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# PERIPHERICAL PROCESSES $2 \to 3$ and $2 \to 4$ IN QED AND QCD IN $p(\bar{p})p$ HIGH-ENERGY COLLISIONS

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Differential cross sections of processes with high-energy $p(\bar{p})p$ collisions — creation of a scalar, a pseudoscalar and a lepton pair — are considered in the Weizsäcker–Williams approximation in QED in the QCD framework, processes with conversion of the initial proton (antiproton) to fermionic jets accompanied with one gluon jet and the state of two gluons and a quark–antiquark pair (without a rapidity gap) are considered in the framework of the effective Reggeon action of Lipatov's theory. The process of creation of a Higgs boson accompanied with two fermionic jets is considered. The azimuthal correlation in the process of two gluon jets separated by a rapidity gap is investigated. The gluon Reggeization effects are taken into account. Some distributions are illustrated by numerical calculations.

## 1. INTRODUCTION

Motivation of this paper is the construction of realistic formulas and the estimation of the cross section with creation of two jets in the proton (antiproton) fragmentation regions and one or two additional jets in a multi-Regge kinematics. Application of QCD methods to the description the peripherical processes in the high-energy proton (antiproton)–proton scattering is based on the proof of the gluon Reggeization phenomenon, which was done in Refs. [1] in 1973-1976. For this, we use the effective Regge action [2, 3] of conversion of two Reggeized gluons $R$ to some set of real particles $P, Q$ (one and two gluons separated by a rapidity gap, two gluons or a quark–antiquark pair, without rapidity gap and a scalar (Higgs) meson).

The paper is organized as follows. After a short review of QED processes in Sec. 2, we consider processes of a single and two gluon production in Sec. 3 and a lepton pair production in Sec. 4, assuming the absence of a rapidity gap between the couple of particles created in the pionization region. Measuring these processes provides the possibility to check the $RRP, RRPP$, and $RRq\bar{q}$ vertices of a effective Regge action theory [3]. In Sec. 5, the azimuthal correlation between two gluon jets separated by some rapidity gap is considered. In Sec. 6, the production of a quark–antiquark pair is considered. In Sec. 7, the Higgs boson production is considered. In the Conclusion we discuss the main topics of our approach and give the results of numerical calculations.

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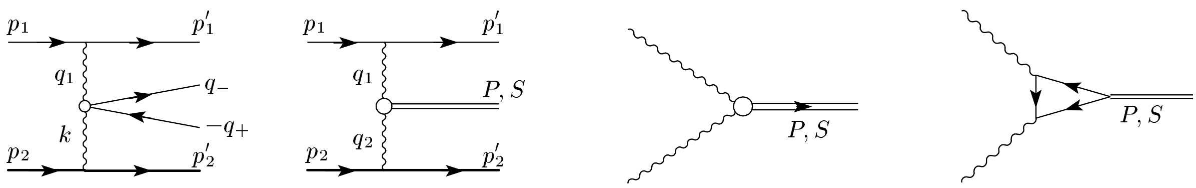
## 2. QED PROCESSES

In the early 1970s, the processes of creation of some set of particles were intensively studied [4, 5]. A different mechanism of pair production in electron–positron collisions was considered. The relevant formulas can in principle be applied to proton–proton (antiproton) collisions. Production of some set of particles in the pionization region in high-energy $p(\bar{p})p$ collisions

$$p(\bar{p}) + p \to p(\bar{p}) + p + F \tag{1}$$

is described by (see [3, 6])




*a* *b* *c*

**Fig. 1.** Feynman diagrams for creation of two jets ($a$) and one jet ($b$) by two Reggeized gluons, creation of $P$- and $S$-mesons by two-Reggeized gluons ($c$)

$$\left(\frac{s_1 d\sigma}{ds_1}\right)^{pp\to ppF} = \left(\frac{\alpha}{2\pi}\right)^2 \ln^2\left(\frac{s}{M_p^2}\right) f\left(\frac{s_1}{s}\right) \times \times \sigma_{tot}^{\gamma\gamma\to F}(s_1)\left(1 + O\left(\frac{1}{\ln(s/M_p^2)}\right)\right), \quad (2)$$
$$f(z) = (2+z)^2 \ln\frac{1}{z} - 2(1-z)(3+z),$$

where $s_1$ is the invariant mass square of the produced set of particles $F$.

In the case where a lepton pair is created outside the proton fragmentation regions, the cross section of the process

$$p(p_1) + p(\bar p)(p_2) \to p(p_1') + p(\bar p)(p_2') + + \mu^+(q_+) + \mu^-(q_-) \quad (3)$$

(see Fig. 1$a$) has the form

$$d\sigma^{p\bar p\to l\bar l p\bar p} = \frac{2\alpha^4}{\pi}\frac{d^2q_1 d^2q_2 d^2k_1 dx}{\pi^3} \times \times \frac{d\beta_1}{\beta_1}\frac{\mathbf{q}_1^2\mathbf{q}_2^2}{(\mathbf{q}_1^2 + M^2\beta_1^2)^2(\mathbf{q}_2^2 + M^2\alpha^2)^2}F, \quad (4)$$

where $M$ is the proton mass,

$$s\alpha = \frac{-c}{\beta_1 x(1-x)}, \quad 0 < x = \frac{\beta_2}{\beta_1} < 1, \quad \beta_1 \ll 1, \quad (5)$$

$$c = m^2 + \mathbf{q}_2^2 + \mathbf{q}_1^2 x + 2\mathbf{q}_1\cdot\mathbf{q}_2 x, \quad (6)$$

$$c_1 = m^2 + (\mathbf{k}_2 - \mathbf{q}_2)^2 + \mathbf{q}_1^2 x + 2\mathbf{q}_1\cdot(\mathbf{q}_2 - \mathbf{k}_2)x, \quad (7)$$

$$\mathbf{q}_1^2\mathbf{q}_2^2 F = \frac{\mathbf{q}_2^2\mathbf{q}_1^2}{cc_1} - \frac{x\bar x}{c^2c_1^2} \times \times \left[(\mathbf{q}_1^2 + 2\mathbf{q}_1\cdot\mathbf{q}_2)(\mathbf{q}_2^2 - 2\mathbf{k}_2\cdot\mathbf{q}_2) + + 2(\mathbf{q}_2\cdot\mathbf{q}_1)(m^2 + \mathbf{k}_2^2)\right]^2. \quad (8)$$

Here $m$ is the lepton mass, $x_1 = 1 - \beta_1 \approx 1$ and $-\mathbf{q}_1$ are the energy fraction of the scattered proton and its momentum transverse to the initial proton direction $\mathbf{p}_1$ (the center of mass of the initial particles understood), $1 + \alpha \approx 1$, $\mathbf{q}_2$ are similar quantities for the scattered proton (antiproton),

$$x\beta_1 + \frac{m^2 + \mathbf{k}_1^2}{s\beta_1 x}, \; -\mathbf{k}_1$$

and

$$(1-x)\beta_1 + \frac{m^2 + \mathbf{k}_2^2}{s\beta_1(1-x)}, \quad \mathbf{k}_2 = \mathbf{q}_1 - \mathbf{q}_2 - \mathbf{k}_1$$

are the corresponding quantities for negative and positive charged leptons from the pair created; $m$ is the mass of the created particle.

