, \tag{2}$$

where $\mathcal{E}$ is some energy component ($\mathcal{E}_\Sigma$, $\mathcal{E}_t$, $\mathcal{E}_m$, etc.), $\mathcal{M}$ is the mass of the cold gas, and $\dot{\mathcal{M}}_-$ is the negative part of the gas mass rate.

• The timescale of the process of formation of ordered magnetic fields from chaotic magnetic fields in this model is parameterized, and is therefore proportional to the period of rotation of the galaxy instead of being equal to it. The latter simpler approach was used in [1].

• Mass gains from mergers are treated explicitly, thus allowing for a detailed investigation of merger-related effects on the evolution of energies of the ordered and random components of galactic magnetic fields.

### 1.2. Equations of energy balance

The rate of change of the total energy of a system with time depends merely on the rate of energy inputs and outputs $\dot{\mathcal{E}}_{io}$, which, after taking (2) into account, yields

$$\dot{\mathcal{E}}_\Sigma = \dot{\mathcal{E}}_{io} - \mathcal{E}_\Sigma\frac{\dot{\mathcal{M}}_-}{\mathcal{M}}. \tag{3}$$

For the rate of change of the energy of ordered magnetic fields $\dot{\mathcal{E}}_o$, everything is somewhat more complicated. When the ordered energy is less than a half of the total energy, it would draw energy from the chaotic magnetic field $\mathcal{E}_c$ and lose it only due to mass losses (2). However, in the case where the total energy decreases and the ordered magnetic field energy is half the total energy, it follows that due to energy equipartition (1), the energy of the ordered magnetic field should decrease along with the total energy without any delays. Both considerations together give

$$\dot{\mathcal{E}}_o = \begin{cases} \dfrac{1}{2}\dot{\mathcal{E}}_\Sigma, & \mathcal{E}_o = \dfrac{1}{2}\mathcal{E}_\Sigma \quad \text{and} \quad \dot{\mathcal{E}}_\Sigma < 0, \\ \dfrac{\mathcal{E}_c}{\tau} - \mathcal{E}_o\dfrac{\dot{\mathcal{M}}_-}{\mathcal{M}} & \text{otherwise}, \end{cases} \tag{4}$$

where $\tau$ is the ordered magnetic field formation timescale, which is proportional to a period of rotation of the considered galaxy, which hence depends indirectly on time.

As a result of the merger, the disk structure and laminar motion of interstellar gas can be disrupted, leading to a partial destruction of large-scale magnetic fields, which can be accounted for by adding more summands to (4), leading to

$$\dot{\mathcal{E}}_o = \begin{cases} \dfrac{1}{2}\dot{\mathcal{E}}_\Sigma, & \mathcal{E}_o = \dfrac{1}{2}\mathcal{E}_\Sigma \quad \text{and} \quad \dot{\mathcal{E}}_\Sigma < 0, \\ \dfrac{\mathcal{E}_c}{\tau} - \mathcal{E}_o\dfrac{\dot{\mathcal{M}}_- + k_{dg}\dot{\mathcal{M}}_{mg} + k_{ds}\dot{\mathcal{M}}_{ms}}{\mathcal{M}} & \text{otherwise}, \end{cases} \tag{5}$$

where $k_{dg}$ and $k_{ds}$ are the efficiencies of the respective mechanisms associated with infall of gas and stars, and $\dot{\mathcal{M}}_{mg}$ and $\dot{\mathcal{M}}_{ms}$ are the corresponding mass infall rates. This, however, does not decrease the total energy of the system.

Finally, all other energies i. e., the turbulent energy, the total energy of magnetic fields, and the chaotic energy, can be obtained from the energy balance equations

$$\dot{\mathcal{E}}_\Sigma = \dot{\mathcal{E}}_t + \dot{\mathcal{E}}_m, \tag{6}$$

$$\dot{\mathcal{E}}_m = \dot{\mathcal{E}}_c + \dot{\mathcal{E}}_o, \tag{7}$$

and equipartition assumption (1).

