ЖЭТФ, 2012, том 141, вып. 2, стр. 267–275

INTERACTION OF MOVING BRANES WITH BACKGROUND MASSLESS AND TACHYON FIELDS IN SUPERSTRING THEORY

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Received May 22, 2011

Using the boundary state formalism, we study a moving Dp-brane in a partially compact spacetime in the presence of background fields: the Kalb–Ramond field $B_{\mu\nu}$, a $U(1)$ gauge field A_α, and the tachyon field. The boundary state enables us to obtain the interaction amplitude of two branes with the above background fields. The branes are parallel or perpendicular to each other. Because of the presence of background fields, compactification of some space–time directions, motion of the branes, and the arbitrariness of the dimensions of the branes, the system is rather general. Due to the tachyon fields and velocities of the branes, the behavior of the interaction amplitude reveals obvious differences from the conventional behavoir.

1. INTRODUCTION

The discovery of the D-branes, as an intrinsic part of string theory [1], gave rise to studies of the properties and interactions of the branes. One of the most applicable methods for this purpose is the boundary state formalism. A boundary state is a BRST-invariant state that describes the creation of closed string from the vacuum.

Among the achievements in this formalism is its extension to the superstring theory and the analysis of the contribution of the conformal and super conformal ghosts to boundary states [2]. There are separate studies that add background fields such as the Kalb–Ramond field $B_{\mu\nu}$, a $U(1)$ gauge field in a compact spacetime [3], and the tachyon field [4–6] to boundary states. These background fields give to the subject a greater generality. Apart from the longitudinal fluctuations of the brane (for instance, the $U(1)$ gauge field and the tachyon field), transverse brane fluctuations [7] should also be considered. This allows interpreting it as a dynamical object. This can be performed by considering velocity for the brane [8, 9]. These observations motivated us to take all background fields and also compactification of some directions of the spacetime into account to study moving branes in the general framework of superstring theory. This general set-up cannot be found in the literature on the boundary state and brane interaction.

Because open strings are quantum excitations of a brane [10], the presence of the open string tachyon reveals an instability of the brane. In the bosonic string theory, this is a natural property, while in the superstring theories, this occurs in special cases. For instance, there are Dp-branes with wrong dimensions in the type-IIA and type-IIB superstring theories; that is, there are Dp-branes with odd dimensions in the type-IIA theory and even dimensions in the type-IIB theory [11], which are unstable. Actually, this instability can be removed by the tachyon rolling toward its minimum potential [12]. During this process, the tachyon energy dissipates to the bulk modes and the unstable system reaches a stable state that consists of lower-dimensional branes or just the closed string vacuum without any D-branes [10]. In the literature, the tachyon field is usually considered in just one dimension, and its effect is studied on a space-filling brane. In this paper, we consider a Dp-brane of an arbitrary dimension, and hence the tachyon field has components along all directions of the brane worldvolume.

We calculate the boundary state corresponding to a moving Dp-brane in the presence of the background fields $B_{\mu\nu}$, a $U(1)$ gauge field, and a tachyon. We use this boundary state to detect the interaction between two moving D-branes. There is no restriction on the brane dimensions, and they can be parallel or per-

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pendicular to each other. To keep the generality, we let some of the spacetime directions be compact. We observe that the presence of the tachyon prevents the closed string from wrapping around the compact directions. Using the boundary state, we calculate the interaction amplitude between two branes in the NS–NS and the R–R sectors. Due to the presence of the velocities and the background tachyon fields, no cancellation between these amplitudes occurs. This is the case even for similar and parallel Dp-branes with the same background fields. We observe that the interaction amplitude vanishes after a long time (or, equivalently, for large distances berween the branes). The origin of this effect is the rolling of the background tachyon field and the decay of the D-branes in this limit.

Putting all this together allows us to study a system in the most general form to obtain considerable results in spite of some mathematical difficulties due to considering longitudinal and transverse fluctuations simultaneously.

2. THE BOUNDARY STATE ASSOCIATED WITH A Dp-BRANE

To obtain the boundary state corresponding to a moving brane in the presence of the antisymmetric field $B_{\mu\nu}$ in the bulk and the tachyon and $U(1)$ gauge fields on the boundary, we consider the following sigma-model action for a closed string:

$$S = -\frac{1}{4\pi\alpha'}\int_{\Sigma} d^2\sigma \times \\ \times \left(\sqrt{-g}g^{ab}G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + \varepsilon^{ab}B_{\mu\nu}\partial_a X^\mu \partial_b X^\nu\right) + \\ + \frac{1}{2\pi\alpha'}\int_{\partial\Sigma} d\sigma \left(A_\alpha \partial_\sigma X^\alpha + V^i X^0 \partial_\tau X^i + \right. \\ \left. + \frac{1}{2}U_{\alpha\beta}X^\alpha X^\beta\right), \quad (1)$$

where the first integral is over the worldsheet of a closed string exchanged by the branes, and the second integral is over the boundary of this worldsheet, which can be at $\tau = 0$ or $\tau = \tau_0$. The $U(1)$ gauge field A_α lives on the Dp-brane worldvolume and V^i is the brane velocity component along X^i direction. The coordinates $\{X^\alpha\}$ and $\{X^i\}$ are respectively directed along and perpendicular to the Dp-brane worldvolume. The term $\frac{1}{2}U_{\alpha\beta}X^\alpha X^\beta$ with a constant symmetric matrix $U_{\alpha\beta}$ specifies the tachyon profile. According to [13], the tachyon field appears in a square form in the action to produce a Gaussian integral. We take the tachyon field to have components along the Dp-brane worldvolume. We consider $G_{\mu\nu}$ to be a flat spacetime metric with the signature

$$\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$$

and the Kalb–Ramond field $B_{\mu\nu}$ to be constant.

