ЖЭТФ, 2013, том 143, вып. 4, стр. 752–758

DECOHERENCE INDUCED BY MAGNETIC IMPURITIES IN A QUANTUM HALL SYSTEM

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Received November 15, 2012

Scattering by magnetic impurities is known to destroy coherence of electron motion in metals and semiconductors. We investigate the decoherence introduced in a single act of electron scattering by a magnetic impurity in a quantum Hall system. For this, we introduce a fictitious nonunitary scattering matrix $\mathcal{S}$ for electrons that reproduces the exactly calculated scattering probabilities. The strength of decoherence is identified by the deviation of eigenvalues of the product $\mathcal{S}\mathcal{S}^\dagger$ from unity. Using the fictitious scattering matrix, we estimate the width of the metallic region at the quantum Hall effect inter-plateau transition and its dependence on the exchange coupling strength and the degree of polarization of magnetic impurities.

DOI: 10.7868/S0044451013040149

1. INTRODUCTION

Scattering by magnetic impurities can affect transport properties of electron systems substantially. Apart from the prominent Kondo effect, magnetic impurities provide a strong source of decoherence at temperatures exceeding the Kondo temperature [1, 2]. The decoherence effect is manifested especially strongly in suppressing the Anderson localization in disordered systems [3, 4]. In particular, scattering by magnetic impurities can create a finite metallic region near the inter-plateaux transition in the integer quantum Hall effect (IQHE) [5]. The characterization of the degree of decoherence introduced by magnetic impurities and evaluation of the corresponding phase coherence length provide an important information for the interpretation of transport experiments. In the presence of decoherence, the dynamics of a physical system ceases to be unitary [6]. In this paper, we introduce a measure of decoherence based on the nonunitarity of a fictitious scattering matrix constructed after averaging the scattering probabilities over magnetic impurities.

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This paper is organized as follows. In Sec. 2, we consider a toy model and show that the nonunitarity of the scattering matrix is related to the uncertainty in the phase of the wave function. The exact scattering matrix for an electron in a saddle-point potential in the quantum Hall regime and in the presence of magnetic impurities is calculated in Sec. 3. Our main results are given in Secs. 4 and 5, where we calculate the fictitious scattering matrix, use it to determine the degree of decoherence induced by magnetic impurities, and finally estimate the width of the inter-plateaux transition. In Sec. 6, we summarize our results and discuss possible further applications of the presented method.

2. NONUNITARITY OF THE SCATTERING MATRIX AS A MEASURE OF DECOHERENCE

In this section, we show with a simple illustrative example that the deviation of eigenvalues of the product $\mathcal{S}\mathcal{S}^\dagger$ (where $\mathcal{S}$ is a fictitious scattering matrix) from unity serves as a measure of the decoherence introduced by scattering. For this, we consider a simple scattering problem with a two-dimensional Hilbert space. Two orthogonal incoming states are parameterized as

$$\psi_1 = \frac{1}{\sqrt{\pi}} \cos\varphi, \quad \psi_2 = \frac{1}{\sqrt{\pi}} \sin\varphi, \qquad (1)$$

and the scalar product is defined as an integral over the angle φ,

$$\langle \psi_i | \psi_j \rangle = \int_0^{2\pi} \psi_i^*(\varphi) \psi_j(\varphi) \, d\varphi. \qquad (2)$$

We assume that in the act of scattering, the states experience both a potential scattering described by the transmission amplitude t and the reflection amplitude r, $(r^2 + t^2 = 1)$, and random phase shifts α_1 and α_2 that describe the decoherence effect. Then the outgoing states are given by

$$\tilde{\psi}_1^{out} = \frac{1}{\sqrt{\pi}} \left\{ r\cos(\varphi + \alpha_1) + t\sin(\varphi + \alpha_2) \right\}, \qquad (3)$$

$$\tilde{\psi}_2^{out} = \frac{1}{\sqrt{\pi}} \left\{ -t\cos(\varphi + \alpha_1) + r\sin(\varphi + \alpha_2) \right\}. \qquad (4)$$

