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ELECTRONIC EXCITATIONS AND TRANSPORT IN APERIODIC SEQUENCES OF QUANTUM DOTS IN EXTERNAL ELECTRIC AND MAGNETIC FIELDS

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The energy spectra and transport of electronic excitations in one-dimensional aperiodic sequences of quantum dots of Thue–Morse and double-periodic type are studied. The influence of external magnetic and electric fields on the energy spectra and transport is considered. For aperiodic sequences of quantum dots, in contrast to aperiodic sequences of atoms, the influence of relatively small magnetic and electric fields is essential, but localization occurs at finite values of the perturbations. The transmission coefficient is determined using the quasiclassical approximation with the Coulomb blockade taken into account. The resonance tunneling is studied.

1. INTRODUCTION

A large class of materials of great importance for quantum electronics, such as the arrays of metallic [1, 2] or semiconducting [3] quantum dots (QDs), consist of weakly coupled nanometer-scale islands. Various physical effects have been investigated in large periodic arrays of QDs [4]. The study of aperiodic sequences of QDs began quite recently [5]. In aperiodic structures, a small perturbation does not localize electrons, and transport is available at finite values of perturbations (external fields), in contrast to periodic one-dimensional structures, where even an infinitesimal perturbation localizes the current states [6].

In this paper, we study the electronic spectra and electronic transport properties in one-dimensional aperiodic sequences of QDs of the Thue–Morse and double-periodic type [7]. Two ways to construct the aperiodic sequences are considered: by defining the confining potential steepness and by defining the distances between QDs. Aperiodic sequences of QDs based on GaAs and its solid solution $Al_xGa_{1-x}As$ and $In_xGa_{1-x}As$ with $x = 0.1$–1 are considered. In the quasiclassical approximation, we obtain the tunneling probability for double QDs and aperiodic sequences of QDs and investigate the influence of control parameters such as external fields and the confining potential steepness on the electronic transport and electronic spectra. The resonance tunneling states appear when energy levels of neighboring QDs become equal as a result of shifting electronic energy levels by the external fields.

2. THE THUE–MORSE AND DOUBLE-PERIODIC APERIODIC SEQUENCES

The Thue–Morse sequence can be defined by the recursive relations $S_n = S_{n-1}S^+_{n-1}$ and $S^+_n = S^+_{n-1}S_{n-1}$ (for $n \geq 1$) with $S_0 = A$ and $S^+_0 = B$. Another way to build this sequence is through the inflation rules $A \to AB$ and $B \to BA$. Generations of the Thue–Morse sequence are

$$S_0 = A, \quad S_1 = AB, \quad S_2 = ABBA,$$

$$S_3 = ABBABAAB.$$

The double-periodic sequence is invariant under the transformation $A \to AB$, $B \to AA$. Generations of the double-periodic sequence are

$$S_0 = A, \quad S_1 = AB, \quad S_2 = ABAA,$$

$$S_3 = ABAAABAB.$$

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We use the two ways to build each aperiodic structure under investigation.

1) By defining the confining potential steepness. This means that A is a QD with the parabolic confining potential steepness α_1 and B is a QD with the parabolic confining potential steepness α_2. The distances between the QDs are equal.

2) By defining the distance between the QDs. This means that A is a pair of QDs with a separation d_A and B is a pair of QDs with a separation distance d_B. The confining potential steepnesses are the same for all QDs.

3. ELECTRON ENERGY SPECTRA

We use the parabolic potential model. The Hamiltonian of a single electron in a two-dimensional QD in a magnetic field with the vector potential $\mathbf{A}$ is [8]

$$\hat{H} = \frac{1}{2m^*}\left(-i\hbar\nabla - \frac{e\mathbf{A}}{c}\right)^2 + \alpha r^2, \tag{1}$$

where $m^* = 0.07m_0$ is the effective electron mass in GaAs, α is the steepness of the confining potential, $\mathbf{A} = (1/2)\mathbf{B}\times\mathbf{r}$, r is the radius vector, and $\mathbf{B}$ is the magnetic field. Hamiltonian (1) leads to the energy spectrum [9]

$$E_{nm} = 2^{3/2}\sqrt{\xi}\left(n + \frac{|m|+1}{2}\right) + \frac{\omega_c m}{4}, \tag{2}$$

where $\xi \equiv e^2B^2/m^* + \alpha$ is the effective confining potential steepness and ω_c is the cyclotron frequency, $n = 0, 1, 2, \ldots$, $m = 0, \pm 1, \ldots$