Equating the variation of action (1) with respect to $X^\mu(\sigma,\tau)$ to zero gives the equations of motion and boundary equations for the emitted (absorbed) closed string.

2.1. Bosonic part of the boundary state

Boundary equations following from action (1) at $\tau = 0$ are given by

$$[\partial_\tau(X^0 - V^i X^i) + \mathcal{F}^0{}_\alpha \partial_\sigma X^\alpha - \\ - U^0{}_\alpha X^\alpha]|B_x, \tau = 0\rangle = 0, \\ (\partial_\tau X^{\bar\alpha} + \mathcal{F}^{\bar\alpha}{}_\beta \partial_\sigma X^\beta - U^{\bar\alpha}{}_\beta X^\beta)|B_x, \tau = 0\rangle = 0, \\ (X^i - V^i X^0 - y^i)|B_x, \tau = 0\rangle = 0. \quad (2)$$

Here, $X^{\overline{\alpha}}$ are the spatial directions of the brane worldvolume (i. e., $\overline{\alpha} \neq 0$) and

$$\mathcal{F}_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha - B_{\alpha\beta}$$

is the total field strength, which contains the B field as well as the $U(1)$ gauge field. We note that we have assumed the mixed elements of the Kalb–Ramond field to be zero, i. e.,

$$B^\alpha{}_i = 0.$$

The solution of the closed string equation of motion is

$$X^\mu(\sigma,\tau) = x^\mu + 2\alpha' p^\mu \tau + 2L^\mu \sigma + \frac{i}{2}\sqrt{2\alpha'} \times \\ \times \sum_{m\neq 0}\frac{1}{m}\left(\alpha^\mu_m e^{-2im(\tau-\sigma)} + \tilde\alpha^\mu_m e^{-2im(\tau+\sigma)}\right). \quad (3)$$

Here, L^μ is zero for noncompact directions and

$$L^\mu = N^\mu R^\mu$$

for the compact direction X^μ with the compactification radius R^μ and the closed string winding number N^μ. The closed string center-of-mass momentum is

$$p^\mu = \frac{M^\mu}{R^\mu},$$

where M^μ is the momentum number. Substituting this solution in boundary equations (2) expresses them in terms of oscillators and zero modes. As a result an interesting condition on the closed string winding is obtained,

$$U^{\alpha}{}_{\overline{\beta}} L^{\overline{\beta}}_{\rm op}|B_x, \tau=0\rangle = 0.$$

We assume that there is no compactification along time direction, and hence

$$L^0 = 0.$$

If the matrix $U^{\overline{\alpha}}{}_{\overline{\beta}}$, is invertible this equation reduces to

$$L^{\overline{\alpha}}_{\rm op}|B_x, \tau=0\rangle = 0.$$

Therefore, the presence of the background tachyon field prevents the closed string from wrapping around compact directions that are parallel to the brane worldvolume.

Using the coherent state method [14] to solve boundary equations (2) for oscillating modes leads to the state

$$|B_{\rm osc}, \tau=0\rangle = \prod_{n=1}^{\infty}[\det M_{(n)}]^{-1} \times \times \exp\left[-\sum_{m=1}^{\infty}\left(\frac{1}{m}\alpha^{\mu}_{-m}\mathcal{S}_{(m)\mu\nu}\widetilde{\alpha}^{\nu}_{-m}\right)\right]|0\rangle, \quad (4)$$

where the matrix $\mathcal{S}_{(m)}$ is defined by

$$\begin{aligned} \mathcal{S}_{(m)} &= S_{(m)} + (S^{-1}_{(-m)})^T, \\ S_{(m)} &= M^{-1}_{(m)} N_{(m)}. \end{aligned} \quad (5)$$

The matrices $M_{(m)}$ and $N_{(m)}$, which are functions of background fields, are defined by

$$M^{\mu}_{(m)\ \nu} = \Omega^{\mu}{}_{\nu} - \frac{i}{2m}U^{\alpha}{}_{\beta}\delta^{\mu}{}_{\alpha}\delta^{\beta}{}_{\nu}, \quad (6)$$

where

$$\begin{aligned} \Omega^0{}_{\mu} &= \delta^0{}_{\mu} - V^i\delta^i{}_{\mu} - \mathcal{F}^0_{\alpha}\delta^{\alpha}{}_{\mu}, \\ \Omega^{\bar{\alpha}}{}_{\mu} &= \delta^{\bar{\alpha}}{}_{\mu} - \mathcal{F}^{\bar{\alpha}}{}_{\beta}\delta^{\beta}{}_{\mu}, \\ \Omega^i{}_{\mu} &= \delta^i{}_{\mu} - V^i\delta^0{}_{\mu}, \end{aligned} \quad (7)$$

and

$$\begin{aligned} N^0_{(m)\mu} &= \delta^0{}_{\mu} - V^i\delta^i{}_{\mu} + \mathcal{F}^0{}_{\alpha}\delta^{\alpha}{}_{\mu} + \frac{i}{2m}U^0{}_{\alpha}\delta^{\alpha}{}_{\mu}, \\ N^{\bar{\alpha}}_{(m)\mu} &= \delta^{\bar{\alpha}}{}_{\mu} + \mathcal{F}^{\bar{\alpha}}{}_{\beta}\delta^{\beta}{}_{\mu} + \frac{i}{2m}U^{\bar{\alpha}}{}_{\beta}\delta^{\beta}{}_{\mu}, \\ N^i_{(m)\mu} &= -\delta^i{}_{\mu} + V^i\delta^0{}_{\mu}. \end{aligned} \quad (8)$$