In this model, the decoherence violates the orthogonality of the outgoing states. The completely coherent scattering is realized in the case $\alpha_1 = \alpha_2$. The degree of decoherence increases with the difference $\alpha_1 - \alpha_2$. It is maximal for $\alpha_1 - \alpha_2 = \pm\pi/2$, when initially orthogonal states become linearly dependent after scattering. In the notation used, a state goes into itself by coherent reflection (the amplitude r), and it goes into the other state by coherent transmission (the amplitude t). We now introduce a (nonunitary) scattering matrix for an incoherent scattering process according to the relation

$$\begin{pmatrix} \tilde{\psi}_1^{out} \\ \tilde{\psi}_2^{out} \end{pmatrix} = \mathcal{S}_{incoh} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \tilde{r}_1 & \tilde{t}_1 \\ -\tilde{t}_2 & \tilde{r}_2 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \qquad (5)$$

The comparison with Eqs. (3) and (4) allows identifying the elements of the matrix $\mathcal{S}_{incoh}$ as

$$\tilde{r}_1 = \langle \tilde{\psi}_1^{out} | \psi_1 \rangle = r\cos\alpha_1 + t\sin\alpha_2, \qquad (6)$$

$$\tilde{t}_1 = \langle \tilde{\psi}_1^{out} | \psi_2 \rangle = t\cos\alpha_2 - r\sin\alpha_1, \qquad (7)$$

$$\tilde{t}_2 = \langle \tilde{\psi}_2^{out} | \psi_1 \rangle = t\cos\alpha_1 - r\sin\alpha_2, \qquad (8)$$

$$\tilde{r}_2 = \langle \tilde{\psi}_2^{out} | \psi_2 \rangle = r\cos\alpha_2 + t\sin\alpha_1. \qquad (9)$$

The deviation of the scattering matrix $\mathcal{S}_{incoh}$ from unitarity can be characterized by the products of this matrix with its hermitian conjugate. We note that for incoherent scattering, the matrices $\mathcal{S}_{incoh}$ and $\mathcal{S}_{incoh}^\dagger$ no longer commute. However, explicit calculation shows that the products $\mathcal{S}_{incoh}\mathcal{S}_{incoh}^\dagger$ and $\mathcal{S}_{incoh}^\dagger\mathcal{S}_{incoh}$ have the same eigenvalues, which are given by

$$\lambda_1 = 1 + \sin(\alpha_1 - \alpha_2), \quad \lambda_2 = 1 - \sin(\alpha_1 - \alpha_2). \qquad (10)$$

Therefore, our toy model shows that the deviation of the eigenvalues of the product $\mathcal{S}\mathcal{S}^+$ from unity is determined by the phase uncertainty after one scattering event, and hence it is directly related to the strength of decoherence. Moreover, those deviations are independent of the parameters r and t characterizing the coherent potential scattering in the chosen model.

3. EXACT SOLUTION FOR THE ELECTRON SCATTERING PROBABILITIES AVERAGED OVER MAGNETIC IMPURITIES

We study the effect of spin-flip scattering by magnetic impurities on the IQHE transition. We adopt the model of point-like exchange interaction between spins of impurities and electron spins $H_{int} = J\mathbf{I} \cdot \mathbf{s}$, where $\mathbf{I}$ and $\mathbf{s}$ respectively denote the spins of impurities and of the electron. Throughout the paper, we assume spin-$1/2$ impurities. In the absence of spin-flip scattering, there are two Zeeman-split critical energies for each Landau level, where the QH delocalization transition occurs. It was found in Ref. [5] that the spin-flip scattering results in the appearance of a finite region of delocalized states around the critical QHE states. In this paper, we estimate the width of the inter-plateaux transition analytically based on the evaluation of the coherence length due to scattering by magnetic impurities.

In general, scattering of electrons by impurity spins induces many-electron Kondo correlations. In this paper, however, we consider the regime when the Kondo temperature is very low and Kondo correlations are suppressed. Scattering of an electron by a saddle-point potential in a strong perpendicular magnetic field and in the presence of a magnetic impurity was studied in Ref. [5].

