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CHARGED FERMIONS TUNNELING FROM REGULAR BLACK HOLES

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We study Hawking radiation of charged fermions as a tunneling process from charged regular black holes, i. e., the Bardeen and ABGB black holes. For this purpose, we apply the semiclassical WKB approximation to the general covariant Dirac equation for charged particles and evaluate the tunneling probabilities. We recover the Hawking temperature corresponding to these charged regular black holes. Further, we consider the back-reaction effects of the emitted spin particles from black holes and calculate their corresponding quantum corrections to the radiation spectrum. We find that this radiation spectrum is not purely thermal due to the energy and charge conservation but has some corrections. In the absence of charge, $e = 0$, our results are consistent with those already present in the literature.

1. INTRODUCTION

Classically, a black hole (BH) is considered to absorb all matter and energy in the surrounding region into it due a strong gravitational field. Bekenstein [1] was the first to discuss the BH thermodynamics. Later, Hawking [2] investigated BH thermodynamical properties and proposed [3] that a BH could emit black-body radiation. According to this, a particle–antiparticle pair appears near the event horizon of a BH due to vacuum fluctuations. In order to preserve the total energy, one member of the pair with negative energy must fall into the BH while the other escapes with positive energy. In this process, the BH loses mass and it appears to an outside observer that the BH has just emitted a particle. This semiclassical process is called quantum tunneling [4, 5]. In this approach, particles follow classically forbidden trajectories from inside the horizon to infinity, for which the action becomes complex. This means that the tunneling probability for the outgoing particle is governed by the imaginary part of this action. Because a particle can classically only fall inside the horizon, the action for the ingoing particle must be real.

There are two different methods to evaluate the imaginary part of the action. One is the semiclassical Wentzel–Kramers–Brillouin (WKB) approximation method, first used in [6, 7], and the other is the radial null-geodesic method [5]. These methods have been used to evaluate the tunneling probabilities of quantum fields passing through an event horizon. Different semiclassical approaches have been adopted to evaluate tunneling of scalar and Dirac particles (charged and uncharged). In Refs. [8, 9], the tunneling of spin-1/2 particles through event horizons of the Rindler spacetime was investigated and Unruh temperature was obtained. In these papers fermion tunneling from the general nonrotating BH as well as the Kerr–Newman BH was also discussed and their corresponding Hawking temperatures was recovered.

Fermions tunneling from the Kerr BH were investigated in [10] by applying the WKB approximation to the general covariant Dirac equation, which allowed finding the Hawking temperature for the Kerr BH. Charged fermion tunneling from dilatonic BHs, the rotating Einstein–Maxwell dilaton–Axion BH, and a rotating Kaluza–Klein BH were studied in [11] and their corresponding Hawking temperatures were recovered. Hawking radiation of spin-1/2 particles from the Reissner–Nordström BH was investigated in [12] using the Dirac equation for charged particles. The tunneling of scalar and Dirac particles from the Kerr–Newman BH was explored in [13] and its Hawking temperature was obtained. The semiclassical fermion tunneling from the Kerr–Newman–Kasuya BH was studied

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in [14] and the Hawking temperature was obtained. Some work has also been done for three-dimensional spacetimes [15].

Tunneling of charged fermions from accelerating and rotating BHs with electric and magnetic charges have been studied in [16–18] using the WKB approximation. Tunneling probabilities of charged fermions and the corresponding Hawking temperature were found. In recent papers [19], the tunneling probabilities of incoming and outgoing scalar and charged/uncharged fermion particles from accelerating and rotating BHs have been investigated. Recently, we have examined the radiation spectrum of an RN-like noncommutative BH [20] by quantum tunneling process (radial null geodesic method). Also, we have investigated quantum corrections of regular BHs [21, 22].