When we solve the boundary equations, the matrix $(S^{-1}_{(-m)})^T$ also appears in Eq. (5). This is because the matrix $S_{(m)}$ is mode dependent and is not orthogonal general. In the absence of the tachyon field, S becomes mode independent and orthogonal, and hence $\mathcal{S} = S$ [3]. The infinite product in Eq. (4), which comes from the path integral, can be regularized [15] as

$$\prod_{n=1}^{\infty}[\det M_{(n)}]^{-1} = \sqrt{\det\Omega}\ \det\left[\Gamma\left(\frac{U}{1+2i\Omega}\right)\right]. \quad (9)$$

From now on, we consider a selected direction X^{i_0} for the motion of the Dp-brane, and hence the other components of the velocity are zero. We also set

$$V^{i_0} = V.$$

Then the zero-mode part of the boundary state becomes

$$\begin{aligned} |B_x, \tau=0\rangle^{(0)} &= \frac{T_p}{2} \times \\ &\times \int_{-\infty}^{\infty}\prod_{\alpha} dp^{\alpha}\left\{\exp\left[-4i\alpha'(U^{-1})_{\alpha\beta}\ \times\right.\right. \\ &\times\left.\left(\left(1-\frac{1}{2}\delta_{\alpha\beta}\right)p^{\alpha}p^{\beta} + Vp^{i_0}p^{\beta}\delta^{\alpha}_0\right)\right] \times \\ &\times \delta(x^{i_0} - Vx^0 - y^{i_0})\prod_{i'\neq i_0}\delta(x^{i'} - y^{i'}) \times \\ &\times \prod_{\alpha}|p^{\alpha}_L = p^{\alpha}_{\rm R}\rangle\prod_{i'\neq i_0}|p^{i'}_L = p^{i'}_{\rm R} = 0\rangle \times \\ &\times \left.\left|p^{i_0}_L = p^{i_0}_{\rm R} = \frac{1}{2}Vp^0\right\rangle\right\}. \end{aligned} \quad (10)$$

The two delta-functions indicate the position of the brane along the perpendicular directions. The integration over the momenta indicates that the effects of all values of the momentum components have been taken into account. In addition, the equality $p^{\alpha}_L = p^{\alpha}_R$ originates from the unwrapping of the closed string around the brane directions and the noncompactness of the time direction.

There are two special limit cases for U. In the limit $U_{\alpha\beta} \to 0$, the oscillating part of the boundary state, i.e., the Eq. (4), reduces to a boundary state corresponding to a moving Dp-brane in the absence of tachyon field [9].

When we send some of the elements of U to infinity, we deal with the boundary state within the concept of tachyon condensation. This condensation can be performed on some or all elements of the tachyon matrix U. Without loss of generality, we regard U as a diagonal matrix. By sending the spatial element $U_{\overline{\alpha}\overline{\alpha}}$ to infinity, we transform the boundary state into the one related to a moving D$(p-1)$-brane that has lost its dimension along the $X^{\bar{\alpha}}$ direction, and is in the presence of a new tachyon field $U'_{(p-1)\times(p-1)}$ that does not include the component $U_{\overline{\alpha}\overline{\alpha}}$.

A notable point here is that although in the process of condensation along the $X^{\overline{\alpha}}$ direction, the matrices M and $\mathcal{S}$ in boundary state (4) change to lower dimensional ones, as expected, the effect of the condensated component remains as a $\sqrt{U_{\overline{\alpha}\overline{\alpha}}}$ factor after regularization of the infinite product

$$\prod_{n=1}^{\infty}[\det M_{(n)}]^{-1}.$$

This result is different from the conventional case in which this factor is canceled by the factor $1/\sqrt{U_{\overline{\alpha}\overline{\alpha}}}$ from zero mode part, which is absent here.

When condensation occurs along the time component of the tachyon matrix, $U_{00}\to\infty$, besides a decrease in the brane worldvolume dimension in the X^0 direction, the brane also loses its velocity. In other words, the tachyon condensation along the temporal direction fixes the Dp-brane in time and space, i. e., produces an instantonic Dp-brane, which has no velocity.

2.2. Fermionic part of the boundary state

The boundary equations for the fermionic degrees of freedom can be found in two ways: 1) by supersymmetrizing action (1) and setting the variation of the fermionic part of the action equal to zero, and 2) by performing the worldsheet supersymmetry on the bosonic boundary in Eqs. (2) and transforming them into fermionic ones, because the supersymmetrized action is invariant under global worldsheet supersymmetry transformations. We choose the second approach here. The fermionic boundary equations are then given by

$$\begin{gathered}[-i\eta(\psi^0_+-V^i\psi^i_+)+(\psi^0_--V^i\psi^i_-)+\mathcal{F}^0{}_{\alpha}(-i\eta\psi^{\alpha}_+-\psi^{\alpha}_-)-\\ -U^0{}_{\nu}(-i\eta\psi^{\nu}_++\psi^{\nu}_-)]|B_\psi,\eta,\tau=0\rangle=0,\end{gathered}$$

$$\begin{gathered}[-i\eta\psi^{\bar{\alpha}}_+-\psi^{\bar{\alpha}}_-+\mathcal{F}^{\bar{\alpha}}_{\beta}(-i\eta\psi^{\beta}_++\psi^{\beta}_-)-\\ -U^{\bar{\alpha}}{}_{\nu}(-i\eta\psi^{\nu}_+-\psi^{\nu}_-)]|B_\psi,\eta,\tau=0\rangle=0,\\ [-i\eta(\psi^i_+-V^i\psi^0_+)-(\psi^i_--V^i\psi^0_-)]\times\\ \times|B_\psi,\eta,\tau=0\rangle=0.\end{gathered}\qquad(11)$$

