

ALGEBRAIC FORM OF THE M3-BRANE ACTION

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We reformulate the bosonic action of an unstable M3-brane to manifest its algebraic representation. It is seen that in contrast to string and M2-brane actions, which are respectively represented only in terms of two- and three-dimensional Lie algebras, the algebraic form of the M3-brane action is a combination of four-, three-, and two-dimensional Lie algebras. Corresponding brackets appear as mixtures of the tachyon field, space-time coordinates X , the two-form field $\hat{\omega}^{(2)}$, and the Born–Infeld one-form \hat{b}_μ .

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1. INTRODUCTION

Algebraic reformulation of known actions in string theory and M-theory shows that string theory is based on the conventional algebra, or a two-dimensional Lie algebra (known as a two-algebra), but a complete description of M-theory requires an extended Lie algebra called a three-algebra [1], which was mainly developed in [2–5]. The numbers two and three are respectively associated with string theory and M-theory. Two is the string worldsheet dimension and also the codimension of D-branes in both type-IIA and type-IIB superstring theories [6]. Three is the membrane worldvolume dimension in M-theory and the codimension of M2- and M5-branes. This means that via two-algebra interactions, some Dp -branes can condense to a $D(p+2)$ -brane [7] and through three-algebra interactions multiple M2-branes condense to a M5-brane [8–16]. These connections between two and three and, respectively, string theory and M-theory become obvious by rewriting Nambu–Goto actions in algebraic form.

By analogy, we can expect to describe p -branes applying a $(p+1)$ -algebra structure [17]. These extended algebras are used to construct worldvolume theories for multiple p -branes in terms of Nambu brackets that are classical approximations to multiple commutators of

these algebras [18]. Nambu n -brackets introduce a way to understand the n -dimensional Lie algebra presented in [19]. Formulation of the p -brane action in terms of a $(p+1)$ -algebra makes it more compact and we are left with algebraic calculations, which are then usually simpler to handle.

In string theory, we are inevitably faced with unstable systems, and studying them deepens our understanding of the string theory. In bosonic string theory, the instability is always present due to the tachyon presence in the open string spectrum. Two examples of unstable states in superstring theories are non-BPS branes (odd (even) dimensional branes in type-IIA (IIB) theory) and brane–anti-brane pairs in both type-IIA and type-IIB theories [20, 21]. An interesting fact about the dynamics of these unstable branes, generally obvious in the effective action formulation, is their dimensional reduction through tachyon condensation [22–27]. During this process, the negative energy density of the tachyon potential at its minimum point cancels the tension of the D-brane (or D-branes) [28], and the final product is a closed-string vacuum without a D-brane or stable lower-dimensional D-branes. On the other hand, stable objects in string theory can be obtained by dimensional reduction of stable branes in M-theory (M2- and M5-branes). Naturally, we can expect to have a preimage of unstable branes in superstring theories by formulating an effective action for unstable branes in M-theory. Among different unstable systems in M-theory [29], the M3-brane is noteworthy

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because it is directly related to the M2-brane. Tachyon condensation of the M3-brane effective action results in the M2-brane action, and its dimensional reduction also leads to a non-BPS D3-brane action in type-IIA string theory [30].

Despite attempts made to formulate the M3-brane action consistent with desired conditions [30], there has been no algebraic approach towards this formulation. The existence of the algebraic form for the action of the M2-brane, as the fundamental object of M-theory, motivated us to search for the algebraic presentation of the M3-brane as the main unstable object in M-theory, whose instability is due to the presence of the tachyon.

What distinguishes the present study from conventional algebraic formulations is the instability of the M3-brane. In other words, the presence of the tachyon and other background fields affect the resultant algebra. It is shown that a pure four-algebra does not occur, as expected, and we are encountered with four-, three-, and two-brackets that are mixtures of the tachyon, space-time coordinates, and other fields.