### 1.3. Sources and sinks

In the recent work [1] on semi-analytic modeling of magnetic field formation and evolution, its authors assumed that the total energy rate consists merely of

• the gravitational energy rate corresponding to the energy brought into the system by accretion;

• a positive energy rate caused by various supernovae feedback mechanisms;

• a negative energy rate due to removal of energy by star formation.

In this model, in addition to those sources, we account for

• energy changes due to mergers including the gravitational energy of infalling matter and in the case where merger causes a disk instability, the negative energy changes due to the transfer of mass from the disk to the bulge;

• supernovae expulsive feedback, which causes all energies of the system to decrease as a result of incurring mass losses.

All the enumerated energy losses are caused by the corresponding mass losses, and hence it is possible to account for all of them just by explicitly defining all the mass rates as

$$\dot{\mathcal{M}} = \dot{\mathcal{M}}_+ - \dot{\mathcal{M}}_-, \tag{8}$$

$$\dot{\mathcal{M}}_+ = \dot{\mathcal{M}}_a + \dot{\mathcal{M}}_{mi} + \dot{\mathcal{M}}_{sfi}, \tag{9}$$

$$\dot{\mathcal{M}}_- = \dot{\mathcal{M}}_{sfo} + \dot{\mathcal{M}}_{sn} + \dot{\mathcal{M}}_{mo} + \dot{\mathcal{M}}_{di}, \tag{10}$$

where $\dot{\mathcal{M}}_+$ is the positive part of the total mass rate, $\dot{\mathcal{M}}_a$ is the positive mass rate due to accretion of matter to the disk, $\dot{\mathcal{M}}_{mi}$ is the positive mass rate due to acquisition of additional mass through mergers, $\dot{\mathcal{M}}_{sfi}$ is the mass input due to gas recycling, $\dot{\mathcal{M}}_{sfo}$ is a negative mass rate due to the effects of star formation, $\dot{\mathcal{M}}_{sn}$ is a negative mass rate due to expulsive supernovae feedback, $\dot{\mathcal{M}}_{mo}$ is a negative mass rate due to mergers, and $\dot{\mathcal{M}}_{di}$ a negative mass rate due to disk instability.

The rest of energy rates should be accounted explicitly,

$$\dot{\mathcal{E}}_{io} = \dot{\mathcal{E}}_{sn} + \dot{\mathcal{E}}_a + \dot{\mathcal{E}}_m, \tag{11}$$

where $\dot{\mathcal{E}}_{sn}$ is the energy input rate due to supernovae feedback, $\dot{\mathcal{E}}_a$ is the energy input rate due to accretion of mass into the galactic disk, and $\dot{\mathcal{E}}_m$ is the energy input rate due to mergers.

We now define model parameters that determine efficiencies of various energy sources and sinks. We account for the efficiency of supernovae with $k_{sn}$, accretion with $k_a$, and mergers with $k_{mg}$ for the gaseous component and $k_{ms}$ for the stellar one; finally, $k_\tau$ defines the relation between a characteristic timescale $\tau$ and the period of rotation of a galaxy.

We now consider the energy sources individually. We begin with accretion, where the energy rate is

$$\dot{\mathcal{E}}_a = k_a G \dot{\mathcal{M}}_a \int\limits_{r_d}^{r_{gh}} \frac{M + \Delta M}{r^2}\, dr = = k_a G \dot{\mathcal{M}}_a \left[ M \left( \frac{1}{r_d} - \frac{1}{r_{gh}} \right) + \int\limits_{r_d}^{r_{gh}} \frac{\Delta M}{r^2}\, dr \right], \tag{12}$$

where $G$ is the universal gravitatinal constant, $M$ is the total mass inside the disk radius, $\Delta M$ is the fraction of mass between the current infall distance and the disk radius, $r_d$ is the radius of the galactic disk, and $r_{gh}$ is the radius of the hot gas halo. This result can be simplified by first assuming

$$M \gg \Delta M \tag{13}$$

and then

$$r_d \ll r_{gh}, \tag{14}$$

which leads to

$$\dot{\mathcal{E}}_a \approx k_a G \dot{\mathcal{M}}_a M \left( \frac{1}{r_d} - \frac{1}{r_{gh}} \right) \approx k_a G \frac{\dot{\mathcal{M}}_a M}{r_d}. \tag{15}$$

