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CONTROLLING CHAOS IN THE BOSE–EINSTEIN CONDENSATE

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The spatial structure of the Bose–Einstein condensate (BEC) is investigated and spatially chaotic distributions of the condensates are revealed. By means of changing the s-wave scattering length with a Feshbach resonance, the chaotic behavior can be well controlled to enter into periodicity. Numerical simulation shows that there are different periodic orbits according to different s-wave scattering lengths only if the Lyapunov exponent of the system is negative.

1. INTRODUCTION

Eighty years after its prediction, the Bose–Einstein condensate (BEC) was observed in trapped gases of rubidium, sodium, and lithium [1]. The mean field theory (the Gross–Pitaevskii (GP) equation) has been quite successful in quantitatively reproducing many experimental observations [2].

The achievement of BEC in dilute alkali vapors has opened the field of weakly interacting degenerate Bose gases. Experimental and theoretical progress has been made in studying the properties of this new state of matter. Several remarkable phenomena, which strongly resemble well-known effects in nonlinear optics, have been observed in the BEC, such as four-wave mixing, vortices, dark and bright solitons, and chaos [3–13]. In a realistic experimental setting, an external electromagnetic field is used to produce, trap, and manipulate the BEC. In early experiments, only the harmonic potential was used, but a wide variety of potentials can now be constructed experimentally. Among the most frequently studied both experimentally and theoretically are periodic lattice potentials. The optical lattice is created as a standing-wave interference pattern of mutually coherent laser beams. With each lattice site occupied by one mass of alkali atoms in its ground state, the BEC in optical lattices shows a number of potential applications, such as an atomic interferometer, detectors for quantum computers, an atom laser, and quantum information processing on the nanometer scale. Optical lattices are therefore of particular interest from the perspective of both fundamental quantum physics and applications [8].

Numerous experimental studies have confirmed the general validity of the time-dependent nonlinear Schrödinger equation, also called the GP equation, used to calculate the ground state and excitations of various BECs of trapped alkali atoms. The dynamics of the system are described by a Schrödinger equation with a nonlinear term that represents many-body interactions in the mean-field approximation. This nonlinearity allows introducing chaos into a quantum system. The existence of BEC chaos has been proved, and the chaos properties have also been extensively investigated in many previous works. Naturally, chaos, which plays a role in the regularity of the system, causes instability of the condensate wave function. The study of chaos in nonlinear deterministic systems has been underway for many years. In addition to addressing basic questions on the mechanisms and predictions of chaos, the ability to control it to a regular state is also an important subject for relevant studies.

For the purpose of applications, the control of chaos has been anticipated. Chaos control in BEC has always been a widely attractive field since the pioneering work [14]. Controlling chaos can be separated into two categories: feedback (active) control and nonfeedback (passive) control. The general method for feedback control is to push the state of a system onto a stable manifold of a target orbit, that is, to stabilize the unstable target orbits embedded within a chaotic attractor. The main purpose is to control the chaos into stable states in the BEC by means of changing the s-wave scattering length by using the Feshbach resonance. We can force the system to a stable periodic orbit.

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2. CONTROLLING CHAOS IN THE BEC

The BEC system considered here is created in a harmonically trapped potential and is then loaded into an optical lattice. For the system considered here, this is similar to the case of the linear junction linking of many BECs. Thus, a damping effect caused by similar elements or other factors may also exist. With these considerations, the system is governed by the quasi-one-dimensional GP equation [15]

$$i\hbar(1-i\lambda)\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + g_0|\psi|^2\psi + v_1\cos^2(k\xi)\psi, \quad (1)$$

where m is the atomic mass, ψ is the macroscopic quantum wave function, and

$$g_0 = \frac{4\pi\hbar^2 a}{m}$$

characterizes the interatomic interaction strength, here a is the s-wave scattering length: $a>0$ corresponds to a repulsive interaction and $a<0$ corresponds to an attractive interaction. Furthermore, the term proportional to γ represents the damping effect that was used in Ref. [15], $\xi = x + v_L t$, and v_L is the velocity of the traveling lattice.

Due to the complexity of Eq. (1), we focus our interest only on the traveling wave solution of this equation and write it in the form

$$\psi = \varphi(\xi)\exp\left[i(\alpha_1 x + \beta_1 t)\right], \quad (2)$$

where α_1 and β_1 are two undetermined real constants.

For simplicity, we use the dimensionless variables and parameters

$$\eta = k\xi, \quad v = \frac{2mv_L}{\hbar k}, \quad \beta = \frac{\hbar\beta_1}{E_r}, \quad \alpha = \frac{\alpha_1}{k}, \quad I_0 = \frac{v_1}{E_r}, \quad (3)$$

where I_0 is the optical intensity, k is the laser wave vector, and $E_r = \hbar^2k^2/2m$ is the recoil energy.