2. ALGEBRAIC M3-BRANE ACTION

The conventional action corresponding to a non-BPS M3-brane is a combination of the DBI (Dirac–Born–Infeld) and WZ (Wess–Zumino) parts [30]

$$\begin{aligned}
 S &= S_{DBI} + S_{WZ}, \\
 S_{DBI} &= - \int d^4 \xi V(T) |\hat{k}|^{1/2} \sqrt{-\det H_{\mu\nu}}, \\
 S_{WZ} &= - \int d^4 \xi V(T) \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \partial_{\mu_1} T \hat{\kappa}_{\mu_2 \mu_3 \mu_4},
 \end{aligned} \tag{2.1}$$

where ξ^μ with $\mu = 0, 1, 2, 3$ labels worldvolume coordinates of the M3-brane, $V(T)$ is the tachyon potential, which is an even function of T and is characterized as $V(T = \pm\infty) = 0$ and $V(T = 0) = \mathcal{T}_{M3}$, \mathcal{T}_{M3} is the M3-brane tension, and $\hat{k}^M(X)$ is the Killing vector, such that the Lie derivatives of all target-space fields vanish with respect to it [30]. Other fields in (2.1) are defined as

$$\begin{aligned}
 H_{\mu\nu} &= \hat{g}_{MN} \hat{D}_\mu \hat{X}^M \hat{D}_\nu \hat{X}^N + \frac{1}{|\hat{k}|} \hat{F}_{\mu\nu} + \frac{1}{|\hat{k}|} \partial_\mu T \partial_\nu T, \\
 \hat{k}^2 &= \hat{k}^M \hat{k}^N \hat{g}_{MN}, \quad \hat{k}^2 = |\hat{k}|^2, \\
 \hat{F}_{\mu\nu} &= \partial_\mu \hat{b}_\nu - \partial_\nu \hat{b}_\mu + \partial_\mu \hat{X}^M \partial_\nu \hat{X}^N (i_{\hat{k}} \hat{C})_{MN},
 \end{aligned}$$

$$\begin{aligned}
 \hat{D}_\mu X^M &= \partial_\mu \hat{X}^M - \hat{A}_\mu \hat{k}^M, \\
 \hat{A}_\mu &= \frac{1}{|\hat{k}|^2} \partial_\mu \hat{X}^M \hat{k}_M, \\
 \hat{\kappa}_{\mu_2 \mu_3 \mu_4} &= \partial_{\mu_2} \hat{\omega}_{\mu_3 \mu_4}^{(2)} - \partial_{\mu_3} \hat{\omega}_{\mu_2 \mu_4}^{(2)} + \partial_{\mu_4} \hat{\omega}_{\mu_2 \mu_3}^{(2)} + \\
 &+ \frac{1}{3!} \hat{C}_{KMN} \hat{D}_{\mu_2} \hat{X}^K \hat{D}_{\mu_3} \hat{X}^M \hat{D}_{\mu_4} \hat{X}^N + \\
 &+ \frac{1}{2!} \hat{A}_{\mu_2} (\partial_{\mu_3} \hat{b}_{\mu_4} - \partial_{\mu_4} \hat{b}_{\mu_3}).
 \end{aligned} \tag{2.2}$$

The tensor $H_{\mu\nu}$ consists of the pullback of the background metric, the field strength $\hat{F}_{\mu\nu}$ of the gauge field A_μ , and the tachyon field T ; M and N represent space-time indices and \hat{D}_μ is the covariant derivative. The field strength itself is expressed in terms of the Born–Infeld 1-form \hat{b}_μ and the R–R sector field \hat{C} . The curvature of the 2-form $\hat{\omega}^{(2)}$ is denoted by $\hat{\kappa}$.

The determinant of the tensor $H_{\mu\nu}$ in the DBI action can be decomposed as

$$\sqrt{-\det H_{\mu\nu}} = \sqrt{-\det(\tilde{G}_{\mu\nu} + \tilde{F}_{\mu\nu})}, \tag{2.3}$$

where

$$\begin{aligned}
 \tilde{F}_{\mu\nu} &= \partial_\mu \hat{b}_\nu - \partial_\nu \hat{b}_\mu, \\
 \tilde{G}_{\mu\nu} &= L_{MN} \partial_\mu X^M \partial_\nu X^N + \frac{1}{|\hat{k}|} \partial_\mu T \partial_\nu T,
 \end{aligned} \tag{2.4}$$

and

$$L_{MN} = g_{MN} + \frac{i_{|\hat{k}|} \hat{C}_{MN}}{|\hat{k}|} - \frac{\hat{k}_M \hat{k}_N}{|\hat{k}|^2}. \tag{2.5}$$

Regarding (2.3), the DBI action can be expanded to the quadratic order [31] as

$$\begin{aligned}
 S_{DBI} &= - \int d^4 \xi V(T) \sqrt{-\det \tilde{G}_{\mu\nu}} \times \\
 &\times \left(1 + \frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \dots \right).
 \end{aligned} \tag{2.6}$$