The energy rate of the source associated with mergers is

$$\dot{\mathcal{E}}_m = k_{mg} G \dot{\mathcal{M}}_{mg} \int\limits_{r_d}^{r_g} \frac{M + \Delta M}{r^2} dr + + k_{ms} G \dot{\mathcal{M}}_{ms} \int\limits_{r_d}^{r_g} \frac{M + \Delta M}{r^2} dr = = G \left( k_{mg} \dot{\mathcal{M}}_{mg} + k_{ms} \dot{\mathcal{M}}_{ms} \right) \times \times \left[ M \left( \frac{1}{r_d} - \frac{1}{r_g} \right) + \int\limits_{r_d}^{r_g} \frac{\Delta M}{r^2} dr \right], \tag{16}$$

where $r_g$ is the distance between merging galaxies, and $\dot{\mathcal{M}}_{mg}$ and $\dot{\mathcal{M}}_{ms}$ are the respective rates of gas and star infall. To simplify the obtained result, in addition to (13), we can assume that

$$r_d \ll r_g \tag{17}$$

which leads to

$$\dot{\mathcal{E}}_m \approx G \left( k_{mg} \dot{\mathcal{M}}_{mg} + k_{ms} \dot{\mathcal{M}}_{ms} \right) M \left( \frac{1}{r_d} - \frac{1}{r_g} \right) \approx \approx G \frac{\left( k_{mg} \dot{\mathcal{M}}_{mg} + k_{ms} \dot{\mathcal{M}}_{ms} \right) M}{r_d}. \tag{18}$$

Further possible simplifications of both these sources include

• equivalence of the merger and accretion efficiency coefficients

$$k_a = k_{mg}, \tag{19}$$

which can be assumed to be true because accretion of gas from a hot halo shares many similarities with the accretion of gas from a satellite galaxy in course of the merger event;

• equivalence of the total negative rate to the overall negative rate

$$\dot{\mathcal{M}}_- \approx \theta \left( -\dot{\mathcal{M}} \right) \dot{\mathcal{M}}, \tag{20}$$

where $\theta$ is the Heaviside step function; this assumption holds when

$$\left|\dot{\mathcal{M}}_-\right| \gg \left|\dot{\mathcal{M}}_+\right| \vee \left|\dot{\mathcal{M}}_-\right| \ll \left|\dot{\mathcal{M}}_+\right|; \tag{21}$$

• equivalence of the total positive rate to the overall positive rate

$$\dot{\mathcal{M}}_{mg} + \dot{\mathcal{M}}_a \approx \theta\left(\dot{\mathcal{M}}\right)\dot{\mathcal{M}}, \tag{22}$$

$$\dot{\mathcal{M}}_{ms} \approx \theta\left(\dot{\mathcal{M}}_{ts}\right)\dot{\mathcal{M}}_{ts}, \tag{23}$$

where $\dot{\mathcal{M}}_{ts}$ is the total stellar mass change in the disk; approximation (22) works only if (19) is applicable, (21) is assumed, and

$$\dot{\mathcal{M}}_+ \approx \dot{\mathcal{M}}_a + \dot{\mathcal{M}}_{mi},$$

i. e., the recycled gas is negligible, and (23) works if (21) is assumed for stellar masses and

$$\dot{\mathcal{M}}_{ts+} \approx \dot{\mathcal{M}}_{ms},$$

i. e., the amount of forming stars is much smaller than the amount of stars incorporated into a galaxy as a result of mergers; thus, the $\dot{\mathcal{M}}_{ts+}$ total positive stellar rate is dominated by stars from the satellite galaxy.

### 1.4. Solutions

Although the mass and energy rates should be determined in order to obtain conclusive results, it is still possible and, moreover, important to obtain analytic solutions for energy balance equations (3), (4), (5), (6), and (7) in the general case.

For the total energy of a system, we obtain

$$\mathcal{E}_\Sigma = \exp\left(-\int_{t_0}^{t}\frac{\dot{\mathcal{M}}_-}{\mathcal{M}}dt'\right) \times \\ \times\left[\mathcal{E}_{\Sigma,0} + \int_{t_0}^{t}\exp\left(\int_{t_0}^{t''}\frac{\dot{\mathcal{M}}_-}{\mathcal{M}}\,dt'\right)\dot{\mathcal{E}}_{io}\,dt''\right], \tag{24}$$

where $\mathcal{E}_{\Sigma,0}$ is the initial total energy at the moment of time $t_0$.

