ЖЭТФ, 2009, том 135, вып. 5, стр. 885–898

SELECTED PROBLEMS OF BARYON SPECTROSCOPY: CHIRAL SOLITON VERSUS QUARK MODELS

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Received December 30, 2008

The inconsistency between the rigid rotator and bound state models at an arbitrary number of colors, the rigid rotator – soft rotator dilemma, and some other problems of baryon spectroscopy are discussed in the framework of the chiral soliton approach (CSA). Consequences of the comparison of CSA results with simple quark models are considered and the $1/N_c$ expansion for the effective strange antiquark mass is presented, as it follows from the CSA. Strong dependence of the effective strange antiquark mass on the $SU(3)$ multiplet is required to fit the CSA predictions. The difference between "good" and "bad" diquark masses, which is about 100 MeV, is in reasonable agreement with other estimates. Multibaryons (hypernuclei) with strangeness are described and some states of interest are also predicted within the CSA.

PACS: 12.39.Dc, 14.20.-c, 14.65.-q, 14.20.Pt

1. INTRODUCTION

In spite of (or possibly due to) recent dramatic events with the (non)observation of narrow pentaquark states, the studies of baryon spectra (nonstrange, strange, and with heavy flavors) preserve their relevance for accelerator physics. A discovery of baryon states besides well-established ones (e.g., octet, decuplet, and certain resonances) could help to achieve progress in understanding the structure of hadrons.

In the absence of the complete theory of strong interactions, there are different approaches and models of hadron structure; each has some advantages and certain drawbacks. Interpretation of hadron spectra in terms of quark models is widely accepted; quark models are the "most successful tool for the classification and interpretation" (R. Jaffe) of hadron spectra. These models are so widely accepted because they probably correspond to our intuitive ideas of how a bigger object–a baryon, for example,–can be made of smaller ones, quarks. However, our intuition, based on the macroscopic experience, may be totally misleading in the world of elementary particles.

Quark models are to a large extent phenomenological because there are no regular methods of solving the relativistic many-body problem. In a true relativistic theory, the number of constituents (e.g., additional $q\bar{q}$ pairs) and their weight should not be fixed as a starting condition, but should be obtained by means of solving relevant relativistic equations (and the quark confinement should be obtained in this way as well).

In view of this global unresolved problem, alternative approaches are of interest. In particular, the chiral soliton approach (CSA) based on few principles represented by the model Lagrangian, has certain advantages. Baryons and baryonic systems are considered on equal footing (the look "from outside"). The CSA has many features of a true theory, but still it is a model: some phenomenological elements are also necessarily present in the CSA. Results obtained within the CSA mimic some features of baryon spectra within quark models due to the Gell-Mann–Okubo relations for the masses of baryons within certain $SU(3)$ multiplet.

2. FEATURES OF THE CSA

The CSA is based on fundamental principles and ingredients incorporated in the truncated effective chiral Lagrangian

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$$L^{eff} = -\frac{F_\pi^2}{16}\operatorname{Tr} l_\mu l_\mu + \frac{1}{32e^2}\operatorname{Tr}[l_\mu, l_\nu]^2 + \\ + \frac{F_\pi^2 m_\pi^2}{8}\operatorname{Tr}\left(U + U^\dagger - 2\right) + \ldots, \quad (1)$$

where $l_\mu = \partial_\mu U U^\dagger$ is the chiral derivative, $U \in SU(2)$ or $\in SU(3)$ is a unitary matrix depending on chiral fields, m_π is the pion mass, F_π is the pion decay constant taken from experiment, $[.,.]$ denotes a commutator, e is the only parameter of the model that defines the weight of the antisymmetric term in the Lagrangian of the 4th order in chiral derivatives (the Skyrme term)[1)]. Effective Lagrangian (1) can be deduced from the underlying QCD Lagrangian [1], with infinitely many terms appearing this way. The terms of higher orders in l_μ are not shown in (1). The 6th-order term is taken into account in a number of calculations, and it does not change the properties of multiskyrmions considerably. The mass term proportional to $F_\pi^2 m_\pi^2$ changes the asymptotic behavior of the profile f and the structure of multiskyrmions at large baryon number B. In the $SU(2)$ case,

$$U = \cos f + i(\mathbf{n}\cdot\boldsymbol{\tau})\sin f, \quad (2)$$

where the unit vector $\mathbf{n}$ depends on two functions α and β, and $\boldsymbol{\tau}$ are the Pauli matrices. Three profiles $\{f, \alpha, \beta\}(x, y, z)$ parameterize a unit vector on the 3-sphere S^3.

