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COSMIC ACCELERATION AND BRANS–DICKE THEORY

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We study the accelerated expansion of the universe by exploring the Brans–Dicke parameter in different eras. For this, we take the FRW universe model with a viscous fluid (without potential) and the Bianchi type-I universe model with a barotropic fluid (with and without a potential). We evaluate the deceleration parameter and the Brans–Dicke parameter to explore cosmic acceleration. It is concluded that accelerated expansion of the universe can also be achieved for higher values of the Brans–Dicke parameter in some cases.

1. INTRODUCTION

The accelerated expansion of the observable universe is one of the most conspicuous recent achievements in modern cosmology. This expansion with positive cosmic acceleration has been confirmed by many astronomical experiments such as Supernova (Ia) [1, 2], WMAP [3], SDSS [4], galactic cluster emission of X-rays [5], large-scale structure [6], weak lensing [7], etc. These results lead to the conclusion that our universe is spatially flat.

The positive cosmic acceleration of the universe has been motivated by a mysterious exotic matter having large negative pressure, known as dark energy. Although general relativity (GR) is an excellent theory to explain the gravitational effects, it is unable to describe the present cosmic acceleration and the reality of dark energy. To explain the nature of this mysterious finding, various models including a Chaplygin gas, phantom, quintessence, cosmological constant, and so on have been constructed [8, 9]. However, none of these models is very successful.

The exploration of scalar–tensor theories of gravity as modified theories of gravity has received much attention due to their vast implications in cosmology [10–14]. The Brans–Dicke (BD) theory of gravity, a special case of scalar–tensor theories, is one of the most viable theories for this purpose. It is the general deformation of GR satisfying the weak equivalence principle, in which gravity effects are mediated by the metric tensor and a scalar field [15]. This provides a direct coupling of the scalar field to geometry. The Brans–Dicke theory is compatible with both Mach's principle [16] and Dirac's large number hypothesis [17]. One of the salient features of this theory is that the gravitational coupling constant, being the inverse of the spacetime scalar field, varies with time. In order to fulfill the solar system experiment constraints, the value of the generic dimensionless BD parameter ω should be very large, $\omega \geq 40.000$ [8, 9].

The Brans–Dicke theory is a successful theory that can tackle many outstanding cosmological problems like inflation, quintessence, late time behavior of the universe, the coincidence problem, the cosmic acceleration [11], and so on. There are different versions of the BD theory available in the literature [20, 21]. In Ref. [22], various BD cosmological models were investigated and it was shown that the Bianchi models are very effective in explaining the evolution of the universe for a perfect fluid. In Ref. [23], different models of the universe with a constant deceleration parameter based on the variation law of the Hubble parameter were discussed. In Ref. [11], it was found that the accelerated expansion of the universe could be obtained with large $|\omega|$ and potential ϕ^2 without considering the positive-energy condition. In Ref. [24], it was shown that the dissipative pressure could support the late-time accelerated expansion of the universe. In Ref. [12], it was found that the present accelerated expansion could be obtained without restoring a cosmological constant or quintessence matter for Friedmann–Robertson–Walker (FRW) model.

In Ref. [25], the observed accelerated expansion of the present universe in this theory for the FRW model

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was explored, and in Ref. [13], exact solutions in different eras of the universe were found and also discussed the possibilities for obtaining cosmic acceleration, inflation, and deceleration for these solutions. The role of a positive power-law potential as regards the accelerated expansion of the universe was investigated in [26]. It was concluded that a self-interacting potential can derive the accelerated expansion in the perfect fluid background with small negative values of the BD parameter. An axially symmetric perfect fluid cosmological model of this theory was found in [27]. To investigate the present accelerated expansion of the universe and different stages of the cosmic evolution, much work has been done using Bianchi models in GR and scalar-tensor theories [28–32]. In a recent paper [14], cosmic acceleration in this theory for the FRW model was investigated. It was shown that the accelerated expansion of the universe with higher values of ω can be achieved only for the closed model.

In this paper, we explore the effect of the BD parameter on the cosmic acceleration by using spatially flat models in the presence of different fluids. The paper is organized as follows. In Sec. 2, we formulate the field equations of the generalized BD theory with a self-interacting potential. Section 3 provides the field equations for the FRW model in the presence of a viscous fluid. We discuss models for both constant and varying bulk viscosity coefficient there. In Sec. 4, we formulate the field equations in the presence of a barotropic fluid for the Bianchi type-I universe model. In that section, we explore all possible choices of the BD parameter ω and the self-interacting potential $V(\phi)$. In Sec. 5, we investigate the observational limit of the gravitational constant for the constructed models. Finally, we discuss the results in the last section.

2. BRANS–DICKE FIELD EQUATIONS

A scalar–tensor theory known as Brans–Dicke theory of gravity [15] is based on the pioneering work of Jordan. A modified version of this theory is the generalized BD theory in which the BD parameter is no longer a constant but is a function of the scalar field. The action for generalized BD theory with a self-interacting potential in the Jordan frame [20, 21] is given by

$$S = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}\phi^{,\alpha}\phi_{,\alpha} - V(\phi) + L_m\right], \quad \alpha = 0,1,2,3, \tag{1}$$

where the BD parameter $\omega(\phi)$ is a modified form of the original BD parameter ω, $V(\phi)$ denotes the self-interacting potential, and L_m is the matter part of the Lagrangian. Here, we set

$$8\pi G_0 = c = 1.$$

Varying this action with respect to the metric tensor $g_{\mu\nu}$ and the scalar field, we obtain the BD field equations [14]

$$G_{\mu\nu} = \frac{\omega(\phi)}{\phi^2}\left[\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}\right] + \frac{1}{\phi}\left[\phi_{,\mu;\nu} - g_{\mu\nu}\Box\phi\right] - \frac{V(\phi)}{2\phi}g_{\mu\nu} + \frac{T_{\mu\nu}}{\phi}, \tag{2}$$

$$\Box\phi = \frac{T}{3+2\omega(\phi)} - \frac{1}{3+2\omega(\phi)}\left[2V(\phi) - \phi\frac{dV(\phi)}{d\phi}\right] - \frac{\frac{d\omega(\phi)}{d\phi}}{3+2\omega(\phi)}\phi_{,\mu}\phi^{,\mu}, \tag{3}$$

where

$$T = g^{\mu\nu}T_{\mu\nu}$$

denotes trace of the energy–momentum tensor and $\Box$ is the d'Alembertian operator. Equation (3) is called the wave equation for the scalar field. We note that the BD theory reduces to GR if $\omega \to \infty$ and the scalar field becomes a constant [33]. However, this is not true in general. It has been pointed out in [30, 34] that the BD theory does not always pass into GR in the limit $\omega \to \infty$ in the case of exact solutions. In this limit, GR can be recovered only if the trace of the energy–momentum tensor $T^{(m)}$ describing all fields other than the BD scalar field does not vanish, i. e., $T^{(m)} \neq 0$ [34–37]. For $T^{(m)} = 0$, the BD solutions do not correspond to respective GR solutions. The Palatini metric $f(R)$ gravity and the metric $f(R)$ gravity are respectively obtained by substituting $\omega = -3/2$ and $\omega = 0$ [38].

3. COSMIC ACCELERATION AND THE FRW MODEL

In this section, we investigate cosmic acceleration by exploring the BD parameter. For this, we consider the FRW model with a viscous fluid. In particular, we discuss two cases according to whether the bulk viscosity is constant or variable. The line element for the FRW model is given by

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right], \tag{4}$$

where $a(t)$ is a scale factor and $k = -1, 0, 1$ respectively indicates an open, flat, and closed universe model. We assume that the universe is filled with a viscous fluid whose energy–momentum tensor is

$$T_{\mu\nu} = (\rho + P_{eff})u_\mu u_\nu - P_{eff} g_{\mu\nu}, \tag{5}$$

where ρ is the energy density, u^μ is the four-vector velocity satisfying the relation $u_\mu u^\nu = 1$, and P_{eff} is the effective pressure defined by

$$P_{eff} = P_I + P_{vis}.$$

Here, P_I denotes the isotropic pressure and P_{vis} is the pressure due to viscosity. The bulk viscous pressure is defined by Eckart's expression in terms of the fluid expansion scalar and is given by

$$P_{vis} = -\xi u^\mu_{;\mu}$$

(see [39]), where $\xi = \xi(t, \rho)$ is the bulk viscosity coefficient. For the FRW model, the viscous pressure is found to be

$$P_{vis} = -\frac{3\xi\dot{a}}{a}$$

and hence the effective pressure becomes

$$P_{eff} = P_I - 3\xi H, \tag{6}$$

where $H = \dot{a}/a$ denotes the Hubble parameter. The corresponding field equations (2) turn out to be

$$\frac{\dot{a}^2 + k}{a^2} + \frac{\dot{a}\dot{\phi}}{a\phi} - \frac{\omega}{6}\frac{\dot{\phi}^2}{\phi^2} = \frac{\rho}{3\phi}, \tag{7}$$

$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{2\dot{a}\dot{\phi}}{a\phi} + \frac{\ddot{\phi}}{\phi} = \frac{-P_{eff}}{\phi}, \tag{8}$$

where the dot denotes the derivative with respect to time. The corresponding wave equation becomes

$$\ddot{\phi} + 3H\dot{\phi} = \frac{\rho - 3P_{eff}}{2\omega + 3} - \frac{\dot{\omega}\dot{\phi}}{2\omega + 3}, \tag{9}$$

where we have set

$$V(\phi) = 0.$$

The equation of state provides a relation between isotropic pressure and energy density and is given by

$$P = \gamma\rho, \tag{10}$$

where γ is the equation of state parameter. The values

$$\gamma = -1, 0, 1/3, 1$$

respectively represent a vacuum-dominated, dust, radiation-dominated era, and a massless scalar field. The continuity equation for viscous fluid (5) can be written as

$$\dot{\rho} + 3H(\rho + P_{eff}) = 0. \tag{11}$$

We can assume the standard expression $\xi = \xi_0\rho^n$ for the bulk viscosity, where n is a nonnegative constant and $\xi_0 > 0$. Different possible values of n are available in literature [40–43], among which two choices $n = 1$ and $n = 3/2$ respectively correspond to the radiative and string-dominated fluids. However, more realistic models can be obtained for $0 \le n \le 1/2$. Here, we evaluate ρ by solving continuity equation (11) in the following two cases.