For the energy of ordered magnetic fields, we obtain two solutions. In the case

$$\mathcal{E}_o = \frac{1}{2}\mathcal{E}_\Sigma \wedge \dot{\mathcal{E}}_\Sigma < 0,$$

the ordered energy is obviously defined as

$$\mathcal{E}_o = \frac{1}{2}\mathcal{E}_\Sigma$$

and in all other cases, as

$$\mathcal{E}_o = \exp\left[-\int_{t_0}^{t}\left(\frac{1}{\tau}+\frac{\dot{\mathcal{M}}_-}{\mathcal{M}}\right)dt'\right] \times \\ \times\left\{\mathcal{E}_{o,0}+\int_{t_0}^{t}\exp\left[\int_{t_0}^{t''}\left(\frac{1}{\tau}+\frac{\dot{\mathcal{M}}_-}{\mathcal{M}}\right)dt'\right]\frac{\mathcal{E}_\Sigma}{2\tau}dt''\right\}, \tag{25}$$

where $\mathcal{E}_{o,0}$ is the initial energy of the ordered magnetic field at the moment of time $t_0$. Appropriate values for $t_0$ and $\mathcal{E}_{o,0}$ should be obtained for each interval where this solution is applicable.

In the case where disruption of ordered magnetic fields (5) is taken into account, instead of (25), we obtain

$$\mathcal{E}_o = \exp\left(-\int_{t_0}^{t} f\,dt'\right) \times \\ \times\left[\mathcal{E}_{o,0} + \int_{t_0}^{t}\exp\left(\int_{t_0}^{t''} f\,dt'\right)\frac{\mathcal{E}_\Sigma}{2\tau}\,dt''\right], \tag{26}$$

where

$$f = \frac{1}{\tau} + \frac{\dot{\mathcal{M}}_- + k_{dg}\dot{\mathcal{M}}_{mg} + k_{ds}\dot{\mathcal{M}}_{ms}}{\mathcal{M}}.$$

## 2. DETAILS OF CURRENT IMPLEMENTATION

### 2.1. Magnetic field model and galaxy formation frameworks

This model is designed to be compatible with most semi-analytic galaxy formation models and to evolve galactic magnetic fields according to their outputs.

A symbiosis of this model with an arbitrary semi-analytic model may be implemented with or without feedback, as one executable or as independent software packages. Additionally, if the feedback is implemented and models are developed as separate packages, it is possible to implement symbiosis in an iterative fashion, when outputs of this model are redirected into the galaxy formation framework until the convergence goal is reached.

Currently, the model for evolution of galactic magnetic fields is implemented as an independent software without feedback. Other implementation options are considered as possible future goals.

### 2.2. Implemented features and dependences

This software is represented by several modules: one for the modeling purposes and several others for data analysis, plotting, etc. It is implemented in C++11 and relies on the Standard Template Library (STL) only.

The current implementation of the modeling module is based on assumptions (13), (14), (17) and (19) and includes all the features except

• disrupted ordered magnetic fields (5), i. e., $k_{dg} = k_{ds} = 0$;

• contributions from stars during mergers (16), i. e., $k_{ms} = 0$.

Results of the modeling, in addition to being processed by other modules of this software, are also placed into a text database and then processed by means of SQLite[1)].

## 3. RESULTS AND CONCLUSION

### 3.1. Model parameters and major obtained results

To produce input data for this model, we used outputs of revision 958 of version 0.9.1 of the Galacticus — a semi-analytic model of galaxy formation and evolution developed by A. Benson [2]. This model have been fitted to strengths of the volume-averaged magnetic fields for a sample of approximately one hundred of late-type galaxies from [3] and stellar masses $M_\star$ from [4] for the same sample (Fig. 1). The correlation coefficient for the derived energy densities and stellar masses in this sample is $-0.119321$, i. e., according to this data, there are no dependences of the density of energy of galactic magnetic fields on the stellar masses of galaxies.