Writing the complex function φ in the form

$$\varphi = R(t)e^{i\theta(t)},$$

where R and θ are real functions, and letting $\gamma = 0$, we obtain two coupled equations

$$\frac{d^2R}{d\eta^2} - R\frac{d\theta}{d\eta} - (v+2\alpha)R\frac{d\theta}{d\eta} - (\beta+\alpha^2)R - gR^3 = I_0\cos^2(\eta)R, \quad (4)$$
$$2\frac{dR}{d\eta}\frac{d\theta}{d\eta} + R\frac{d^2\theta}{d\eta^2} + (v+2\alpha)\frac{dR}{d\eta} = 0.$$

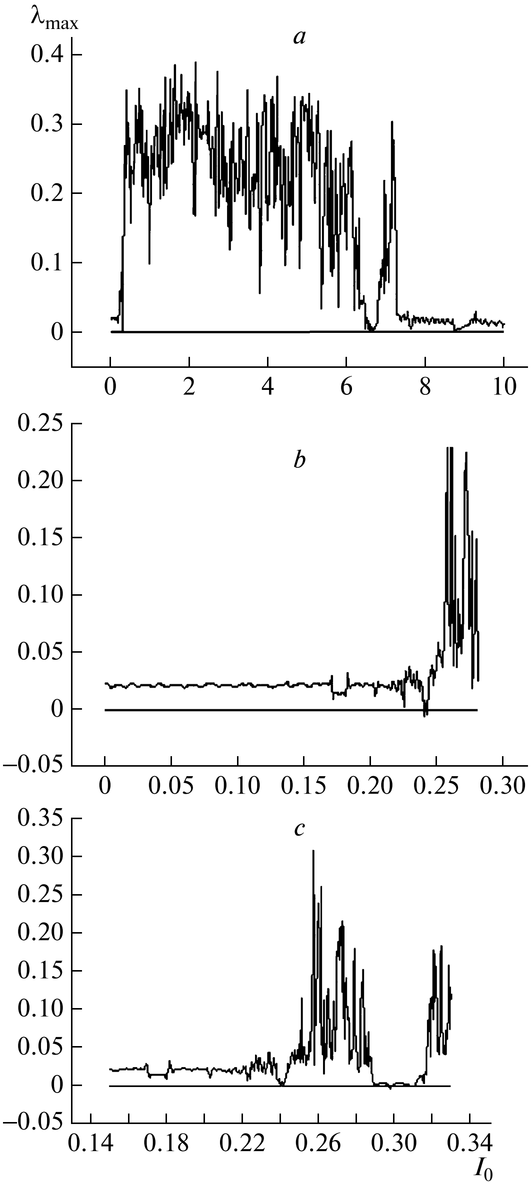

Fig. 1. The maximum Lyapunov exponent λ_{max} as a function of the optical intensity I_0 with $v+2\alpha = 0.0001$, $g=-0.5$, $\beta+\alpha^2=2$, $c_1=0$

Clearly, the square of the amplitude R is just the particle number density because

$$|R| = |\varphi| = |\psi|,$$

and θ is the phase of φ. It is not difficult to see that when the phase is related to the space-time variable linearly, i. e.,