2.1. DBI part of the M3-brane action

To find the algebraic form of the DBI action, we start with the first term in (2.6), $\sqrt{-\det \tilde{G}_{\mu\nu}}$, which is the determinant of a 4×4 matrix and all its elements are sums of a tachyonic part and a space-like part ($\partial X \partial X + \partial T \partial T$). This determinant totally consists of 48×8 terms. These terms can be classified into sixteen 4×4 determinants such that the elements of these determinants are only $\partial X \partial X$ or $\partial T \partial T$ and not sums of them. Hence, each determinant has 24 terms such that adding them leads to the same number of terms (16×24) as in the initial main determinant. These 16 determinants can be categorized as: one

determinant with $\partial X \partial X$ elements (four combinations from the 4 states $\binom{4}{4} = 1$), one determinant with elements $\partial T \partial T$ ($\binom{4}{4} = 1$), four determinants with three rows of $\partial X \partial X$ elements and one row of $\partial T \partial T$ elements ($\binom{4}{1} = 4$), four determinants with three rows of $\partial T \partial T$ elements and one row of $\partial X \partial X$ elements ($\binom{4}{1} = 4$), and, finally, six determinants with two rows of $\partial T \partial T$ elements and two rows of $\partial X \partial X$ elements ($\binom{4}{2} = 6$). It follows that determinants with more than one row of $\partial T \partial T$ are zero. We are therefore left with two kinds of determinants: a determinant consisting of only $\partial X \partial X$ entities and those with three rows of $\partial X \partial X$ elements and one row of $\partial T \partial T$ entities. Because a determinant does not change under exchanging of rows, considering all possible permutations (4!) of rows for each of the remaining determinants yields the form of the four-algebra in accordance with (A.5). Eventually, after a tedious calculation, the algebraic form of $\sqrt{-\det \tilde{G}_{\mu\nu}}$ is obtained as

$$\begin{aligned} &\sqrt{-\det \tilde{G}_{\mu\nu}} \rightarrow \\ &\rightarrow \left\{ - \left(L_{MN} L_{OP} L_{QR} L_{ST} [X^M, X^O, X^Q, X^S] \times \right. \right. \\ &\quad \times [X^N, X^P, X^R, X^T] + \frac{4}{|\hat{k}|} L_{MN} L_{OP} L_{QR} \times \\ &\quad \left. \left. \times [T, X^M, X^O, X^Q][T, X^N, X^P, X^R] \right) \right\}^{1/2}. \end{aligned} \quad (2.7)$$

The 4-bracket of the space-time coordinates X corresponds to the algebraic action derived in [1, 17] for $p = 3$ case and with the fermionic fields turned off. The new term here is the mixed four-bracket of the X and T .

Presenting a general algebraic form for the term $\tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}$ in the DBI action is not possible, but in some special cases it acquires a simple form. For example, one can consider a selfdual (anti-selfdual) field strength that corresponds to instanton. An instanton is a static (solitonic) solution of pure Yang–Mills theories [32]. They are important in both supersymmetric field theories and superstring theories, mostly because of their nonperturbative effects. They also play a role in M-theory, for instance, in applying the M2-brane actions to the M5-brane [33]. The solution of field equations in the Yang–Mills theory corresponding to an instanton has a selfdual (anti-selfdual) field strength [32]. Considering this property gives the following expression

for $\text{tr} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu}$ in the case of a regular one-instanton solution [32]:

$$\text{tr} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} = -96 \frac{\rho^4}{((x-x_0)^2 + \rho^2)^4}, \quad (2.8)$$

where x_0 and ρ are arbitrary parameters called collective coordinates. Hence, in the instantonic case, the full algebraic form of the DBI part of the action

$$\begin{aligned} S_{DBI} = & - \int d^4 \xi V(T) \left(1 - 24 \frac{\rho^4}{((x-x_0)^2 + \rho^2)^4} \right) \times \\ & \times \left\{ - \left(L_{MN} L_{OP} L_{QR} L_{ST} [X^M, X^O, X^Q, X^S] \times \right. \right. \\ & \quad \times [X^N, X^P, X^R, X^T] + \frac{4}{|\hat{k}|} L_{MN} L_{OP} L_{QR} \times \\ & \quad \left. \left. \times [T, X^M, X^O, X^Q][T, X^N, X^P, X^R] \right) \right\}^{1/2}. \end{aligned} \quad (2.9)$$