In this paper, we use the procedure in [8] to investigate the tunneling probabilities of charged fermions for charged regular BHs, i. e., the regular Bardeen and regular Ayón-Beato–García–Bronnikov (ABGB) BHs. We recover the corresponding Hawking temperatures for charged massive as well as massless fermions. Also, we explore the radiation spectrum by using the radial null-geodesic method [12]. This paper is organized as follows. In Sec. 2, we review the basic formalism for the pure thermal spectrum of charged fermions using the Dirac equation for charged particles. Section 3 is devoted to the study of fermion tunneling from the regular Bardeen and ABGB BHs. In Sec. 4, we discuss the correction spectrum of charged fermions due to back-reaction effects. Finally, Sec. 5 summarizes the results.

2. REVIEW: TUNNELING OF CHARGED FERMIONS

In this section, we briefly review some basic material used to evaluate the tunneling probabilities of charged fermions. For this purpose, we apply the WKB approximation to the general covariant Dirac equation for charged particles. The line element of a spherically symmetric BH can be written as

$$ds^2 = -Fdt^2 + F^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2, \quad (2.1)$$

where

$$F = 1 - 2\frac{M(r)}{r}.$$

This metric can be reduced to well-known BHs for special choices of $M(r)$. The Dirac equation with electric charge q is given by [9]

$$i\gamma^\mu \left(D_\mu - \frac{iq}{\hbar}A_\mu\right)\Psi + \frac{m}{\hbar}\Psi = 0, \quad \mu,\nu = 0,1,2,3, \quad (2.2)$$

where m is the mass of fermion particles, A_μ is the 4-potential, Ψ is the wave function, and γ^μ are the Dirac matrices [14]. The antisymmetric property of the Dirac matrices, i. e.,

$$[\gamma^\alpha, \gamma^\beta] = \begin{cases} 0, & \alpha = \beta, \\ -[\gamma^\beta, \gamma^\alpha], & \alpha \neq \beta, \end{cases}$$

reduces Dirac equation (2.2) to the form

$$i\gamma^\mu \left(\partial_\mu - \frac{iq}{\hbar}A_\mu\right)\Psi + \frac{m}{\hbar}\Psi = 0. \quad (2.3)$$

The spinor wave function Ψ has two spin states: spin-up (radially outward, i. e., in positive r-direction) and spin-down (radially inward, i. e., in negative r-direction). The solutions for spin-up and spin-down particles are respectively given by [8]

$$\Psi_\uparrow(t,r,\theta,\phi) = \begin{bmatrix} A(t,r,\theta,\phi) \\ 0 \\ B(t,r,\theta,\phi) \\ 0 \end{bmatrix} \times \exp\left[\frac{i}{\hbar} I_\uparrow(t,r,\theta,\phi)\right], \quad (2.4)$$

$$\Psi_\downarrow(t,r,\theta,\phi) = \begin{bmatrix} 0 \\ C(t,r,\theta,\phi) \\ 0 \\ D(t,r,\theta,\phi) \end{bmatrix} \times \exp\left[\frac{i}{\hbar} I_\downarrow(t,r,\theta,\phi)\right], \quad (2.5)$$

where $I_{\uparrow/\downarrow}$ is the action of the emitted spin-up/spin-down particles. In what follows, we discuss the spin-up case in detail; the spin-down case follows in a similar fashion. Using Eq. (2.4) in Dirac equation (2.3), we obtain the set of equations

$$-\left[\frac{iA}{\sqrt{F(r)}}\partial_t I_\uparrow + B\sqrt{F(r)}\partial_r I_\uparrow - \frac{iA}{\sqrt{F(r)}} qA_0\right] + mA = 0, \quad (2.6)$$

$$-B\left[\frac{1}{r}\partial_\theta I_\uparrow + \frac{i}{r\sin\theta}\partial_\phi I_\uparrow\right] = 0, \quad (2.7)$$

$$\left[\frac{iB}{\sqrt{F(r)}}\partial_t I_\uparrow - A\sqrt{F(r)}\partial_r I_\uparrow - \frac{iB}{\sqrt{F(r)}}qA_0\right] + mB = 0, \quad (2.8)$$

$$-A\left[\frac{1}{r}\partial_\theta I_\uparrow + \frac{i}{r\sin\theta}\partial_\phi I_\uparrow\right] = 0. \quad (2.9)$$