$$\frac{d\theta}{d\eta} = \left(\frac{c_1}{R^2} - \frac{v}{2} - \alpha\right),$$

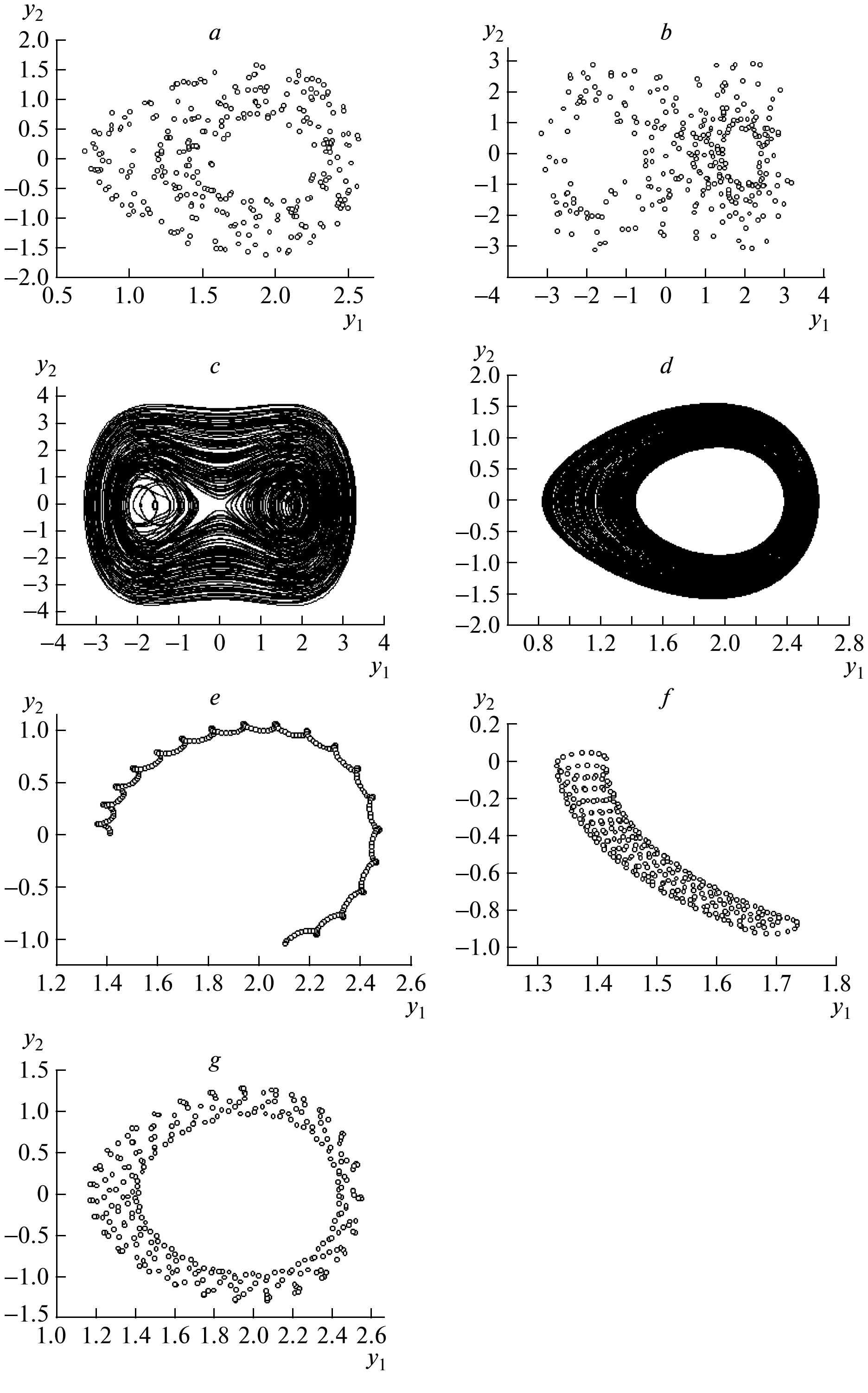

Fig. 2. The attractors with the initial conditions $(R, dR/d\eta) = (\sqrt{2}, 0)$, $c_1 = 0$, $\alpha = 0$, $\beta = 2.0$, $g = -0.5$

where c_1 is a constant, we obtain the uncoupled equation

$$\frac{d^2R}{d\eta^2} - R\left(\frac{c_1}{R^2} - \frac{v}{2} - \alpha\right)^2 - (v + 2\alpha)\frac{c_1}{R} + \\ + \frac{(v+2\alpha)^2}{2}R - (\beta+\alpha^2)R - gR^3 = I_0\cos^2(\eta)R. \quad (5)$$

Here $g = 8\pi ak$ is the dimensionless strength [16].

3. NUMERICAL SIMULATION

We calculate the Lyapunov exponent of the BEC system. Figure 1 shows the maximum Lyapunov exponent λ_{max} as a function of the optical intensity I_0. The middle point-drawing line stands for zero value. The parameters are:

$$v + 2\alpha = 0.0001, \quad g = -0.5, \quad \beta + \alpha^2 = 2, \quad c_1 = 0.$$

The initial conditions are

$$R(0) = \sqrt{2}\,, \quad \frac{dR(0)}{d\eta} = 0. \quad (6)$$

Using the fourth Runge–Kutta algorithm, we solve Eq. (5) numerically, and illustrate the attractors in the equivalent phase of $(R, dR/d\eta)$. The initial conditions are $(R, dR/d\eta) = (\sqrt{2}, 0)$. The three Lyapunov exponents are different.

Figure 2*a*: $I_0 = 0.3$, $v = 0.0001$, $\lambda_1 = 0.03122$, $\lambda_2 = 0$, and $\lambda_3 = -0.03122$.

Figure 2*b*: $I_0 = 0.28$, $v = 0.0001$, $\lambda_1 = 0.15013$, $\lambda_2 = 0$, and $\lambda_3 = -0.15013$.

Figure 2*c*: $I_0 = 0.95$, $v = 0.0001$, $\lambda_1 = 0.3122$, $\lambda_2 = 0$, and $\lambda_3 = -0.3122$.