The values of parameters corresponding to the chosen observational sample are $k_a = k_{mg} = 3.87 \cdot 10^{-7}$, $k_{sn} = 4.99 \cdot 10^{-4}$, and $k_\tau = 10.53$. These values show that

• the fraction of supernovae energy is three orders of magnitude larger then the fraction of gravitational energy, and hence accretion and mergers have a smaller impact on formation and evolution of galactic magnetic fields per unit of produced energy;

• the efficiency coefficients are much smaller than their analogs, for example, in [1], showing that explicit inclusion of mergers leads to a significant increase in the total energy inputs and outputs;

• a characteristic timescale is an order of magnitude larger than the corresponding period of rotation, which leads to significantly lower rates of formation and evolution of ordered magnetic fields and, thus, results in a qualitatively different long term behavior in contrast with predictions of earlier models.

The results obtained with these parameters can be divided into several groups in domains of cosmic time and stellar mass vs energy of ordered magnetic fields. In the cosmic time domain, galactic magnetic fields evolve through three distinctive epochs:

• the early epoch (formation of the first galaxies, 2.5 Gyr);

• the intermediate epoch (2.5–6.75 Gyr);

• the late epoch (6.75 Gyr — present days).

In the domain stellar mass vs energy of ordered magnetic fields, galaxies form two major qualitatively different groups with two minor subgroups (Fig. 2).

Group I is represented by rapidly evolving galaxies that have much more energy in their magnetic fields due to various intensive processes than they can maintain in a long run, and hence at a later stage of their evolution, they evolve into the second group. These magnetic fields form at the earliest epochs and exist up to the present day. However, their diagram of location in the stellar mass vs energy of ordered magnetic fields moves in the direction of high stellar masses over time.

Group II is galaxies with a sustainable value of energy of ordered magnetic fields. Their magnetic fields evolve with their stellar masses along a curve with a constant ratio $(\lg \mathcal{E}_o - \text{const})/\lg M_\star$, showing an explicit power law dependence on the latter. This group forms at the beginning of the intermediate epoch, and during the next epoch it is divided into two subgroups, which exhibit two qualitatively different sustainable relations between energies of ordered and random magnetic fields. Group IIA has most of its magnetic fields energy in ordered magnetic fields, and group IIB conserves most of its magnetic fields energy in the chaotic fields.

### 3.2. Conclusion

In this work, we discovered:

• three epochs of evolution of galactic magnetic fields;

[1)] SQLite is a free implementation of SQL data base engine (http://www.sqlite.org).

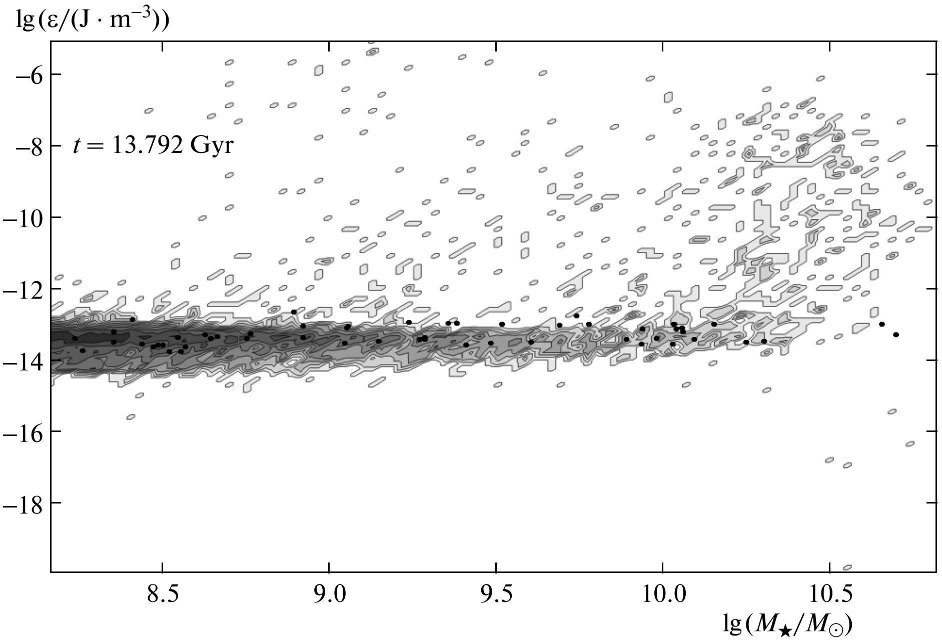


**Fig. 1.** Energy density $\varepsilon$ of ordered magnetic fields: contours represent number counts of modelled galaxies; black dots represent observations