To find the action from the above equations, we use separation of variables in accordance with

$$I_\uparrow = -Et + W(r) + J(\theta, \phi), \quad (2.10)$$

where E and J denote the energy and angular momentum of the emitted particle, and W is an arbitrary function of r. Inserting this value of the action in Eqs. (2.6)–(2.9), we also use Taylor's expansion to expand $F(r)$ near the outer horizon r_+, neglecting squares and higher powers. Substituting the values of $A_0(r_+)$ and setting $iA = B$ and $iB = A$ in the above set of equations, we obtain

$$-B\left[\frac{-E - qA_0}{\sqrt{(r - r_+)F'(r_+)}} + \sqrt{(r - r_+)F'(r_+)}W'\right] + mA = 0, \quad (2.11)$$

$$-B\left[\frac{1}{r}\partial_\theta J + \frac{i}{r\sin\theta}\partial_\phi J\right] = 0, \quad (2.12)$$

$$A\left[\frac{-E - qA_0}{\sqrt{(r - r_+)F'(r_+)}} - \sqrt{(r - r_+)F'(r_+)}W'\right] + mB = 0, \quad (2.13)$$

$$-A\left[\frac{1}{r}\partial_\theta J + \frac{i}{r\sin\theta}\partial_\phi J\right] = 0, \quad (2.14)$$

where the prime denotes the derivative with respect to r. Equations (2.12) and (2.14) yield

$$\frac{1}{r}\partial_\theta J + \frac{i}{r\sin\theta}\partial_\phi J = 0 \quad (2.15)$$

which implies

$$J = \exp[ik\phi]\left[c_1\int \csc\theta\, d\theta + c_2\right], \quad (2.16)$$

where k, c_1, and c_2 are arbitrary functions of θ and ϕ. This quantity must be same for both outgoing and incoming cases. As a result, it cancels from the formula for the tunneling probability from inside to outside the horizon (which is the ratio of outgoing and incoming modes [8]).

In the massless case ($m = 0$), Eqs. (2.11) and (2.13) yield the respective solutions

$$W'(r) = W'_+(r) = \frac{E + qA_0}{(r - r_+)F'(r_+)}, \quad (2.17)$$

$$W'(r) = W'_-(r) = -\frac{E + qA_0}{(r - r_+)F'(r_+)}, \quad (2.18)$$

where $W_{+/-}$ correspond to the outgoing/incoming solutions. The tunneling probability of a particle going from outside to inside the horizon is equal to unity [9]. Also, Eqs. (2.17) and (2.18) lead to

$$\operatorname{Im} W_+ = -\operatorname{Im} W_-.$$

Hence, the overall tunneling probability of the outgoing particle turns out to be

$$\Gamma = \frac{\text{Prob[out]}}{\text{Prob[in]}} = \frac{\exp[-2(\operatorname{Im} W_+)]}{\exp[-2(\operatorname{Im} W_-)]} = \exp[-4\operatorname{Im} W_+]. \quad (2.19)$$

We can recover the Hawking temperature T_H from the relation as

$$\Gamma = \exp[-\beta E], \quad \beta = \frac{1}{T_H}.$$

In the massive case ($m \neq 0$), Eqs. (2.11) and (2.13) no longer decouple. We eliminate the function W' from these two equations by respectively multiplying Eqs. (2.11) and (2.13) with A and B. After some manipulations, it follows that

$$\frac{A}{B} = \frac{-(E + qA_0) \pm \sqrt{(E + qA_0)^2 + m^2(r - r_+)F'(r_+)}}{m\sqrt{(r - r_+)F'(r_+)}}. \quad (2.20)$$