For the two-photon processes with creation of a pseudoscalar and scalar particle, we use the corresponding subprocess

$$\gamma(q_1, \mu) + \gamma(q_2, \nu) \to P(S)$$

(see Fig. 1$b,c$) with matrix elements described in terms of triangle Feynman loop diagrams with quarks as internal fermions:

$$M^{\gamma\gamma P} = \frac{2\alpha N_P g_p}{\pi m_q}(q_1e_1q_2e_2)I_P,$$
$$(q_1e_1q_2e_2) = \epsilon^{\alpha\beta\gamma\sigma}q_{1\alpha}e_{1\beta}q_{2\gamma}e_{2\sigma}, \quad (9)$$
$$M^{\gamma\gamma S} = \frac{2\alpha N_S g_S}{\pi m_q}[(q_1q_2)(e_1e_2) - (e_1q_2)(e_2q_1)]I_S,$$

where $e_{1,2}(q_{1,2})$ are the polarization vectors of photons and $N_{P,S}$ are the color factors:

$$N_P = N_c\left(\frac{4}{9} - \frac{1}{9}\right) = 1,$$
$$N_S = N_c\left(\frac{4}{9} + \frac{1}{9}\right) = \frac{5}{3}. \quad (10)$$

Performing the loop momentum integration, we obtain

$$I_{P,S} = \int_0^1 dx \int_0^1 \frac{y\,dy}{d_{P,S}}(1, 1-4y^2x(1-x)),$$
$$d_{P,S} = 1 - y^2x(1-x)\frac{M_{P,S}^2}{m_q^2} - - y(1-y)\left[x\frac{q_1^2}{m_q^2} + (1-x)\frac{q_2^2}{m_q^2}\right], \quad (11)$$

where $M_{P,S}$ and $m_q$ are the masses of the produced particles and the quark mass. We can use the Goldberger–Treiman relation

$$\frac{g_P}{m_q} = \frac{1}{F_\pi},$$

with

$$F_\pi = 93\ \text{MeV}$$

being the decay constant of a charged pion, and a similar relation

$$\frac{g_S}{m_q} = \frac{1}{F_\sigma},$$
$$F_\sigma \approx F_\pi.$$

In inserting these matrix elements into the matrix element of $2 \to 3$ the process, the combination

$$M^{\gamma\gamma F}(e_1 \to p_1, e_2 \to p_2)/s = m^{\gamma\gamma F}$$

is used. We obtain

$$m^{\gamma\gamma P} = \frac{\alpha N_P}{\pi F_\pi}[\mathbf{q}_1, \mathbf{q}_2]_z I_P,$$
$$m^{\gamma\gamma S} = \frac{\alpha N_S}{\pi F_\sigma}(\mathbf{q}_1, \mathbf{q}_2) I_S, \quad (12)$$

where we consider the four-momenta of the virtual photons to be essentially transverse two-component Euclidean vectors:

$$\mathbf{p}_1 \cdot \mathbf{q}_{1,2} = 0, \quad q_{1,2}^2 = -\mathbf{q}_{1,2}^2 < 0.$$

The cross sections of the processes of a single meson production in the pionization region are

$$d\sigma^{pp\to ppP} = \frac{2\alpha^4}{\pi}\frac{d\beta_1}{\beta_1} dN_1 dN_2 C_P \sin^2\theta,$$
$$d\sigma^{pp\to ppS} = \frac{2\alpha^4}{\pi}\frac{d\beta_1}{\beta_1} dN_1 dN_2 C_S \cos^2\theta, \quad (13)$$

where $\theta$ is the azimuthal angle between two-dimensional vectors $\mathbf{q}_1$ and $\mathbf{q}_2$,

$$C_P = \left|\frac{N_P}{F_\pi} I_P\right|^2, \quad C_S = \left|\frac{N_S}{F_\sigma} I_S\right|^2, \quad (14)$$

and the Weizsäcker–Williams enhanced factors are

$$dN_1 = \frac{\mathbf{q}_1^2 d^2\mathbf{q}_1}{(\mathbf{q}_1^2 + m_p^2\beta_1^2)^2},$$
$$dN_2 = \frac{\mathbf{q}_2^2 d^2\mathbf{q}_2}{(\mathbf{q}_2^2 + m_p^2\alpha_2^2)^2}, \quad (15)$$
$$|s\alpha_2\beta_1| = M_{P,S}^2 + (\mathbf{q}_1 + \mathbf{q}_2)^2.$$

We use the expression of the squared four-vectors of momenta transferred to the a lepton pair:

$$q_1^2 \approx -(\mathbf{q}_1^2 + m_p^2\beta_1^2),$$
$$q_2^2 \approx -(\mathbf{q}_2^2 + m_p^2\alpha_2^2). \quad (16)$$

These factors, being integrated, produce the "large logarithmic" factors

$$\frac{1}{\pi}\int_0^{Q^2} dN_1 = \ln\frac{q^2}{m_p^2\beta_1^2} - 1, \quad m_p^2 \ll Q^2 \ll s. \quad (17)$$

## 3. QCD PROCESSES. CHECK OF *RRP* VERTEX

Using Gribov's prescription for the Green's function of the exchanged gluon in the process

$$p(\bar p)(p_1) + p(p_2) \to \text{jet}(X_1) + \text{jet}(X_2),$$

we express the matrix element in form

$$M^{pp\to j_1 j_2} = \frac{4\pi\alpha_s}{q^2}\langle X_1|J_\mu t^a|p_1\rangle\langle X_2|J_\nu t^a|p_2\rangle \times \times \frac{2}{s}p_2^\mu p_1^\nu = \frac{8\pi\alpha_s s}{q^2}\Phi_1^a\Phi_2^a, \quad (18)$$

$$\Phi_1^a = \frac{1}{-s\alpha}\langle X_1|\mathbf{J}\mathbf{q}t^a|p_1\rangle,$$
$$\Phi_2^a = \frac{1}{s\beta}\langle X_2|\mathbf{J}\mathbf{q}t^a|p_2\rangle, \quad s = (p_1+p_2)^2 \gg M_p^2, \quad (19)$$

where $t^a$ are generators of the color $SU(N)$ group, and we use the gauge conditions

$$q^\mu\langle X_{1,2}|J_\mu|p_{1,2}\rangle = 0$$

and the Sudakov parameterization

$$q = \alpha p_2 + \beta p_1 + q_\perp$$

for the four-momentum of the exchanged gluon.

