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OPEN STRING IN THE PRESENCE OF THE *pp*-WAVE, LINEAR DILATON, AND KALB–RAMOND BACKGROUNDS

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We study open strings attached to a Dp-brane in the presence of the pp-wave background along with a constant antisymmetric B-field and the linear dilaton. The noncommutativity structure of this system is also investigated.

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1. INTRODUCTION

String theory in various backgrounds has been profoundly studied. Some of these backgrounds admit a solvable string theory. One of them is the pp-wave spacetime [1], which is supported by a null, constant 5-form flux and can be obtained from the $AdS_5 \times S^5$ metric by taking the Penrose limit. The pp-wave background is a maximal supersymmetric space in which closed string theory is exactly solvable in the light-cone gauge [2, 3]. Another popular background is the constant antisymmetric B-field, which has been extensively studied in the literature. It leads to nontrivial physics on the branes. The noncommutativity of the open string end points, which are attached to a D-brane [4], is a consequence of the mixed boundary condition in the B-field background. In addition, we have the linear dilaton field as a background, which is the simplest background for noncritical string theory [5]. Among the various conformal field theories (CFTs), the linear dilaton CFT has some interesting applications in string theory [6]. For example, the D-brane noncommutativity is investigated in various background fields such as the dilaton [7–10].

In this article, we consider all the three open-string background fields mentioned above and investigate on the solvability of the theory. Besides, it has been demonstrated that in the light-cone formulation of strings in the pp-wave, the momentum space also becomes noncommutative, which leads to a fully noncommutative phase space. This fact also motivated us to extend the problem by adding the above background fields and see whether any new kind of quantum geometry arises.

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2. OPEN STRING IN A SET OF BACKGROUND FIELDS

The pp-wave background consists of a plane wave metric, accompanied by a homogeneous R–R 5-form flux

$$
\begin{aligned}
ds^2 &= -f^2 X^i X^i (dX^+)^2 + 2dX^+ dX^- + dX^I dX^I, \\
&\qquad\qquad I = 1, 2, \ldots, 8, \\
F_5 &= f dX^+ \wedge \left(dX^1 \wedge dX^2 \wedge dX^3 \wedge dX^4 + \right. \\
&\qquad \left. + \, dX^5 \wedge dX^6 \wedge dX^7 \wedge dX^8\right).
\end{aligned}
\tag{1}
$$

We consider an open string attached to a Dp-brane in the presence of the following background fields: the pp-wave metric, a constant Kalb–Ramond tensor field $B_{\mu\nu}$, and the dilaton field Φ. In the light-cone formalism, the coordinates are decomposed as

$$
\{X^+, X^-\} \bigcup \{X^I | I = 1, 2, \ldots, p-1\} \bigcup \bigcup \{X^i | i = p+1, \ldots, 9\},
$$

where $X^{\pm} = (X^0 \pm X^p)/\sqrt{2}$ and $X^+ = x^+ + \alpha' p^+ \tau$. The string sigma-model action in the above backgrounds is

$$S = -\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma \times$$
$$\times \left[g_{IJ}\left(\sqrt{-h}h^{ab}\partial_a X^I \partial_b X^J + m^2 X^I X^J\right) + \right.$$
$$\left. + \epsilon^{ab} B_{IJ}\partial_a X^I \partial_b X^J + \alpha' \sqrt{-h}\,\Phi R^{(2)}\right], \quad (2)$$

where Σ is the string worldsheet with the metric h_{ab}, and $h = \det h_{ab}$. The scalar curvature $R^{(2)}$ is constructed from the metric h_{ab}. The spacetime metric is also $g_{\mu\nu}$, which is given by Eq. (1). The mass parameter m, i. e., the mass of the worldsheet fields X^I, is defined as $m := \alpha' p^+ f$.

In the conventional case, the dilaton is usually a general arbitrary function of the spacetime coordinates, but considering only a linear dilaton gives rise to simplified equations. We suppose that the dilaton field has a linear form along the brane worldvolume, i. e., $\Phi = a_\alpha X^\alpha$, where the parameters $\{a_\alpha | \alpha = 0, 1, \ldots, p\}$ are constant. Regarding the diffeomorphism invariance of action (1), we are able to choose a conformally flat form for the worldsheet metric,

$$h_{ab}(\sigma,\tau) = e^{\rho(\sigma,\tau)}\eta_{ab}.$$

Because the dilaton field removes the Weyl symmetry, we are not allowed to set $\rho(\sigma,\tau)$ equal to zero. We finish our setup by setting $a_0 = a_p = 0$ to avoid the presence of the coordinates $X^\pm$ in the action.