The limit $r \to r_+$ yields either $A/B \to 0$ or $A/B \to -\infty$, i. e., either $A \to 0$ or $B \to 0$. For $A \to 0$, we can evaluate the value of m from Eq. (2.13) as

$$m = -\frac{A}{B}\left[-\sqrt{(r - r_+)F'(r_+)}W'(r) + \frac{-(E + qA_0)}{\sqrt{(r - r_+)F'(r_+)}}\right]. \quad (2.21)$$

Inserting this value in Eq. (2.11) and simplifying, we obtain the same value of $W'_+(r)$ as in Eq. (2.17). Similarly, for $B \to 0$, the same expression for $W'_-(r)$ is found as in (2.18). Consequently, the Hawking temperature turns out to be the same as in the massless case. In the spin-down case, for both massive and massless fermions, the Hawking temperature remains the same as for the spin-up case. Thus, both spin-up and spin-down particles are emitted at the same rate, i. e., as many spin-up fermions are emitted as spin-down. We note that for the tunneling of charged massive fermions, the tunneling probability is independent of the mass but depends only on the charge. This is because the massive case reduces to the massless case as $r \to r_+$, and hence the tunneling probability is the same as in the massless case.

3. REGULAR BLACK HOLES

Singularities exist in all known physical exact solutions of BHs. In order to remove these singularities, some regular BH models have been proposed. These models represent singularity-free solutions of the field equations coupled to a suitable nonlinear electrodynamics satisfying the weak energy condition. Here, we consider the Bardeen and ABGB regular BH solutions to discuss tunneling process.

3.1. Bardeen regular black hole

Ayón-Beato and García [23] gave a physical interpretation of the Bardeen regular BH [24] by showing that the charge associated with it acts as a magnetic monopole charge. This is described by metric (2.1) with

$$M(r) = \frac{Mr^3}{(r^2+e^2)^{3/2}}. \tag{3.1}$$

Here, M and e stand for the mass and monopole charge of a self-gravitating magnetic field of a nonlinear electrodynamic source. This solution exhibits a BH behavior for $e^2 \le (16/27)m^2$ and has a spherical event horizon at $r_+ = 2M(r_+)$. For $e = 0$, it reduces to the Schwarzschild solution.

We compute the tunneling probability of a charged particle for this solution by using the fermion tunneling approach developed in the previous section. The derivative of $F(r_+)$ takes the form

$$F'(r_+) = \frac{2Mr_+(r_+^2-2e^2)}{(r_+^2+e^2)^{5/2}}. \tag{3.2}$$

In the massless case, using Eq. (3.2) in Eq. (2.17) leads to

$$W'_+(r) = \frac{[E+qA_0](r_+^2+e^2)^{5/2}}{(r-r_+)2Mr_+(r_+^2-2e^2)}, \tag{3.3}$$

where

$$A_0 = -\frac{3e}{2r_+^2}(r_+^2+e^2)^{1/2}$$

(see [21]). Similarly, the solution for incoming particles can be obtained by setting the values in Eq. (2.18),

$$W'_-(r) = -\frac{[E+qA_0](r_+^2+e^2)^{5/2}}{(r-r_+)2Mr_+(r_+^2-2e^2)}. \tag{3.4}$$

The imaginary part of W_+ is

$$\operatorname{Im} W_+ = \frac{\pi(E+qA_0)(r_+^2+e^2)^{5/2}}{2Mr_+(r_+^2-2e^2)}. \tag{3.5}$$

Similarly, the imaginary part of W_- becomes

$$\operatorname{Im} W_- = -\frac{\pi(E+qA_0)(r_+^2+e^2)^{5/2}}{2Mr_+(r_+^2-2e^2)}. \tag{3.6}$$

Equations (3.5) and (3.6) imply that

$$\operatorname{Im} W_+ = -\operatorname{Im} W_-.$$

Consequently, tunneling probability (2.19) becomes

$$\Gamma = \exp\left[-\frac{2\pi(E+qA_0)(r_+^2+e^2)^{5/2}}{Mr_+(r_+^2-2e^2)}\right]. \tag{3.7}$$