Quantities $(-s\alpha)$ and $s\beta$ can be interpreted in terms of the invariant mass squared of fermionic jets created by the initial protons:

$$(p_1 - q)^2 \approx M_1^2 \approx -\mathbf{q}^2 - s\alpha, \quad (20)$$

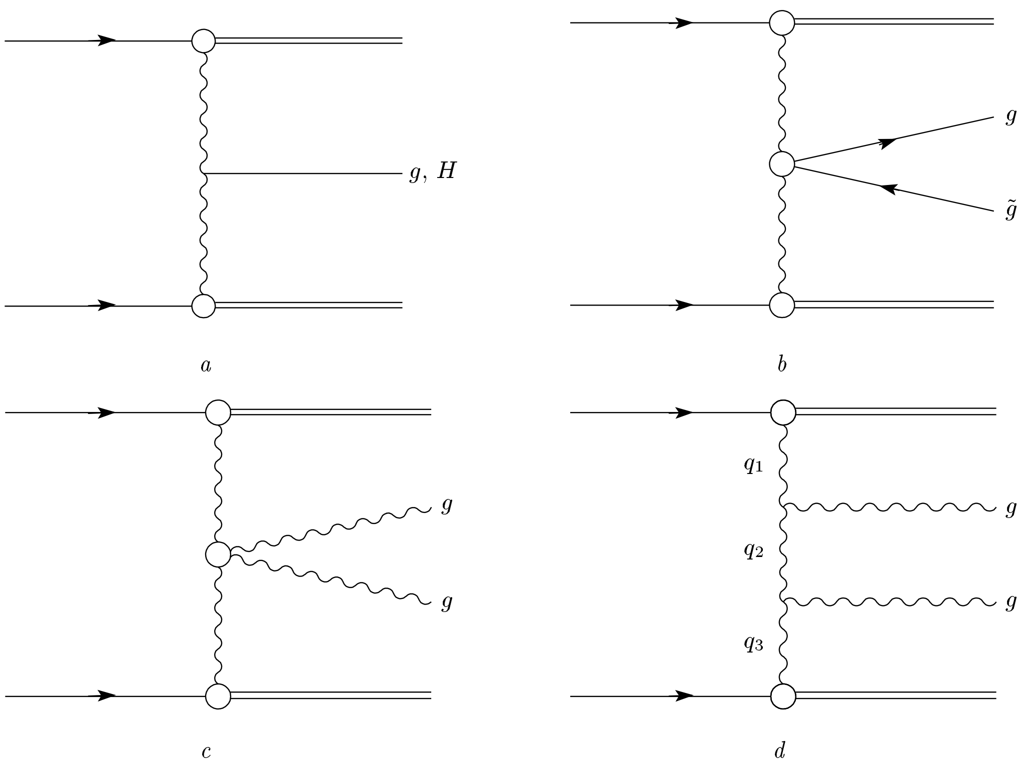



**Fig. 2.** Feynman diagrams of a single-gluon jet and a Higgs boson ($a$), quark–antiquark jets ($b$), two-gluon jets ($c$), two gluons separated by a rapidity gap ($d$)

$$(p_2 + q)^2 \approx M_2^2 \approx -\mathbf{q}^2 - s\beta. \tag{21}$$

As was shown in Refs. [7] the phenomenon of Reggeization of gluon Green's function occurs in the kinematics $s \gg |q^2|$ which amounts to replacing the ordinary gluon with the Reggeized gluon with the same quantum numbers except of the "moving" gluon spin (its Regge trajectory). The matrix element of processes with a Reggeized gluon exchange acquire the Regge factor

$$R = \left(\frac{s}{s_0}\right)^{\alpha(q^2)},$$

where

$$\alpha(q^2) = 1 - \alpha' \mathbf{q}^2$$

is the trajectory of the gluon Regge pole (specified below).

The matrix element of the creation of an additional gluon has the form (see Fig. 2$a$)

$$M^{PP \to j_1 j_2 g} = s \frac{(4\pi\alpha_s)^{3/2}}{q_1^2 q_2^2} \frac{\langle X_1 | \mathbf{J} \mathbf{q}_1 t^a | p_1 \rangle}{-s\alpha_1} \times \\ \times \frac{\langle X_2 | \mathbf{J} \mathbf{q}_2 t^b | p_2 \rangle}{s\beta_2} f^{abc} C_\mu e_\mu^c(k), \quad k = q_1 - q_2, \tag{22}$$

with

$$C_\mu = n_-^\alpha n_+^\beta \Gamma_{\alpha\beta\mu}, \quad n_- = \frac{2p_1}{\sqrt{s}}, \quad n_+ = \frac{2p_2}{\sqrt{s}},$$

$$n_+^2 = n_-^2 = 0, \quad n_+ n_- = 2,$$

where

$$C_\mu = 2\left[(n_-)_\mu \left(q_1^+ + \frac{\mathbf{q}_1^2}{q_2^-}\right) + \right. \\ \left. + (n_+)_\mu \left(q_2^- + \frac{\mathbf{q}_2^2}{q_1^+}\right) - (q_1 + q_2)_\mu\right] \tag{23}$$

is the effective vertex of conversion of two Reggeized gluons to a real gluon, with the properties

$$C_\mu(q_1, q_2)(q_1 - q_2)_\mu = 0, \quad C_\mu^2 = \frac{16\mathbf{q}_1^2 \mathbf{q}_2^2}{(-q_1^+ q_2^-)}, \\ (q_1 - q_2)^2 = M_g^2 = -q_1^+ q_2^- - (\mathbf{q}_1 - \mathbf{q}_2)^2. \tag{24}$$