For a one-dimensional sequence of QDs and $m = 0$, the energy spectrum is

$$E = \hbar\sqrt{\frac{\xi}{m^*}}\left(n + \frac{1}{2}\right). \tag{3}$$

The expression for the energy spectrum of an electron in a QD in the electric field F is

$$E = \sqrt{\frac{\alpha}{m^*}}\left(n + \frac{1}{2}\right) - \frac{e^2F^2}{2\alpha}. \tag{4}$$

We obtain the electron energy spectra of aperiodic sequences in the pairwise interaction approximation. We consider the energy of the Nth generation of the sequence in the form

$$E_N = N_AE_A + N_BE_B + + N_{AB}E_{AB}^{(1)} + N_{AA}E_{AA}^{(1)} + N_{BB}E_{BB}^{(1)}. \tag{5}$$

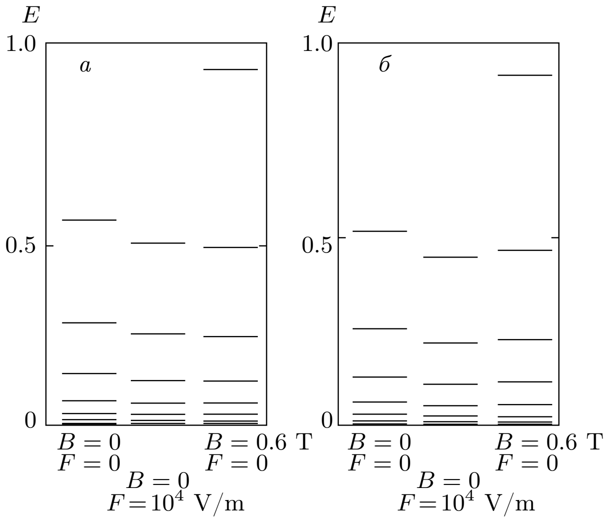

Fig. 1. The electron energy spectra of the nth generation of the aperiodic sequences and energy levels shifts without external fields, in the magnetic field $B = 0.6$ T (with $F = 0$), and in the electric field $F = 10^4$ V/m (with $B = 0$): a — Thue–Morse sequence; b — double-periodic sequence

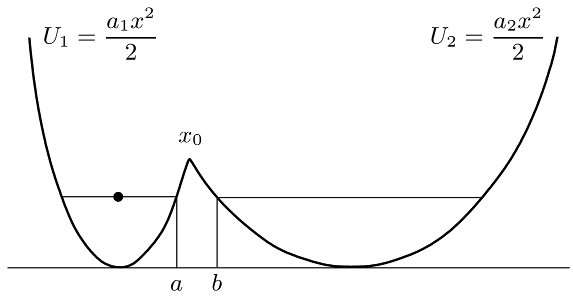

Fig. 2. The potential barrier generated by parabolic confining potentials on neighboring QDs

The number of type-A dots is N_A and of type-B dots is N_B, and the numbers of pairs of QDs of the types AB, AA, and BB are N_{AB}, N_{AA}, and N_{BB}; E_A and E_B are the particle energies in the QDs A and B; and $E_{AB}^{(1)}$, $E_{AA}^{(1)}$, and $E_{BB}^{(1)}$ are the first-order corrections to the energy of a pair of QDs of the corresponding type. First-order corrections are obtained from the perturbation theory:

$$E_V^{(1)} = \frac{H_{11} + H_{22} \pm \sqrt{4H_{12}^2 + (H_{11} - H_{21})^2}}{2}, \tag{6}$$