Equating the variation of the action to zero gives the equations of motion for the worldsheet fields X^I and ρ in the form

$$(\partial^2 - m^2)X^I + \frac{1}{2}\alpha' a^I \partial^2\rho = 0, \quad (3)$$

$$a_I \partial^2 X^I = 0, \quad (4)$$

where $\partial^2 = -\partial_\tau{}^2 + \partial_\sigma{}^2$. In a noncritical string theory, i. e., for $a^2 = a_I a^I \propto d - 26 \neq 0$, these equations can be written as

$$(\partial^2 - m^2)X^I + \mathcal{A}^I_J X^J = 0, \quad (5)$$

$$\partial^2\rho = \frac{2m^2 a_I X^I}{\alpha' a^2}, \quad (6)$$

where the matrix is defined by $\mathcal{A}_{IJ} := m^2 a_I a_J / a^2$. The first equation reveals that the worldsheet fields X^I effectively feel the potential

$$V(X) = \frac{1}{2}\mathcal{A}_{IJ} X^I X^J + V_0,$$

where V_0 is the potential at the origin of the coordinates. We observe that the presence of the linear dilaton and *pp*-wave background simultaneously is the origin of this potential. However, the vanishing of the variation of the action also defines boundary conditions for the open string. For example, for the open string end at $\sigma = 0$, we obtain the equations

$$(\partial_\sigma X^I - B^I_J \partial_\tau X^J)|_{\sigma=0} = 0, \quad (7)$$

$$(a_I \partial_\sigma X^I)|_{\sigma=0} = 0 \quad (8)$$

for X^I and ρ.

It is not very easy to solve Eq. (5) in the general case. Therefore, we consider the situation where the only nonzero components of the vector a_I are a_1 and a_2, which gives $\Phi = a_1 X^1 + a_2 X^2$. We also apply the block diagonal form of the B-field

$$B = \begin{pmatrix} 0 & b & 0 & 0 \\ -b & 0 & 0 & 0 \\ 0 & 0 & 0 & b' \\ 0 & 0 & -b' & 0 \end{pmatrix}, \quad (9)$$

where the nonzero elements are $B_{12} = b$ and $B_{34} = b'$. Now, rewriting Eq. (5), we obtain

$$\begin{aligned} \Delta_1 X^1 + k X^2 = 0, \\ \Delta_2 X^2 + k X^1 = 0, \end{aligned} \quad (10)$$

where the operators $\Delta_{\{1,2\}}$ and the constant k are defined by

$$\begin{aligned} \Delta_{\{1,2\}} &:= \partial^2 - m^2 \frac{a^2{}_{\{2,1\}}}{a^2}, \\ k &:= m^2 \frac{a_1 a_2}{a^2}. \end{aligned} \quad (11)$$

By combining Eqs. (10), we obtain the equations

$$\Delta_2\Delta_1 X^1 - k^2 X^1 = 0, \quad (12)$$

$$X^2 = -\frac{1}{k}\Delta_1 X^1. \quad (13)$$

The general solution of partial differential equation (12), which has rank four, with boundary condition (7) can be written as

$$X^1(\sigma,\tau) = \left(x^1 \cos\omega_0\tau + 2\alpha' p^1 \frac{\sin\omega_0\tau}{\omega_0}\right)\mathrm{ch}(\omega_0 b\sigma) -$$
$$- \frac{1}{\omega_0}\left(-2\alpha' p^2 \cos\omega_0\tau + x^2\omega_0 \sin\omega_0\tau\right)\mathrm{sh}(\omega_0 b\sigma) +$$
$$+ \sqrt{2\alpha'}\sum_{n\neq 0}\exp(-i\omega_n\tau) \times$$
$$\times \left(i\frac{\alpha_n{}^1}{\omega_n}\cos n\sigma + \frac{\alpha_n{}^2}{n} b \sin n\sigma\right). \quad (14)$$

Then Eq. (13) implies that

$$X^2(\sigma,\tau) = -\frac{a_1}{a_2}X^1(\sigma,\tau). \quad (15)$$

Here, the frequencies are

$$\omega_0 = \pm\frac{m}{\sqrt{1+b^2}},\qquad \omega_n = \operatorname{sign}(n)\sqrt{m^2+n^2}. \quad (16)$$