In the framework of the fermion-jet model, we replace the set of particles consisting of the jet developed by the initial proton with an on-shell proton and additionally modify the vertex of its interaction with the (Reggeized) gluon,

$$\langle X_1|J_\mu t^a|p_1\rangle = J^a_\mu = \bar{u}(p'_1+P_1)t^a V_\mu u(p_1),\quad V_\mu = \gamma_\mu - \frac{p_2^\mu}{q_1p_2}\hat{q}_1, \tag{25}$$

where the following notation enters

$$\hat{a} \equiv a_\mu\gamma^\mu.$$

This vertex function obeys the gauge condition

$$J_1^\mu q_1^\mu = 0.$$

We also have

$$\frac{1}{s}J^a_\mu p_2^\mu = \frac{1}{-s\alpha_1}\mathbf{J^a q_1^a},\quad \int d(s\alpha_1)\,d\gamma_1\sum\left(\frac{\mathbf{J}^a_\mu p_2^\mu}{s}\right)\left(\frac{\mathbf{J}^b_\nu p_2^\nu}{s}\right) = \frac{1}{2}\delta_{ab}\frac{2\mathbf{q}_1^2}{\bar{M}^2+\mathbf{q}_1^2}, \tag{26}$$

where $d\gamma_1$ is the phase volume of the proton jet defined in (29) and $\bar{M}$ is the average value of the invariant mass of the proton jet and the summation goes over initial particles spin projections.

For the matrix element squared averaged over the final states in the fermion-jet model we obtain

$$\int d(s\beta_2)\,d\gamma_2\int d(s\alpha_1)\,d\gamma_1\sum|M|^2 = \frac{s^2 2^6\pi^3\alpha_s^3}{\mathbf{q}_1^2\mathbf{q}_2^2}\times \frac{N(N^2-1)}{M_g^2+(\mathbf{q}_1-\mathbf{q}_2)^2}\frac{\mathbf{q}_1^2}{M^2+\mathbf{q}_1^2}\frac{\mathbf{q}_2^2}{M^2+\mathbf{q}_2^2}. \tag{27}$$

Considering the phase volume of the process

$$p(\bar{p})p\to j_1j_2F$$

we introduce two auxiliary variables the four-momenta of the exchanged gluons

$$\int d^4q_1d^4q_2\delta^4(p_1-q_1-p'_1-P_1)\times \delta(q_2+p_2-p'_1-P_2) = 1. \tag{28}$$

We then have

$$d\Gamma_3 = (2\pi)^{-2}d^4q_1d^4q_2d\gamma_1d\gamma_2d\gamma_j, \tag{29}$$

$$d\gamma_1 = \frac{d^3p'_1}{2\varepsilon'_1}\prod_i\frac{d^3r_i}{2\varepsilon_i(2\pi)^3}\delta^4(p_1-q_1-p'_1-P_1),\quad P_1 = \sum_i r_i,$$

$$d\gamma_2 = \frac{d^3p'_2}{2\varepsilon'_2}\prod_i\frac{d^3v_i}{2\varepsilon_i(2\pi)^3}\delta^4(p_2+q_2-p'_2-P_2),\quad P_2 = \sum_i v_i,$$

$$d\gamma_j = \prod_i\frac{d^3l_i}{2\varepsilon_i(2\pi)^3}\delta^4(q_1-q_2-P_j),\quad P_j = \sum_i l_i. \tag{30}$$

Using

$$d^4q_1d^4q_2 = \frac{s}{2}d\alpha_1d\beta_1d^2q_1\frac{s}{2}d\alpha_2d\beta_2d^2q_2 = \frac{\pi^2}{4s}d(s\alpha_1)\,d(s\beta_2)\frac{d\beta_1}{\beta_1}d(s\alpha_2\beta_1)\frac{d^2q_1d^2q_2}{\pi^2}, \tag{31}$$

we write $d\Gamma_3$ in the form

$$d\Gamma_3 = \frac{1}{2^7\pi^3s}\frac{d\beta_1}{\beta_1}\cdot d\Phi_1\cdot d\Phi_2\cdot d\Phi_g\frac{d^2q_1d^2q_2}{\pi^2},\quad d\Phi_{1,2} = dM^2_{1,2}d\gamma_{1,2},\quad d\Phi_g = dM_g^2d\gamma_j. \tag{32}$$

In the fermion-jet model approximation, we obtain

$$d\Gamma^{(3)} = (2\pi)^{-5}\frac{\pi^2}{4s}\frac{d\beta_1}{\beta_1}\frac{d^2q_1d^2q_2}{\pi^2}. \tag{33}$$

For the cross section of the process

$$pp\to j_1j_2j_g,$$

we obtain

$$d\sigma = \frac{\alpha_s^3}{16M_g^2}N(N^2-1)R_2dL_1I(\rho),\quad dL_1 = \frac{d\beta_1}{\beta_1}, \tag{34}$$

$$R_2 = \left(\frac{s_1}{s_0}\right)^{2(\alpha(\vec{q}_1^2)-1)}\left(\frac{s_2}{s_0}\right)^{2(\alpha(\vec{q}_0^2)-1)} \approx \left(\sqrt{s}\ [\mathrm{GeV}]\right)^{-4\frac{\alpha_s}{\pi}\mathbf{q}^2\ [\mathrm{GeV}^2]}, \tag{35}$$

$$I(\rho) = \int_0^\infty\int_0^\infty\frac{dx_1d_2}{(x_1+\rho)(x_2+\rho)\sqrt{(1+x_1+x_2)^2-4x_1x_2}},\quad \rho = \frac{\bar{M}^2}{M_g^2}. \tag{36}$$

The function $R_2$ is tabulated in the Table, the function $I(\rho)$ is plotted in Fig. 3.