2.2. WZ part of the M3-brane action

The integrand of the WZ action in (2.1) can be divided into three parts by replacing \hat{k} from (2.2):

$$\begin{aligned} S_{WZ} \rightarrow & \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \partial_{\mu_1} T \hat{k}_{\mu_2 \mu_3 \mu_4} = \\ = & \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \partial_{\mu_1} T \left(\partial_{\mu_2} \hat{\omega}_{\mu_3 \mu_4}^{(2)} - \partial_{\mu_3} \hat{\omega}_{\mu_2 \mu_4}^{(2)} + \partial_{\mu_4} \hat{\omega}_{\mu_2 \mu_3}^{(2)} + \right. \\ & \quad + \frac{1}{3!} \hat{C}_{KMN} \hat{D}_{\mu_2} \hat{X}^K \hat{D}_{\mu_3} \hat{X}^M \hat{D}_{\mu_4} \hat{X}^N + \\ & \quad \left. + \frac{1}{2!} \hat{A}_{\mu_2} (\partial_{\mu_3} \hat{b}_{\mu_4} - \partial_{\mu_4} \hat{b}_{\mu_3}) \right), \end{aligned} \quad (2.10)$$

where we now deal with each part separately.

By expanding the first part, three terms of $\hat{\omega}^{(2)}$ derivatives, and considering all possible permutations of the four-dimensional Levi-Civita symbol $\varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4}$, we come to a view of a two-algebra. The reason is that according to (A.5), having two derivative factors signals a two-algebra carrying its two-dimensional Levi-Civita symbol. But because only different permutations of $\varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4}$ give correct signs to the terms here, multiplying the resultant two-algebra by another two-dimensional Levi-Civita symbol and using the relation

$$\varepsilon^{\alpha\beta} \varepsilon_{\gamma\delta} = \delta_{\gamma}^{\alpha} \delta_{\delta}^{\beta} - \delta_{\delta}^{\alpha} \delta_{\gamma}^{\beta}$$

leads to the correct form. The first part of the WZ action is therefore reformulated in terms of the two-bracket as

$$\begin{aligned} S_{WZ,1} \rightarrow & \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \times \\ & \times \partial_{\mu_1} T \left(\partial_{\mu_2} \hat{\omega}_{\mu_3 \mu_4}^{(2)} - \partial_{\mu_3} \hat{\omega}_{\mu_2 \mu_4}^{(2)} + \partial_{\mu_4} \hat{\omega}_{\mu_2 \mu_3}^{(2)} \right) = \\ = & 3 \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \varepsilon_{\mu_1 \mu_2} [T, \omega_{\mu_3 \mu_4}]. \end{aligned} \quad (2.11)$$

In the second part of the WZ action, three X derivatives, ∂X , and one tachyon derivative, ∂T , appear such that they obviously form a four-algebra:

$$\begin{aligned} S_{WZ,2} &\rightarrow \frac{1}{3!} \hat{C}_{KMN} \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \times \\ &\quad \times \partial_{\mu_1} T \hat{D}_{\mu_2} \hat{X}^K \hat{D}_{\mu_3} \hat{X}^M \hat{D}_{\mu_4} \hat{X}^N = \\ &= \frac{1}{3!} \hat{C}_{KMN} \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \left(1 - \frac{\hat{k}^P \hat{k}_P}{|\hat{k}|^2} \right)^3 \partial_{\mu_1} T \times \\ &\quad \times \partial_{\mu_2} X^K \partial_{\mu_3} X^M \partial_{\mu_4} X^N = \\ &= \frac{1}{3!} \hat{C}_{KMN} \left(1 - \frac{\hat{k}^P \hat{k}_P}{|\hat{k}|^2} \right)^3 [T, X^K, X^M, X^N]. \end{aligned} \quad (2.12)$$

Substituting A_μ in the last part of the WZ action, we are faced with terms consisting of ∂X , ∂T , and ∂b , which according to (A.5) indicates a three-algebra. Similarly to the argument made for the first part of the WZ action, multiplying this three-bracket by a three-dimensional Levi-Civita symbol and using the identity

$$\begin{aligned} \varepsilon^{\alpha\beta\gamma} \varepsilon_{\delta\eta\lambda} &= \delta_\delta^\alpha (\delta_\eta^\beta \delta_\lambda^\gamma - \delta_\lambda^\beta \delta_\eta^\gamma) - \delta_\eta^\alpha (\delta_\delta^\beta \delta_\lambda^\gamma - \delta_\lambda^\beta \delta_\delta^\gamma) + \\ &\quad + \delta_\lambda^\alpha (\delta_\delta^\beta \delta_\eta^\gamma - \delta_\eta^\beta \delta_\delta^\gamma), \end{aligned}$$