Comparing this with

$$\Gamma = \exp[-\beta E], \quad \beta = \frac{1}{T_H},$$

we recover the Hawking temperature of the regular Bardeen BH [25] as

$$T_H = \frac{Mr_+(r_+^2-2e^2)}{2\pi(r_+^2+e^2)^{5/2}}. \tag{3.8}$$

In the massive case, Eqs. (2.11) and (2.13) provide the outgoing and incoming particle solutions corresponding to $A \to 0$ and $B \to 0$. These solutions turn out to be the same as in the massless case for outgoing and incoming particles given in Eqs. (3.5) and (3.6). Consequently, the Hawking temperature of massive fermion tunneling takes the same form as for the massless fermion tunneling.

3.2. The ABGB regular black hole

A solution of the coupled system of equations of nonlinear electrodynamics and gravity representing a class of BHs was formulated in [26, 27]. It is given by metric (2.1) with

$$M(r) = M\left[1 - \operatorname{th}\left(\frac{e^2}{2Mr}\right)\right], \tag{3.9}$$

where M is the mass and e is either the electric or the magnetic charge. The ABGB regular BH solution has a spherical event horizon at $F(r_+) = 0$ or $r_+ = 2M(r_+)$. This solution describes a regular static spherically symmetric configuration that reduces to the Schwarzschild solution for $e = 0$.

For this BH, we find

$$F'(r_+) = 2\left(\frac{M}{r_+^2} - \frac{e^2}{r_+^3} + \frac{e^6}{6M^2r_+^5}\right). \tag{3.10}$$

In the massless case, Eqs. (2.11) and (2.13) yield

$$W'_+(r) = \\ = \frac{[E + qA_0]}{2(r - r_+)\left[M/r_+^2 - e^2/r_+^3 + e^6/6M^2r_+^5\right]}, \tag{3.11}$$

$$W'_-(r) = \\ = -\frac{[E + qA_0]}{2(r - r_+)\left[M/r_+^2 - e^2/r_+^3 + e^6/6M^2r_+^5\right]}, \tag{3.12}$$

where

$$A_0 = -\left[-4.8\frac{e^5}{r_+^5} + 2.8\frac{e^3}{r_+^3} + 1.8\frac{e}{r_+}\right]$$

(see [22]). The imaginary parts of W_+ and W_- become

$$\operatorname{Im} W_+ = -\operatorname{Im} W_- = \\ = \frac{\pi(E + qA_0)}{2\left[M/r_+^2 - e^2/r_+^3 + e^6/6M^2r_+^5\right]}. \tag{3.13}$$

The tunneling probability turns out to be

$$\Gamma = \exp\left[-\frac{2\pi(E + qA_0)}{M/r_+^2 - e^2/r_+^3 + e^6/6M^2r_+^5}\right]. \tag{3.14}$$

The corresponding Hawking temperature can be recovered by

$$T_H = \frac{M/r_+^2 - e^2/r_+^3 + e^6/6M^2r_+^5}{2\pi}. \tag{3.15}$$

In the massive case, Eqs. (2.11) and (2.13) lead to the same results as in the massless case.

4. TUNNELING CORRECTIONS

In this section, we examine the tunneling process of charged massive fermions through the quantum horizon of regular BHs by using the radial null-geodesic method [12]. Due to vacuum fluctuations, the mass and charge of the BH fluctuate as the BH accretes a small negative energy, which decreases its mass. If a particle with energy E and charge q tunnels through the horizon, the total mass and charge of the BH become $M - E$ and $e - q$, and the radius of the horizon shrinks. Consequently, the imaginary part of the action becomes

$$\operatorname{Im} W_+ = \\ = -\frac{1}{4}\int\limits_{(0,0)}^{(E,q)} \frac{2\pi\left[d\tilde{E} - \tilde{A}_0(M - \tilde{E}, e - \tilde{q})\, d\tilde{q}\right]}{\kappa(M - \tilde{E}, e - \tilde{q})}. \tag{4.1}$$