Following Ref. [7], we introduce the dimensionless measure of energy $\epsilon = (E + J/4)/E_1$, where E_1 is the energy parameter characterizing the shape of the saddle-point potential. Furthermore, we let $\delta = J/E_1$ denote the dimensionless strength of exchange interaction. This interaction results in two exchange-split energies $\epsilon_{1,2} = \epsilon \mp \delta/2$. Using the expressions for transmission and reflection coefficients, we construct the scattering matrix at the node relating the incoming and outgoing waves as (see Fig. 1)

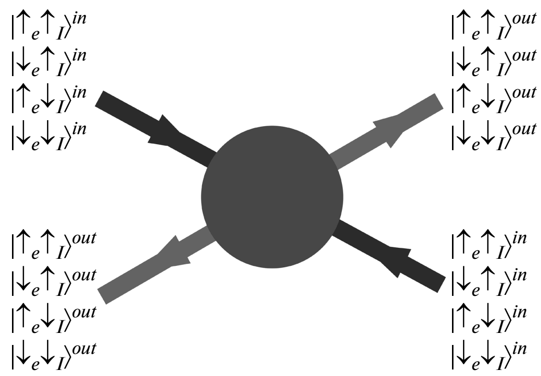

Fig. 1. Incoming and outgoing states at a single node. Up and down arrows indicate z-components of the electron (subscript e) and impurity (subscript I) spins correspondingly

$$S = \begin{pmatrix} \mathcal{R} & \mathcal{T} \\ -\mathcal{T} & \mathcal{R} \end{pmatrix} = \begin{pmatrix} r_1 & 0 & 0 & 0 & t_1 & 0 & 0 & 0 \\ 0 & s_{22} & s_{23} & 0 & 0 & s_{26} & s_{27} & 0 \\ 0 & s_{23} & s_{22} & 0 & 0 & s_{27} & s_{26} & 0 \\ 0 & 0 & 0 & r_1 & 0 & 0 & 0 & t_1 \\ -t_1 & 0 & 0 & 0 & r_1 & 0 & 0 & 0 \\ 0 & -s_{26} & -s_{27} & 0 & 0 & s_{22} & s_{23} & 0 \\ 0 & -s_{27} & -s_{26} & 0 & 0 & s_{23} & s_{22} & 0 \\ 0 & 0 & 0 & -t_1 & 0 & 0 & 0 & r_1 \end{pmatrix}, \quad (11)$$

where the 4×4 blocks $\mathcal{R}$ and $\mathcal{T}$ describe the reflection and transmission amplitudes. Here, we use the notation

$$t_{1,2} = \frac{1}{\sqrt{1 + e^{-\pi \epsilon_{1,2}}}}, \quad r_{1,2} = \sqrt{1 - t_{1,2}^2}, \quad (12)$$

$$s_{22} = (r_1 + r_2)/2, \quad s_{23} = (r_1 - r_2)/2, \quad (13)$$

$$s_{26} = (t_1 + t_2)/2, \quad s_{27} = (t_1 - t_2)/2. \quad (14)$$

The absolute value squared of a scattering matrix element in Eq. (11) gives the quantum scattering probability between the corresponding initial and final states of the electron and impurity. Given the density matrix of the impurity spin, we can calculate the scattering probability for the *electron only*, averaged over the impurity states. In what follows, we assume the density matrix of the magnetic impurity to have the diagonal form

$$\rho_I = \mathrm{diag}(w_\uparrow, w_\downarrow). \quad (15)$$

The difference $w_\uparrow - w_\downarrow$ denotes the polarization degree of the magnetic impurity. After averaging over magnetic impurities, the resulting system loses quantum coherence, and it can be described in terms of scattering probabilities. Using the density matrix in Eq. (15), we can write the averaged probability of the electron entering in the state with spin σ to be reflected (transmitted) into the state with spin σ' as

$$R_{\sigma'\sigma} = \sum_{s,s'} \rho_I^{ss} |\mathcal{R}_{\sigma' s', \sigma s}|^2, \qquad T_{\sigma'\sigma} = \sum_{s,s'} \rho_I^{ss} |\mathcal{T}_{\sigma' s', \sigma s}|^2, \quad (16)$$