We note that we can write Eqs. (10) as

$$\Delta_2\Delta_1 X^2 - k^2X^2 = 0,$$

which reveals that X^2 has a solution similar to the one in Eq. (14), with the indices 1 and 2 interchanged. Comparing this solution for X^2 and Eq. (14) leads to $b = 0$. In this case, the mixed boundary conditions for X^1 and X^2 reduce to the Neumann boundary conditions. Thus, the mode expansion for X^1 is

$$X^1(\sigma,\tau) = x^1\cos m\tau + 2\alpha' p^1\frac{\sin m\tau}{m} + i\sqrt{2\alpha'}\sum_{n\neq 0}\exp(-i\omega_n\tau)\frac{\alpha_n{}^1}{\omega_n}\cos n\sigma, \quad (17)$$

and again $X^2(\sigma,\tau) = -(a_1/a_2)X^1(\sigma,\tau)$.

Next with a different approach, we demonstrate that our setup is consistent only for $b=0$. It is noteworthy that the dilaton term of the action cannot be treated just classically, but it has a quantum worldsheet correction that modifies the energy–momentum tensor. String action (2) with the linear dilaton $\Phi = a_\mu X^\mu$ defines a family of CFTs with the energy–momentum tensor

$$T(z) = -\frac{1}{\alpha'} : g_{\mu\nu}\partial X^\mu\partial X^\nu : + a_\mu\partial^2X^\mu,\qquad \tilde{T}(\bar{z}) = -\frac{1}{\alpha'} : g_{\mu\nu}\bar{\partial} X^\mu\bar{\partial} X^\nu : + a_\mu\bar{\partial}^2X^\mu. \quad (18)$$

Recalling that X^- is related to the other coordinates except X^+ and using the *pp*-wave metric (1), we obtain

$$T(z) = -\frac{1}{\alpha'} : \left(\frac{1}{2}\partial X^K\partial X_K + \frac{m^2}{8z^2}X^KX_K + \partial X^i\partial X_i\right) : + a_K\partial^2X^K,$$
$$\tilde{T}(\bar{z}) = -\frac{1}{\alpha'} : \left(\frac{1}{2}\bar{\partial} X^K\bar{\partial} X_K + \frac{m^2}{8\bar{z}^2}X^KX_K + \bar{\partial} X^i\bar{\partial} X_i\right) : + a_K\bar{\partial}^2X^K, \quad (19)$$

where

$$K\in\{1,2,3,4\},\quad i\in\{5,\ldots,p-1\}\bigcup\{p+1,\ldots,9\}.$$

We note that we have assumed the only nontrivial dilaton coefficients to be a_1 and a_2. Because momentum does not flow at the boundary, we must have $T(z) = \tilde{T}(\bar{z})$ (at the boundary, $z=\bar{z}$), hence we obtain the conditions

$$a^k\alpha_n{}^lB_{kl} = 0,\qquad a^kp_0{}^lB_{kl} = 0,\qquad a^kx_0{}^lB_{kl} = 0,\quad k,l = 1,2. \quad (20)$$

These equations imply that $B_{12} = b = 0$.

3. QUANTIZATION

For the string coordinates X^3 and X^4, we can write the solutions as

$$X^{I'}(\sigma,\tau) = X_0^{I'}(\sigma,\tau) + X_1^{I'}(\sigma,\tau),$$
$$X_0^{I'}(\sigma,\tau) = \left(x^{I'}\cos\omega_0\tau + 2\alpha' p^{I'}\frac{\sin\omega_0\tau}{\omega_0}\right)\times \operatorname{ch}(\omega_0 b'\sigma) + \frac{1}{\omega_0 b'}B^{I'}_{J'}\times\left(-x^{J'}\omega_0\sin\omega_0\tau + 2\alpha' p^{J'}\cos\omega_0\tau\right)\operatorname{sh}(\omega_0 b'\sigma),$$
$$X_1^{I'}(\sigma,\tau) = \sqrt{2\alpha'}\sum_{n\neq 0}\exp(-i\omega_n\tau)\times\left(i\frac{\alpha_n^{I'}}{\omega_n}\cos n\sigma + \frac{\alpha_n^{J'}}{n}B^{I'}_{J'}\sin n\sigma\right), \quad (21)$$

where $X_0^{I'}$ is the zero-mode part and $X_1^{I'}$ is the oscillating part, and $\{I', J' = 3,4\}$. We note that both signs of ω_0 determine only one solution for X^{I_1}. According to our setup, we see that only the coordinates X^3 and X^4 contain the *B*-field elements. Therefore, only the X^3X^4 plane is noncommutative, and we now investigate it.