(i) Constant bulk viscosity, i. e., $\xi = \xi_0$ (for $n = 0$).

(ii) Variable bulk viscosity, i. e., $\xi = \xi(t, \rho)$ with $n = 1/2, 1$.

In both cases, we choose $k = 0$, i. e., a flat FRW model.

3.1. Constant bulk viscosity coefficient

Energy conservation equation (11), in terms of a constant bulk viscosity, can be written as

$$\dot{\rho} + 3H(1 + \gamma)\rho = 9\xi_0 H^2, \tag{12}$$

where we use the equation of state given by Eq. (10). We assume that the scale factor $a(t)$ has the form of an expanding solution (power-law form)

$$a(t) = a_0 t^\alpha, \quad \alpha > 0, \tag{13}$$

where a_0 is the present value of the scale factor. The deceleration parameter is given by

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right). \tag{14}$$

We note that the deceleration parameter q suggests $\alpha > 1$ for cosmic acceleration. Equation (12) leads to

$$\rho(t) = \frac{9\xi_0\alpha^2}{-1+3\alpha(1+\gamma)}\, t^{-1} + \rho_0 a_0^{-3(1+\gamma)} t^{-3\alpha(1+\gamma)}. \tag{15}$$

The scalar field can be found from Eq. (9) by setting

$$\omega(t) = \omega_0 \text{ (constant)}$$

as follows:

$$\phi(t) = \frac{(1 - 3\gamma)\rho_0 a_0^{-3(1+\gamma)} t^{2-3\alpha(1+\gamma)}}{(3 + 2\omega_0)(1 - 3\alpha\gamma)[2 - 3\alpha(1 + \gamma)]} + \\ + \frac{3\xi_0(1 - 4\alpha)t}{(3 + 2\omega_0)[1 - 3\alpha(1 + \gamma)]}.$$

This equation suggests that the scalar field can be taken in a power-law form when the scale factor is given in the expanding form.

We next discuss the time-dependent BD parameter ω, which satisfies the field equations as well as the wave equation. For this, we assume a simple power-law form for the scalar field,

$$\phi(t) = \phi_0 t^{\beta}, \tag{16}$$

where ϕ_0 is the present value of the scalar field and β is any nonzero constant. Field equation (7) can be rearranged in the form

$$\frac{\dot{a}}{a} = -\frac{\dot{\phi}}{2\phi} \pm \sqrt{\frac{\Omega(t)\dot{\phi}^2}{12\phi^2} + \frac{\rho}{3\phi}}, \tag{17}$$

where

$$\Omega(t) = 2\omega(\phi(t)) + 3.$$

Using Eqs. (13) and (16) in Eq. (17), we obtain

$$\rho(t) = 3\phi_0 t^{\beta}\left[\frac{(2\alpha+\beta)^2}{4t^2} - \frac{\Omega(t)\beta^2}{12t^2}\right]. \tag{18}$$

The comparison of Eqs. (15) and (18) yields

$$\Omega(t) = \frac{3}{\beta^2}(2\alpha+\beta)^2 - \frac{36\xi_0\alpha^2}{\phi_0\beta^2[3\alpha(1+\gamma)-1]}t^{(1-\beta)} - \frac{4\rho_0 a_0^{-3(1+\gamma)}t^{-3\alpha(1+\gamma)-\beta+2}}{\phi_0\beta^2}. \tag{19}$$

The corresponding expression for $\omega(t)$ becomes

$$\omega(\phi(t)) \approx \omega(t) = \frac{-18\xi_0\alpha^2 t^{(1-\beta)}}{\phi_0\beta^2[3\alpha(1+\gamma)-1]} - \frac{2\rho_0 a_0^{-3(1+\gamma)}t^{2-\beta-3\alpha(1+\gamma)}}{\phi_0\beta^2}. \tag{20}$$

Here, we consider time-dependent terms only.

To verify the consistency of these solutions with the wave equation, we substitute these values in (9). This leads to the two consistency relations

$$\beta[4(\beta-1+3\alpha) + \beta(1-3\gamma) + 2[2-3\alpha(1+\gamma)-\beta]] = 0, \tag{21}$$

$$4\alpha(\beta-1+3\alpha) + \alpha(1-3\gamma)\beta + [3\alpha(1+\gamma)-1]\beta + 2\alpha(1-\beta) = 0. \tag{22}$$

Equation (21) implies that either $\beta = 0$ or $\beta = -2\alpha$, while Eq. (22) is satisfied for either $\beta = -2\alpha$ or $\alpha = 1/6$. For cosmic acceleration, $\alpha = 1/6$ is not an interesting value and we therefore ignore it. When $\beta = 0$, the BD parameter yields

$$\omega(t) \to -\infty$$

and the scalar field becomes a constant, $\phi = \phi_0$. This leads to GR, and hence it is not the interesting case. For $\beta = -2\alpha$, ω takes the form

$$\omega(t) = -\frac{9\xi_0 t^{(1+2\alpha)}}{2\phi_0[3\alpha(1+\gamma)-1]} - \frac{\rho_0 a_0^{-3(1+\gamma)}t^{2-\alpha(1+3\gamma)}}{2\phi_0\alpha^2}. \tag{23}$$

The power-law expression for the scalar field turns out to be

$$\phi(t) = \phi_0 t^{-2\alpha}.$$

In what follows, we evaluate the BD parameter at different epochs of the universe.

In the vacuum-dominated era ($\gamma = -1$), the BD parameter is

$$\omega(t) = \frac{9\xi_0}{2\phi_0}t^{(1+2\alpha)} - \frac{\rho_0}{2\phi_0\alpha^2}t^{2(1+\alpha)}. \tag{24}$$

In the radiation-dominated era ($\gamma = 1/3$), the BD parameter becomes

$$\omega(t) = \frac{9\xi_0}{2\phi_0(1-4\alpha)}t^{(1+2\alpha)} - \frac{\rho_0 a_0^{-4}t^{2(1-\alpha)}}{2\phi_0\alpha^2}. \tag{25}$$

In the matter-dominated era or the dust case ($\gamma = 0$), the BD parameter takes the form

$$\omega(t) = \frac{9\xi_0}{2\phi_0(1-3\alpha)}t^{(1+2\alpha)} - \frac{\rho_0 a_0^{-3}t^{(2-\alpha)}}{2\phi_0\alpha^2}. \tag{26}$$

In the massless scalar field era ($\gamma = 1$), the BD parameter turns out to be

$$\omega(t) = \frac{9\xi_0}{2\phi_0(1-6\alpha)}t^{(1+2\alpha)} - \frac{\rho_0 a_0^{-6}t^{(2-4\alpha)}}{2\phi_0\alpha^2}. \tag{27}$$

Finally, for the present time, $t = t_0$, the BD parameter can be calculated from the dust case, i. e., matter with negligible pressure. Equation (26) leads to the present value of the BD parameter ω_0 given by

$$\omega_0 = -\frac{9\xi_0}{2(3\alpha-1)} - \frac{1}{2\alpha^2}. \tag{28}$$

Here, we normalize the constants as

$$\phi_0 = a_0 = t_0 = 1, \quad \rho_0 = 0, \quad \alpha \geq 1,$$

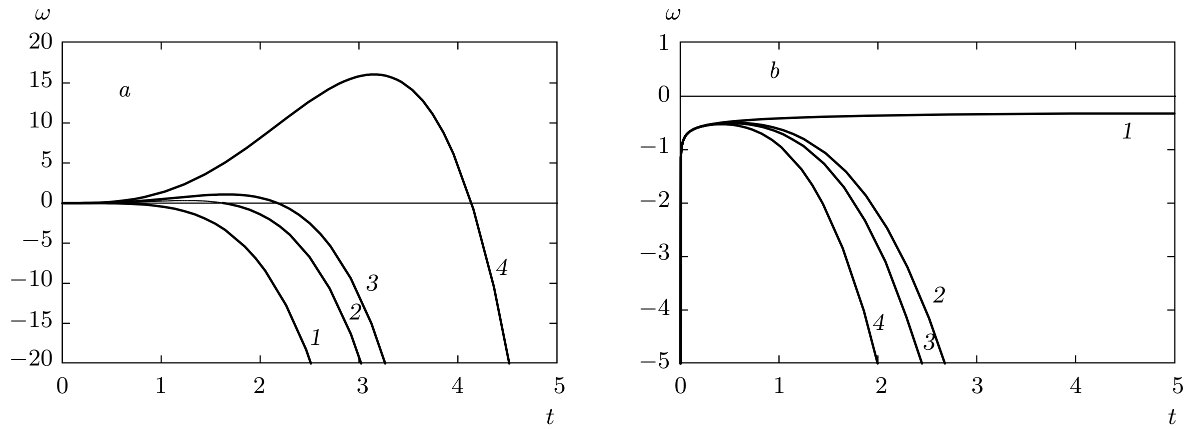

Fig. 1. ω versus t for $\gamma = -1$ (a) and $1/3$ (b) with $\alpha = 1.1$ and $\xi_0 = 0.0001$ (*1*), 0.15 (*2*), 0.2 (*3*), 0.38 (*4*)