Using the first law of BH thermodynamics,

$$dM = T\,dS - A_0\,de,$$

we write this equation as

$$\operatorname{Im} W_+ = -\frac{1}{4}\int\limits_{S_i(M,e)}^{S_f(M-E,e-q)} dS = -\frac{\Delta S}{4}, \tag{4.2}$$

where

$$\Delta S = S(\tilde{r}_+) - S(r_+)$$

is the change of the Bekenstein–Hawking entropy, with $S(\tilde{r}_+)$ and $S(r_+)$ being the BH entropies after and before the radiation. Then the total tunneling probability of the emitted spin particle is

$$\Gamma \propto \exp[\Delta S] = \exp[S(M-E, e-q) - S(M, e)], \tag{4.3}$$

implying that the tunneling rate is related to the change in the Bekenstein–Hawking entropy

$$S = \frac{A}{4} = \pi r_+^2.$$

It follows that the emission spectrum cannot be precisely thermal. The entropy difference of the BH can be expanded using Taylor's expansion as

$$\Delta S = \frac{dS}{dr_+}\Delta r_+ + \frac{1}{2!}\frac{d^2S}{dr_+^2}(\Delta r_+)^2 + \\ + \frac{1}{3!}\frac{d^3S}{dr_+^3}(\Delta r_+)^3 + \ldots, \tag{4.4}$$

where

$$\frac{dS}{dr_+} = 2\pi r_+, \quad \Delta r_+ = r_+(M-E, e-q) - r_+(M,e).$$

Using this value of ΔS in Eq. (4.3) and considering the changes of the BH mass and charge,

$$\Delta M = -E, \quad \Delta e = -q,$$

we obtain

$$\Gamma \propto \exp(\Delta S) = \exp\left[-\beta(E+qA_0) \times \right.$$
$$\left. \times \left(1 - \frac{1}{2!\beta(E+qA_0)}\frac{d^2S}{dr_+^2}(\Delta r_+)^2\right)\right]. \quad (4.5)$$

When higher-order terms in $(E+qA_0)$ is ignored, the purely thermal spectrum of the regular BH can be obtained.

We next evaluate the correction spectrum of fermions for the Bardeen and ABGB regular BHs. For the Bardeen regular BH, the surface gravity is

$$\kappa = \frac{Mr_+(r_+^2 - 2e^2)}{(r_+^2+e^2)^{5/2}}. \quad (4.6)$$

Inserting this value in Eq. (4.1), we obtain

$$\operatorname{Im} W_+ = -\frac{1}{4}\int_{(0,0)}^{(E,q)} 2\pi\left[d\tilde{E} - \tilde{A}_0(M-\tilde{E}, e-\tilde{q})\, d\tilde{q}\right] \times$$
$$\times \left[\frac{(M-\tilde{E})\tilde{r}_+(\tilde{r}_+^2 - 2(e-\tilde{q})^2)}{(\tilde{r}_+^2 + (e-\tilde{q})^2)^{5/2}}\right]^{-1}, \quad (4.7)$$

where

$$\tilde{A}_0 = -\frac{3(e-q)}{2\tilde{r}_+^2}(\tilde{r}_+^2 + (e-q)^2)^{1/2},$$
$$\tilde{r}_+ = r_+(M-E, e-q). \quad (4.8)$$

The equation for the spherical event horizon leads to

$$\Delta r_+ = \frac{3Mer_+^2\Delta e - r_+^2(r_+^2+e^2)\Delta M}{Mr_+(2e^2 - r_+^2)}. \quad (4.9)$$