With the solution of the equations of motion for the fermions

$$\begin{gathered}\psi^{\mu}_-=\sum_k\psi^{\mu}_k e^{-2ik(\tau-\sigma)},\\ \psi^{\mu}_+=\sum_k\widetilde{\psi}^{\mu}_k e^{-2ik(\tau+\sigma)},\end{gathered}\qquad(12)$$

the boundary state in Eqs. (11) can be represented as

$$(\psi^{\mu}_k-i\eta S^{\mu}{}_{(k)\nu}\tilde{\psi}^{\nu}_{-k})|B_\psi,\eta,\tau=0\rangle=0.\qquad(13)$$

We note that in Eqs. (12) and (13), k is an integer number m for the R–R sector, with $\psi^{\mu}_m=d^{\mu}_m$ and $\widetilde{\psi}^{\mu}_m=\widetilde{d}^{\mu}_m$, while in the NS–NS sector, k is a half-integer number r, with $\psi^{\mu}_r=b^{\mu}_r$ and $\widetilde{\psi}^{\mu}_r=\widetilde{b}^{\mu}_r$. The constant number η can be $+1$ or -1. It is irrelevant whether we choose $+1$ or -1, because to obtain the interaction of the branes, we need to use the boundary state that has been affected by the GSO projector. As we see in what follows, this projection operator causes both states with $\eta=+1$ and $\eta=-1$ to contribute to the interaction.

Similarly to the bosonic part, we should also consider the part of the superconformal ghosts in the fermionic boundary state. The superghosts include the commuting fields β, γ, $\widetilde{\beta}$, and $\widetilde{\gamma}$.

2.2.1. The NS–NS sector

According to Eq. (13), the resultant NS–NS sector boundary state of the fermions is given by

$$\begin{gathered}|B_\psi,\eta,\tau=0\rangle_{\rm NS}=\prod_{r=1/2}^{\infty}[\det M_{(r)}]\times\\ \times\exp\left[i\eta\sum_{r=1/2}^{\infty}(b^{\mu}_{-r}\mathcal{S}_{(r)\mu\nu}\widetilde{b}^{\nu}_{-r})\right]|0\rangle_{\rm NS}.\end{gathered}\qquad(14)$$

When the path integral is computed, the determinant is inversed in comparison to the bosonic case, Eq. (4). This is due to the Grassmann nature of the integration variables [2]. Because r is half-integer, the regularization of this infinite product is

$$\prod_{r=1/2}^{\infty}[\det M_{(r)}]=\det\left(\frac{\sqrt{\pi}}{\Gamma\left[\dfrac{U}{2i\Omega}+\dfrac{1}{2}\right]}\right).\qquad(15)$$

2.2.2. The R–R sector

To derive the boundary state in the R–R sector, we have to follow the same procedure as in the NS–NS sector with little differences that require some care. Because $k=m$ in Eq. (13) runs over integers in the R–R sector, there is a zero mode that affects the boundary state. Solving Eq. (13) in the R–R sector yields the boundary state

$$|B_\psi,\eta,\tau=0\rangle_{\rm R}=\prod_{m=1}^{\infty}[\det M_{(m)}]\times \\ \times\exp\left[i\eta\sum_{m=1}^{\infty}(d^\mu_{-m}\mathcal{S}_{(m)\mu\nu}\widetilde{d}^\nu_{-m})\right]|B_\psi,\eta\rangle^{(0)}_{\rm R}. \quad (16)$$

Because m is an integer number, the regularization of the infinite product is exactly similar to the one in bosonic case (of course, the determinant is here the inverse of one in the bosonic case)

$$\prod_{m=1}^{\infty}[\det M_{(m)}]= \\ =\left\{\sqrt{\det\Omega}\,\det\left(\Gamma\left[1+\frac{U}{2i\Omega}\right]\right)\right\}^{-1}. \quad (17)$$

The state $|B_\psi,\eta\rangle^{(0)}_{\rm R}$ in Eq. (16) is the zero-mode boundary state

$$|B_\psi,\eta\rangle^{(0)}_{\rm R}=\left[C\Gamma_{11}\left(\frac{1+i\eta\Gamma_{11}}{1+i\eta}\right)\times \\ \times\exp\left(\frac{1}{2}\Phi_{\mu\nu}\Gamma^\mu\Gamma^\nu\right)\right]^{AB}|A\rangle|\widetilde{B}\rangle, \quad (18)$$

where $|A\rangle|\widetilde{B}\rangle$ is the vacuum of the zero modes d^μ_0 and $\widetilde{d}^\mu_0$, C is the charge conjugate matrix, and the antisymmetric matrix Φ is defined in terms of the matrix $\mathcal{S}$:

$$\mathcal{S}=(1-\Phi)^{-1}(1+\Phi). \quad (19)$$

The details of obtaining Eqs. (18) and (19) are given in Appendix A. Because the matrix $\mathcal{S}$ should be orthogonal,

$$\mathcal{S}^{-1}=\mathcal{S}^T,$$

its definition

$$\mathcal{S}_{(m)}=S_{(m)}+[(S_{(-m)})^{-1}]^T$$

implies that the matrix S should satisfy the relation

$$S^T_{(m)}-S^{-1}_{(m)}=S^T_{(-m)}+S^{-1}_{(-m)}. \quad (20)$$

According to Eqs. (5)–(8), S is defined in terms of the background fields. Thus, Eq. (20) imposes a relation on these background fields. When the tachyon and velocity are put to zero in action (1), we obtain $\Phi=\mathcal{F}$, and hence the term $\exp(\frac{1}{2}\Phi_{\alpha\beta}\Gamma^\alpha\Gamma^\beta)$ reduces to the known form $\exp(\frac{1}{2}\mathcal{F}_{\alpha\beta}\Gamma^\alpha\Gamma^\beta)$ [3].