The soliton is a configuration of chiral fields having a topological charge identified with the baryon number B as proposed by T. H. R. Skryme near 50 years ago:

$$B = -\frac{1}{2\pi^2}\int \sin^2 f \sin\alpha I\left[(f,\alpha,\beta)/(x,y,z)\right] d^3r, \quad (3)$$

where $I\left[(f,\alpha,\beta)/(x,y,z)\right]$ is the Jacobian of the coordinate transformation. Therefore, the quantity B shows how many times S^3 is covered when integration is performed over R^3. We recall that surface of the unit sphere S^3 equals

$$\int \sin^2 f \sin\alpha df\, d\alpha\, d\beta = 2\pi^2. \quad (4)$$

Minimization of the classical mass functional M_{cl} for each value of the baryon number provides three profiles $\{f, \alpha, \beta\}$, and the static configuration mass, and allows calculating binding energies of classical configurations, the moments of inertia Θ_π (isotopical), Θ_J (orbital), and Θ_K (kaonic or strange), and some other characteristics of chiral solitons that implicitly contain information about the interaction between baryons and are necessary to perform the quantization procedure, i.e., to obtain the spectrum of baryon states with definite quantum numbers.

[1)] In some papers, the constant F_π and even the mass m_π are considered to be parameters, although they are fixed by the existing data. Such an approach is useful, however, in investigating some global properties of chiral soliton models and multiskyrmions.

3. SKYRMION QUANTIZATION AND THE SPECTRUM OF BARYONS

The observed spectrum of states is obtained by means of a quantization procedure and depends on quantum numbers of baryons and the above-mentioned properties of classical configurations, the moments of inertia, the Σ-term (Γ), etc. In the $SU(2)$ case, the rigid rotator model (RRM) [2] is most effective and successfull in describing the properties of nucleons, the Δ-isobar, some properties of light nuclei [3], and the so-called "symmetry energy" of nuclei with the atomic number $A \lesssim 20$ [4].

In the $SU(3)$ case, different quantization models have been developed. Probably, most common way to obtain the spectrum of baryons is to place an established $SU(2)$ classical configuration (e.g., the so called "hedgehog" for the $B = 1$ skyrmion) in the upper left corner of the $SU(3)$ matrix of chiral fields and to quantize the $SU(3)$ zero modes corresponding to rotations in the $SU(3)$ configuration space [5]. The following mass formula is valid in this rigid rotator model:

$$M(p,q,Y,I,J) = M_{cl} + \frac{K(p,q,I_R)}{2\Theta_K} + \\ + \frac{J(J+1)}{2\Theta_\pi} + \delta M_{(p,q)}(Y,I), \quad (5)$$

where the four terms in the right-hand side are respectively proportional to N_c, 1, N_c^{-1}, and 1, where N_c is the number of colors in the underlined QCD. This formula is in fact an expansion in powers of $1/N_c$. There,

$$K(p,q,I_R) = C_2(SU3) - I_R(I_R+1) - N_c^2 B^2/12,$$
$$C_2(SU3) = (p^2 + q^2 + pq)/3 + p + q,$$

p and q are the numbers of upper and lower indices in the spinor describing the $SU(3)$ multiplet, Y, I, and J are respectively the hypercharge, isospin, and spin of the quantized state, I_R is the so called "right" isospin, and $I_R = J$ is the value of spin of the $B = 1$ state. Somewhat of a paradox is the fact that the total splitting of the entire multiplet is proportional to N_c.