Using Eqs. (4.4) and (4.9) in (4.3) and ignoring higher-order terms in $(E+qA_0)$, we find the emission rate

$$\Gamma \propto \exp\left[-\beta(E+qA_0)\right] \approx$$
$$\approx \exp\left[-\beta\frac{r}{2M}(E+qA_0)\right], \quad (4.10)$$

where

$$\beta = \frac{2\pi(r_+^2+e^2)^{5/2}}{Mr_+(r_+^2 - 2e^2)}.$$

For the ABGB regular BH, the surface gravity is

$$\kappa = \frac{M}{r_+^2} - \frac{e^2}{r_+^3} + \frac{e^6}{6M^2r_+^5}. \quad (4.11)$$

Substituting this value in Eq. (4.1) gives

$$\operatorname{Im} W_+ = -\frac{1}{4}\int_{(0,0)}^{(E,q)} 2\pi\left[d\tilde{E} - \tilde{A}_0(M-\tilde{E}, e-\tilde{q})\, d\tilde{q}\right] \times$$
$$\times \left[\frac{M-\tilde{E}}{\tilde{r}_+^2} - \frac{(e-\tilde{q})^2}{\tilde{r}_+^3} + \frac{(e-\tilde{q})^6}{6(M-\tilde{E})^2\tilde{r}_+^5}\right]^{-1}, \quad (4.12)$$

where

$$\tilde{A}_0 = -\left[-4.8\frac{(e-q)^5}{\tilde{r}_+^5} + \right.$$
$$\left. + 2.8\frac{(e-q)^3}{\tilde{r}_+^3} + 1.8\frac{(e-q)}{\tilde{r}_+}\right], \quad (4.13)$$

$$\tilde{r}_+ = r_+(M-E, e-q). \quad (4.14)$$

The spherical event horizon equation yields

$$\Delta r_+ = \left[-r_+\left(-2 + \frac{e^6}{6M^3r_+^3}\right)\Delta M - \right.$$
$$\left. - \left(2e - \frac{e^5}{2M^2r_+^2}\right)\Delta e\right] \times$$
$$\times \left[2M - \frac{2e^2}{r_+} + \frac{e^6}{3M^2r_+^3}\right]^{-1}. \quad (4.15)$$

Inserting Eqs. (4.4) and (4.15) in (4.3) and ignoring higher-order terms in $(E+qA_0)$, we obtain

$$\Gamma \propto \exp\left[-\beta(E+qA_0)\right] \approx$$
$$\approx \exp\left[-\beta\left(E\left(1 - \frac{e^6}{12m^3r_+^3}\right) + \right.\right.$$
$$\left.\left. + q\left(\frac{A_0}{1.8} + \frac{2.8e^3}{1.8r_+^3}\right)\right)\right], \quad (4.16)$$

where

$$\beta = 2\pi\left[\frac{M}{r_+^2} - \frac{e^2}{r_+^3} + \frac{e^6}{6M^2r_+^5}\right]^{-1}.$$

5. OUTLOOK

The first regular BH solution was proposed in [24]. In this paper, the idea of the central matter core as a singular region, was introduced, by deriving a solution of the Einstein equations with horizons and without singularities [28]. The Bardeen model is a regular BH model obeying the weak energy condition. All the subsequent regular BH solutions are based on Bardeen's scenario, which is an incredible development in the implementation and analysis of the properties of regular BH solutions. Nonlinear fields and sources generating a four-parameter solution [29] were found in [23]. For the extremal limit of the regular BH solutions, a regular ABGB BH solution was constructed in [30].

There exists a direct correspondence between the laws of BH physics and the laws of thermodynamics. The temperature, energy, and entropy of the thermodynamical system respectively correspond to the surface gravity at the horizon, the BH mass, and the area of the BH horizon. For a distant observer who stays at a fixed distance from the BH event horizon, the BH seems to radiate particles with the thermal spectrum at the Hawking temperature [31]. In the semiclassical tunneling picture (Hamilton–Jacobi equations), the Hawking temperature apparently depends on the coordinate system. The Hawking temperature obtained from the Kerner and Mann technique is coordinate independent, which provides the expected Hawking temperature.