Figure 2*d*: $I_0 = 0.001$, $v = 0.2$, $\lambda_1 = 0.00734$, $\lambda_2 = 0$, and $\lambda_3 = -0.00734$.

Figure 2*e*: $I_0 = 0.0095$, $v = 0.0001$, $\lambda_1 = 0.00793$, $\lambda_2 = 0$, and $\lambda_3 = -0.00793$.

Figure 2*f*: $I_0 = 0.015$, $v = 0.0001$, $\lambda_1 = 0.00778$, $\lambda_2 = 0$, and $\lambda_3 = -0.00778$.

Figure 2*g*: $I_0 = 0.057$, $v = 0.0001$, $\lambda_1 = 0.00745$, $\lambda_2 = 0$, and $\lambda_3 = -0.00745$.

The BEC system is in a chaotic state.

To control the chaos in the BEC, we adjust the interaction by changing the *s*-wave scattering length, that is, changing the value of g. In this paper, we only consider the effect of the *s*-wave.

Figure 3 shows the maximum Lyapunov exponent as a function of the *s*-wave scattering length g. The middle point-drawing line stands for the zero value. We find that in many ranges, for example,

$$-0.161 < g < -0.16, \quad \text{and} \quad -0.1582 < g < -0.1578,$$

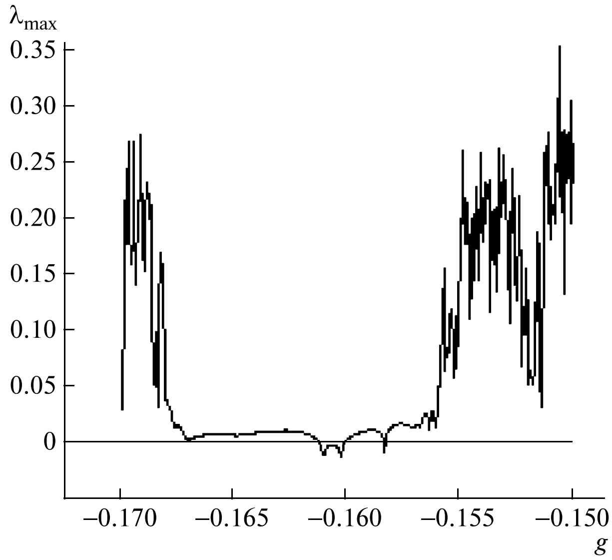

Fig. 3. The maximum Lyapunov exponent λ_{max} as a function of the *s*-wave scattering length g with $c_1 = 0$, $\alpha = 0$, $\beta = 2.0$, $I_0 = 0.95$, $v = 0.0001$

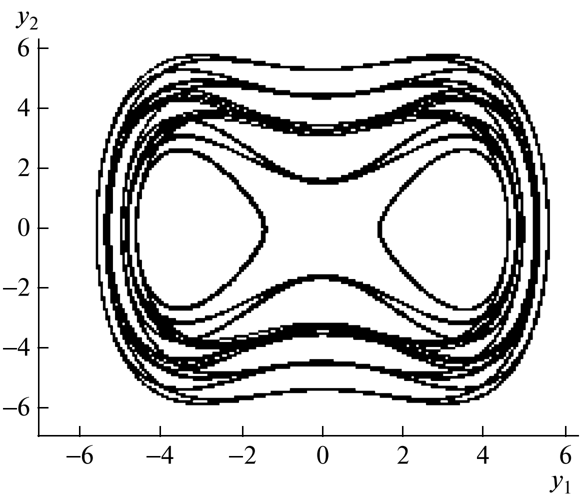

Fig. 4. The attractor projection on the y_1y_2 plane with $c_1 = 0$, $\alpha = 0$, $\beta = 2.0$, $I_0 = 0.95$, $v = 0.0001$, $g = -0.1602$

the maximum Lyapunov exponent is negative. If g takes a value in these ranges, the BEC is in a periodic state. The BEC is in a periodic state when g takes the values -0.1602.

We solve Eq. (5) numerically by using the fourth Runge–Kutta algorithm. The initial conditions are $y_1 = \sqrt{2}$ and $y_2 = 0.023$. Figure 4 shows the attractor projected onto the y_1y_2 plane. The parameters are the same as in Fig. 2, the other parameter being $g =$

= 0.1602. We can therefore transform a chaotic state into a periodic regular state by modulating the s-wave length g.

4. CONCLUSIONS

We have investigated chaotic behavior in the BEC. The stationary features of the system have been studied analytically and numerically. In the recent advancements in applications of the BEC, quantum computation with BEC atoms in Mott-insulating states is an interesting subject [18]. However, chaos is associated with quantum entanglement [19] and quantum error corrections [20], which are all key subject in quantum computation; therefore, investigating the chaos in BEC is very important.

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