where s and s' denote the initial and final spin states of the impurity. We note that the averaging applies only to the initial spin state of the impurity. Finally, the averaged probability matrix for the electron takes the form

$$P = \begin{pmatrix} R & T \\ T & R \end{pmatrix}, \quad (17)$$

where

$$R = \begin{pmatrix} w_\uparrow r_1^2 + w_\downarrow s_{22}^2 & w_\uparrow s_{23}^2 \\ w_\downarrow s_{23}^2 & w_\downarrow r_1^2 + w_\uparrow s_{22}^2 \end{pmatrix}, \qquad T = \begin{pmatrix} w_\uparrow t_1^2 + w_\downarrow s_{26}^2 & w_\uparrow s_{27}^2 \\ w_\downarrow s_{27}^2 & w_\downarrow t_1^2 + w_\uparrow s_{26}^2 \end{pmatrix}. \quad (18)$$

4. INTRODUCTION OF A FICTITIOUS SCATTERING MATRIX

We now define a fictitious scattering matrix for the quantum mechanical amplitude of the electron, which corresponds to the exact probability matrix obtained after averaging over the magnetic impurity states. For this, we construct a scattering matrix with elements satisfying the following condition: the squared modulus of each element must be equal to the corresponding probability of matrix (17). Furthermore, we choose the opposite signs of the elements in the two off-diagonal blocks, which ensures that the scattering matrix becomes unitary in the absence of spin–spin interaction, that is, for $\delta = 0$. Schematically, the scattering matrix acquires the form

$$\mathcal{S} = \begin{pmatrix} \sqrt{R} & \sqrt{T} \\ -\sqrt{T} & \sqrt{R} \end{pmatrix}, \quad (19)$$

where the square root is taken element-wise.

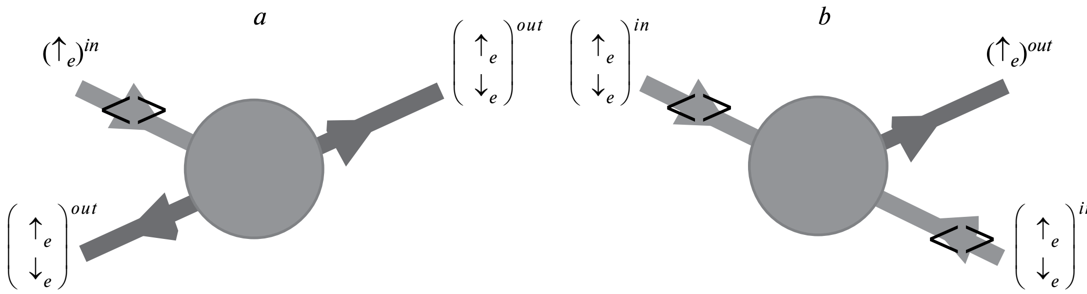

Fig. 2. a) Probability conservation as a sum of the elements squared in the first column in Eq. (19). The incoming wave is scattered in the four (including spin) outgoing channels. b) Loss of the probability conservation by the sum of the elements squared in the first row in Eq. (19). The single outgoing wave is not the sum of the four incoming channels. Angular brackets symbolize the averaging over the initial distribution of impurity spins. Because of the angular brackets, it is impossible to map panel (b) onto panel (a), which is in contradistinction to the reversibility of quantum mechanics

Being nonunitary in general, the fictitious scattering matrix still has some properties of a unitary matrix that follow from the conservation of probability. For example, it follows from Eqs. (17), (18), and (19) that the sum of the elements squared in each column in Eq. (19) is equal to 1, which describes the total probability for an electron entering the node to be scattered (see Fig. 2a). For instance, the first column gives

$$w_\uparrow r_1^2 + w_\downarrow s_{22}^2 + w_\downarrow s_{23}^2 + w_\uparrow t_1^2 + w_\downarrow s_{26}^2 + w_\downarrow s_{27}^2 = \\ = w_\uparrow (r_1^2 + t_1^2) + w_\downarrow (s_{22}^2 + s_{23}^2 + s_{26}^2 + s_{27}^2) = \\ = w_\uparrow + w_\downarrow = 1. \quad (20)$$