Bekenstein [1] suggested that BHs must have a finite temperature. Hawking found that particles could escape from BHs as they escape from the center of an atom. This leads to a quantum mechanical phenomenon in which particles tunnel through the event horizon. The rate at which particles escape is related to the measure of the BH temperature. Massive BHs have an extremely low surface temperature while low-mass BHs (Hawking miniature BHs) are superhot. The Hawking temperature provides information about the BH mass and allows understanding behavior of the universe containing celestial objects from its birth to its end [32].

In quantum tunneling, virtual particles (charged fermions) face a barrier regardless of whether they move from the inside to the outside or from the outside to the inside across the barrier. Classically, a particle can easily cross the horizon, i. e., particles have 100 % chances when going inward. Hence, their probabilities are equal to 1. Semiclassically, a particle faces the barrier when crossing the horizon in the outward direction. However, in the tunneling process (in the semiclassical approach), a pair of negative–positive-energy particles is created due to vacuum fluctuations near the horizon. For a pair of particles inside the horizon, the positive-energy particle must tunnel out of the horizon while the negative-energy component goes inward. For a pair outside the horizon, the negative-energy component must tunnel into the horizon, with the positive-energy component going outward. In this approach, the horizon represents a two-way barrier for the pairs of virtual particles and contradicts the classical approach. We have considered both the incoming and outgoing particles and the horizon as the tunneling barrier.

Hawking radiation can be defined as a semiclassical quantum tunneling phenomenon of BHs. We have used the formulation in [8] to study quantum tunneling of charged fermions from charged regular BHs. To apply the WKB approximation, we used the assumption of spin-up particles in the general covariant Dirac equation for charged particles. We have computed the tunneling probabilities for the outgoing and incoming charged fermion particles across the horizon. Also, we have obtained Hawking temperature corresponding to these BHs. Interestingly, the tunneling probabilities of charged fermions are independent of the mass of the fermions but depend only on its charge. The Hawking temperature depends on the mass and electric charge of the BH. The equations for the spin-down case are of the same form as for the spin-up case except for a negative sign. In both massive and massless cases, the Hawking temperature implies that both spin-up and spin-down particles are emitted at the same rate. The temperatures of these BHs coincide with the corresponding temperatures given in [21, 22, 25]. In the absence of charge, the temperature of the Bardeen and ABGB BHs reduces to the Schwarzschild temperature [33].

Finally, we have used the radial null-geodesic method to explore tunneling probabilities. For this, we took gravitational self-interaction and back-reaction effects of the emitted spin charged fermions into account. We note here that when the back reaction effects are taken into account, the tunneling probability for charged massive fermions is related to the Bekenstein–Hawking entropy. This radiation spectrum is not precisely thermal. When higher-order terms in $E + qA_0$ are ignored, we can obtain the pure thermal spectrum similar to that for the Bardeen and ABGB regular BHs.

This tunneling approach provides new physical insight into the emission of spin-1/2 fermions as the BH radiation. Also, this offers an effective way to compute the surface gravity for a wide range of BH solutions. The scattering of spin-1/2 particles could lead to a violation of the weak cosmic censorship conjecture [34]. For example, the results in [35] show the creation of a naked singularity by the quantum tunneling of spin-1/2 charged fermions. Within the semiclassical WKB approach, the tunneling probability refers to local aspects and is also more general than the standard one [36]. In this paper, the tunneling probability of charged fermions and the Hawking temperature at the horizon are correlated with the energy of post-radiating regular BHs.

There are subtle technical issues involved in choosing an appropriate ansatz for the Dirac field consistent with the choice of gamma matrices, and the failure to make such a choice makes the method break down. Some difficulties must also be overcome in calculating the real radiation spectrum. The first is how to understand an electromagnetic field with a source of electric and magnetic charges. The second is related to the formation of the Dirac equation according to the tunneling nature of the charged particles. In order to take the effects of an electromagnetic field into account, we can consider the BH and the electromagnetic field outside it as a system [14].

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