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MUON SPIN ROTATION IN HEAVY-ELECTRON PAULI-LIMIT SUPERCONDUCTORS

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The formalism for analyzing the magnetic field distribution in the vortex lattice of Pauli-limit heavy-electron superconductors is applied to the evaluation of the vortex lattice static linewidth relevant to the Muon Spin Rotation (μSR) experiment. Based on the Ginzburg–Landau expansion for the superconductor free energy, we study the evolution with respect to the external field of the static linewidth both in the limit of independent vortices (low magnetic field) with a variational expression for the order parameter and in the near $H_{c2}^P(T)$ regime with an extension of the Abrikosov analysis to Pauli-limit superconductors. We conclude that in the Ginzburg–Landau regime in the Pauli-limit, anomalous variations of the static linewidth with the applied field are predicted as a result of the superconductor spin response around a vortex core that dominates the usual charge-response screening supercurrents. We propose the effect as a benchmark for studying new puzzling vortex lattice properties recently observed in $CeCoIn_5$.

1. INTRODUCTION

As an example of a superconductor in the Pauli limit, the heavy-electron system $CeCoIn_5$ has special properties regarding its response to the external magnetic field. Notably, the Muon Spin Rotation (μSR) experiment [1] has revealed anomalous variations of the vortex lattice static linewidth σ_s^{VL} with respect to the magnetic field oriented along the tetragonal crystal c-axis. In Ref. [1], the static linewidth measured at temperature $T = 20$ mK showed an increase with the applied field from zero field to about 95 % of the upper critical field and eventually decreased just before the first-order superconductor-to-metal transition. The decrease in σ_s^{VL} with respect to the external field usually observed and analyzed [2] is the hallmark of a diminution in the vortex lattice local field contrast due to the decrease in the intervortex spacing with increasing field.

Here, we show that in the Ginzburg–Landau regime, an increasing behavior of the static linewidth is predicted. This results from the Zeeman interaction of the electron spin with the superconductor internal field, which dominates the usual charge-response supercurrents. As a result, the field distribution is modified on a distance of the order of ξ (ξ is the coherence length obtained in the Ginzburg–Landau formulation) from the center of each vortex [3]. The existence of the effect was pointed out in Ref. [4] in the context of magnetism of the FFLO (Fulde–Ferrel–Larkin–Ovchinnikov) state. Parallel to this, a numerical approach to Eilenberger equations was undertaken in Ref. [5] and effects of strong Pauli paramagnetism were highlighted in the vortex lattice state of Pauli-limit superconductors.

We first discuss the properties of Pauli-limit heavy-electron superconductors qualitatively [3, 4]. This class is characterized by a larger-than-one Maki parameter defined as

$$\alpha_{M0} = H_{c20}^{orb}/H_{c20}^{P} > 1$$

(an alternative definition includes the factor $\sqrt{2}$, which is not assumed here for clarity). We set

$$H_{c20}^{orb} = \frac{\phi_0}{\xi_0^2}, \quad H_{c20}^{P} = \frac{T_c}{\mu},$$

the zero-temperature scales for orbital and Pauli-limit fields respectively (we use units where $\hbar = c = 1$ throughout). Here, $\phi_0 = \pi/e \approx 2.07 \cdot 10^7$ G·cm^2 is the vortex fluxoid quantum, e is the absolute value of the electron charge, $\xi_0 = v_F/T_c$ is the $T = 0$ Cooper pair radius or coherence length, $v_F = k_F/m^*$ is the Fermi

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velocity, k_F is the Fermi momentum, m^* is the renormalized electron mass, T_c is the superconductor critical temperature, $\mu = g\mu_B/2$ is the electron magnetic moment absolute value, g is the Landé factor, $\mu_B = e/2m$ is the Bohr magneton, and m is the electron bare mass.

There are three characteristic lengths in the problem: the zero-temperature coherence length ξ_0 defined above, the intervortex distance $L(H_{c20}^P) = \sqrt{\mu\phi_0/T_c}$ in a square vortex lattice in the Pauli limit at temperature $T = 0$ and the field H_{c20}^P (more generally, we let $L(B) = \sqrt{\phi_0/B}$ denote the intervortex spacing of a square vortex lattice with the internal field B), and the London penetration depth $\lambda_L = \sqrt{m^*/4\pi ne^2}$ with the electron density n in the superconductor (at $T = 0$ and for a cylindrical Fermi surface, this is the electron density $n = k_F^2/2\pi l_c$ in the $2D$ metal with the spacing l_c between the planes of the tetragonal crystal). Hence,