3. INTERACTION OF THE BRANES

The interaction amplitude between Dp_1- and Dp_2-branes in each sector is defined as

$$\mathcal{A}_{\rm NS-NS,R-R}= \\ =2\alpha'\int_0^\infty dt\;{}_{\rm NS,R}\langle B^1,\tau=0|e^{-tH_{\rm NS,R}}|B^2,\tau=0\rangle_{\rm NS,R}.$$

The total Hamiltonian $H_{\rm NS,R}$ is the sum of the Hamiltonians of the X^μ, the ψ^μ, and the ghosts and superghosts in each sector. To calculate the interaction amplitude, we need the total projected boundary state. The total boundary state of each sector is

$$|B,\eta,\tau=0\rangle_{\rm NS,R}=|B_X,\tau=0\rangle|B_{\rm gh},\tau=0\rangle\times \\ \times|B_\psi,\eta,\tau=0\rangle_{\rm NS,R}|B_{\rm sgh},\eta,\tau=0\rangle_{\rm NS,R}.$$

In Appendix B, the projection process is discussed. The total projected boundary states are given by Eqs. (40) and (41).

3.1. Interaction amplitude in the NS–NS sector

Using boundary state (40) for NS–NS sector, after a long calculation, we obtain the total interaction amplitude in this sector as

$$\mathcal{A}_{\rm NS-NS}=\frac{\alpha' V_{\overline{u}}}{8(2\pi)^{d_{\bar i}}}\frac{T_{p_1}T_{p_2}}{|V_1-V_2|}\times \\ \times\prod_{m=1}^{\infty}\frac{\det[M_{(m-1/2)1}M_{(m-1/2)2}]}{\det[M_{(m)1}M_{(m)2}]}\times \\ \times\int_0^\infty dt\left\{\prod_{\bar i_c}\Theta_3\left(\frac{y_1^{\bar i_c}-y_2^{\bar i_c}}{2\pi R_{\bar i_c}}\mid\frac{i\alpha' t}{\pi(R_{\bar i_c})^2}\right)\times\right. \\ \times\left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d_{\bar i_n}}\exp\left(-\frac{1}{4\alpha' t}\sum_{\bar i_n}(y_1{}^{\bar i_n}-y_2{}^{\bar i_n})^2\right)\times \\ \times\frac{1}{q}\left(\prod_{m=1}^{\infty}\left[\left(\frac{1-q^{2m}}{1+q^{2m-1}}\right)^2\times\right.\right. \\ \left.\times\frac{\det(1+\mathcal{S}_{(m-1/2)1}\mathcal{S}^T_{(m-1/2)2}q^{2m-1})}{\det(1-\mathcal{S}_{(m)1}\mathcal{S}^T_{(m)2}q^{2m})}\right]- \\ -\prod_{m=1}^{\infty}\left[\left(\frac{1-q^{2m}}{1-q^{2m-1}}\right)^2\times\right. \\ \left.\left.\times\frac{\det(1-\mathcal{S}_{(m-1/2)1}\mathcal{S}^T_{(m-1/2)2}q^{2m-1})}{\det(1-\mathcal{S}_{(m)1}\mathcal{S}^T_{(m)2}q^{2m})}\right]\right)\times \\ \times\frac{1}{\sqrt{\det Q\;\det K_1\;\det K_2}}\times \\ \times\exp\left[-\frac{1}{4}\left(E^TQ^{-1}E+\sum_{\alpha'_1,\beta'_1}y_2^{\alpha'_1}y_2^{\beta'_1}(K_1^{-1})_{\alpha'_1\beta'_1}+\right.\right. \\ \left.\left.\left.+\sum_{\alpha'_2,\beta'_2}y_1^{\alpha'_2}y_1^{\beta'_2}(K_2^{-1})_{\alpha'_2\beta'_2}\right)\right]\right\}, \quad (21)$$

where $q=e^{-2t}$ and $V_{\bar u}$ is the common volume of the branes. The set $\{\bar i\}$ comprises directions perpendicular to both branes except i_0, $\{\overline{u}\}$ comprises the directions along both branes except 0, $\{\alpha'_1\}$ is used for the directions along the Dp_1-brane and perpendicular to the Dp_2-brane and $\{\alpha'_2\}$ indicates the directions along the Dp_2-brane and perpendicular to the Dp_1-brane. $\bar{i}_c$ and $\bar{i}_n$ are respectively related to the compact and noncompact parts of $\bar{i}$. The matrices Q, K_1, K_2 and the doublet E are defined is terms of their elements as

$$\begin{aligned} Q_{11} &= \frac{\alpha' t}{2(V_2 - V_1)^2}(1 + {V_1}^2)(1 - {V_2}^2) + \\ &\quad + 2i\alpha'(U_1^{-1})^{00}(1 - V_2^2)^2, \\ Q_{22} &= \frac{\alpha' t}{2(V_2 - V_1)^2}(1 + {V_2}^2)(1 - {V_1}^2) - \\ &\quad - 2i\alpha'(U_2^{-1})^{00}(1 - V_1^2)^2, \\ Q_{12} = Q_{21} &= \frac{\alpha' t}{(V_2 - V_1)^2}(1 + {V_1}^2) \times \\ &\quad \times (1 + {V_2}^2)(1 - V_1 V_2), \end{aligned} \tag{22}$$

$$\begin{aligned} E_1 &= \frac{i}{2(V_2-V_1)}\left[{y_2}^{i_0}(1+{V_1}^2)^2 - {y_1}^{i_0}(1+V_1V_2)\right], \\ E_2 &= \frac{i}{2(V_2-V_1)}\left[{y_1}^{i_0}(1+{V_2}^2)^2 - {y_2}^{i_0}(1+V_1V_2)\right], \end{aligned} \tag{23}$$

$$\begin{aligned} K_1^{\alpha'_1\beta'_1} &= 4i\alpha'\left(1 - \frac{1}{2}\delta_{\alpha'_1\beta'_1}\right) \times \\ &\quad \times (U_1^{-1})^{\alpha'_1\beta'_1} - \alpha' t \delta^{\alpha'_1\beta'_1}, \\ K_1^{\overline{uv}} &= 4i\alpha'\left(1 - \frac{1}{2}\delta_{\overline{uv}}\right)(U_1^{-1})^{\overline{uv}} - \frac{1}{2}\alpha' t \delta^{\overline{uv}}, \\ K_1^{\alpha'_1\overline{u}} &= K_1^{\overline{u}\alpha'_1} = 4i\alpha'(U_1^{-1})^{\alpha'_1\overline{u}}. \end{aligned} \tag{24}$$