The sum of the elements squared in each raw, which would correspond to the probability of a time-reversed scattering process, differs from 1 (see Fig. 2b). This is due to the breaking of the time-reversal invariance introduced by averaging only over the initial states of the magnetic impurity. For example, the sum of the elements in the first raw gives

$$w_\uparrow r_1^2 + w_\downarrow s_{22}^2 + w_\uparrow s_{23}^2 + w_\uparrow t_1^2 + w_\downarrow s_{26}^2 + w_\uparrow s_{27}^2 = \\ = w_\uparrow (r_1^2 + t_1^2) + w_\downarrow (s_{22}^2 + s_{23}^2 + s_{26}^2 + s_{27}^2) + \\ + (w_\uparrow - w_\downarrow)(s_{23}^2 + s_{27}^2) = 1 + (w_\uparrow - w_\downarrow)(s_{23}^2 + s_{27}^2). \quad (21)$$

We note that Eq. (21) gives unity in the case $w_\uparrow = w_\downarrow = 1/2$, which corresponds to a completely unpolarized magnetic impurity. In that case, the time reversal symmetry seams to be restored. We can relate the restoration of time reversalbility to the maximal possible entropy of the impurity spin, which, therefore, remains unchanged by the scattering and corresponds to a time-reversible process in terms of thermodynamics.

However, even in the case of an unpolarized impurity, the fictitious scattering matrix is not unitary because of the decoherence introduced by averaging over the magnetic impurity. Formally, the different rows and columns of the matrix $\mathcal{S}$ are not orthogonal. This is a manifestation of the violation of the orthogonality of two quantum states by phase decoherence (the toy model for that process is discussed in Sec. 2).

Now we apply the analysis in Sec. 2 to the fictitious scattering matrix Eq. (19). The nonunitary matrix $\mathcal{S}$ does not commute with its hermitian conjugate $\mathcal{S}^\dagger$. However, it is easy to show that the products $\mathcal{S}^\dagger\mathcal{S}$ and $\mathcal{S}\mathcal{S}^\dagger$ have the same eigenvalues. Calculating the eigenvalues of $\mathcal{S}^\dagger\mathcal{S}$, we obtain two doubly degenerate eigenvalues that can be written as

$$\lambda_{1,2} = 1 \pm \sqrt{a^2 + b^2}, \quad (22)$$

where $a = \left(\mathcal{S}^\dagger\mathcal{S}\right)_{12}$ and $b = \left(\mathcal{S}^\dagger\mathcal{S}\right)_{14}$. We note that the eigenvalues are symmetric with respect to unity. In the limit of a weak spin–spin interaction, $\delta \ll 1$, the deviation of the eigenvalues from unity is given by

$$c = \sqrt{a^2 + b^2} \approx \frac{\pi\delta}{4} r_0 t_0 \left[\left(\sqrt{w_\uparrow} - \sqrt{w_\downarrow}\right)^2 + \right. \\ \left. + \frac{\pi^2\delta^2}{16} r_0^2 t_0^2 \left(w_\uparrow^{3/2} + w_\downarrow^{3/2}\right)^2 \right]^{1/2}, \quad (23)$$

where r_0 and t_0 denote the reflection and transmission amplitudes in Eq. (12) calculated for $\delta = 0$. According to the arguments given in Sec. 2, the parameter c serves as a measure of the decoherence introduced by the magnetic impurity. Moreover, comparing Eqs. (23) and (10), we conclude that c measures the phase uncertainty acquired after a single incoherent scattering