$$\alpha_{M0} = \left[\frac{L(H_{c20}^P)}{\xi_0}\right]^2 = \frac{\mu\phi_0 T_c}{v_F^2} \sim \frac{m^* T_c}{mE_F}, \qquad (1)$$

and we define the Ginzburg–Landau ratio

$$\kappa = \frac{\lambda_L}{\xi_0} \sim \sqrt{\frac{m^*}{mr_e k_F}}\,\frac{T_c}{E_F}, \qquad (2)$$

where $r_e = e^2/m$ is the classical radius of the electron and $r_e k_F \sim 10^{-5}$.

The orders of magnitudes are as follows. In a classical, nonheavy electron superconductor, $m^* \sim m$ and $E_F \sim 10^3 T_c$ give $\kappa \sim 1$ and $\alpha_{M0} \sim 10^{-3}$. In $CeCoIn_5$, however, $T_c \approx 2.3$ K, $\xi_0 \sim 50$ Å, and $\lambda_L \sim 5000$ Å yield $m^* \sim 100m$, $E_F \sim 50T_c$, $\kappa \sim 100$, and $\alpha_{M0} \sim 1$–5, which, as we see in what follows, is the origin of special magnetic properties of the vortex lattice. The large Ginzburg–Landau parameter implies [6] that at $T = 0$, the ratio between the field at which the first vortex nucleates in the bulk of the sample and the orbital upper critical field $H_{c10}/H_{c20}^{orb} \sim \kappa^{-2}\ln\kappa \ll 1$, whence $B \approx H$ for a broad magnetic field range. In a Pauli-limit superconductor,

$$\frac{H_{c10}}{H_{c20}^P} \sim r_e k_F \frac{E_F}{T_c}\ln\kappa \sim 10^{-3},$$

and the same property applies.

We now study the electrodynamics of the vortex lattice that results from the large values for parameters in (1) and (2). The vortex lattice static linewidth is defined as

$$\sigma_s^{VL} = \frac{\gamma_\mu}{\sqrt{2}}\sqrt{\overline{\delta h(\mathbf{r})^2}}, \qquad (3)$$

where $\gamma_\mu = 2\pi\cdot 135.5342$ MHz/T is the muon gyromagnetic ratio, $h(\mathbf{r})$ is the component of the internal local field parallel to the applied field H, $\delta h(\mathbf{r}) = h(\mathbf{r}) - B$, the macroscopic internal field (or induction) is $B = \overline{h(\mathbf{r})}$, and the overline means averaging over the vortex-lattice unit cell. Equation (3) can be expressed as a sum involving all-order Fourier components F_{mn} of the field distribution in the vortex lattice,

$$\sigma_s^{VL} = \frac{\gamma_\mu}{\sqrt{2}}\sqrt{\sum_{(m,n)\neq(0,0)}(F_{mn})^2}. \qquad (4)$$

The components F_{mn} are called vortex-lattice form factors [3, 7] in the context of the Small Angle Neutron Scattering (SANS) experiment [7][1)].

2. MUON STATIC LINEWIDTH IN THE LOW-FIELD HIGH-TEMPERATURE REGIME

Here, we use results of the Ginzburg–Landau formulation [3, 4] to evaluate the static linewidth in Eq. (4). The near-T_c Ginzburg–Landau regime in the Pauli limit is accessible since the crossover temperature T^* from orbitally limited to the Pauli-limit superconductivity is in the range $(T_c - T^*)/T_c \sim 1/\alpha_{M0}^2$ (we see below that this follows from the relation $H_{c2}^{orb}(T)/H_{c2}^P(T) \sim \alpha_{M0}\sqrt{1 - T/T_c}$). In the independent-vortex approximation (low magnetic field) and high-κ limit, the form factors can be decomposed as a sum of two distinct contributions [3]:

$$F_{mn} = F_{mn}^{orb} + F_{mn}^{Z}. \qquad (5)$$

The first term is the usual charge response, which gives rise to orbital supercurrents. It is given by [3]

$$F_{mn}^{orb} = \frac{B\xi_v}{q_{mn}\lambda^2}K_1(q_{mn}\xi_v), \qquad (6)$$

where $\xi_v = \sqrt{2}\,\xi$ is a variational parameter that minimizes the superconductor free energy, $q_{mn} = [2\pi/L(B)](m^2 + n^2)^{1/2}$ for a square vortex lattice, and $K_n(z)$ is the nth-order modified Bessel function of the second kind (or the MacDonald function) [8]. The near-T_c coherence length and the penetration depth depend on the symmetry of the superconducting gap [3]. The expressions for d-wave pairing are

$$\frac{1}{\xi^2} = \frac{32\pi^2 T_c^2}{7\zeta(3)v_F^2}\left[\frac{T_c - T}{T_c} - 7\zeta(3)\left(\frac{\mu B}{2\pi T_c}\right)^2\right], \qquad (7)$$