By changing $1 \to 2$ and $i \to -i$ in the elements of the matrix K_1, we obtain the elements of the matrix K_2. Obviously Q, E, and K are velocity and tachyon dependent.

In amplitude (21), the theta function comes from the compact part of the set $\{X^{\bar{i}}\}$, while the exponential and its pre-factor in the third line originate from the noncompact part. In fact, the exponential is a damping factor with respect to the distance of the branes. If all directions $\{X^{\bar{i}}\}$ are compact, the exponential and its pre-factor disappear. In this case, $\bar{i}_c$ takes all values of $\bar{i}$. In the case where all directions $\{X^{\bar{i}}\}$ are noncompact, the Θ_3 factor is removed, and hence $\bar{i}_n$ takes all values of $\bar{i}$. The next two lines, which contain the $\mathcal{S}$ matrix, reflect the part of the oscillators, conformal ghosts and superconformal ghosts. The remaining part, which is obtained by integration over the momenta, Eq. (10), is due to the presence of the velocities and the background tachyon fields. In the absence of the velocities and tachyon fields, this factor is also absent, and hence the interaction amplitude resembles the one in [3].

3.2. Interaction amplitude in the R–R sector

For interaction amplitude in the R–R sector, we use the total GSO-projected boundary state for the R–R sector, the Eq. (41), and follow the same procedure as in the NS–NS sector, whence

$$\begin{aligned} \mathcal{A}_{\text{R–R}} &= \frac{\alpha' V_{\overline{u}}}{8(2\pi)^{d_{\bar{i}}}}\frac{T_{p_1}T_{p_2}}{|V_1 - V_2|} \times \\ &\times \int_0^\infty dt \left\{ \prod_{\bar{i}_c} \Theta_3\left(\frac{y_1^{\bar{i}_c} - y_2^{\bar{i}_c}}{2\pi R_{\bar{i}_c}} \mid \frac{i\alpha' t}{\pi (R_{\bar{i}_c})^2}\right) \times \right. \\ &\times \left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d_{\bar{i}_n}} \exp\left(-\frac{1}{4\alpha' t}\sum_{\bar{i}_n}({y_1}^{\bar{i}_n} - {y_2}^{\bar{i}_n})^2\right) \times \\ &\times \left(\zeta \prod_{m=1}^{\infty}\left[\left(\frac{1-q^{2m}}{1+q^{2m}}\right)^2 \frac{\det(1+\mathcal{S}_{(m)1}\mathcal{S}^T_{(m)2}q^{2m})}{\det(1-\mathcal{S}_{(m)1}\mathcal{S}^T_{(m)2}q^{2m})}\right] + \zeta'\right) \times \\ &\times \frac{1}{\sqrt{\det Q \ \det K_1 \ \det K_2}} \times \\ &\times \exp\left[-\frac{1}{4}\left(E^T Q^{-1} E + \sum_{\alpha'_1,\beta'_1} y_2^{\alpha'_1} y_2^{\beta'_1}(K_1^{-1})_{\alpha'_1\beta'_1} + \right.\right. \\ &\left.\left.\left. + \sum_{\alpha'_2,\beta'_2} y_1^{\alpha'_2} y_1^{\beta'_2}(K_2^{-1})_{\alpha'_2\beta'_2}\right)\right]\right\}, \end{aligned} \tag{25}$$

where

$$\zeta \equiv -\frac{1}{2}\text{Tr}[G_1 C^{-1} G_2^T C], \tag{26}$$

$$\zeta' \equiv -i\text{Tr}[G_1 C^{-1} G_2^T C \Gamma_{11}], \tag{27}$$

and

$$G_{1,2} = \exp\left[\frac{1}{2}(\Phi_{(1,2)})_{\mu\nu}\Gamma^\mu\Gamma^\nu\right].$$

We note that the variables ζ and ζ' implicitly depend on the brane dimensions through Φ_1 and Φ_2 in G_1 and G_2.

We now study the total amplitude, i. e., the combination of the amplitudes in the NS–NS and R–R sectors. We consider the following special case: there is no compactification, the two Dp-branes are parallel and have the same dimensions, and the same fields live on them. As in the literature, this interaction amplitude vanishes due to the cancelation of the attractive and repulsive forces in the NS–NS and R–R sectors.

In the case under study, in addition to the fields living on the branes, velocities are also present, which are transverse fluctuations of the branes. In amplitudes

(21) and (25), the relative speed appears in the denominators. This imposes a constraint on the system that the velocities of the branes should be different; otherwise, the total amplitude becomes infinite. In this case, we cannot check the vanishing of the interaction amplitude for identical parallel branes with the same fields. Therefore, even if all the fields are identical, the velocities should be different. This causes the branes to have different Φ's and consequently different $\mathcal{S}$'s. Then the NS–NS and R–R amplitudes cannot cancel the effect of each other.