where

$$H_{ij} = \int_{-\infty}^{\infty} \phi_i^{(0)} V \phi_j^{(0)} dx,$$

the perturbation

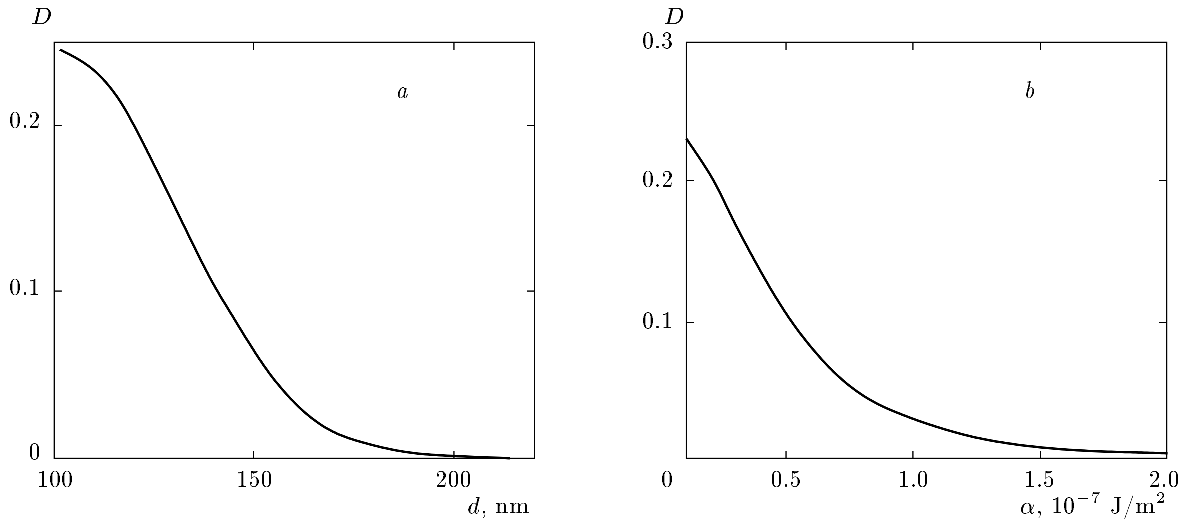

Fig. 3. The tunneling probability through a double QD versus (a) the distance between QDs (the confining potential steepness $\alpha = 0.5 \cdot 10^{-7}$ J/m^2) and (b) the confining potential steepness (the distance between QDs $d = 130$ nm); $n = 0$ for both QDs

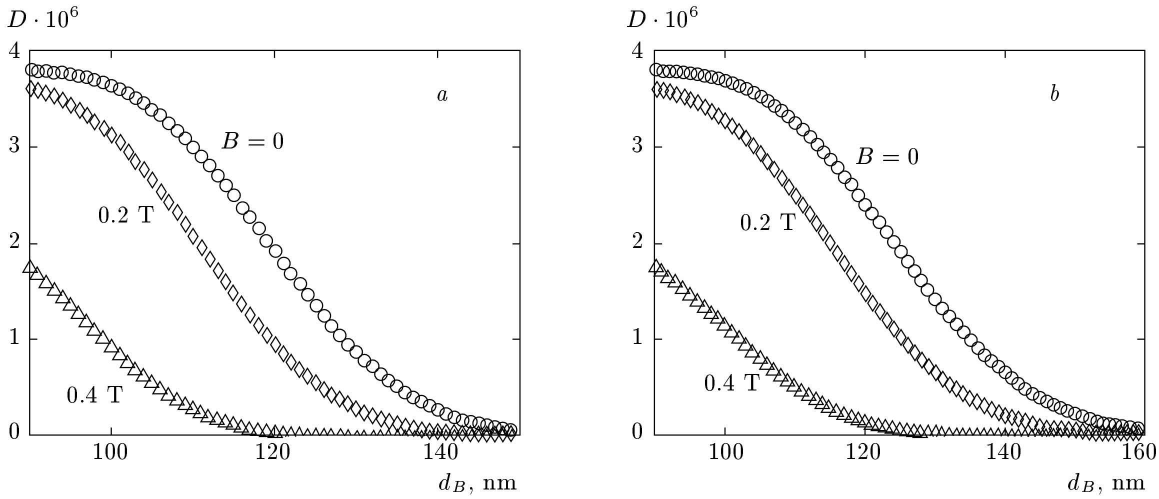

Fig. 4. The electron tunneling probability through three generations of aperiodic (a) Thue–Morse and (b) double-periodic sequences versus the distance d_B between QDs in an external magnetic field B. The distance $d_A = 90$ nm, the confining potential steepness $\alpha = 0.5 \cdot 10^{-7}$ J/m^2

$$V = \frac{e^2}{4\pi\varepsilon\varepsilon_0 |d - x|}$$

is the Coulomb interaction, $\phi^{(0)}$ is the nonperturbed wave function of the ith QD, $\varepsilon = 12.9$ is the dielectric constant of GaAs, d is distance between QDs.