which is consistent with Eq. (17). The minimum value of ω_0 is

$$\omega_0 = -\frac{9\xi_0}{4} - \frac{1}{2}.$$

Clearly, the minimum value of ω_0 depends on the value of the constant bulk viscosity coefficient ξ_0. In the BD theory, the gravitational coupling constant and the scalar field density should be positive in the present universe, which can be achieved for $\omega > -3/2$ [12]. In our case, the bulk viscosity coefficient must be $\xi_0 < 4/9$ with $\alpha > 1$ for consistency. The present observational range for the deceleration parameter is

$$-1 < q_0 < 0$$

(see [1, 2]), which restricts $\alpha > 1$. A more general form of the model for the present universe can be obtained by taking $\alpha = 1+\epsilon$, $\epsilon > 0$ (for small values of ϵ), which gives

$$\omega(t) = -\frac{9\xi_0 t^{(3+2\epsilon)}}{2(2+3\epsilon)} - \frac{t^{(1-\epsilon)}}{2(1+\epsilon)^2},$$

$$\phi = \phi_0 t^{-2(1+\epsilon)}, \quad a(t) = a_0 t^{(1+\epsilon)}.$$

We now discuss the BD parameter for the vacuum and matter-dominated eras. In the vacuum-dominated era, the graphs indicate that $\omega(t)$ is a decreasing function starting from zero for $0 < \xi_0 \leq 0.11$. For $\xi_0 > 0.11$, the graphs represent functions that first increase and then, after some particular points, become decreasing again, as is shown in Fig. 1. Therefore, for this range of the constant viscosity ξ_0 with $\alpha > 1$, it is possible to achieve the cosmic acceleration with positive values of $\omega(t)$. In all other eras of the universe, $\omega(t)$ is a decreasing function of time with smaller negative values. For $0 < \alpha < 1$, in the radiation-dominated era, $\omega(t)$ is a decreasing function and the universe undergoes a decelerated expansion. Hence the role of ξ_0 is to control the time dependence of $\omega(t)$. In the radiation and matter-dominated eras, and the massless scalar field era, the BD parameter approaches $-\infty$ for $\alpha = 1/6, 1/3$, and $1/4$. For cosmic acceleration, we must have $\alpha > 1$, and hence these values are not interesting.

3.2. Variable bulk viscosity coefficient

For simplicity, we set

$$n = 1/2, \quad \text{i.e.,} \quad \xi(t,\rho) = \xi_0 \rho^{1/2}(t).$$

Using this value of the bulk viscosity coefficient along with Eq. (13) in (11) yields

$$\dot{\rho(t)} + \left[\frac{3\alpha}{t}(1+\gamma) - \frac{9\xi_0\alpha^2}{t^2}\right]\rho(t)^{1/2} = 0.$$

This has the solution

$$\rho(t) = \left[\frac{9\xi_0\alpha^2}{t[3\alpha(1+\gamma)-2]} + \rho_0 t^{-3\alpha(1+\gamma)/2}\right]^2, \qquad (29)$$

where ρ_0 is an integration constant. Comparing this equation with Eq. (18), we obtain

$$\Omega(t) = 3\left(\frac{2\alpha+\beta}{\beta}\right)^2 - \frac{4t^2}{\phi_0\beta^2 t^\beta} \times \left[\frac{9\xi_0\alpha^2}{t[3\alpha(1+\gamma)-2]} + \rho_0 t^{-3\alpha(1+\gamma)/2}\right]^2.$$

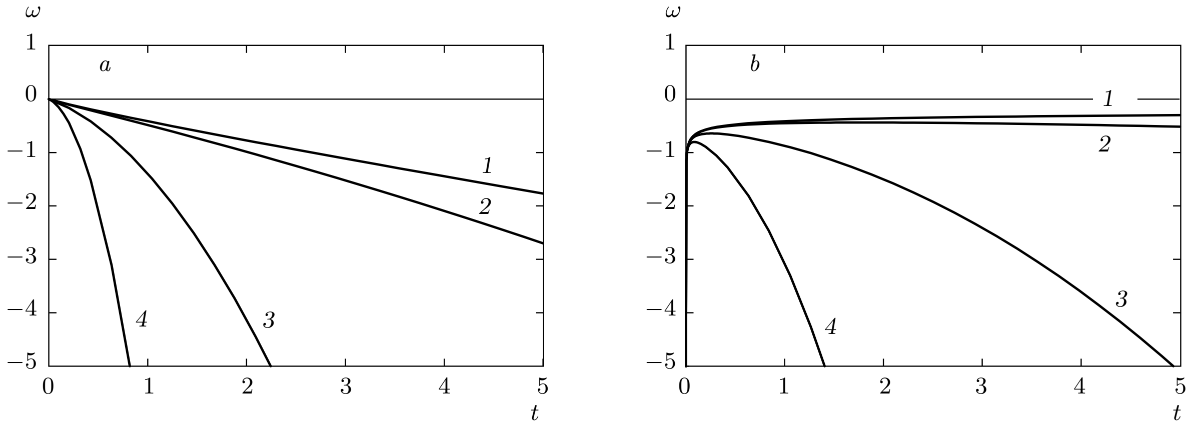

Fig. 2. ω versus t for $\gamma = 0$ (a) and $1/3$ (b) with $\alpha = 1.1$ and $\xi_0 = 0.0001$ (*1*), 0.01 (*2*), 0.1 (*3*), 0.38 (*4*)

The corresponding BD parameter becomes

$$\omega(t) = -\frac{2t^2}{\phi_0\beta^2 t^\beta} \times \left[\frac{9\xi_0\alpha^2}{t[3\alpha(1+\gamma)-2]} + \rho_0 t^{-3\alpha(1+\gamma)/2}\right]^2. \quad (30)$$

To verify the consistency of this solution with the wave equation, we substitute all these values in wave equation (9), which leads to

$$\alpha = 1/3, \quad \beta = -2\alpha, \quad \gamma = 1.$$

For $\beta = -2\alpha$, we obtain

$$\omega(t) = -\frac{t^{2(1+\alpha)}}{2\phi_0\alpha^2} \times \left[\frac{9\xi_0\alpha^2}{t(3\alpha(1+\gamma)-2)} + \rho_0 t^{-3\alpha(1+\gamma)/2}\right]^2. \quad (31)$$

The choice $\alpha = 1/3$ is not feasible for obtaining cosmic acceleration, while $\gamma = 1$ corresponds to a massless scalar field, which is discussed below. We now evaluate the BD parameter in the different eras.

In the vacuum-dominated era, the BD parameter is

$$\omega(t) = -\frac{t^{2(1+\alpha)}}{2\phi_0\alpha^2}\left[\frac{-9\xi_0\alpha^2}{2t} + \rho_0\right]^2. \quad (32)$$

In the radiation-dominated era, the BD parameter turns out to be

$$\omega(t) = -\frac{t^{2(1+\alpha)}}{2\phi_0\alpha^2}\left[\frac{9\xi_0\alpha^2}{t(4\alpha-2)} + \rho_0 t^{-2\alpha}\right]^2. \quad (33)$$

In the matter-dominated era the BD parameter is

$$\omega(t) = -\frac{t^{2(1+\alpha)}}{2\phi_0\alpha^2}\left[\frac{9\xi_0\alpha^2}{t(3\alpha-2)} + \rho_0 t^{-3\alpha/2}\right]^2. \quad (34)$$

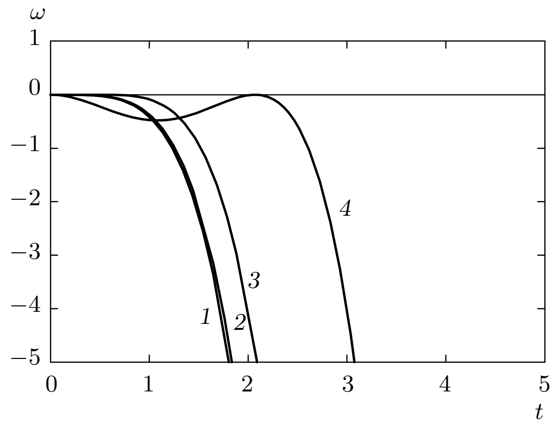

Fig. 3. ω versus t for $\alpha = 1.1$ and $\gamma = -1$ with $\xi_0 = 0.0001$ (*1*), 0.01 (*2*), 0.1 (*3*), 0.38 (*4*)

In the massless scalar field era, the BD parameter is given by

$$\omega(t) = -\frac{t^{2(1+\alpha)}}{2\phi_0\alpha^2}\left[\frac{9\xi_0\alpha^2}{t(6\alpha-2)} + \rho_0 t^{-3\alpha}\right]^2. \quad (35)$$

The expressions for $\omega(t)$ correspond to a function decreasing as $-t^2$ for increasing values of the viscosity coefficient ξ_0 and $-1 \le \gamma \le 1$ except for the vacuum-dominated era. This gives rise to accelerated expansion of the universe for $\alpha > 1$ as shown in Figs. 2 and 3. For $\alpha = 1/2, 3/2$, and $1/3$, the BD parameter approaches $-\infty$. In the matter-dominated era, $\alpha = 3/2$ lies in the range $\alpha > 1$ allowed for the accelerated expansion of the universe.