The mass splittings δM are due to the term

$$\mathcal{L}_M \approx -\tilde{m}_K^2 \Gamma \frac{\sin^2\nu}{2} \quad (6)$$

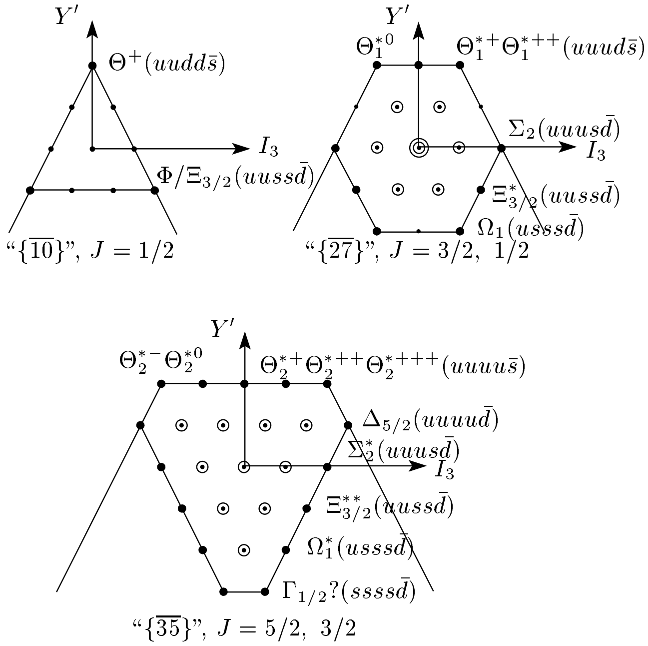


Fig. 1. The I_3–Y' diagrams ($Y' = S + 1$) for multiplets of pentaquark baryons, i.e., the antidecuplet with $[p, q] = [0, (N_c + 3)/2]$, the $\{27\}$-plet with $[p, q] = [2, (N_c + 1)/2]$, and the $\{35\}$-plet with $[p, q] = [4, (N_c - 1)/2]$. For $N_c > 3$, these diagrams should be extended within long lines, as shown in the picture. The quark content is given for manifestly exotic states denoted by full circles (components with the maximal value of I_3), when $N_c = 3$

in the Lagrangian, where ν is the angle of rotation into the "strange" direction and $\tilde{m}_K^2 = F_K^2 m_K^2 / F_\pi^2 - m_\pi^2$ includes the $SU(3)$-symmetry violation in the flavor decay constants. For the accepted values of the model parameters, numerical values of some important characteristics of the $B = 1$ skyrmion are $\Gamma \approx 6$ GeV^{-1} proportional to the Σ-term, the moments of inertia $\Theta_\pi \approx$ 5–6 GeV^{-1}, and $\Theta_K \approx$ 2–3 GeV^{-1}. All moments of inertia and Γ are proportional to the number of colors, $\Theta \propto N_c$.

The multiplets of exotic baryons are shown in Fig. 1. We recall that $[p, q] = [1, (N_c - 1)/2]$ for the "octet"[2], $[p, q] = [3, (N_c - 3)/2]$ for the "decuplet", and $p + 2q = N_c$. For exotic multiplets[3] shown in Fig. 1, $p + 2q = N_c + 3$. The lower index in the notation for states indicates the isospin of the state, e.g.,

$$\Phi/\Xi_{3/2} = |\overline{10}, S = -2, I = 3/2\rangle,$$

$$\Sigma_2 = |27, S = -1, I = 2\rangle, \quad \Omega_1 = |27, S = -3, I = 1\rangle,$$

where S is the strangeness of the state. The "strangeness content"

$$C_S = \left\langle \frac{1}{2} \sin^2 \nu \right\rangle_B \tag{7}$$

can be calculated exactly with the help of wave functions in the $SU(3)$ configuration space, for an arbitrary number of colors N_c [6, 7].

Table 1. Strangeness content of the "octet", "decuplet", and "antidecuplet" of baryons at an arbitrary $N = N_c$, for unmixed states, $Y' = S + 1$. Few states (marked by an asterisk) are shown that appear only if $N > 3$; they are mostly states with the maximal possible value of isospin at a fixed Y'