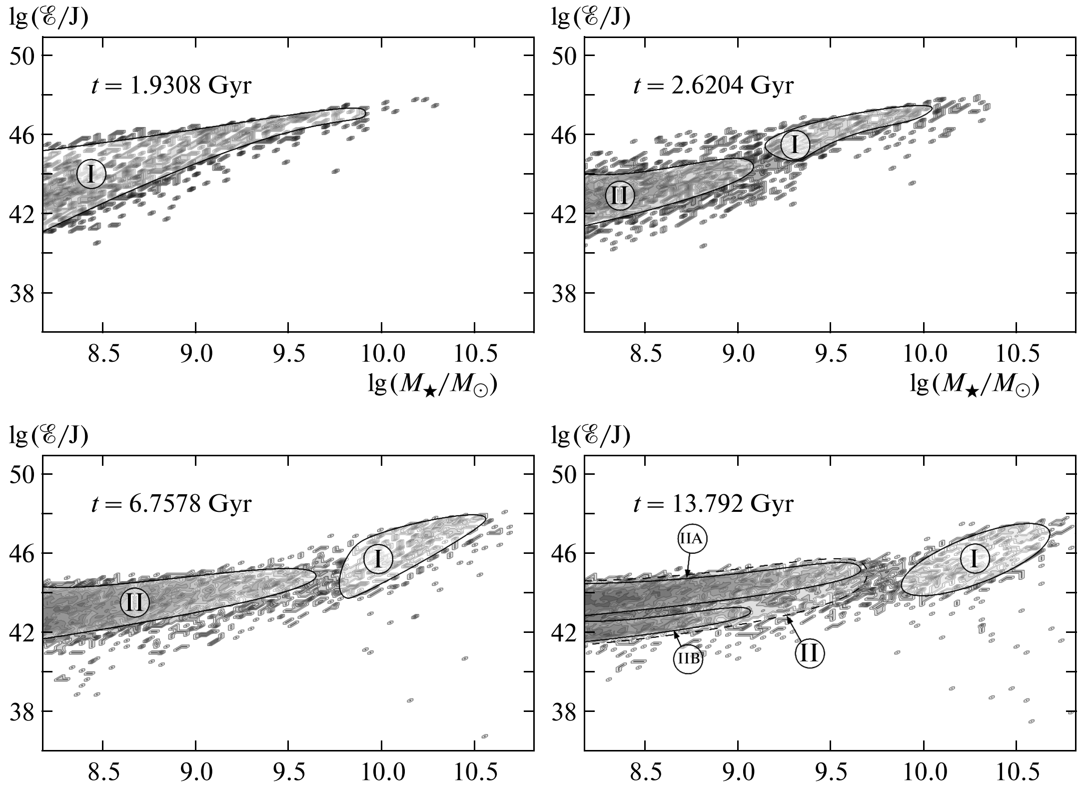


**Fig. 2.** Ordered magnetic fields evolution diagram

• two major groups of evolving galaxies;

• a power-law dependence between the total stellar masses and energies of ordered magnetic fields of galaxies;

Major innovations of this model are as follows:

• all mass losses, including mass losses due to mergers, starformation, etc., are taken into account and they affect both the total energy and the magnetic fields energy;

• the timescale of formation of ordered magnetic fields is parameterized, which gives additional control over the way galactic magnetic fields in the model evolve in time;

• new energy inputs and outputs have been introduced, while old energy inputs and outputs are treated in a much more elaborated manner;

• the new model may account for shapes of gravitational potential wells when energy inputs are being calculated;

• simplified versions of this new model, which are computationally less demanding and require less inputs, are also introduced and justified.

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