We now discuss the radiative fluid case ($n = 1$). Here, we take

$$\xi(t,\rho) = \xi_0\rho(t).$$

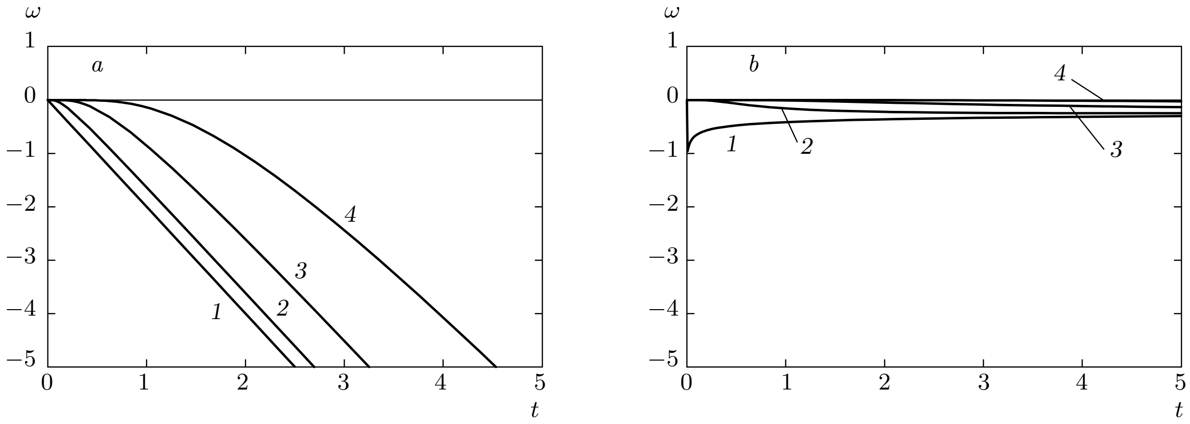

Fig. 4. ω versus t for $\alpha = 1.1$ (a) and 0.5 (b) with $\xi_0 = 0.0001$ (*1*), 0.09 (*2*), 0.38 (*3*), 1.2 (*4*)

Consequently, the continuity equation yields

$$\rho(t) = \rho_0 t^{-3\alpha(1+\gamma)} \exp\left(\frac{-9\xi_0\alpha^2}{t}\right). \qquad (36)$$

The BD parameter $\omega(t)$ turns out to be

$$\omega(t) = -\frac{2\rho_0 \exp\left(-9\xi_0\alpha^2/t\right)}{\phi_0\beta^2}\, t^{2-3\alpha(1+\gamma)-\beta}.$$

Here, $\beta = 0$ and $\beta = -2\alpha$ are the corresponding consistency relations. Choosing $\beta = 0$ provides no interesting insights, while $\beta = -2\alpha$ leads to the expression

$$\omega(t) = -\frac{\rho_0 \exp\left(-9\xi_0\alpha^2/t\right)}{2\phi_0\alpha^2}\, t^{2-\alpha(1+3\gamma)}. \qquad (37)$$

For the radiation-dominated era, the BD parameter takes the form

$$\omega(t) = -\frac{\rho_0 \exp\left(-9\xi_0\alpha^2/t\right)}{2\phi_0\alpha^2}\, t^{2(1-\alpha)}. \qquad (38)$$

We see that the coefficient of viscosity appears only in the exponential function. In the radiation-dominated era, for small values of ξ_0 and $\alpha > 1$, we have

$$\exp(-9\xi_0\alpha/t) \to 1,$$

providing small negative values of $\omega(t)$ as shown in Fig. 4. If $\xi_0 \to \infty$ with $\alpha > 1$, then

$$\exp(-9\xi_0\alpha/t) \to 0,$$

which implies $\omega(t) \to 0$. Therefore, this model may correspond to that of the metric $f(R)$ gravity. However, it is not physically possible. Also in this case, for $0 < \alpha < 1$, the values of $\omega(t)$ are constrained within the range $-3/2 < \omega(\phi) < 0$, which shows that the universe undergoes a transition to the decelerated phase.

4. COSMIC ACCELERATION WITH A BAROTROPIC FLUID AND THE BIANCHI-I UNIVERSE MODEL

Here, we investigate expansion of the universe by using the LRS Bianchi type-I model in the barotropic fluid background. The line element of the Bianchi type-I universe model is described by [44]

$$ds^2 = dt^2 - A^2(t)\,dx^2 - B^2(t)(dy^2 + dz^2), \qquad (39)$$

where A and B are scale factors. This model has one transverse direction x and two equivalent longitudinal directions y and z. We assume that matter contents of the universe are described by the perfect fluid with the energy–momentum tensor

$$T_{\mu\nu} = (\rho + P)u_\mu u_\nu - P g_{\mu\nu}. \qquad (40)$$

The corresponding field equations (2) and (3) can be written as

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = \frac{\rho}{\phi} + \frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} + \frac{V(\phi)}{2\phi} - \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\frac{\dot{\phi}}{\phi}, \qquad (41)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -\frac{P}{\phi} - \frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} - 2\frac{\dot{B}}{B}\frac{\dot{\phi}}{\phi} - \frac{\ddot{\phi}}{\phi} + \frac{V(\phi)}{2\phi}, \qquad (42)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} = -\frac{P}{\phi} - \frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\ddot{\phi}}{\phi} + \frac{V(\phi)}{2\phi} - \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\frac{\dot{\phi}}{\phi}. \qquad (43)$$

The wave equation is

$$\ddot{\phi} + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{\phi} = \frac{\rho - 3P}{2\omega(\phi)+3} - \\ - \frac{2V(\phi) - \phi dV(\phi)/d\phi}{2\omega(\phi)+3} - \frac{\dot{\phi}^2 d\omega(\phi)/d\phi}{2\omega(\phi)+3}. \quad (44)$$

For this model, the average scale factor and the mean Hubble parameter are

$$a^3(t) = AB^2, \quad H(t) = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right).$$

The energy conservation equation for the energy–momentum tensor in Eq. (40) is

$$\dot{\rho} + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)(\rho + P) = 0. \quad (45)$$

We assume that the universe is filled with a barotropic fluid. The barotropic equation of state [14] is given by

$$P = \gamma\rho, \quad -1 \le \gamma \le 1.$$

The expansion scalar for Bianchi type-I model is given by

$$\theta = u^a_{;a} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}$$

while the shear scalar is

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right).$$

It is known [45] that for a spatially homogeneous metric, the normal congruence to homogeneous expansion yields a constant ratio σ/θ, i.e., the expansion scalar θ is proportional to the shear scalar σ. This physical condition leads to the relation

$$A = B^m \quad (46)$$

between the scale factors, where $m \neq 1$ is any positive constant (for $m = 1$, it reduces to the flat FRW model). In the literature [44–49], this condition has been widely used to find exact cosmological models. Using this assumption in Eq. (45), we obtain

$$\dot{\rho} + (1+\gamma)(m+2)\frac{\dot{B}}{B}\rho(t) = 0$$

whence

$$\rho(t) = \rho_0 B^{-(1+\gamma)(m+2)}. \quad (47)$$

We now discuss the various possible choices for $\omega(\phi)$ and $V(\phi)$.

4.1. Model without potential, $V(\phi) = 0$

We consider the following two cases according to whether ω is constant or $\omega = \omega(\phi)$.

4.1.1. Case (i)

We first take BD parameter to be a constant, $\omega(\phi) = \omega_0$. For the solution of the field equations, we consider the power law

$$B(t) = b_0 t^\alpha, \quad \alpha \ge 0. \quad (48)$$

Using Eqs. (46) and (48) and the mean Hubble parameter H, we can write the deceleration parameter as

$$q = -\left[1 - \frac{3}{\alpha(m+2)}\right].$$

We note that $q < 0$, $q = 0$, and $q > 0$ respectively indicate an accelerated expansion, uniform expansion, and the decelerating phase of the universe. For the accelerated expansion of the universe, we must have the following condition on α:

$$\alpha > \frac{3}{(m+2)}, \quad m \neq 1. \quad (49)$$

Substituting Eqs. (46) and (48) in (44), we express the scalar field as

$$\phi(t) = \\ = \frac{(1-3\gamma)\rho_0 b_0^{-(m+2)(1+\gamma)} t^{2-\alpha(1+\gamma)(m+2)}}{(3+2\omega_0)[1-\alpha\gamma(m+2)][2-\alpha(1+\gamma)(m+2)]}. \quad (50)$$

The BD parameter is obtained from field equations (41)–(43) as

$$\omega_0 = -\frac{1}{(1-\gamma)}\left[\frac{(m+3)\alpha(\alpha-1)}{[2-\alpha(m+2)(1+\gamma)]^2} + \right. \\ + \alpha^2\frac{(m^2+1)+2\gamma(2m+1)}{[2-\alpha(m+2)(1+\gamma)]^2} + \frac{\alpha[m+3+2\gamma(m+2)]}{2-\alpha(m+2)(1+\gamma)} + \\ \left. + \frac{1-\alpha(m+2)(1+\gamma)}{2-\alpha(m+2)(1+\gamma)}\right]. \quad (51)$$

For a massless scalar field, $\gamma = 1$, we have $\omega \to -\infty$, which leads to GR. We have seen that the BD parameter depends on the parameters α, γ, and m. These parameters are constrained using some physical conditions. The possible ranges for m are $0 < m < 1$ and $m > 1$ and γ is allowed to be in the range $-1 \le \gamma \le 1$. By taking different possible choices for these parameters, it can be seen that the BD parameter takes small negative values as well as positive values for $-1 \le \gamma < 0$, as shown in Figs. 5–7. This gives rise to cosmic acceleration for this range of γ. We note here that for ceratin ranges of α allowed for cosmic acceleration and $-1 \le \gamma < 0$, ω_0 can take larger values that would be compatible with the solar system experiment constraints.

Solving Eq. (51) for α, we obtain the quadratic equation

$$\alpha^2[(m+2)^2(1+\gamma)^2[(\gamma-1)\omega_0-2]-(m+3)- \\ -[m^2+1+2\gamma(2m+1)]+(m+2)(1+\gamma)\times \\ \times[m+3+2\gamma(m+2)]]+\alpha[(m+2)(1+\gamma)\times \\ \times[-4(\gamma-1)\omega_0+6](m+3)- \\ -2[m+3+2\gamma(m+2)]]+4[\omega_0(\gamma-1)-1]=0 \quad (52)$$

which has two roots. These values for $m = 1/2$ and $\gamma = 0$ (the present universe) are given by

$$\alpha = \frac{23/2+10\omega_0 \pm \sqrt{-15/4-6\omega_0}}{17+25/2\omega_0}. \quad (53)$$

Because $-2 \le \omega_0 \le -3/2$ is the observed range for cosmic acceleration, the choice of $\omega_0 = -5/3$ leads to following values of α:

$$\alpha_1 = 16/23, \quad \alpha_2 = 2.$$

Here, α_1 gives $q > 0$, and we hence leave it, while $\alpha_2 = 2$ yields $q < 0$, leading to accelerating expansion. Also, it yields $\phi(t) = t^{-3}$, which provides a positive coupling constant. In our case, $\phi(t)$ is decreasing more rapidly than $\phi(t) = t^{-2}$ [11] and $\phi(t) = t^{-5/2}$ [12], and it therefore corresponds to a greater rate of the accelerated expansion of the universe.