(Y', I)	$C_S(N)$	$C_S(N = 3)$
	$[p, q] = [1, (N - 1)/2]$	
$(1, 1/2)$	$2(N + 4)/[(N + 3)(N + 7)]$	$7/30$
$(0, 0)$	$3/(N + 7)$	$9/30$
$(0, 1)$	$(3N + 13)/[(N + 3)(N + 7)]$	$11/30$
$*(-1, 3/2)$	$(4N + 18)/[(N + 3)(N + 7)]$	–
$(-1, 1/2)$	$4/(N + 7)$	$12/30$
	$[p, q] = [3, (N - 3)/2]$	
$(1, 3/2)$	$2(N + 4)/[(N + 1)(N + 9)]$	$7/24$
$(0, 1)$	$(3N + 7)/[(N + 1)(N + 9)]$	$8/24$
$*(-1, 5/2)$	$(4N + 22)/[(N + 1)(N + 9)]$	–
$(-1, 1/2)$	$(4N + 6)/[(N + 1)(N + 9)]$	$9/24$
$*(-2, 3)$	$(5N + 29)/[(N + 1)(N + 9)]$	–
$(-2, 0)$	$5/(N + 9)$	$10/24$
	$[p, q] = [0, (N + 3)/2]$	
$(2, 0)$	$3/(N + 9)$	$6/24$
$(1, 1/2)$	$(4N + 9)/[(N + 3)(N + 9)]$	$7/24$
$(0, 1)$	$(5N + 9)/[(N + 3)(N + 9)]$	$8/24$
$(-1, 3/2)$	$(6N + 9)/[(N + 3)(N + 9)]$	$9/24$
$*(-2, 2)$	$(7N + 9)/[(N + 3)(N + 9)]$	–

[2] The notations of the $SU(3)$ multiplet in inverted commas refer to the case of arbitrary $N_c > 3$, without iverted commas — to the case $N_c = 3$.

[3] This particular choice of $[p, q]$ values is actually a result of convention for a large-N_c generalization of the model. For this choice, the upper states within each $SU(3)$ multiplet at arbitrary N_c coincide with those at $N_c = 3$ (see Fig. 1).

Table 2. Strangeness content for unmixed states of the "{27}"-plet (spin $J = 3/2$) and the "{35}"-plet ($J = 5/2$) of baryons, for an arbitrary $N = N_c$ and numerically for $N_c = 3$. As in Table 1, some states that exist only for $N_c > 3$ (with the maximal isospin) are marked with an asterisk

(Y', I)	$C_S(N)$	$C_S(N=3)$
	$[p,q] = [2, (N+1)/2]$	
$(2,1)$	$(3N+23)/[(N+5)(N+11)]$	$32/112$
$(1,3/2)$	$(4N^2+65N/2-3/2)/[(N+1)(N+5)(N+11)]$	$33/112$
$(1,1/2)$	$(4N+24)/[(N+5)(N+11)]$	$36/112$
$(0,2)$	$(5N^2+39N-26)/[(N+1)(N+5)(N+11)]$	$34/112$
$(0,1)$	$(5N^2+33N+8)/[(N+1)(N+5)(N+11)]$	$38/112$
$(0,0)$	$5/(N+11)$	$5/14$
$*(-1,5/2)$	$(6N^2+91N/2-101/2)/[(N+1)(N+5)(N+11)]$	–
$(-1,3/2)$	$(6N^2+38N-8)/[(N+1)(N+5)(N+11)]$	$40/112$
$(-1,1/2)$	$(6N+7/2)/[(N+1)(N+11)]$	$43/112$
$*(-2,3)$	$(7N^2+52N-75)/[(N+1)(N+5)(N+11)]$	–
$(-2,1)$	$(7N+2)/[(N+1)(N+11)]$	$46/112$
	$[p,q] = [4, (N-1)/2]$	
$(2,2)$	$(3N+25)/[(N+3)(N+13)]$	$34/96$
$(1,5/2)$	$(4N^2+85N/3-79)/[(N-1)(N+3)(N+13)]$	$21/96$
$(1,3/2)$	$(4N+24)/[(N+3)(N+13)]$	$36/96$
$*(0,3)$	$(5N^2+104N/3-133)/[(N-1)(N+3)(N+13)]$	–
$(0,2)$	$(5N^2+74N/3-67)/[(N-1)(N+3)(N+13)]$	$26/96$
$(0,1)$	$(5N+23)/[(N+3)(N+13)]$	$38/96$
$*(-1,7/2)$	$(6N^2+41N-187)/[(N-1)(N+3)(N+13)]$	–
$(-1,3/2)$	$(6N^2+21N-55)/[(N-1)(N+3)(N+13)]$	$31/96$
$(-1,1/2)$	$(6N+22)/[(N+3)(N+13)]$	$40/96$
$*(-2,4)$	$(7N^2+142N/3-241)/[(N-1)(N+3)(N+13)]$	–
$(-2,1)$	$(7N^2+52N/3-43)/[(N-1)(N+3)(N+13)]$	$36/96$
$(-2,0)$	$7/(N+13)$	$42/96$
$*(-3,9/2)$	$(8N^2+161N/3-295)/[(N-1)(N+3)(N+13)]$	–
$(-3,1/2)$	$(8N-31/3)/[(N-1)(N+13)]$	$41/96$

Some examples of the values of C_S at an arbitrary number of colors N_c taken from Ref. [7][4] are presented in Tables 1 and 2.