[1)] The measurement of the first-order form factor F_{10} at $T = 50$ mK [7] has revealed a similar behavior to the one obtained in μSR experiment [1]: F_{10} increases with field up to 4.7 T and eventually decreases while approaching the (first-order) superconductor to metal transition.

and

$$\frac{1}{\lambda^2} = \frac{16}{3}\pi e^2 v_F^2 N_0 \left[\frac{T_c - T}{T_c} - 7\zeta(3)\left(\frac{\mu B}{2\pi T_c}\right)^2\right], \quad (8)$$

where N_0 is the density of states with dimension $[\text{energy}]^{-1} \times [\text{length}]^{-3}$ and $\zeta(z)$ is the Riemann zeta function, $\zeta(3) \approx 1.2021$. The field at which these two lengths diverge is defined as the near-T_c Pauli-limit upper critical field

$$H_{c2}^P = 2\pi\frac{T_c}{\mu}\sqrt{\frac{T_c - T}{7\zeta(3)T_c}}. \quad (9)$$

The near-T_c Zeeman spin contribution in Eq. (5) for the superconducting gap with a d-wave symmetry is given by [3]

$$F_{mn}^Z = \frac{28\zeta(3)v_F^2 N_0}{3\phi_0}\left(\frac{\mu B}{T_c}\right)^2 K_0(q_{mn}\xi_v). \quad (10)$$

We scale the internal field, the form factors, and the coherence length such that Eq. (5) in dimensionless units becomes

$$f_{mn} = \frac{q}{m^2 + n^2}K_1(q) + 4\pi b^2 K_0(q), \quad (11)$$

where $b = B/H_{c2}^P$ and

$$q = q_{mn}\xi_v = \sqrt{\frac{\pi\sqrt{7\zeta(3)}}{2}\frac{b}{\alpha_M}(m^2 + n^2)} \quad (12)$$

with

$$\alpha_M = \alpha_{M0}\sqrt{1 - \frac{T}{T_c}}, \quad (13)$$

α_{M0} is given in Eq. (1), and $f_{mn} = F_{mn}(2\pi\lambda)^2/\phi_0$.

Near T_c, dimensionless form factors (11) take a simple, universal form where only the parameter α_M remains that controls the relative contributions of the spin response with respect to the charge response. The static linewidth variations in the independent vortex limit with a dimensionless internal field b and different values of α_M are shown in Fig. 1. We note the low-field regime where all curves meet, which follows from the limit

$$\sqrt{\sum_{(m,n)\neq(0,0)}(f_{mn})^2} \xrightarrow{\text{as } b\to 0} \times \sqrt{\sum_{(m,n)\neq(0,0)}\frac{1}{(m^2+n^2)^2}} \approx 2.455. \quad (14)$$

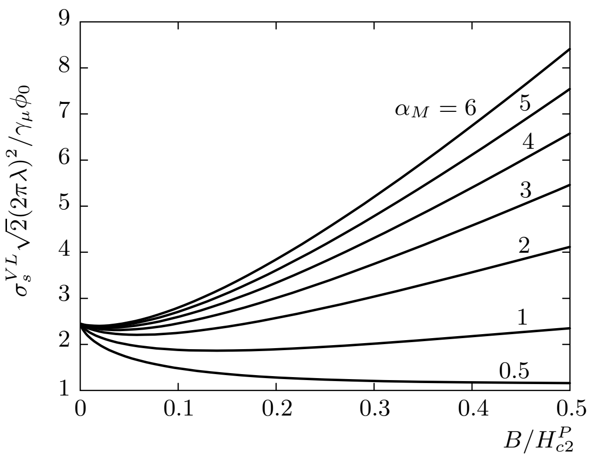

Fig. 1. Variations of the dimensionless μSR static linewidth $\sigma_s^{VL}\sqrt{2}\,(2\pi\lambda)^2/\gamma_\mu\phi_0$ with field where σ_s^{VL} is taken from Eq. (4) and the form factors from Eq. (11). Different values for the temperature Maki parameter (13) were used as indicated near the curves