4.1.2. Case (ii)

In this case, the BD parameter is not constant, but is a function of ϕ. Using Eqs. (13), (46), (41)–(43), and (48), we can write the BD parameter as

$$\omega(\phi) = \frac{1}{\beta^2}\left[\frac{(3m-m^2-2)}{2}\alpha^2+\frac{(m+3)\alpha}{2}+ \right. \\ \left. +\frac{(m+1)\alpha\beta}{2}-\beta^2+\beta\right]- \\ -\frac{1}{\beta^2}\left[\rho_0 b_0^{-(m+2)(1+\gamma)}(1+\gamma)\phi^{(-\alpha(m+2)(1+\gamma)-\beta+2)/\beta}\times \right. \\ \left. \times\,\phi_0^{(-\alpha(m+2)(1+\gamma)-2)/\beta}\right]. \quad (54)$$

Substituting this value in Eq. (44), we obtain the consistency relation

$$\beta = -\frac{(m+2)\alpha(1+\gamma)}{2}, \quad m \ne 1. \quad (55)$$

This shows that β remains negative for all

$$0 < m < 1, \quad m > 1, \quad \alpha > \frac{3}{m+2}, \quad -1 \le \gamma \le 1.$$

The consistency of this solution with the dynamical equations (the requirement that each term in the dynamical equations have the same time dependence), results in another constraint given by

$$\beta = 2-\alpha(m+2)(1+\gamma).$$

Using this value of β in Eq. (55), it can be seen that the parameter β is restricted to -2. We now discuss the BD parameter and cosmic acceleration in different phases of the universe by using this value of β. The expressions for the BD parameter in matter and radiation-dominated eras with $\beta = -2$, $\alpha > 6/5$, and $m = 1/2$ turn out to be

$$\omega(\phi) = \frac{1}{4}\left[-\frac{3\alpha^2}{8}+\frac{\alpha}{4}-6\right]-\frac{1}{4}\phi^{-2+5\alpha/4},$$

$$\omega(\phi) = \frac{1}{4}\left[-\frac{3\alpha^2}{8}+\frac{\alpha}{4}-6\right]-\frac{1}{3}\phi^{-2+5\alpha/3}.$$

By taking different choices for these parameters, we see that for all phases of the universe, the BD parameter $\omega(\phi)$ has small negative values and lies in the range $\omega \le -3/2$, as shown in Figs. 8 and 9, which corresponds to an accelerated expansion of universe. This result is in agreement with [14] for a spatially flat model.

4.2. Model with potential $V(\phi) \ne 0$

Again, we discuss two cases depending on the value of the BD parameter ω.

4.2.1. Case (i)

First, we discuss the case of a constant BD parameter, $\omega(\phi) = \omega_0$. We then consider the power-law form of the scalar field in terms of the scale factor $B(t)$

$$\phi = \phi_0 B^\alpha, \quad \alpha > 0. \quad (56)$$

Using this value of ϕ in field equations (41)–(43) leads to

$$\frac{\ddot{B}}{B}+A\left(\frac{\dot{B}}{B}\right)^2 = -CB^{-\alpha(1+\gamma)(m+2)},$$

where

$$A = \frac{\alpha^2-3\alpha-2+2m^2+\alpha m-2m+\omega_0\alpha^2}{3\alpha+2m},$$

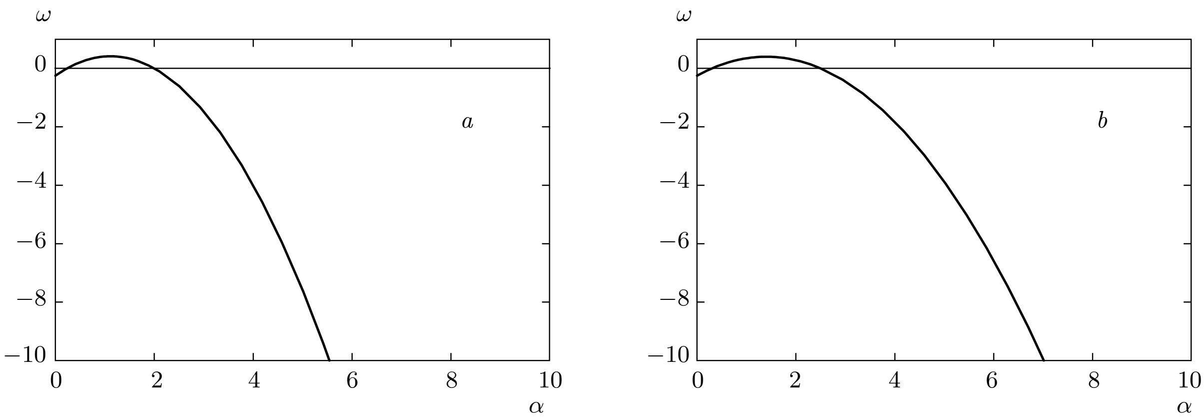

Fig. 5. ω_0 versus α for $m = 3/2$ (a), $4/5$ (b), and $\gamma = -1$. The corresponding ranges for α are $\alpha > 6/7$ and $\alpha > 15/14$

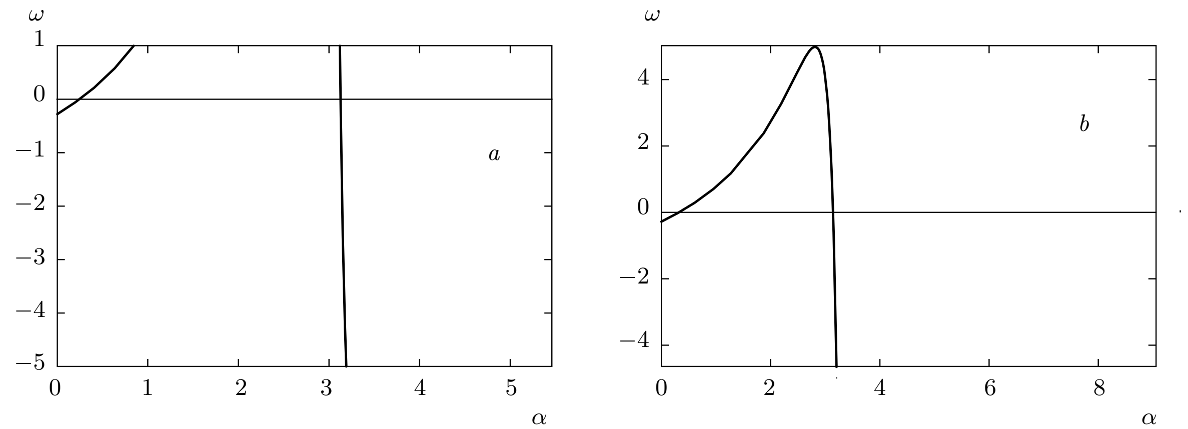

Fig. 6. ω_0 versus α for $m = 3/2$ (a), $4/5$ (b), and $\gamma = -4/5$. The corresponding ranges for α are $\alpha > 6/7$ and $\alpha > 15/14$

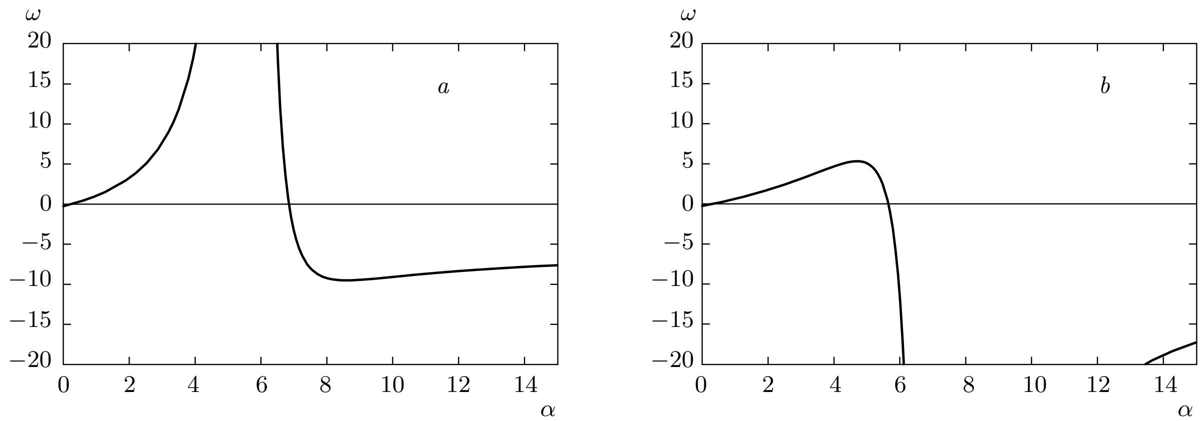

Fig. 7. ω_0 versus α for $m = 3/2$ (a), $4/5$ (b), and $\gamma = -9/10$. The corresponding ranges for α are $\alpha > 6/7$ and $\alpha > 15/14$