**Table.** Estimation of the gluon reggeization factor $R_2$

| $\sqrt{s}$ [GeV] | $q^2$ [GeV$^2$] | | | |
|---|---|---|---|---|
| | 0.5 | 1 | 3 | 5 |
| 1000 | 0.2512 | 8.0631 | 0.00025 | $10^{-6}$ |
| 7000 | 0.17021 | 0.0289 | 0.000024 | $2.041 \cdot 10^{-8}$ |
| 14000 | 0.1482 | 0.02196 | 0.0000106 | $5.102 \cdot 10^{-9}$ |

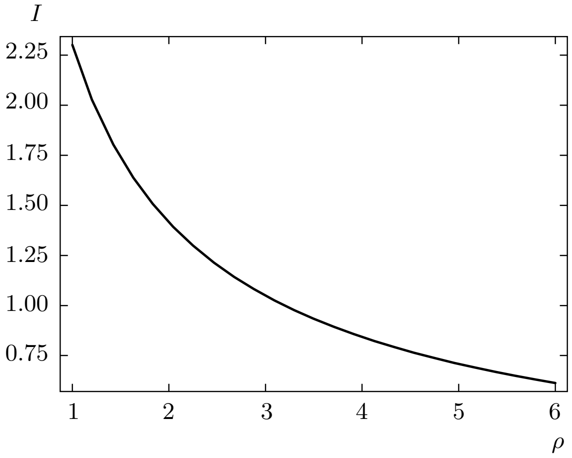



**Fig. 3.** $I(\rho)$ (see (36)) for the process $pp \to j_1 j_2 j_g$

## 4. QCD PROCESSES. CHECK OF *RRPP* VERTEX

We now consider the process of the creation of two gluons not separated by a rapidity gap,

$$p(p_1) + p(p_2) \to \mathrm{jet}_1(X_1) + \mathrm{jet}_2(X_2) + g(k_1) + g(k_2). \quad (37)$$

For the case of production of two particles

$$RR \to a(k_1) + b(k_2)$$

(with no rapidity gap between $a$ and $b$), we obtain the phase volume

$$d\Gamma_4 = \frac{1}{2^{11}\pi^5} \frac{dx}{x\bar{x}} \frac{d^2q_1 d^2q_2 d^2k_1}{\pi^3} \quad (38)$$

with

$$k_i = b_i p_1 + a_i p_2 + k_\perp, \quad x = \frac{b_1}{\beta_1}, \quad y = -\frac{a_1}{\alpha_2}.$$

The differential cross section of $ab$ pair production ($ab = gg, q\bar{q}$) in the fermion-jet model can be written as

$$d\sigma^{pp\to j_1 j_2 ab} = \frac{\alpha_s^4}{2^6\pi} dLR_2 \frac{dx}{x(1-x)} \frac{d^2k_1}{\pi} \times \frac{d^2q_1 d^2q_2}{\pi^2} \Phi^{ab}, \quad \Phi^{ab} = \frac{\sum |M^{RRab}|^2}{\mathbf{q}_1^2 \mathbf{q}_2^2}. \quad (39)$$

The explicit expression for $\Phi^{gg}$ is [3, 8] (with $M^{RRgg} = M^{RRPP}$):

$$\Phi^{gg} = \frac{\sum |M^{RRPP}|^2}{\mathbf{q}_1^2 \mathbf{q}_2^2}.$$

It was obtained in Refs. [2, 3, 7–9] that

$$\sum |M^{RRPP}|^2 = G_1 (a^{\nu_1\nu_2}(k_1,k_2))^2 + G_2 \Omega_{\sigma\sigma'}(k_1) \times \Omega_{\rho\rho'}(k_2) a^{\sigma\rho}(k_1,k_2) a^{\rho'\sigma'}(k_2,k_1) + (k_1 \leftrightarrow k_2), \quad (40)$$

where

$$G_1 = (f_{d_1 d_2 r} f_{cdr})^2 = N^2(N^2-1), \quad G_2 = f_{d_1 d_2 r} f_{cdr} f_{d_2 cr} f_{d_1 dr} = -\frac{1}{2} N^2 (N^2-1), \quad (41)$$

the projection operator is

$$\Omega_{\sigma\sigma'}(k) = -g^\perp_{\sigma\sigma'} - \frac{2}{\mathbf{k}^2} k_{\sigma\perp} k_{\sigma'\perp}, \quad (42)$$

and

$$a^{\nu_1\nu_2}(k_1,k_2) = 4\left[\frac{1}{t} q_\perp^{\nu_1} q_\perp^{\nu_2} - \frac{1}{\chi} q_\perp^{\nu_1}\left(k_1 - \frac{x}{\bar{x}} k_2\right)^{\nu_2} + \frac{1}{\chi} q_\perp^{\nu_2}\left(k_2 - \frac{\bar{y}}{y} k_2\right)^{\nu_1} - \frac{x\mathbf{q}_2^2}{\chi \mathbf{k}_1^2} k_1^{\nu_1} k_1^{\nu_2} - \frac{\bar{y}\mathbf{q}_1^2}{\chi \mathbf{k}_2^2} k_2^{\nu_1} k_2^{\nu_2} - \frac{1}{\chi}\left(1 + \frac{tx}{\bar{x}\mathbf{k}_1^2}\right) k_1^{\nu_1} k_2^{\nu_2} + \frac{1}{\chi} k_1^{\nu_1} k_2^{\nu_2} - 2D g_\perp^{\nu_1\nu_2}\right], \quad (43)$$

with

$$D = 1 + \frac{t}{\chi} + \frac{\bar{x}\mathbf{k}_1^2}{tx} + \frac{1}{\chi}\left[\frac{\bar{x}}{x}\mathbf{k}_1^2 - \frac{x}{\bar{x}}\mathbf{k}_2^2\right] + \frac{\mathbf{q}_1^2}{\chi}\bar{y} + \frac{\mathbf{q}_2^2}{\chi} x. \quad (44)$$

Using the relations

$$s a_i b_i = \mathbf{k}_i^2 + m^2, \quad (45)$$

$$q = q_1 - k_1 = q_2 + k_2, \quad t = q^2, \quad \chi = (k_1 + k_2)^2,$$
$$t = -(\mathbf{q}_1 - \mathbf{k}_1)^2 - \frac{\bar{x}}{x}\mathbf{k}_1^2, \quad \chi = \frac{1}{x\bar{x}}(\bar{x}\mathbf{k}_1 - x\mathbf{k}_2)^2,$$

we can verify that the gauge conditions

$$D|_{\mathbf{q}_1\to 0} = D|_{\mathbf{q}_2\to 0} = 0, \quad (46)$$
$$a^{\nu_1\nu_2}(k_1,k_2)|_{\mathbf{q}_1\to 0} = 0,$$
$$a^{\nu_1\nu_2}(k_1,k_2)|_{\mathbf{q}_2\to 0} = 0,$$

are satisfied. Due to the gauge properties of $a^{\nu_1\nu_2}$, the quantity $\Phi^{gg}$ is finite as $\mathbf{q}_1, \mathbf{q}_2 \to 0$, which provides the convergence of

$$I^{gg}(\mathbf{k}_1) = \frac{1}{\pi^2}\int \frac{d^2\mathbf{q}_1 d^2\mathbf{q}_2}{(\mathbf{q}_1^2+\bar{M}^2)(\mathbf{q}_2^2+\bar{M}^2)}\Phi^{gg}, \qquad (47)$$

which is presented in Fig. 4*a*.