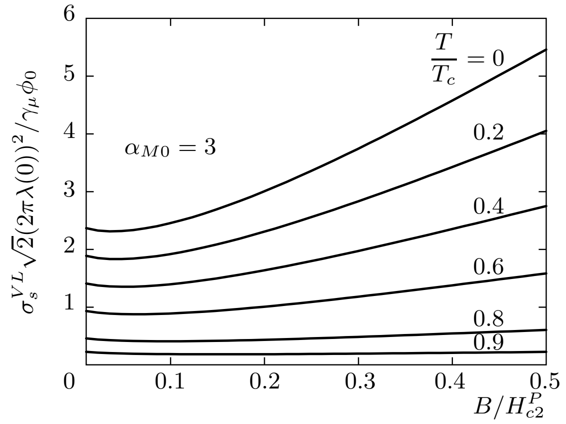

Fig. 2. Variations of the static linewidth $\sigma_s^{VL}\sqrt{2} \times [2\pi\lambda(0)]^2/\gamma_\mu\phi_0$ with field at different temperatures. We considered the parameter $\alpha_{M0} = 3$

We now turn to the effect of temperature on the form factors in Eq. (5). We fix the value $\alpha_{M0} = 3$ and plot $\sigma_s^{VL}\sqrt{2}[2\pi\lambda(0)]^2/\gamma_\mu\phi_0$ for different T/T_c, where $\lambda(0)$ is the Ginzburg–Landau penetration depth, Eq. (8), taken at $T = 0$. The results are shown in Fig. 2. We note that we have extended the temperature domain to very low T/T_c, which is not as justified as it is in the near-T_c region, but is expected to give qualitatively meaningful variations.

The MacDonald functions have the limits $K_0(q) \to -\ln(q/2) - C$ and $K_1(q) \to 1/q$ as $q \to 0$, where $C \approx 0.5772$ is the Euler constant. Having SANS experiment in mind in the large-α_M limit, it is useful to consider Eq. (11) with $(m,n) = (1,0)$:

$$f_{10} = 1 - 2\pi b^2 \ln\left(\frac{\pi\sqrt{7\zeta(3)}}{8\alpha_M} b e^{2C}\right). \qquad (15)$$

3. MUON STATIC LINEWIDTH CLOSE TO THE SECOND-ORDER TRANSITION CRITICAL FIELD

The form factors in Eqs. (6) and (10) are found in the independent vortex approximation. The derivation does not work at high field near the transition to the nonsuperconducting metal. In the high-field limit close to the transition line, the main source of magnetic field inhomogeneity in the vortex lattice comes from the Zeeman spin response [3, 4]

$$\delta h(\mathbf{r}) = -4\pi\varepsilon\left(|\Delta(x,y)|^2 - \overline{|\Delta(x,y)|^2}\right), \qquad (16)$$

where

$$\varepsilon = \frac{N_0\mu}{2\pi T}\,\mathrm{Im}\,\Psi^{(1)}\left(\frac{1}{2} - i\frac{\mu B}{2\pi T}\right), \qquad (17)$$

the overline again denotes averaging over a vortex lattice unit cell, and $\Psi^{(n)}(z)$ is the polygamma function [8] of order n. In a square vortex lattice, the Fourier decomposition of the square of the gap magnitude is given by [3]

$$|\Delta(x,y)|^2 = \overline{|\Delta(x,y)|^2} \sum_{m,n=-\infty}^{\infty} (-1)^{m+n+mn} \times \exp\left[-\frac{\pi}{2}(m^2+n^2)\right] \exp\left(\frac{2\pi i m x}{L(B)}\right) \times \exp\left(\frac{2\pi i n y}{L(B)}\right). \qquad (18)$$

Therefore, the form factors corresponding to Bragg peaks with indices $(m,n) \neq (0,0)$ take the form

$$F_{mn} = -4\pi\varepsilon\overline{|\Delta(x,y)|^2}(-1)^{m+n+mn} \times \exp\left[-\frac{\pi}{2}(m^2+n^2)\right], \qquad (19)$$

and the vortex lattice static linewidth is simply given by

$$\sigma_s^{VL} = \frac{4\pi s}{\sqrt{2}}\gamma_\mu \varepsilon \overline{|\Delta(x,y)|^2}, \qquad (20)$$

where

$$s = \sqrt{\left(\sum_{n=-\infty}^{\infty} \exp\left(-\pi n^2\right)\right)^2 - 1} \approx 0.4247. \qquad (21)$$