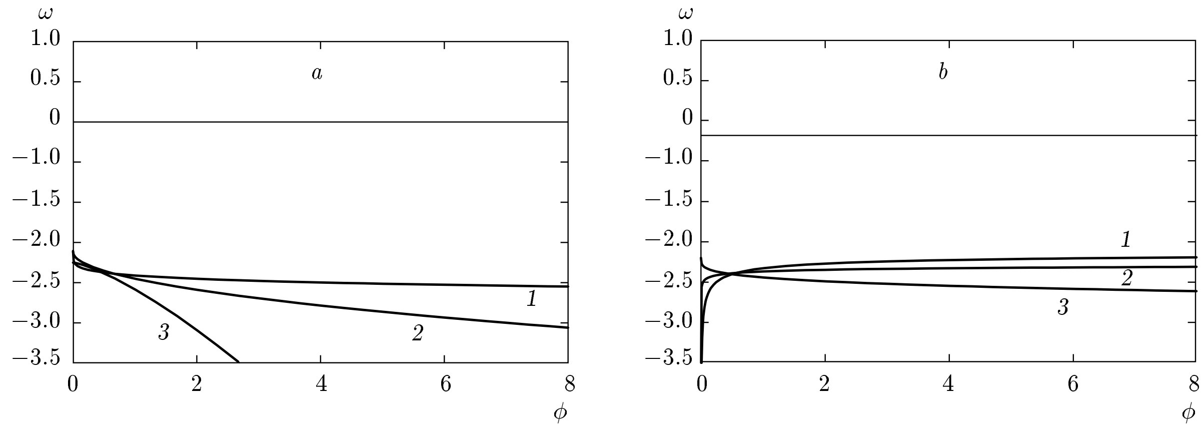

Fig. 8. ω versus ϕ for $\gamma = 1/3$ (a) and 0 (b) with $m = 1/2$ and $\alpha > 6/5$: $\alpha = 1.3$; $\alpha = 1.5$, $\alpha = 2$

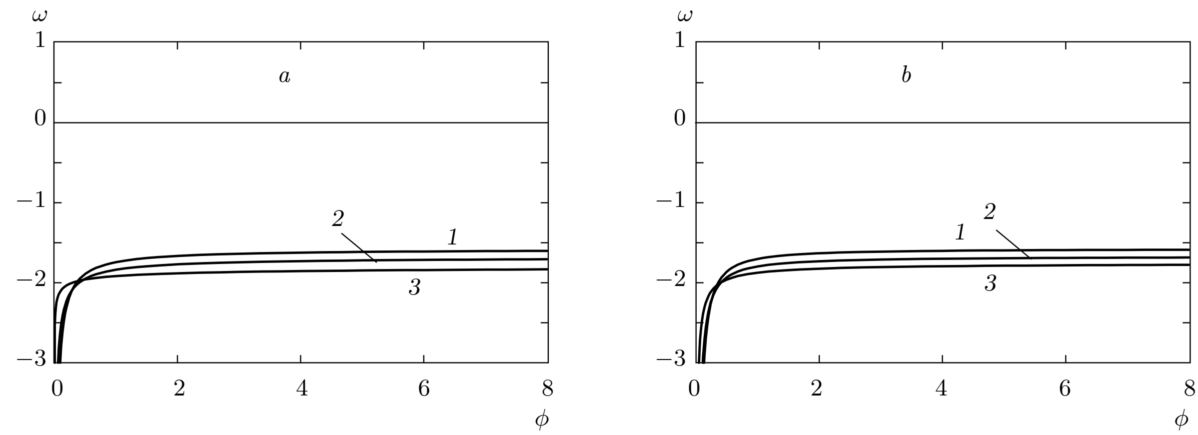

Fig. 9. ω versus ϕ for $\gamma = -1/3$ (a) and $-1/2$ (b) with $m = 1/2$ and $\alpha > 6/5$: $\alpha = 1.3$; $\alpha = 1.5$, $\alpha = 2$

$$C = \frac{(1+\gamma)\rho_0}{\phi_0(3\alpha + 2m)}.$$

The expression for $\dot{B}(t)$ can be written as

$$\dot{B}(t) = \sqrt{\frac{2(1+\gamma)\rho_0}{\phi_0}} B(t)^{(2-\alpha-(1+\gamma)(m+2))/2} \times \times [(3\alpha + 2m)[(1+\gamma)(m+2) + (\alpha - 2)] - -2\alpha^2 + 6\alpha + 4 - 4m^2 - 2m\alpha + 4m - 2\omega_0\alpha^2]^{-1/2}, \quad (57)$$

which yields

$$B(t) = A' t^{2/[\alpha + (1+\gamma)(m+2)]}, \quad (58)$$

where

$$A' = \left[\frac{\rho_0}{2\phi_0}[\alpha + (1+\gamma)(m+2)]^2 (1+\gamma) \times \times [(3\alpha + 2m)[(1+\gamma)(m+2) + (\alpha - 2)] - - 2\alpha^2 + 6\alpha + 4 - 4m^2 - 2m\alpha + 4m - - 2\omega_0\alpha^2]^{-1}\right]^{1/[\alpha + (1+\gamma)(m+2)]}. \quad (59)$$

The value of the scale factor $A(t)$ can be obtained using value of $B(t)$ in Eq. (46).

The corresponding expression for the scalar field is

$$\phi(t) = \theta_0 t^{2\alpha/[\alpha + (1+\gamma)(m+2)]}, \quad (60)$$

where

$$\theta_0 = \phi_0 A'^{\alpha}.$$

Equation (58) yields the following constraint on α:

$$3\alpha \leq -(1 + 3\gamma)(m+2). \quad (61)$$

The deceleration parameter q turns out to be

$$q = -1 + \frac{3[\alpha + (1+\gamma)(m+2)]}{2(m+2)}.$$

It can be easily seen that for all positive constant m ($m \neq 1$), $\alpha > 1$, and $-1 \leq \gamma \leq 1$, the deceleration parameter remains negative, $q < 0$. Hence, the universe is in the state of accelerated expansion. From wave equation (44), the potential can be written as

$$V(\phi) = \frac{B'}{\phi^{(1+\gamma)(m+2)/\alpha}}, \tag{62}$$

where

$$B' = \frac{-\alpha\theta_0^{2/(\alpha p)}}{(1+\gamma)(m+2)[\alpha + (1+\gamma)(m+2)]^2} \times \times [4(1+\gamma)(m+2)(\omega_0\alpha - m - 2) - 8(m+2)\alpha\omega_0 + 16m + 8m^2 + 24 - 4m\alpha - 8\alpha].$$

4.2.2. Case (ii)

We take the BD parameter as a function of the scalar field ϕ, i.e, $\omega(\phi)$. We consider the power-law forms of the scalar field and the scale factor given by (48) and (13). Using field equations (41)–(43), we express the scalar potential as

$$V(\phi) = \phi_0^{2/\beta}\phi^{(\beta-2)/\beta}\left[\frac{(m^2+5m+6)\alpha^2}{2} - \frac{(m+3)\alpha}{2} + \beta^2 - \beta + \frac{3m+7}{2}\alpha\beta\right] - (1-\gamma) \times \rho_0 b_0^{-(m+2)(1+\gamma)}\phi^{-\alpha(m+2)(1+\gamma)/\beta} \times \phi_0^{\alpha(m+2)(1+\gamma)/\beta}. \tag{63}$$

The BD parameter turns out to be the same as in (54). Substituting these values in Eq. (44), we obtain the consistency relations

$$\beta = 0, \quad \beta = -2 \quad \beta = -\frac{\alpha}{2}(m+3), \quad \beta = 1 - \alpha(m+2). \tag{64}$$

The consistency of this solution with the dynamical equation implies that

$$\beta = 2 - \alpha(m+2)(1+\gamma).$$

We now discuss the behavior of the self-interaction potential $V(\phi)$ for these values of β in different eras of the universe. The choice $\beta = 0$ is not feasible, and we therefore neglect it. For $\beta = -2$, the self-interaction potential can be written as

$$V(\phi) = \phi^2\left[\left(\frac{m^2+5m+6}{2}\right)\alpha^2 - \left(\frac{m+3}{2}\right)\alpha + 6 - (3m+7)\alpha\right] - (1-\gamma)\phi^{\alpha(m+2)(1+\gamma)/2},$$

where $m \neq 1$ is a positive constant and

$$\alpha > \frac{3}{m+2}.$$

For

$$\beta = -9\alpha/4, \quad m = 3/2, \quad \alpha > 6/7,$$

we obtain

$$V(\phi) = \phi^{1+8/9\alpha}(-3\alpha^2) - (1-\gamma)\phi^{14(1+\gamma)/9}.$$

For

$$m = 2, \quad \alpha > 3/4, \quad \beta = -5\alpha/2,$$

the potential turns out to be

$$V(\phi) = -(1-\gamma)\phi^{8(1+\gamma)/5},$$

where $-1 \leq \gamma \leq 1$. The expression for the self-interaction potential for the radiation-dominated era with

$$\beta = 2(1 - 5\alpha/3), \quad \alpha > 6/5, \quad m = 1/2$$

is given by

$$V(\phi) = \left(\frac{95\alpha^2}{72} - \frac{13\alpha}{4} + \frac{4}{3}\right)\phi^{(-5\alpha/3)/(1-5\alpha/3)}.$$

The self interaction potential for the matter-dominated era with

$$\beta = 2 - 5\alpha/2, \quad \alpha > 6/5, \quad m = 1/2$$

takes the form

$$V(\phi) = \left(1 - \frac{3\alpha}{4}\right)\phi^{-5\alpha/2(2-5\alpha/2)}.$$

For the first three consistency relations for β in Eq. (64), we see that $V(\phi)$ is a decreasing function starting from zero with the increasing values for ϕ except in the case $\beta = -2$. In that case, only $\gamma = -1$ and $\gamma = 1$ with $\alpha \geq 1.3$ provide a positive potential energy because for these ranges, they are increasing functions of ϕ as shown in Figs. 10, 11, 13. Figure 12*a* shows that $V(\phi)$ attains negative values starting from zero, but at larger values of α, it is an increasing function with positive values. Figure 12*b* shows that $V(\phi)$ attains positive increasing values for $\alpha > 6/5$. Therefore, we conclude that these cases provide positive potential energy because they result in increasing functions of ϕ for particular values of α.