## 5. AZIMUTHAL CORRECTION IN THE PROCESS OF CREATION OF TWO GLUON-JETS, SEPARATED BY A RAPIDITY GAP

We now consider the process of two-gluon production, with the gluons separated by a rapidity gap. The corresponding matrix element contains three gluon Regge factors

$$R_3 = \left(\frac{s_1}{s_0}\right)^{\alpha(q_1)}\left(\frac{s_2}{s_0}\right)^{\alpha(q_2)}\left(\frac{s_3}{s_0}\right)^{\alpha(q_3)}$$

with the momenta of exchanged gluons

$$q_i = \alpha_i p_2 + \beta_i p_1 + q_{1\perp},$$

$$s_1 \approx -s\alpha_2, \quad s_2 \approx -s\alpha_3\beta_1,$$

$$s_3 = s\beta_2,$$

$$1 \gg \beta_1 \gg \beta_2 \gg \beta_3, \quad 1 \gg \alpha_3 \gg \alpha_2 \gg \alpha_1,$$

$$s_1 s_2 s_3 = [M_1^2 + (\bar{q}_1 - \bar{q}_2)^2][M_2 + (\bar{q}_2 - \bar{q}_3)^2]s, \qquad (48)$$

where $M_1^2, M_2^2$ are the invariant squared masses of the created gluon jets.

The matrix element has the form

$$M^{pp\to j_1 j_2 j_{g_1} j_{g_2}} = \frac{s(4\pi\alpha_s)^2}{2q_1^2q_2^2q_3^2}\frac{\langle X_1|\mathbf{J}\mathbf{q}_1 t^a|p_1\rangle}{(-s\alpha_1)} \times \\ \times \frac{\langle X_2|\mathbf{J}\mathbf{q}_2 t^b|p_2\rangle}{s\beta_3} \times \\ \times f_{adc_1} f_{bdc_2} C^{\mu}(q_1,q_2)e_{\mu}^{c_1} C^{\lambda}(q_2,q_3)e_{\lambda}^{c_2}. \qquad (49)$$

The phase volume of the process

$$pp \to j_1 j_2 j_{g_1} j_{g_2}$$

can be written in the form (in the fermion-jet model)

$$d\Gamma_4 = (2\pi)^{-8}\frac{\pi^3}{8s}\frac{d\beta_1}{\beta_1}\frac{d\beta_2}{\beta_2}\frac{d^2q_1 d^2q_2 d^2q_3}{\pi^3}. \qquad (50)$$

For the cross section, we obtain

$$d\sigma^{pp\to 4j} = \frac{4\alpha_s^4}{\pi}dL_1\Delta Y \frac{d\bar{q}_1^2 d\bar{q}_3^2 (d\varphi/2\pi)(d^2q_2/\pi)R_3 N^2(N^2-1)}{(\bar{q}_1^2+\bar{M}^2)(\bar{q}_3^2+\bar{M}^2)(M_{g_1}^2+(\bar{q}_1-\bar{q}_2)^2)(M_{g_2}^2+(\bar{q}_2-\bar{q}_3)^2)}, \qquad (51)$$

with

$$M_{p_1}^2 = -s\alpha_1, \quad M_{p_2}^2 = -s\beta_3, \quad M_{j_1}^2 = -s\alpha_2\beta_1,$$

$$M_{j_1}^2 = -s\alpha_3\beta_2, \quad L_1 = \ln\frac{s\beta_1}{M^2},$$

$$\Delta Y = \ln\frac{\beta_1}{\beta_2}$$

is the rapidity gap of gluon jets.

For the azimuthal correlation, (performing the integration over $d^2q_2$ and setting)

$$M_{g_1}^2 = M_{g_2}^2 = M_g^2$$

we obtain

$$\frac{2\pi d\sigma^{pp\to j_1 j_2 j_{g_1} j_{g_2}}}{dY d\varphi} = L_1\sigma_0 F(\varphi), \quad L_1 = \ln\frac{s\beta_1}{M^2}, \\ Y = \ln\frac{\beta_1}{\beta_2}, \quad \sigma_0 = \frac{4\alpha_s^4}{\pi M_g^2}N^2(N^2-1), \qquad (52)$$

$$F(\varphi) = \int\limits_0^{\infty}\frac{dx_1}{x_1+\rho}\int\limits_0^{\infty}\frac{dx_2}{x_2+\rho}\psi(z), \\ z = x_1 + x_2 - 2\sqrt{x_1x_2}\cos\varphi, \qquad (53)$$

$$\psi(z) = \frac{2}{\sqrt{z(4+z)}}\ln\frac{\sqrt{4+z}+\sqrt{z}}{\sqrt{4+z}-\sqrt{z}}, \quad \rho = \frac{\bar{M}^2}{M_g^2}.$$

The function $F(\varphi)$ is plotted in Fig. 5.

## 6. THE $pp \to jjq\bar{q}$ PROCESS. CHECK OF $RRqq$ VERTEX

The matrix element of the subprocess of conversion of two Reggeized gluons into a quark–antiquark pair (see Fig. 2*c*)

$$R(a,q_1) + R(b,-q_2) \to q(k_1) + \bar{q}(k_2) \qquad (54)$$

is described by two different mechanisms: direct interaction and the production of a gluon with its subsequent conversion into the quark pair [10]