At large N_c, approximately,

$$C_S \approx \frac{2+|S|}{N_c}. \tag{8}$$

The Gell-Mann–Okubo formula holds in the form

$$C_S = -A(p,q)\,Y - B(p,q)\left[Y^2/4 - \mathbf{I}^2\right] + C(p,q), \tag{9}$$

where $A(p,q)$, $B(p,q)$, and $C(p,q)$ depend on the particular $SU(3)$ multiplet. For the "octet", for example [7],

[4] In the case of a "nucleon", the strangeness content at an arbitrary N_c was first presented in Ref. [8].

$$A(\text{“}\{8\}\text{”}) = \frac{N_c+2}{(N_c+3)(N_c+7)},$$
$$B(\text{“}\{8\}\text{”}) = \frac{2}{(N_c+3)(N_c+7)}, \tag{10}$$
$$C(\text{“}\{8\}\text{”}) = \frac{3}{N_c+7}.$$

For the “decuplet”,

$$A(\text{“}\{10\}\text{”}) = \frac{N_c+2}{(N_c+1)(N_c+9)},$$
$$B(\text{“}\{10\}\text{”}) = \frac{2}{(N_c+1)(N_c+9)}, \tag{11}$$
$$C(\text{“}\{10\}\text{”}) = \frac{3}{N_c+9},$$

and for the “antidecuplet”, where the relation $I = (1-S)/2$ holds for each isomultiplet, it was possible to obtain the relations

$$A(\text{“}\{\overline{10}\}\text{”}) + \frac{3}{2}B(\text{“}\{\overline{10}\}\text{”}) = \frac{N_c}{(N_c+3)(N_c+9)},$$
$$C(\text{“}\{10\}\text{”}) - 2B(\text{“}\{\overline{10}\text{”}\}) = \frac{5N_c+9}{(N_c+3)(N_c+9)}. \tag{12}$$

If we try to expand in $1/N_c$, then the parameter is $7/N_c$ for the “octet”. For the “decuplet” and “antidecuplet”, the expansion parameter is $9/N_c$, and it becomes worse for higher multiplets, the “$\{27\}$”-plet, the “$\{35\}$”-plet, etc. Apparently, for real world with $N_c = 3$, the $1/N_c$ expansion does not work.

Any chain of states connected by the relation $I = C' \pm Y/2$ reveals a linear dependence on the hypercharge (strangeness). Interpretation of these results in terms of strange quark/antiquark masses should be done with great care. For multiplets such as the “octet” and the “decuplet”, the CSA mimics the quark model with the effective strange quark mass

$$m_s^{eff} \sim \tilde{m}_K^2\Gamma\left[A(p,q) \mp 3B(p,q)/2\right]. \tag{13}$$

This is valid if the flavor symmetry breaking is included in the lowest order of the perturbation theory. At large N_c,

$$m_s^{eff} \sim \bar{m}_K^2\Gamma/N_c \tag{14}$$

is too large, about 0.6–0.7 GeV, if extrapolated to $N_c = 3$.

If we make expansion in the RRM for the “octet” of baryons, we obtain the contribution to the mass proportional to $\bar{m}_K^2$,

$$\delta M_N = 2\tilde{m}_K^2\frac{\Gamma}{N_c}\left(1-\frac{6}{N_c}\right),$$
$$\delta M_\Lambda = \bar{m}_K^2\frac{\Gamma}{N_c}\left(3-\frac{21}{N_c}\right),$$
$$\delta M_\Sigma = \bar{m}_K^2\frac{\Gamma}{N_c}\left(3-\frac{17}{N_c}\right), \tag{15}$$
$$\delta M_\Xi = \bar{m}_K^2\frac{\Gamma}{N_c}\left(4-\frac{28}{N_c}\right),$$

and for decuplet,

$$\delta M_\Delta = 2\bar{m}_K^2\frac{\Gamma}{N_c}\left(1-\frac{6}{N_c}\right),\ \ldots,$$
$$\delta M_\Omega = \bar{m}_K^2\frac{\Gamma}{N_c}\left(5-\frac{45}{N_c}\right), \tag{15'}$$

equidistantly for all four components. We note that for the “nucleon” and “Δ”, these contributions to the mass coincide in the leading and next-to-leading orders of the $1/N_c$ expansion, and can be regarded as the contribution of the “sea” of $s\bar{s}$ pairs. The effective strange quark masses estimates and their $1/N_c$ expansion follow from Eq. (15) immediately (see Sec. 6).