The energy spectrum is shown in Fig. 1. The external fields were used to tune the electron energy levels. In contrast to sequences of atoms, the influence of external fields becomes essential for magnetic fields B about 1 T and electric fields F about 10^4 V/m.

4. THE TUNNELING PROBABILITY

We consider the tunneling probability in quasiclassical approximation [8]. We study the influence of several control parameters on the tunneling probability. The layered structure of GaAs and its solid solutions $Al_xGa_{1-x}As$ and $In_xGa_{1-x}As$ with $0.1 \le x \le 1.0$ is considered as a sequence of QDs separated by a GaAs barrier layer. The probability of tunneling through the potential barrier generated by parabolic confining potentials of neighboring QDs (Fig. 2) is equal to

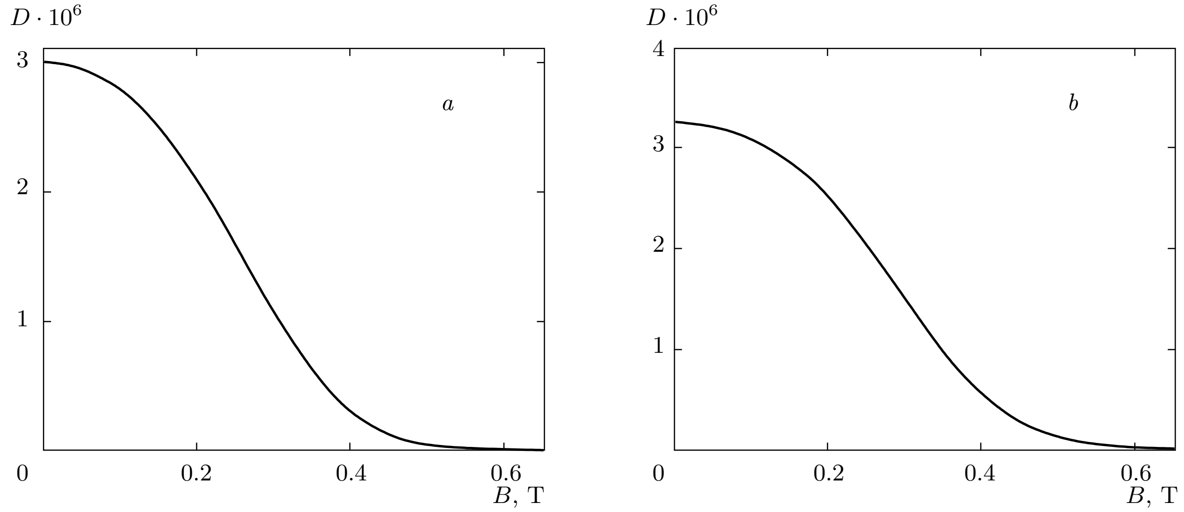

Fig. 5. The electron tunneling probability through three generations of aperiodic (a) Thue–Morse and (b) double-periodic sequences versus the external magnetic field B. The distances are $d_A = 90$ nm and $d_B = 110$ nm, and the confining potential steepness is $\alpha = 0.5 \cdot 10^{-7}$ J/m^2