$$M^{q\bar{q}} = \bar{u}(k_1)[At_at_b - Bt_bt_a]v(k_2), \qquad (55)$$

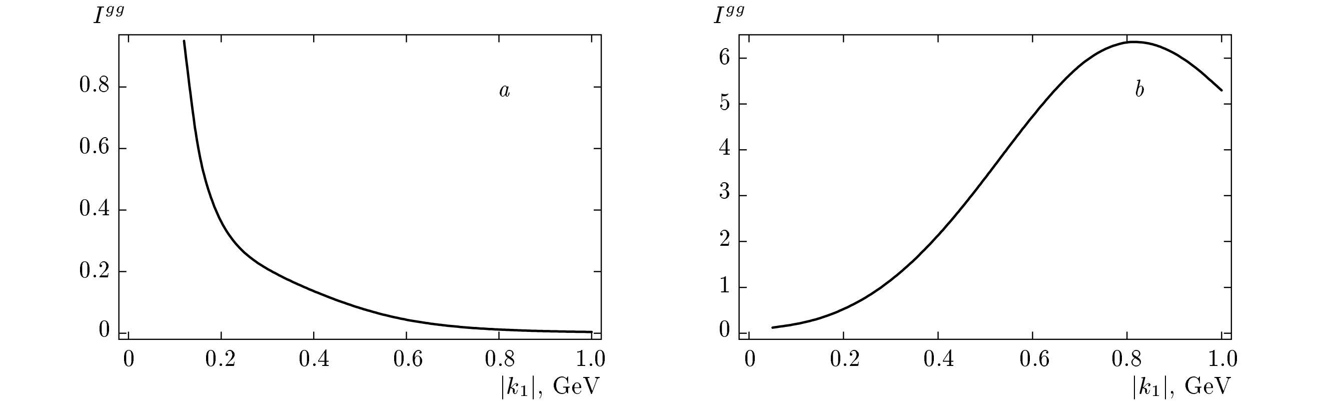



**Fig. 4.** Values $I^{gg}$ (47) ($a$) and $I^{qq}$ (61) ($b$) as functions of the transverse momentum modulus $|\mathbf{k}_1|$ of one of the gluons in the produced gluon pair in the case where $M_1 = M_2 = 1$ GeV, $x = 0.2$, and $y = 0.3$

with $t_a$ being the generator of the color $SU(N)$ group in the fermion representation,

$$\sum_a (t_a^2)^2 = I \frac{N^2-1}{2N},$$
$$\sum_{a,b} \mathrm{Tr}\,(t_a t_b t_a t_b) = -\frac{N^2-1}{4N}, \qquad (56)$$
$$\sum_{a,b} \mathrm{Tr}\,(t_a t_a t_b t_b) = \frac{(N^2-1)^2}{4N},$$

and [9, 8]

$$A = \gamma^- \frac{\hat{q}_1 - \hat{k}_1 - m}{(q_1-k_1)^2 - m^2}\gamma^+ - \frac{1}{q^2}\hat{C}, \quad \gamma_\pm = n_\pm^\mu \gamma_\mu,$$
$$B = \gamma^+ \frac{\hat{q}_1 - \hat{k}_2 - m}{(q_1-k_2)^2 - m^2}\gamma^- - \frac{1}{q^2}\hat{C}, \quad q = k_1 + k_2, \qquad (57)$$

where $m$ is the quark mass and the four-vector $C_\mu$ describes the conversion of two Reggeized gluons into the ordinary gluon which was given in (23). The gauge properties of $M^{q\bar{q}}$, i. e. its vanishing in the limit $\mathbf{q}_1 \to 0$ as well as in the limit $\mathbf{q}_2 \to 0$ can be seen explicitly. These properties provide convergence of the relevant integrals over $\mathbf{q}_{1,2}$. We obtain

$$\Phi^{q\bar{q}} = \frac{4M_{q\bar{q}}^4}{\mathbf{q}_1^2 \mathbf{q}_2^2}[N_1(S_A + S_B) - 2N_2 S_{AB}],$$
$$N_1 = \frac{(N^2-1)^2}{4N}, \quad N_2 = -\frac{(N^2-1)}{4N}, \qquad (58)$$

$$M_{q\bar{q}}^2 = \frac{1}{x\bar{x}}[m^2 + (\mathbf{k}_1 + x(\mathbf{q}_2 - \mathbf{q}_1))^2] \qquad (59)$$

with

$$S_A = \frac{1}{4}\mathrm{Sp}(\hat{k}_1 + m)A(\hat{k}_2 - m)\tilde{A},$$

$$S_B = \frac{1}{4}\mathrm{Sp}(\hat{k}_1 + m)B(\hat{k}_2 - m)\tilde{B},$$

$$S_{AB} = \frac{1}{4}\mathrm{Sp}(\hat{k}_1 + m)A(\hat{k}_2 - m)\tilde{B}, \qquad (60)$$

where we have used the notation

$$\tilde{A} = \gamma_0 A^\dagger \gamma_0, \quad \tilde{B} = \gamma_0 B^\dagger \gamma_0,$$

$A^\dagger$ and $B^\dagger$ are the Hermitian conjugations of $A$ and $B$. We note that the value $\Phi^{q\bar{q}}$ is finite in both limits $\mathbf{q}_1 \to 0$ and $\mathbf{q}_2 \to 0$.

The result of numerical integration of

$$I^{qq} = \int \frac{d^2\mathbf{q}_1 d^2\mathbf{q}_2}{\pi^2} \frac{M^4 \Phi^{q\bar{q}}}{(M^2 + \mathbf{q}_1^2)(M^2 + \mathbf{q}_2^2)} \qquad (61)$$

is presented in Fig. 4$b$.

The cross section of the process

$$pp \to j_1 j_2 q\bar{q}$$

is

$$d\sigma^{pp \to j_1 j_2 q\bar{q}} = \frac{\alpha_s^4}{2^6 \pi M^2} \frac{d^2 k_1}{\pi M^2} \frac{dx}{x(1-x)} I^{qq} \frac{d\beta_1}{\beta_1} \bar{R}_2. \qquad (62)$$

## 7. HIGGS BOSON PRODUCTION

We assume the Higgs boson to be produced in collision of two Reggeized gluons through the intermediate heavy top quark–antiquark state. By analogy with QED, we have the matrix element

$$M^{pp \to j_1 j_2 H} = \frac{\pi \alpha_s}{q_1^2 q_2^2} \frac{2\alpha_s N g_H}{\pi m_t} I_s \mathbf{q}_1 \cdot \mathbf{q}_2 \times$$
$$\times \frac{\langle X_1 | \mathbf{J} \mathbf{q}_1 t^a | p_1 \rangle}{-s\alpha_1} \frac{\langle X_2 | \mathbf{J} \mathbf{q}_2 t^a | p_2 \rangle}{s\beta_2}. \qquad (63)$$