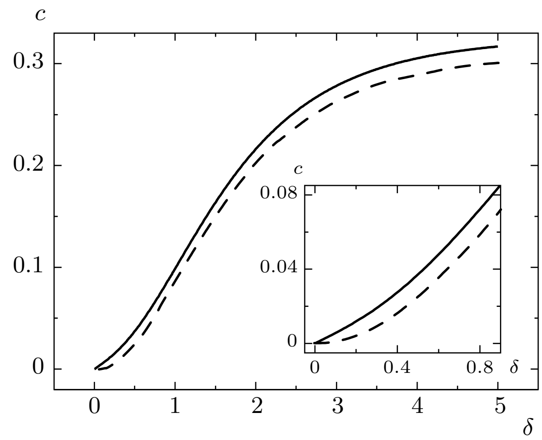

Fig. 3. Decoherence parameter c as a function of the exchange coupling strength δ. Energy $\epsilon = 0$. Solid line: polarization $w_\uparrow - w_\downarrow = 0.2$. Dashed line: polarization $w_\uparrow - w_\downarrow = 0$. The inset shows details of the behavior of $c(\delta)$ at small δ

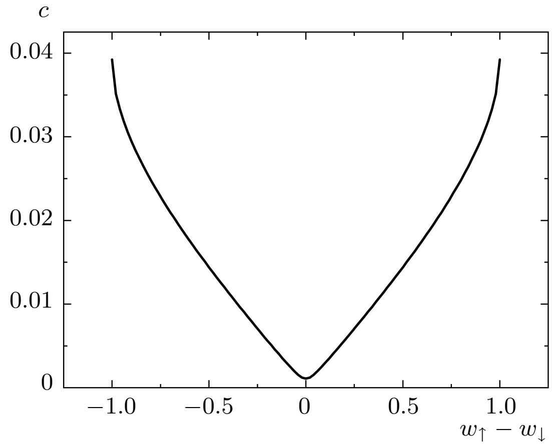

Fig. 4. Decoherence parameter c as a function of the impurity polarization $w_\uparrow - w_\downarrow$. Energy $\epsilon = 0$, exchange coupling $\delta = 0.1$

event. For a finite polarization of the impurity, the decoherence parameter c increases linearly with δ,

$$c \approx \frac{\pi\delta}{4} r_0 t_0 \left(\sqrt{w_\uparrow} - \sqrt{w_\downarrow}\right). \tag{24}$$

The dependence on δ becomes stronger with the degree of polarization of the impurity.

By contrast, for the completely unpolarized impurity ($w_\uparrow = w_\downarrow = 1/2$), the decoherence parameter c increases with δ much slower, as δ^2,

$$c \approx \frac{\pi^2 \delta^2 r_0^2 t_0^2}{16\sqrt{2}}. \tag{25}$$

This result is in accord with the restoration of the time reversal invariance of fictitious scattering matrix (19) for an unpolarized impurity, which decreases the decoherence. Figure 3 shows the dependence of the decoherence parameter c given by Eq. (23) on the exchange strength δ for the completely unpolarized ($w_\uparrow - w_\downarrow = 0$, dashed line) and a weakly polarized ($w_\uparrow - w_\downarrow = 0.2$, solid line) magnetic impurity. The dependence for small $\delta \ll 1$ is shown in the inset in detail. According to Eq. (25), there is a purely quadratic dependence for the unpolarized impurity (dashed line). For a weak polarization, a solid line exhibits a transition from the linear part in accordance with Eq. (24) to the nonlinear behavior at larger δ, described by Eq. (23). Figure 3 shows that the decoherence parameter c saturates at large values of δ.

The dependence of the decoherence parameter c on the polarization of the magnetic impurity in shown in Fig. 4. According to the foregoing, the decoherence is minimal for the completely unpolarized impurity, and it increases monotonically with the impurity polarization.

5. PHASE COHERENCE LENGTH AND THE INTER-PLATEAUX TRANSITION BROADENING DUE TO MAGNETIC IMPURITIES

We now apply the results in the preceding section to the estimation of the phase coherence length due to scattering by magnetic impurities. In what follows, we evaluate the energy width of the metallic region appearing at the inter-plateaux transition in the integer quantum Hall effect.