4. LONG-DISTANCE BEHAVIOR OF THE AMPLITUDE

We now find the interaction between the branes when they are far from each other. That is, we find the behavior of interaction amplitudes (21) and (25) as time tends to infinity. Conventionally, in the large-distance limit, only, the massless states of a closed string contribute to the brane interaction.

The large-distance amplitude is equivalent to the long-time behavior of the branes. It can be obtained by sending q to zero in Eqs. (21) and (25). Hence, the interaction amplitudes due to massless states in the NS–NS and R–R sectors are

$$\lim_{q\to 0}\mathcal{A}_{\mathrm{NS-NS}} = \frac{V_{\bar{u}}T_{p_1}T_{p_2}}{4(2\pi)^{d_{\bar{i}}}}\times \frac{i(-1)^{(p_1+p_2)/2}\,2^{d_{\bar{u}}+1/2}}{\alpha'^{(p_1+p_2)/2}(1+{V_1}^2)(1+{V_2}^2)}\times \prod_{m=1}^{\infty}\frac{\det[M_{(m-1/2)1}M_{(m-1/2)2}]}{\det[M_{(m)1}M_{(m)2}]}\times \int^{\infty} dt\left\{\left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d_{\bar{i}_n}}\exp\left(-\frac{1}{4\alpha' t}\sum_{\bar{i}_n}({y_1}^{\bar{i}_n}-{y_2}^{\bar{i}_n})^2\right)\times \lim_{t\to\infty}\frac{2[\mathrm{Tr}(\mathcal{S}_{(1)1}\mathcal{S}^T_{(1)2})-2]}{t^{1+(p_1+p_2)/2}}\right\}, \quad (28)$$

and

$$\lim_{q\to 0}\mathcal{A}_{\mathrm{R-R}} = \frac{V_{\bar{u}}\,T_{p_1}T_{p_2}}{8(2\pi)^{d_{\bar{i}}}}\times \frac{i(-1)^{(p_1+p_2)/2}\,2^{d_{\bar{u}}+1/2}}{\alpha'^{(p_1+p_2)/2}(1+{V_1}^2)(1+{V_2}^2)}\times \int^{\infty} dt\left\{\left(\sqrt{\frac{\pi}{\alpha' t}}\right)^{d_{\bar{i}_n}}\exp\left(-\frac{1}{4\alpha' t}\sum_{\bar{i}_n}({y_1}^{\bar{i}_n}-{y_2}^{\bar{i}_n})^2\right)\times \lim_{t\to\infty}\frac{1}{t^{1+(p_1+p_2)/2}}\right\}. \quad (29)$$

We do not extend the limit to the exponential part and its pre-factor in Eqs. (28) and (29) because these factors are related to the positions of the branes, and closed string emission is independent of the location of the branes. When there is no tachyonic background [3], the last factors in Eqs. (28) and (29) do not have the factor $1/t^{1+(p_1+p_2)/2}$. Thus, due to the presence of tachyon fields, the interaction amplitude decreases with time. In fact, the statement that the massless closed string states dominate in the interaction at large distances berween the branes is valid until there is no tachyon background in the system.

There is an interpretation for this unusual behavior. In fact, the open string tachyon background causes an instability in the system. Therefore, after a sufficiently long time, by the tachyon rolling [12] toward its minimum potential, unstable D-branes decay to the bulk modes and their dimensions decrease to reach a stable system. Final products of this process are branes with lower dimensions or the closed string vacuum [10]. This implies that there are no physical perturbative open-string states around the minimum of the potential. This is because the open string states live only on the branes. Thus, in the concept of interacting branes, as time passes, leading to the tachyon rolling and a decrease in their dimensions, the brane configuration distorts and prevents them from interacting.

The amplitude $\mathcal{A}_{\mathrm{NS-NS}}$ in Eq. (28) depends on the background fields through the factor $\mathrm{Tr}(\mathcal{S}_{(1)1}\mathcal{S}^T_{(1)2})$ and the determinants of the matrices $\{M_{(m-1/2)}|m = 1,2,3,\ldots\}$, while such a dependence is absent in the amplitude $\mathcal{A}_{\mathrm{R-R}}$ in Eq. (29). In other words, when the branes are far from each other, the R–R amplitude becomes background independent.

Another interesting feature of the long-time amplitude is its time-dependent behavior on the brane dimensions. An exception here is the D-instanton. When two D-instantons, which have the dimension $p_1 = p_2 = -1$, interact, the factor $1/t^{1+(p_1+p_2)/2}$ is removed and the long-time amplitude behavior resembles that of a system without a tachyon. For this system, the presence of the tachyon does not affect the conventional behavior of the large-distance interaction.

5. CONCLUSIONS

The boundary state of a closed superstring traveling between two moving branes in the presence of $B_{\mu\nu}$, a tachyon, and a $U(1)$ gauge field was calculated. A notable feature in the boundary state equations is the prevention of closed string wrapping around the compact directions of spacetime, which is due to the presence of the tachyon field. Also, the boundary state includes a momentum-dependent exponential factor, which is absent in the conventional boundary states. This factor originates from the zero-mode parts of the velocity and tachyon terms in the boundary action.

The interaction amplitude of the branes via an exchange by a closed string was calculated for the NS–NS and R–R sectors. It is shown that even for codimension parallel branes with similar external fields, the total amplitude is not zero. This is due to the presence of the velocities and tachyon fields in the system.

The long-distance behavior of the interaction amplitude was studied. In this domain, the instability of the branes due to the background tachyon fields weakens the interaction. This decreasing behavior can be understood in terms of dissipation of the branes to the bulk modes because of the rolling of the tachyon to its minimum potential in the long-time regime. The interaction of two D-instantons obviates this decreasing behavior. The long-time amplitude in this case behaves as in the conventional case, in which the massless states dominate.