4. THE BOUND-STATE MODEL OF SKYRMION QUANTIZATION

In the bound-state model (BSM) [9], the antikaon or the kaon is bound by the $SU(2)$ skyrmion. The mass formula for the states with strangeness S is then given by

$$M = M_{cl} + \omega_S + \omega_{\bar{S}} + |S|\omega_S + \Delta M_{HFS}, \tag{16}$$

where flavor (ω_S) and antiflavor ($\omega_{\bar{S}}$) excitation energies are

$$\omega_S = \frac{N_c(\mu-1)}{8\Theta_K} \approx \frac{\bar{m}_K^2\Gamma}{N_c},$$
$$\omega_{\bar{S}} = \frac{N_c(\mu+1)}{8\Theta_K} \approx \frac{N_c}{4\Theta_K} + \frac{\bar{m}_K^2\Gamma}{N_c}, \tag{17}$$

$$\mu = \sqrt{1+\frac{\bar{m}_K^2}{M_0^2}} \approx 1 + \frac{\bar{m}_K^2}{2M_0^2} = 1 + 8\frac{\bar{m}_K^2\Gamma\Theta_K}{N_c^2},$$
$$\omega_S + \omega_{\bar{S}} = \frac{\mu N_c}{4\Theta_K} \approx \frac{N_c}{4\Theta_K} + \frac{\bar{m}_K^2 N_c}{8\Theta_K M_0^2}, \tag{18}$$
$$M_0^2 = N_c^2/16\Gamma\Theta_K \sim N_c^0, \quad \mu \sim N_c^0 \sim 1.$$

The expansion of μ written above does not work well even for strangeness, but it is very useful for comparison of the BSM and RRM.

The hyperfine splitting (HFS) correction depending on the hyperfine splitting constants c and $\bar{c}$, the isospin I and spin J of the state, and the "strange isospin" $I_S = |S|/2$ is given by [9]

$$\Delta M_{HFS} = \frac{J(J+1)}{2\Theta_\pi} + \\ + \frac{(c_S-1)[J(J+1)-I(I+1)]+(\bar{c}_S-c_S)I_S(I_S+1)}{2\Theta_\pi}, \quad (19)$$

$$c_S = 1 - \frac{\Theta_\pi}{2\mu\Theta_K}(\mu-1) \approx 1 - 4\frac{\Theta_\pi \Gamma m_K^2}{N_c^2}, \\ \bar{c}_S = \frac{\Theta_\pi}{\mu^2\Theta_K}(\mu-1) \approx 1 - 8\frac{\Theta_\pi \Gamma m_K^2}{N_c^2}. \quad (20)$$

The approximate equalities shown in the right-hand sides are valid when the expansion in m_K^2 is possible. In this approximation, $\bar{c}_S \approx c_S^2$, as mentioned in the literature. It is a point of principle that baryon states in the BSM are labeled by their strangeness (flavor), spin, and isospin, but do not apriori belong to a definite $SU(3)$ multiplet (p,q). They can be a mixture of different $SU(3)$ multiplets, indeed.

For flavor (negative strangeness or beauty, positive charm), the HFS correction disappears if $\bar{m}_K = 0$, and we can rewrite the mass formula for flavored states as

$$M(I,J,S) \approx M_{cl} + \frac{N_c}{4\Theta_K} + \frac{J(J+1)}{2\Theta_\pi} + \frac{\bar{m}_K^2\Gamma}{N_c} \times \\ \times \left\{ 2+|S| - \frac{2}{N_c}\left[J(J+1)-I(I+1)+I_S(I_S+1)\right]\right\}. \quad (21)$$

It is clear from this expression that the energy is minimal when the "strange isospin" is maximal, i.e., $I_S = -S/2$. For the decuplet isospin $I = (3+S)/2$ and $I_S(I_S+1) - I(I+1) = -5(3+2S)/4$, therefore, equidistant location of the decuplet components is reproduced.