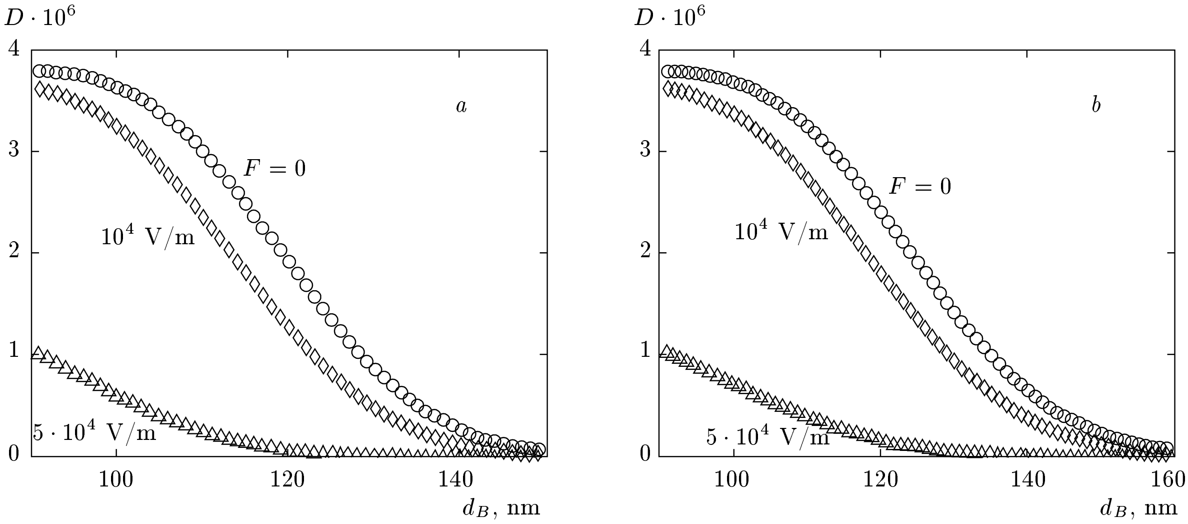

Fig. 6. The electron tunneling probability through three generations of aperiodic (a) Thue–Morse and (b) double-periodic sequences versus the distance d_B between QDs in an external electrical field F. The distance $d_A = 90$ nm, the confining potential steepness $\alpha = 0.5 \cdot 10^{-7}$ J/m^2

$$D = e^{-2\gamma}\left(1 + \frac{1}{4}e^{-2\gamma}\right)^{-2}, \tag{7}$$

where $\gamma = \hbar^{-1}\left|\int_a^b p\,dx\right|$, a and b are turning points, $p = \sqrt{2m^*(E-U)}$ is the momentum of the electron, $U_1 = \alpha_1 x^2/2$ for $x \in [a, x_0)$, and $U_2 = \alpha_1 x^2/2$ for $x \in [x_0, b)$.

For a wide and high potential barrier treated in the quasiclassical approximation, we have $e^{-2\gamma} \ll 1$ and

$$D = e^{-2\gamma}. \tag{7$'$}$$

In Fig. 3, we show the dependence of the tunneling probability for double QDs with equal confining potentials on the confining potential steepness and the distance between the QDs. It can be seen that an increase in the distance leads to a decrease in the tunneling probability. The tunneling probability decreases as the confining potential steepness increases. We estimated the tunneling probability through several generations of Thue–Morse and double-periodic sequences with a definite distance between the QDs. The influence of differ-

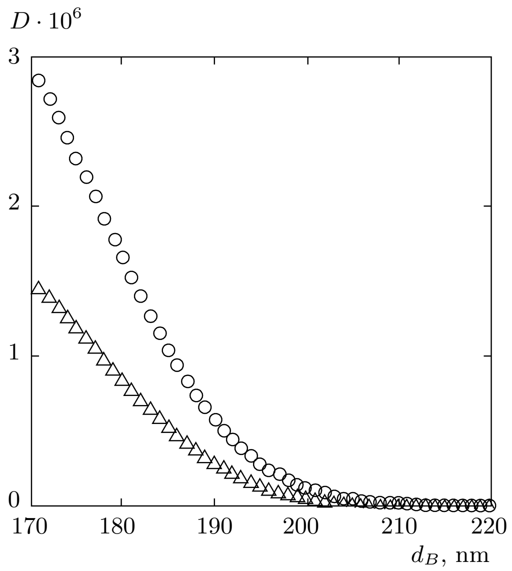

Fig. 7. The electron tunneling probability through three generations of the Thue–Morse aperiodic sequence vs. the distance between QDs at $T = 273$ K, the confining potential steepness $\alpha = 10^{-7}$ J/m^2, the distance $d_A = 150$ nm, and $n = 1$ for all QDs. Circles correspond to the absence of the Coulomb blockade, and triangles correspond to the Coulomb blockade taken into account