gives the convenient three-algebra. Different permutations of the four-dimensional Levi-Civita symbol are responsible for correct signs of different terms in the three-algebra. Hence, the algebraic form of this part is

$$\begin{aligned} S_{WZ,3} &\rightarrow \frac{1}{2!} \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \partial_{\mu_1} T \hat{A}_{\mu_2} (\partial_{\mu_3} \hat{b}_{\mu_4} - \partial_{\mu_4} \hat{b}_{\mu_3}) = \\ &= \frac{\hat{k}_M}{2! |\hat{k}|^2} \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \varepsilon_{\mu_1 \mu_2 \mu_3} [T, X^M, b_{\mu_4}]. \end{aligned} \quad (2.13)$$

Therefore, the WZ action of the M3-brane is presented in terms of two-, three-, and four-brackets as

$$\begin{aligned} S_{WZ} &= - \int d^4 \xi V(T) \left\{ 3 \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \varepsilon_{\mu_1 \mu_2} [T, \omega_{\mu_3 \mu_4}] + \right. \\ &\quad + \frac{1}{3!} C_{KMN} \left(1 - \frac{\hat{k}^P \hat{k}_P}{|\hat{k}|^2} \right)^3 [T, X^K, X^M, X^N] + \\ &\quad \left. + \frac{\hat{k}_M}{2! |\hat{k}|^2} \varepsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \varepsilon_{\mu_1 \mu_2 \mu_3} [T, X^M, b_{\mu_4}] \right\}. \end{aligned} \quad (2.14)$$

We see that the tachyon field respectively couples to space-time coordinates, the Born–Infeld one-form \hat{b}_μ , and the two-form $\hat{\omega}^{(2)}$ through four-, three-, and two-brackets.

3. SUMMARY AND CONCLUSION

We have presented an algebraic form for the bosonic M3-brane action by reformulating this action in terms

of brackets. In the literature, p -branes are described by a $(p + 1)$ -algebra [17], and we could therefore expect a four-algebra structure for the M3-brane. But it was shown that the algebraic representation of the M3-brane is a combination of four-, three-, and two-algebras. Generally, this difference stems from the instability of the system that the tachyon is responsible for. Except the four-bracket of space-time coordinates in the DBI part, the tachyon field is present in all other brackets and forms four-, three-, and two-brackets with space-time coordinates, the two-form, $\hat{\omega}^{(2)}$, and the Born–Infeld one-form \hat{b}_μ . In the future, we will try to study the dimensional reduction of this algebraic action.

APPENDIX

Fillipov’s n -Lie algebra

Fillipov’s n -Lie algebra [19], as a natural generalization of a Lie algebra, is defined by an n -bracket satisfying the total antisymmetry property

$$\begin{aligned} [X_1, \dots, X_i, \dots, X_j, \dots, X_n] &= \\ &= -[X_1, \dots, X_j, \dots, X_i, \dots, X_n], \end{aligned} \quad (A.1)$$

and the Leibniz rule

$$\begin{aligned} [X_1, \dots, X_{n-1}, [Y_1, \dots, Y_n]] &= \\ &= \sum_{j=1}^n [Y_1, \dots, [X_1, \dots, X_{n-1}, Y_j], \dots, Y_n]. \end{aligned} \quad (A.2)$$

The n -Lie algebra is equipped with an invariant inner product

$$\langle X, Y \rangle = \langle Y, X \rangle, \quad (A.3)$$

and the invariance under the n -bracket transformation

$$\begin{aligned} \langle [X_1, \dots, X_{n-1}, Y], Z \rangle + \\ + \langle Y, [X_1, \dots, X_{n-1}, Z] \rangle = 0. \end{aligned} \quad (A.4)$$

When $n = 2$, the definition reduces to the usual Lie algebra and the inner product can be given by the trace.

The n -Lie algebra can be realized in terms of the Nambu n -bracket defined over a functional space on an n -dimensional manifold [18]:

$$\begin{aligned} [X_1, X_2, \dots, X_n] &\Leftrightarrow \{X_1, X_2, \dots, X_n\}_{NB} := \\ &:= \frac{1}{\sqrt{\mathcal{G}}} \varepsilon^{l_1 l_2 \dots l_n} \partial_{l_1} X_1 \partial_{l_2} X_2 \dots \partial_{l_n} X_n, \end{aligned} \quad (A.5)$$

where \mathcal{G} is the determinant of the metric of the manifold and can be chosen arbitrarily since properties (A.1)–(A.4) hold irrespective of the presence of the local factor [1].

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