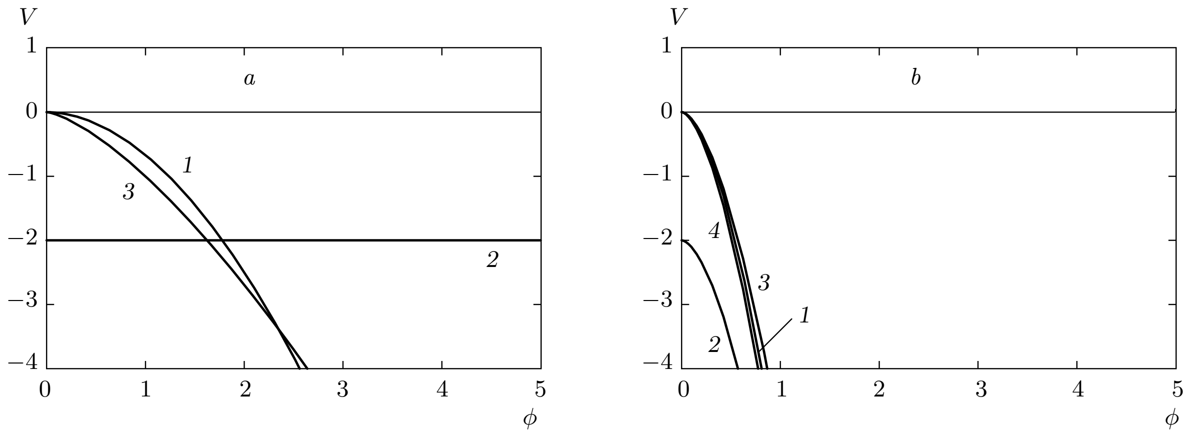

Fig. 10. a — V versus ϕ for $m = 1/2$, $\alpha = 1.3$, $\beta = -7\alpha/4$, and $\gamma = 1/3$ (*1*), -1 (*2*), 0 (*3*). b — V versus ϕ for $m = 3/2$, $\alpha = 1.3$, $\beta = -9\alpha/4$, and $\gamma = 1/3$ (*1*), -1 (*2*), 0 (*3*), 1 (*4*)

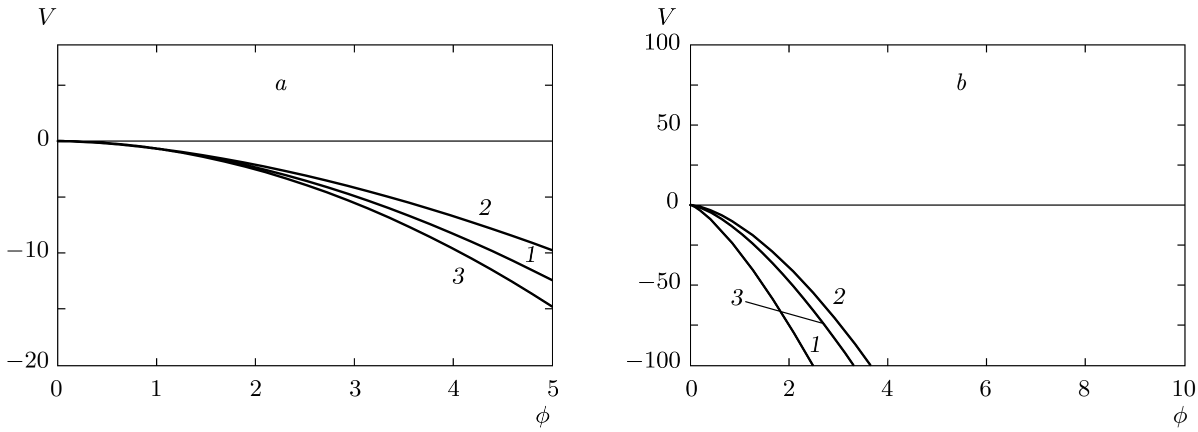

Fig. 11. a — V versus ϕ for $m = 1/2$, $\gamma = 1/3$, $\beta = 1 - 5\alpha/2$, and $\alpha = 1.5$ (*1*), 2 (*2*), 1.3 (*3*). b — V versus ϕ for $m = 3/2$, $\gamma = 0$, $\beta = 1 - 7\alpha/2$, and $\alpha = 2$ (*1*), 1.3 (*2*), 1.5 (*3*)

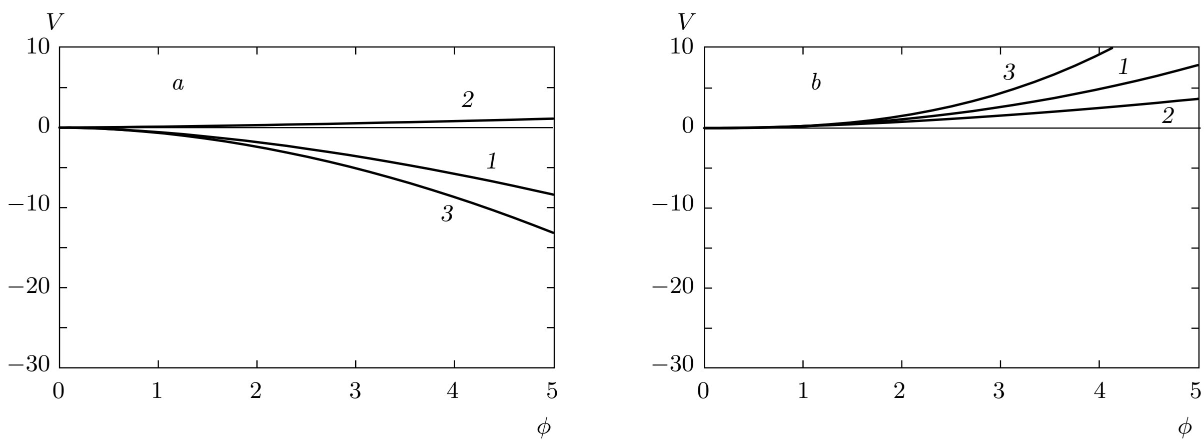

Fig. 12. a — V versus ϕ for $m = 1/2$, $\gamma = 1/3$, $\beta = 2(1 - 5\alpha/3)$, and $\alpha = 1.5$ (*1*), 2 (*2*), 1.3 (*3*). b — V versus ϕ for $m = 1/2$, $\gamma = 0$, $\beta = 2 - 5\alpha/2$, and $\alpha = 1.5$ (*1*), 2 (*2*), 1.3 (*3*)

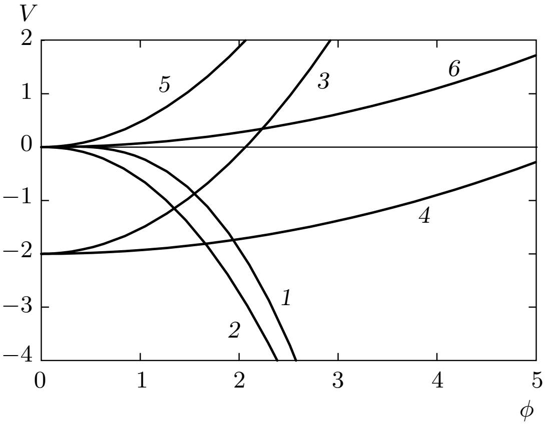

Fig. 13. V versus ϕ for $m = 1/2$, $\beta = -2$, and $\alpha = 1.5$, $\gamma = 1/3$ (*1*); $\alpha = 1.3$, $\gamma = 1/3$ (*2*); $\alpha = 1.5$, $\gamma = -1$ (*3*); $\alpha = 1.3$, $\gamma = -1$ (*4*); $\alpha = 1.5$, $\gamma = 1$ (*5*); $\alpha = 1.3$, $\gamma = 1$ (*6*)

5. VARIATION FOR THE NEWTON'S GRAVITATIONAL CONSTANT IN THE GENERALIZED BD THEORY

A well-known fact about the BD theory of gravity is that it provides very small variations for the gravitational constant. However, the generalized BD theory suggests various possibilities for variation of G. In the generalized BD theory, the expression for G is found to be [20]

$$G(t) = \frac{4 + 2\omega(\phi)}{\phi(3 + 2\omega(\phi))}.$$

The present rate of variation of the gravitational constant is given by

$$\left(\frac{\dot G}{G}\right)_0 = -\left(\frac{\dot\phi}{\phi}\right)_0 - \frac{2(\dot{\omega_0})}{(3 + 2\omega_0)(4 + 2\omega_0)}. \qquad (65)$$

Here, the subscript indicates the present values of the corresponding parameters. Using Eq. (23),

$$\beta = -2\alpha, \quad \alpha > 1, \quad \xi_0 = 0.0001$$

and the estimated age of the universe $t_0 = 14 \pm 2$ Gyrs, we obtain the rate of variation of $(\dot G/G)_0$ to be $1.5714 \cdot 10^{-19}$ yrs. It lies clearly within the allowed range of variation of G for cosmic acceleration, that is,

$$(\dot G/G)_0 < 4 \cdot 10^{-10} \text{ yrs}$$

(see [11, 12]).

For the Bianchi type-I model, by using expression for $\omega(\phi)$ given by Eq. (54) in Eq. (65) along with values

$$\beta = -2, \quad \alpha > 6/5, \quad \gamma = 0,$$

$$t_0 = 14 \pm 2 \text{ Gyrs}, \quad m = 1/2,$$

we obtain

$$(\dot G/G)_0 = 1.4287 \cdot 10^{-10} \text{ yrs}.$$

This also safely lies within the allowed range of variation of G for cosmic acceleration. Hence, our obtained models satisfy the observational limit of G for cosmic acceleration.