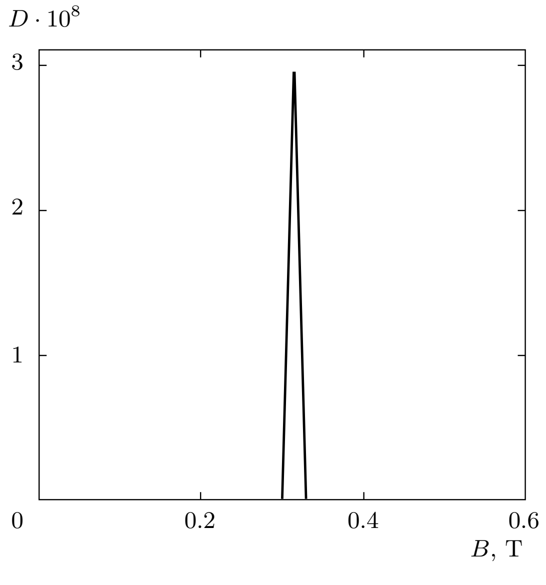

Fig. 8. Resonance tunneling through a double QD. The electron energy levels are shifted by the magnetic field and aligned at $B = 0.31$ T. The distance between the QDs is $d = 110$ nm

ent control parameters such as the distance between the QDs, the confining potential steepness, and the values of external fields were considered. Three generations of each sequence were investigated. The elastic cotunneling was considered at zero temperature [10], because the crossover temperature between elastic and inelastic cotunneling in our case is about 10 K. The resulting transmission coefficient is then equal to the product of transmission coefficients between pairs of QDs (the possible oscillations of the transmission coefficient were not taken into account):

$$D = D_A^{N_A} D_B^{N_B}, \tag{8}$$

where D_A and D_B are the tunneling probabilities of pairs of QDs separated by the respective distances d_A and d_B, N_A and N_B are the numbers of type A and type B QDs. The results are shown in Figs. 4–6. In contrast to arrays of atoms, the magnetic fields of about 0.1 T and electric fields of about 10^4 V/m essentially affect the tunneling probability. Also in contrast to one-dimensional periodic sequences, the tunneling is possible at finite values of fields (0–0.6 T in Fig. 4).

We estimated the influence of the Coulomb blockade on the tunneling probability in aperiodic sequences of QDs. A spherical QD was considered with the capacitance $C = 4\pi\varepsilon_0\varepsilon r$, where $\varepsilon = 12.9$, and r is the QD radius. The energy of such a spherical capacitor is $E = e^2/2C$. The tunneling probability with the Coulomb blockade was estimated as

$$D_C = DW, \tag{9}$$

where $W = \exp(-E/kT)$ and D is defined in Eq. (7′).

The results for the Thue–Morse sequence are shown in Fig. 7. It can be seen that the Coulomb blockade essentially decreases the tunneling probability through aperiodic sequence of QDs.

5. THE RESONANCE TUNNELING EFFECT

In general, when the confining potential steepness is defined, the electron energy levels corresponding to the same n for different QDs are not equal (see Eq. (2)). We can use external fields to shift and align the electron energy levels in neighboring QDs. When energy levels become equal, the resonance tunneling states (current states) appear. This elastic tunneling is the most essential contribution to electron transport [11]. The obtained resonance peak corresponds to electron tunneling from $n = 1$ of the first QD with $\alpha_1 = 5 \cdot 10^{-8}$ J/m^2 to $n = 0$ of the second QD with $\alpha_2 = 7.7 \cdot 10^{-7}$ J/m^2, when these levels are aligned by the external magnetic