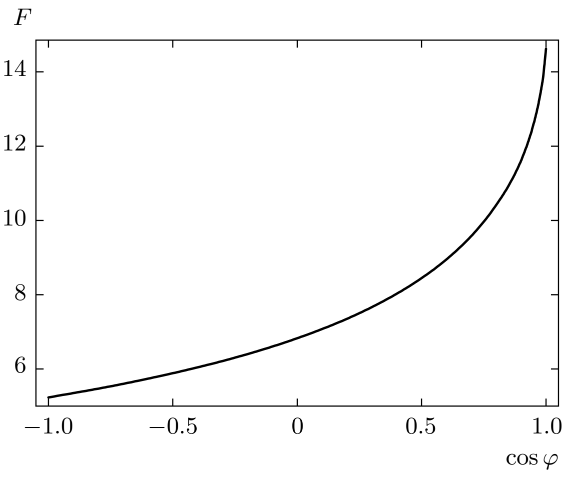



**Fig. 5.** Azimuthal correlation of two-gluon jet production with the gluons separated by a rapidity gap (see (53))

For the differential cross section, we obtain

$$\frac{d\sigma}{dL} = \sigma_0 \gamma, \quad \sigma_0 = \frac{\alpha_s^4 N^2 (N^2-1)}{2^9 \pi^3 m_t^2} |I_s|^2, \\ \gamma = \int \frac{d^2 q_1 d^2 q_2 / \pi^2}{(\bar{M}^2 + \mathbf{q}_1^2)(\bar{M}^2 + \mathbf{q}_2^2)} R_2 \tag{64}$$

with

$$I_s \left( \left| \frac{M_H}{m_t} \right|^2, \frac{\mathbf{q}_1^2}{m_t^2}, \frac{\mathbf{q}_2^2}{m_t^2} \right) \approx I_s(0,0,0) = \frac{1}{3}.$$

The quantity $\gamma$ turns out to be small, $\gamma \sim 10^{-2}$ for $\alpha_s/\pi = 0.1$, $\sqrt{s} \sim (10^3–10^4)$ GeV. Hence, the differential of the Higgs boson rapidity cross section $\sigma_0$ is rather small: $\gamma\sigma_0 \sim 1$ fb, $\sqrt{s} = 14000$ GeV.

## 8. DISCUSSION

We have considered the channel of peripherical processes with creation of some state in $s_0$, called the pionization region in proton (antiproton)–proton collisions at high energies. We assume that the initial–state proton (antiproton) develops the protonic (antiprotonic) jets resulting from the interaction of the initial proton (antiproton) with a Reggeized (colored) gluon.

The "Reggeization" of a gluon, i. e., the replacement of the Green's function of the exchanged gluon by the exchange of a Regge pole with gluon quantum numbers except the spin, which is replaced by the gluon Regge trajectory

$$\alpha(q^2) = 1 + \frac{\alpha_s q^2}{\pi} \int \frac{d^2 k}{k^2 (q-k)^2}, \tag{65}$$

was proved in Ref. [1]. The gluon Regge trajectories suffer from infrared singularities, which can be regularized by introducing a fictitious gluon mass $m$:

$$k^2 \to k^2 + m^2, \quad (q-k)^2 \to (q-k)^2 + m^2.$$

It was shown in Refs. [11] that the singular dependence of the gluon mass disappears in an experimental set-up with the emission of (arbitrary) "real" gluons, also having a mass. This means taking the inelastic processes with creation of so-called "mini-jets" into account in the multi-Regge kinematics. In this way, the expression for the cross section in terms of a Pomeron-pole exchange was developed. The corresponding forward scattering amplitude was shown in [1] to obey the so-called BFKL equation.

The statement of gluonic "mini-jets" is currently somewhat problematic. According to common knowledge, the gluon color must manifest itself in developing a hadronic jet consisting of pions.

In this paper, we used the theoretical result about the existence of a gluon trajectory and used the phenomenologic approach in describing its form as [12]

$$\alpha(q) = 1 - \frac{\alpha_s}{\pi} \frac{\mathbf{q}^2}{q_0^2} C, \quad q^2 = -\mathbf{q}^2 < 0, \\ q_0^2 \approx 1 \text{ GeV}^2, \quad C \approx 1. \tag{66}$$

We also used some simplified version of generalized parton distribution (GPD) describing the interaction of a Reggeized gluon with proton (antiproton) and converging it to a proton jet.

The main feature of this "fermion-jet" model consist in absence of evolution effects, which is the essential part of GPD approach. Simultaneously applying the BFKL and evolution mechanisms seems to suffer from double counting.

The Regge factor written in the form (successive two-Reggeon and three-Reggeon exchanges)

$$R_2 = \left(\frac{s_1}{s_0}\right)^{2(\alpha(q_1)-1)} \left(\frac{s_2}{s_0}\right)^{2(\alpha(q_2)-1)}, \tag{67}$$

$$R_3 = \left(\frac{s_1}{s_0}\right)^{2(\alpha(q_1)-1)} \left(\frac{s_2}{s_0}\right)^{2(\alpha(q_2)-1)} \times \\ \times \left(\frac{s_3}{s_0}\right)^{2(\alpha(q_3)-1)}, \tag{68}$$

where

$$s_1 s_2 = M_F^2 s, \quad s_1 s_2 s_3 = M_{1F}^2 M_{2F}^2 s,$$

and $M_F^2$ is the invariant mass created in pionization regions, turns out to be a rather efficient suppression factor. In Refs. [13] the of gluon Reggeization effects was omitted.

In Ref. [14], the $pp \to ppH$ channel of Higgs boson production was investigated. Here, the exchange by at least two (parallel) (Reggeized) gluons must be applied to provide a colorless $ppH$ final state. Introducing the Sudakov-type formfactors also seems to be illegetimate.

The cross section of Higgs boson production is rather small in our approach, $d\sigma/dL \sim 1$ fb, but can be measured at the LHC. This result is in agreement with those obtained in [14, 15]. Tests of the effective Regge action theory developed in Ref. [3] provided by the processes of creation of a single gluonic jet and production of two gluons and a quark–antiquark pair in the pionization region (without a rapidity gap between the created gluon or quarks). In these experiments, the form of the $RRP$, $RRPP$, and $RRq\bar{q}$ vertices of the effective Regge action can be tested.

Measuring the azimuthal correlation in the process of production of two-gluon jets separated by a rapidity gap as a test of the theory prediction for the multi-Regge kinematics can also be realized at the RHIC or LHC.

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