The canonical momentum corresponding to the open string coordinate X^I is given by

$$P^I(\sigma,\tau) = \frac{1}{2\pi\alpha'}\left(\partial_\tau X^I - B^I{}_J\partial_\sigma X^J\right). \quad (22)$$

For the directions X^3 and X^4, the conjugate momenta also split into two pieces, the zero-mode part and the oscillating part:

$$P^{I'}(\sigma,\tau) = P_0^{I'}(\sigma,\tau) + P_1^{I'}(\sigma,\tau),$$
$$P_0^{I'}(\sigma,\tau) = \frac{1}{2\pi\alpha'}\left[M_{J'}^{I'}\left(-x^{J'}\omega_0\sin\omega_0\tau + \right.\right.$$
$$\left. + 2\alpha' p^{J'}\cos\omega_0\tau\right)\mathrm{ch}(\omega_0 b'\sigma) - \frac{1}{b}(BM)_{J'}^{I'}\times$$
$$\left.\times\left(2\alpha' p^{J'}\sin\omega_0\tau + x^{J'}\omega_0\cos\omega_0\tau\right)\mathrm{sh}(\omega_0 b'\sigma)\right], \quad (23)$$
$$P_1^{I'}(\sigma,\tau) = \frac{1}{\pi\sqrt{2\alpha'}}\sum_{n\neq 0}\exp\left(-i\omega_n\tau\right)\times$$
$$\times\left(\alpha_n^{J'} M_{J'}{}^{I'}\cos n\sigma + i\frac{m^2}{n\omega_n}\alpha_n^{J'} B_{J'}^{I'}\sin n\sigma\right).$$

The symmetric matrix M is given by

$$M_{I'J'} = (1+b^2)\delta_{I'J'}. \quad (24)$$

Again, both signs of ω_0 specify one value for each of the momentum components P^3 and P^4.

It is known that in a D-brane with the B-field background, the spatial coordinates of the brane do not commute. We now investigate this in our setup. To quantize the open string theory, we use the symplectic form

$$\Omega = \int_0^\pi d\sigma\left(\sum_{I'=3}^4\sum_{J'=3}^4 g_{I'J'}\mathbf{d}P^{I'}\wedge\mathbf{d}X^{J'}\right). \quad (25)$$

This can be justified by analyzing the constraint structure of the theory (see, e.g., Refs. [11,12]). With Eqs. (21) and (23), this differential form becomes

$$\Omega = \sum_{I'=3}^4\sum_{J'=3}^4\left(\frac{\mathrm{sh}(2\pi\omega_0)}{2\pi\omega_0 b}M_{I'J'}\mathbf{d}p^{I'}\wedge\mathbf{d}x^{J'} - \right.$$
$$- \frac{\mathrm{sh}^2(\pi\omega_0 b)}{2\pi\alpha' b^2}(MB)_{I'J'}\mathbf{d}x^{I'}\wedge\mathbf{d}x^{J'} -$$
$$- \frac{2\alpha'\,\mathrm{sh}^2(\pi\omega_0 b)}{\pi{\omega_0}^2 b^2}(MB)_{I'J'}\mathbf{d}p^{I'}\wedge\mathbf{d}p^{J'} +$$
$$\left. + i\sum_{n=1}^\infty\frac{M_{(n)I'J'}}{\omega_n}\mathbf{d}\alpha_n^{I'}\wedge\mathbf{d}\alpha_{-n}^{J'}\right), \quad (26)$$

where the symmetric matrices $M_{(n)I'J'}$ are defined by

$$M_{(n)I'J'} = \left(1 + \frac{\omega_n^2 b^2}{n^2}\right)\delta_{I'J'}. \quad (27)$$

This symplectic form enables us to obtain the nontrivial commutation relations

$$\begin{aligned}
&[x^{I'}, x^{J'}] = 2i\pi\alpha'(BM^{-1})^{I'J'},\\
&[x^{I'}, p^{J'}] = iM^{I'J'}\frac{\pi\omega_0 b'}{\mathrm{th}(\pi\omega_0 b')},\\
&[p^{I'}, p^{J'}] = i\frac{\pi{\omega_0}^2}{2\alpha'}(BM^{-1})^{I'J'},\\
&[\alpha_n^{I'}, \alpha_s^{J'}] = \omega_n M_{(n)}^{I'J'}\delta_{n+s,0},
\end{aligned} \quad (28)$$

The matrix $M_{(n)}^{I'J'}$ is inverse to $M_{(n)I'J'}$, and the matrix (BM^{-1}) is antisymmetric. The matrix $M_{(n)}^{I'J'}$ may be interpreted as a mode-dependent open string metric for the oscillators.