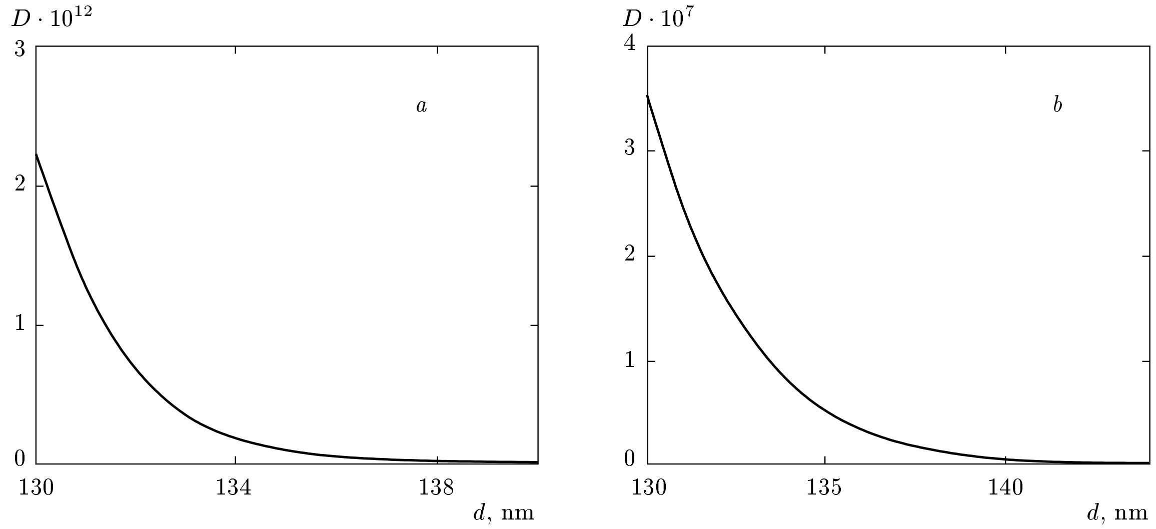

Fig. 9. The electron tunneling probability through three generations of (a) Thue–Morse and (b) double-periodic sequences versus the distance between the QDs. Steepnesses of the confining potential are $\alpha_1 = 7.7 \cdot 10^{-8}$ J/m^2 and $\alpha_2 = 7.7 \cdot 10^{-7}$ J/m^2. The electron energy levels are aligned by the magnetic field $B = 0.155$ T

field (Fig. 8). In this case, the tunneling probability is given by

$$D_n = D_{AA}^{N_{AA}} D_{BB}^{N_{BB}} D_{AB}^{N_{AB}}, \quad (10)$$

where D_{AA}, D_{BB}, and D_{AB} are the tunneling probabilities through double QDs AA, BB, and AB; N_{AA} and N_{BB} are numbers of pairs of types AA and BB; and N_{AB} is total number of pairs of types AB and BA for the sequence. This means that the tunneling conductivity corresponds to an independent sequential tunneling from one QD to another, as is usually the case with elastic cotunneling [10]. We obtained the dependence of the tunneling probability for each type of QD pairs on the distance between the QDs. The tunneling probabilities for different types of QDs differ by several orders of magnitude, and the smallest tunneling probability is through a pair of BB-type QDs in the case $\alpha_1 < \alpha_2$. The tunneling probability through the Thue–Morse and double-periodic sequences in the case $\alpha_1 < \alpha_2$ is presented on Fig. 9. The electron tunneling probability in the double-periodic sequence is several orders of magnitude higher than in the Thue–Morse sequence, because the double-periodic sequence does not contain BB pairs. If $\alpha_1 > \alpha_2$, then the lower tunneling probability is through the AA pair. In this case, the tunneling probability in the Thue–Morse sequence is higher than in the double-periodic sequence. As we can see, the aperiodic structure type (e. g., the Thue–Morse or double-periodic) and the control parameters (e. g., the confining potential) have an essential effect on the tunneling probability.

6. CONCLUSIONS

We have investigated the influence of the electric and magnetic fields on the electron energy spectrum and electron transport in the Thue–Morse and double-periodic sequences of quantum dots. Unlike with sequences of atoms, relatively small fields about 1 T and 10^4 V/m essentially affect the energy spectrum and transport properties. The tunneling probability was estimated in the quasiclassical approximation and the Coulomb blockade effect was taken into account. In contrast to periodic sequences, the current states survive at finite values of external fields. The magnetic and electric fields shift the electron energy levels, and resonance tunneling occurs when the energy levels become equal. Increasing the external electric and magnetic fields leads to localization of excitations, but the localization of excitations occurs at finite values of the perturbations in contrast to the case of periodic one-dimensional sequences.

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