APPENDIX A

Zero-mode boundary state in the R–R sector

The state $|B_\psi, \eta\rangle_{\rm R}^{(0)}$ in Eq. (16) is the zero-mode boundary state that obeys the equation

$$|B_\psi, \eta\rangle_{\rm R}^{(0)} = \mathcal{M}^{(\eta)AB}|A\rangle|\widetilde{B}\rangle, \tag{30}$$

where $|A\rangle|\widetilde{B}\rangle$ is the vacuum of the zero modes d_0^μ and $\widetilde{d}_0^\mu$. The matrix $\mathcal{M}^{(\eta)}$ has to satisfy the equation

$$(\Gamma^\mu)^T \mathcal{M}^{(\eta)} - i\eta \mathcal{S}^\mu_{(m)\nu}\Gamma_{11}\mathcal{M}^{(\eta)}\Gamma^\nu = 0. \tag{31}$$

We consider a solution of the form

$$\mathcal{M}^{(\eta)} = C\Gamma_{11}\left(\frac{1+i\eta\Gamma_{11}}{1+i\eta}\right)G, \tag{32}$$

where C is the charge conjugation matrix. Substitution of Eq. (32) in Eq. (31) leads to the following equation for the matrix G:

$$\Gamma^\mu G = \mathcal{S}^\mu{}_\nu G\Gamma^\nu. \tag{33}$$

There is a conventional solution for G in the form

$$G = \exp\left(\frac{1}{2}\Phi_{\mu\nu}\Gamma^\mu\Gamma^\nu\right). \tag{34}$$

Indeed, we must expand the exponential with the convention that all gamma matrices anticommute, and therefore there are a finite number of terms. The antisymmetric matrix Φ is defined in terms of the matrix $\mathcal{S}$, see Eg. (19).

APPENDIX B

GSO-projected and ghosts boundary states

The GSO-projected boundary states are given by

$$|B,\tau=0\rangle_{\rm NS} = \frac{1-(-1)^{F+G}}{2}\,\frac{1-(-1)^{\widetilde{F}+\widetilde{G}}}{2}\,\times \\ \times\,|B,\eta=+1,\tau=0\rangle_{\rm NS}, \tag{35}$$

$$|B,\tau=0\rangle_{\rm R} = \frac{1+(-1)^n(-1)^{F+G}}{2}\,\frac{1-(-1)^{\widetilde{F}+\widetilde{G}}}{2}\,\times \\ \times\,|B,\eta=+1,\tau=0\rangle_{\rm R}, \tag{36}$$

where n is an even number for the type-IIA superstring theory and an odd number for the type-IIB superstring theory. The definitions of F and G are

$$F = \sum_{r=1/2}^{\infty} b^\mu_{-r}b_{r\mu}, \\ G = -\sum_{r=1/2}^{\infty}(\gamma_{-r}\beta_r + \beta_{-r}\gamma_r) \tag{37}$$

in the NS–NS sector, and

$$(-1)^F = \Gamma_{11}(-1)^{\sum_{m=1}^{\infty} d^\mu_{-m}d_{m\mu}}, \\ G = -\gamma_0\beta_0 - \sum_{m=1}^{\infty}(\gamma_{-m}\beta_m + \beta_{-m}\gamma_m) \tag{38}$$

in the R–R sector. Similar definitions also hold for $\tilde{F}$ and $\tilde{G}$. Hence, the total projected boundary states are

$$|B,\tau=0\rangle_{\rm NS} = \\ = \frac{1}{2}\left(|B,+,\tau=0\rangle_{\rm NS} - |B,-,\tau=0\rangle_{\rm NS}\right), \tag{39}$$

$$|B,\tau=0\rangle_{\rm R} = \\ = \frac{1}{2}\left(|B,+,\tau=0\rangle_{\rm R} + |B,-,\tau=0\rangle_{\rm R}\right). \tag{40}$$

Since the bulk action in the Eq. (1) preserves conformal symmetry, working in the covariant formalism requires including conformal ghosts [2, 16]. In fact, we need a part of the ghosts (i. e., anticommuting fields b, c, $\widetilde{b}$, and $\widetilde{c}$) in the bosonic boundary state. This part is independent of the background fields and is expressed as

$$|B_{\rm gh},\tau=0\rangle = \\ = \exp\left[\sum_{m=1}^{\infty} e^{4im\tau_0}\left(c_{-m}\widetilde{b}_{-m} - b_{-m}\widetilde{c}_{-m}\right)\right]\times \\ \times\,\frac{c_0+\widetilde{c}_0}{2}|q=1\rangle|\widetilde{q}=1\rangle. \tag{41}$$

In superstring theory, in addition to the conformal ghosts, we should also consider the superconformal ghosts. Then, the boundary states, corresponding to the superconformal ghosts in the NS–NS and R–R sectors, are given by

$$|B_{\rm sgh},\eta,\tau=0\rangle_{\rm NS} = = \exp\left[i\eta\sum_{r=1/2}^{\infty}\left(\gamma_{-r}\widetilde{\beta}_{-r}-\beta_{-r}\widetilde{\gamma}_{-r}\right)\right]\times \times|P=-1\rangle|\widetilde{P}=-1\rangle, \quad (42)$$

$$|B_{\rm sgh},\eta,\tau=0\rangle_{\rm R} = = \exp\left[i\eta\sum_{m=1}^{\infty}\left(\gamma_{-m}\widetilde{\beta}_{-m}-\beta_{-m}\widetilde{\gamma}_{-m}\right)+i\eta\gamma_0\tilde{\beta}_0\right]\times \times\left|P=-\frac{1}{2}\right\rangle\left|\widetilde{P}=-\frac{3}{2}\right\rangle. \quad (43)$$

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