The above results allows calculating intrinsic commutation relations for the open string coordinates and their conjugate momenta:

$$\left[X^{I'}(\sigma,\tau),\ X^{J'}(\sigma',\tau)\right] = 2\pi i\alpha'(BM^{-1})^{I'J'}, \quad (29)$$

$$\left[P^{I'}(\sigma,\tau), P^{J'}(\sigma',\tau)\right] = i\frac{m^2}{2\pi\alpha'}B^{I'J'}, \quad (30)$$

$$\left[X^{I'}(\sigma,\tau), P^{J'}(\sigma',\tau)\right] = i\delta^{I'J'}\ \delta(\sigma-\sigma'), \quad (31)$$

where the first and the second equations are established on the brane, i.e., at $\sigma=\sigma'=0$. The left-hand sides of Eqs. (29) and (30) define the noncommutativity parameters respectively associated with the space and momentum parts of the 4-dimensional phase space. Equations (28)–(31) clarify the fact that the open string zero modes as well as the string coordinates $X^{I'}$ and the momenta $P^{I'}$ feel a noncommutative phase space.

4. CONCLUSIONS

We have investigated the behavior of an open string attached to a Dp-brane in the presence of massless fields: the Kalb–Ramond field and the linear dilaton in the *pp*-wave background. We chose a suitable configuration of the background fields such that they be appropriate for the light-cone-gauge formalism. The presence of the linear dilaton effectively deforms the equations of motion, the boundary conditions and, due to the lack of the Weyl invariance, introduces a new worldsheet field ρ into the theory.

Separation of the variables elaborates a differential equation for the string coordinates. By solving this equation and the boundary equations, we established that the noncommutativity is extremely influenced by the dilaton field. If there is a magnetic field along two specific directions of the brane, then the positions of the string endpoints in that plane are expected to be noncommutative. However, it is possible to suppress the noncommutativity by turning on a linear dilaton field such that its vector components fall into that plane, which quenches the magnetic field. In fact, by adding the elements of the magnetic field, without altering the dilaton vector, one can turn on the noncommutativity in some other directions. The momentum components of the open string in these directions also acquire a noncommutative structure.

The equations in this paper also are valid for a general block-diagonal magnetic field. It may be interesting to extend the problem beyond this consideration with more components, and find out whether new results would follow.

REFERENCES

1. R. R. Metsaev, Nucl. Phys. B **625**, 70 (2002).

2. R. R. Metsaev, Nucl. Phys. B **625**, 70 (2002).

3. R. R. Metsaev and A. A. Tseytlin, Phys. Rev. D **65**, 126004 (2002).

4. N. Seiberg and E. Witten, JHEP 09(1999)032.

5. R. Myers, Phys. Lett. B **199**, 371 (1987).

6. P. Di Francesco, P. Mathieu, and D. Senechal, *Conformal Field Theory*, Springer, New York (1997); P. Ginsparg and G. Moore, arXiv:hep-th/9304011; N. Prezas and K. Sfetsos, JHEP 06(2008)080; C. G. Callan, J. A. Harvey, and A. Strominger, arXiv:hep-th/9112030; A. Rajaraman and M. Rozali, JHEP 12(1999)005; J. Kluson, JHEP 11(2003)068; J. Polchinski, *String Theory*, Vol. I, Cambridge Univ. Press, Cambridge, UK (1998); D. Kamani, Braz. J. Phys. **40**, 375 (2010).

7. S. Bhattacharyya, A. Kumar, and S. Mahapatra, Mod. Phys. Lett. A **16**, 2263 (2001).

8. A. Sen, JHEP 04(2002)048.

9. B. Nikolić and B. Sazdović, Facta Universitatis, ser. Phys. Chem. Tech. **4**(2), 405 (2006).

10. M. Zoghi and D. Kamani, Braz. J. Phys. **44**, 102 (2014).

11. C. Chu and P. Ho, Nucl. Phys. B **636**, 141 (2002).

12. M. Daouda and A. Hamama, Int. J. Mod. Phys. A **23**, 2591 (2008).