The phase coherence length can be defined as the length of path after which the phase uncertainty from multiple collisions becomes of the order of 1. Because the phase uncertainty in a single act of scattering is a random quantity, the parameter c evaluated in Eq. (23) should be understood as the dispersion of the distribution of random scattering phases,

$$c = \sqrt{\langle \delta\phi^2 \rangle}. \tag{26}$$

The total phase uncertainty after multiple scattering events is evaluated as a sum of random phases, and it is given by

$$\left\langle \delta\phi^2 \right\rangle_N = N \left\langle \delta\phi^2 \right\rangle = N c^2, \tag{27}$$

where N denotes the number of scattering events. Therefore, the number of scattering events needed to reach a complete decoherence is determined by the relation $Nc^2 \sim 1$, whence $N \sim 1/c^2$. The corresponding phase coherence time can be estimated analogously to the calculation of the spin relaxation time by spin–orbit scattering due to the Elliot–Yafet mechanism [8]

$$\tau_\phi \sim N\tau_0 \sim \tau_0/c^2, \tag{28}$$

where τ_0 denotes the time between two consecutive scatteting events. The time τ_0 is proportional to the distance between impurities. For a two-dimensional quantum Hall system, $\tau_0 \propto n_{imp}^{-1/2}$, where n_{imp} is the concentration of magnetic impurities.

We note that it follows from Eqs. (24) and (25) that the inverse phase coherence time $1/\tau_\phi \propto c^2$ exhibits a crossover as a function of the exchange strength δ from the behavior $1/\tau_\phi \propto \delta^4$ for unpolarized magnetic impurities to $1/\tau_\phi \propto \delta^2$ if the magnetic polarization is finite. The crossover from the δ^4 behavior in the unpolarized system to the δ^2 dependence for a finite spin polarization ($w_\uparrow \neq w_\downarrow$) is in accord with the previous findings in [1, 9]. The corresponding phase coherence length can be calcuated as the length of diffusion during the time τ_ϕ

$$L_\phi = \sqrt{D\tau_\phi} \sim \frac{1}{|c| n_{imp}^{1/4}}. \tag{29}$$

The region of delocalized states in IQHE appears when the phase coherence length for the electron becomes smaller than its localization length, which leads to the metallic behavior [10–12]. The phase coherence length of the electron corresponds to the length at which the phase uncertainty of its wave function becomes of the order of 1. At the same time, close to the quantum Hall inter-plateaux transition, the localization length is known to scale with the deviation ϵ from the critical energy as $\xi \sim |\epsilon|^{-\nu}$ ($\nu \approx 2.6$) [13]. Equating L_ϕ and ξ, we obtain an estimate for the energy width Δ of the metallic phase,

$$\Delta \sim \left(|c| n_{imp}^{1/4}\right)^{1/\nu}. \tag{30}$$

Therefore, using the results in Eqs. (23), (24), (25) we obtain the dependence of the width of the metallic region on both the spin–spin interaction strength and the polarization of magnetic impurities.

6. SUMMARY AND CONCLUSIONS

In this paper, we proposed a method for evaluating the phase coherence length of an electron due to scattering by magnetic impurities. The method is based on the introduction of a fictitious nonunitary scattering matrix that describes the electron motion averaged over the dynamics of magnetic impurities. The degree of nonunitarity is characterized by a single parameter c, which is the deviation of eigenvalues of the product $\mathcal{S}^\dagger\mathcal{S}$ from unity. The nonunitarity parameter is related to the phase uncertainty acquired in a single act of scattering, and it is inversely proportional to the phase coherence length. Our calculation revealed a change in the dependence of the nonunitarity parameter c on the exchange coupling from a linear dependence at strong magnetic polarization to a quadratic one for unpolarized magnetic impurities.

With the help of the proposed method, we estimate the width of the metallic region at the IQHE interplateau transition and its dependence on the exchange coupling strength and the degree of polarization of magnetic impurities. We believe that our method will be especially useful for other systems that allow the description in terms of scattering matrices and network models, such as topological insulators, graphene, quantum networks, etc. [14–16].

We are grateful to I. Burmistrov for the illuminating discussions. We acknowledge financial support from the Collaborative Research Center (SFB) 668. This work was supported by the SCE under internal grant No. 5368911113.

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