6. SUMMARY AND DISCUSSION

We have investigated the possibility of obtaining cosmic acceleration by using the role of the BD parameter in the presence of viscous and barotropic fluids. For this purpose, we considered the FRW and Bianchi type-I universe models. The constructed models entirely depend on the values of the parameters α, β, γ, and m. First, we discussed the FRW model in the presence of a viscous fluid. We have seen that the total effective pressure contains a negative factor associated with bulk viscosity, which leads to negative effective pressure. Consequently, the fluid acts as a dark energy candidate and can explain many aspects of evolution of the universe. The deceleration parameter constraints the parameter α for cosmic acceleration, i. e., $\alpha > 1$.

For a constant bulk viscosity coefficient, we obtain $\beta = 0$ and $\beta = -2\alpha$. The first case leads to GR, while for the second choice, in all eras of the universe except the vacuum-dominated era, the BD parameter $\omega(t)$ is a decreasing function of time with small negative values. In the vacuum-dominated era, we see that for the viscosity greater than 0.11, it is possible to achieve cosmic acceleration with positive values of $\omega(t)$. For the variable bulk viscosity coefficient with $n = 1/2$, $\omega(t)$ corresponds to a function decreasing as $-t^2$ with small negative values for different small values of the viscosity coefficient ξ_0 and $-1 \le \gamma \le 1$. This gives rise to accelerated expansion of the universe for $\alpha > 1$. For the radiative fluid, we have found that the viscosity coefficient appears in the exponential function. In that case, $\omega(t)$ is a decreasing function with negative values for both the accelerated and decelerated phases of the universe $(-3/2 \le \omega \le 0)$.

Second, we have taken the Bianchi type-I universe model in the presence of a perfect fluid with a barotropic equation of state. Here, we considered two cases: $V(\phi) = 0$ and $V(\phi) \ne 0$. In the first case, when $\omega(\phi) = \omega_0$, by taking different possible choices for the parameters, we have seen that the BD parameter takes small negative as well as positive values for $-1 \le \gamma < 0$

and certain ranges of α. It follows that cosmic acceleration can be achieved for positive larger values of ω with different values of α. Also, for the present universe with $\omega_0 = -5/3$ (taken from the negative observed range for cosmic acceleration $-2 \leq \omega_0 \leq -3/2$), we have found

$$\phi(t) \propto t^{-3}.$$

In this case, the acceleration rate of the universe is higher than in [11],

$$\phi(t) \propto t^{-2},$$

and in [12],

$$\phi(t) \propto t^{-5/2}.$$

When $\omega = \omega(\phi)$, taking different values of the parameters, we have seen that for all phases of the universe, the BD parameter $\omega(\phi)$ takes small negative values and lies in the range $\omega \leq -3/2$, which corresponds to cosmic acceleration and agrees with the already found results [14].

For $V(\phi) \neq 0$ and $\omega(\phi) = \omega_0$, we have evaluated the values of scale factors A and B, the scalar field, and $V(\phi)$. We have found that in all phases of the universe, these values of scale factors $A(t)$ and $B(t)$ lead to $q < 0$ for all positive constant m with $m \neq 1$ and $\alpha > 1$, which corresponds to cosmic acceleration. Finally, for $V(\phi) \neq 0$ and $\omega = \omega(\phi)$, we see that $V(\phi)$ is a decreasing function starting from zero with increasing ϕ except for the choice $\beta = -2$ with particular values of γ and $\alpha \geq 1.3$. These values provide positive potential energy and result in an increasing function of ϕ. However, for the constraint

$$\beta = 2 - \alpha(m+2)(1+\gamma),$$

it is possible to have a positive potential energy for larger values of α in the matter-dominated era and for smaller values of α in the radiation-dominated era.

It is worthwhile to mention that all models discussed here satisfy the observational constraints for the variation of Newton's gravitational constant available in literature [11, 12], which supports our results. In each case, we have explained the phenomenon of cosmic acceleration for different ranges of the corresponding parameters. But these ranges of the BD parameter, except a few cases, are incompatible with solar system constraints, which require $\omega \geq 40.000$. This is the generic problem noted in the context of scalar–tensor theories. It would be of great interest to see whether this problem can be resolved using other Bianchi models.

In order to check the viability of dark energy models based on modified theories of gravity, the evolution of cosmological perturbations and the background expansion history of the universe may be studied. This can be done using the Chameleon and Vainshtein mechanisms, which suppress the propagation of the fifth force and provide consistency with local gravity experiments [50, 51]. These procedures can be used to check the viability of above-discussed models.

REFERENCES

1. S. Perlmutter, S. Gabi, G. Goldhaber, A. Goobar et al., Astrophys. J. **483**, 565 (1997); S. Perlmutter, G. Aldering, M. D. Valle, S. Deustua et al., Nature **391**, 51 (1998); S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop et al., Astrophys. J. **517**, 565 (1999).

2. A. G. Riess, A. V. Filippenko, P. Challis, A. ClocChiatti et al., Astron. J. **116**, 1009 (1998).

3. C. L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik et al., Astrophys. J. Suppl. **148**, 1 (2003).

4. M. Tegmark, M. A. Strauss, M. R. Blanton, K. Abazajian et al., Phys. Rev. D **69**, 03501 (2004).

5. S. W. Allen, R. W. Schmidt, H. Ebeling, A. C. Fabian, and L. V. Speybroeck, Mon. Not. Roy. Astron. Soc. **353**, 457 (2004).

6. E. Hawkins, S. Maddox, S. Cole, O. Lahav et al., Mon. Not. Roy. Astr. Soc. **346**, 78 (2003).

7. B. Jain and A. Taylor, Phys. Rev. Lett. **91**, 141302 (2003).

8. R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).

9. A. S. Al-Rawaf and M. O. Taha, Gen. Relativ. Gravit. **28**, 935 (1996).

10. A. H. Guth, Phys. Rev. D **23**, 347 (1981).

11. O. Bertolami and P. J. Martins, Phys. Rev. D **61**, 064007 (2000).

12. N. Banerjee and D. Pavon, Phys. Rev. D **63**, 043504 (2001).

13. B. K. Sahoo and L. P. Singh, Mod. Phys. Lett. A **18**, 2725 (2003).

14. W. Chakraborty and U. Debnath, Int. J. Theor. Phys. **48**, 232 (2009).

15. C. H. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

16. S. Weinberg, *Gravitation and Cosmology*, Wiley (1972).

17. P. A. M. Dirac, Proc. R. Soc. Lond. A **165**, 199 (1938).

18. B. Bertotti, L. Iess, and P. Tortora, Nature **425**, 374 (2003).

19. A. D. Felice, G. Mangano, P. D. Serpico, and M. Trodden, Phys. Rev. D **74**, 103005 (2006).

20. K. Nordtvedt, Jr., Astrophys. J. **161**, 1059 (1970).

21. R. V. Wagoner, Phys. Rev. D **1**, 3209 (1970).

22. T. Singh and L. N. Rai, Gen. Relativ. Gravit. B **15**, 875 (1983).

23. M. S. Bermann, Nuovo Cimento B **74**, 192 (1983).

24. S. Sen and A. A. Sen, Phys. Rev. D **63**, 124006 (2001).

25. B. K. Sahoo and L. P. Singh, Mod. Phys. Lett. A **17**, 2409 (2002).

26. S. Sen and T. R. Seshadri, Int. J. Mod. Phys. D **12**, 445 (2003).

27. D. R. K. Reddy and M. V. S. Rao, Astrophys. Space. Sci. **305**, 183 (2006).

28. J. P. Singh and P. S. Baghel, Elect. J. Theor. Phys. **6**, 85 (2009).

29. M. K. Verma, M. Zeyauddin, and S. Ram, Rom. J. Phys. **56**, 616 (2011).

30. M. Sharif and S. Waheed, Eur. Phys. J. C **72**, 1876 (2012); A. K. Yadav and B. Saha, Astrophys. Space. Sci. **337**, 759 (2012).

31. C. P. Singh, Brazil. J. Phys. **39**, 619 (2009).

32. S. Fay, Gen. Relativ. Gravit. **32**, 203 (2000).

33. S. K. Rama and S. Gosh, Phys. Lett. B **383**, 32 (1996); S. K. Rama, Phys. Lett. B **373**, 282 (1996).

34. C. Romero and A. Barros, Phys. Lett. A **173**, 243 (1993).

35. N. Banerjee and S. Sen, Phys. Rev. D **56**, 1334 (1997).

36. A. Bhadra and K. K. Nandi, Phys. Rev. D **64**, 087501 (2001).

37. C. H. Brans, Phys. Rev. **125**, 2194 (1962).

38. T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010).

39. C. Eckart, Phys. Rev. **58**, 919 (1940).

40. G. L. Murphy, Phys. Rev. D **8**, 4231 (1973).

41. S. Weinberg, Astrophys. J. **168**, 175 (1971).

42. U. A. Belinskii and I. M. Khalatnikov, Sov. Phys. JETP **42**, 205 (1976).

43. G. P. Singh, S. G. Ghosh, and A. Beesham, Aust. J. Phys. **50**, 903 (1997).

44. M. Sharif and M. Zubair, Astrophys. Space Sci. **330**, 399 (2010).

45. C. B. Collins, E. N. Glass, and D. A. Wilkinson, Gen. Relativ. Gravit. **12**, 805 (1980).

46. K. S. Throne, Astrophys. J. **148**, 51 (1967).

47. J. Kristian and R. K. Sachs, Astrophys. J. **143**, 379 (1966).

48. C. B. Colins, Phys. Lett. A **60**, 397 (1977).

49. S. R. Roy and S. K. Banerjee, Class. Quantum Grav. **11**, 1943 (1995).

50. G. Radouane, M. Bruno, F. M. David, P. David, T. Shinji, and A. W. Hans, Phys. Rev. D **82**, 124006 (2010).

51. T. Shinji, Lect. Notes Phys. **800**, 99 (2010).