Equation (20) shows explicitly that the vortex-lattice contribution to the static linewidth vanishes when the transition is of the second order but shows a discontinuity where the transition is of the first order. In the former case, the gap average is [3, 4]

$$\overline{|\Delta(x,y)|^2} = \frac{|\alpha|}{2\beta_A\beta}, \qquad (22)$$

where

$$\alpha = N_0\left[\ln\left(\frac{T}{T_c}\right) + \mathrm{Re}\,\Psi\left(\frac{1}{2} - i\frac{\mu B}{2\pi T}\right) - \Psi\left(\frac{1}{2}\right)\right], \qquad (23)$$

and

$$\beta = -\frac{3N_0}{64\pi^2 T^2}\,\mathrm{Re}\,\Psi^{(2)}\left(\frac{1}{2} - i\frac{\mu B}{2\pi T}\right) \qquad (24)$$

are the respective quadratic and quartic coefficients of the Ginzburg–Landau free energy [3, 4], $\Psi(z)$ is the digamma function [8],

$$\beta_A = \frac{\overline{|\Delta(x,y)|^4}}{\left(\overline{|\Delta(x,y)|^2}\right)^2}$$

is the Abrikosov parameter equal to $\beta_A^{\square} = 1.18$ for a square-vortex lattice and $\beta_A^{\triangle} = 1.16$ for a triangular lattice. It then follows that

$$\sigma_s^{VL} = \frac{2\pi s\gamma_\mu}{\sqrt{2}\beta_A}\frac{|\alpha|\varepsilon}{\beta}, \qquad (25)$$

which is shown in Fig. 3.

4. CONCLUSION

Based on the Ginzburg–Landau expansion for the superconductor free energy in the Pauli limit, we have studied the evolution with respect to the external field of the muon spin rotation vortex lattice static linewidth both in the limit of independent vortices (low magnetic field) near T_c, and in the near-$H_{c2}^P(T)$ regime. In the first case, we have found a simple form of the total form factor, which is a function of the internal field scaled with the temperature-dependent upper critical field in the Pauli limit and includes a single parameter $\alpha_M = \alpha_{M0}\sqrt{1 - T/T_c}$ with $\alpha_{M0} = \mu\phi_0 T_c/v_F^2$. In the regime near $H_{c2}^P(T)$, we have used an extension of the Abrikosov analysis to Pauli-limit superconductivity

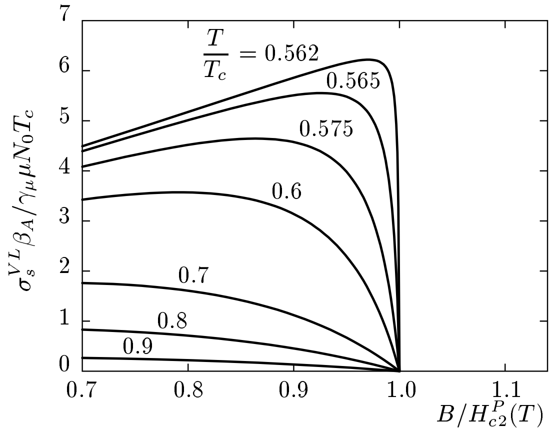

Fig. 3. The μSR static linewidth close to the second-order transition line $H_{c2}^P(T)$ as obtained from Abrikosov's analysis in the Pauli limit, Eq. (20), for a temperature range as indicated near the curves. We have scaled the internal field with respect to $H_{c2}^P(T)$ defined here as the curve solution of $\alpha(T, B) = 0$ [3, 4]. We note the rapid increase in the absolute value of the slope of $\sigma_s^{VL}(B)$ while approaching the first-order transition at $T/T_c \approx 0.5615$ and $\mu H_{c2}^P/T_c \approx 1.0728$

and observed a transformation from the second-order to the first-order transition to the metal (this occurs at $T/T_c \approx 0.5615$ and $\mu H_{c2}^P/T_c \approx 1.0728$) with a sharp increase in the absolute value of the slope of $\sigma_s^{VL}(B)$ in approaching $H_{c2}^P(T)$. Such an analysis allows a simple modeling of the effect of heavy electron superconductor strong paramagnetism on the vortex lattice electrodynamics. It is proposed as a benchmark for studying new puzzling vortex lattice properties in $CeCoIn_5$ [9].

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