In this way, for the "octet" and the "decuplet", we obtain the contributions depending on m_K^2:

$$\delta M_N = 2\tilde{m}_K^2\frac{\Gamma}{N_c}, \quad \delta M_\Lambda = \tilde{m}_K^2\frac{\Gamma}{N_c}\left(3 - \frac{3}{N_c}\right), \\ \delta M_\Sigma = \tilde{m}_K^2\frac{\Gamma}{N_c}\left(3 + \frac{1}{N_c}\right), \\ \delta M_\Xi = \tilde{m}_K^2\frac{\Gamma}{N_c}\left(4 - \frac{4}{N_c}\right), \\ \delta M_\Delta = 2\tilde{m}_K^2\frac{\Gamma}{N_c} \approx \delta M_N, \\ \delta M_\Omega = \tilde{m}_K^2\frac{\Gamma}{N_c}\left(5 - \frac{15}{N_c}\right). \quad (22)$$

It is instructive to compare the total splitting of the "octet" and "decuplet" in the BSM and in the RRM:

$$\Delta_{tot}(\text{“}\{8\}\text{”}, \mathrm{BSM}) = \tilde{m}_K^2\frac{\Gamma}{N_c}\left(2 - \frac{4}{N_c}\right), \\ \Delta_{tot}(\text{“}\{8\}\text{”}, \mathrm{RRM}) = \tilde{m}_K^2\frac{\Gamma}{N_c}\left(2 - \frac{16}{N_c}\right), \\ \Delta_{tot}(\text{“}\{10\}\text{”}, \mathrm{BSM}) = \tilde{m}_K^2\frac{\Gamma}{N_c}\left(3 - \frac{15}{N_c}\right), \ldots, \\ \Delta_{tot}(\text{“}\{10\}\text{”}, \mathrm{RRM}) = \tilde{m}_K^2\frac{\Gamma}{N_c}\left(3 - \frac{33}{N_c}\right). \quad (23)$$

In the BSM, mass splittings are bigger than in the RRM.

It follows already from this comparison that the RRM used for prediction of pentaquarks [10] is different from the BSM model used in [11][5)] to disavow the Θ^+.

For antiflavor (positive strangeness or beauty, or negative charm), the changes $\omega_S \to \omega_{\bar{S}}$ and $c_S \to c_{\bar{S}}$ should be made in Eqs. (16) and (19). It is crucially important that the hyperfine splitting constants are different for the antiflavor; they can be obtained by means of the change $\mu \to -\mu$ in the above formulas (see, e.g., a detailed evaluation in Ref. [7]):

$$c_{\bar{S}} = 1 - \frac{\Theta_\pi}{2\mu\Theta_K}(\mu+1) \approx \\ \approx 1 - \frac{\Theta_\pi}{\Theta_K} + 4\frac{\Theta_\pi\Gamma m_K^2}{N_c^2} + O(m_K^4), \\ \bar{c}_{\bar{S}} = 1 + \frac{\Theta_\pi}{\mu^2\Theta_K}(\mu+1) \approx \\ \approx 1 + 2\frac{\Theta_\pi}{\Theta_K} - 24\frac{\Theta_\pi\Gamma m_K^2}{N_c^2} + O(m_K^4), \quad (24)$$

and even an approximate equality of the type $\bar{c}_S \approx c_S^2$ does not hold for positive strangeness.

As a result, the mass formula for antiflavored states becomes

$$M(I,J,S>0) \approx M_{cl} + \frac{N_c(1+S)}{4\Theta_K} + \frac{J(J+1)}{2\Theta_\pi} + \\ + \frac{1}{2\Theta_K}\left[I(I+1) - J(J+1) + 3I_S(I_S+1)\right] + \\ + \frac{\bar{m}_K^2\Gamma}{N_c}\left\{2 + |S| + \frac{2}{N_c}\left[J(J+1) - I(I+1) - \\ - 7I_S(I_S+1)\right]\right\}. \quad (25)$$

5) Intense discussion of the CSA predictions validity for exotic baryon states was initiated in Ref. [12]. However, the explicit difference between the RRM and BSM in the next-to-leading terms in the $1/N_c$ expansion of contributions $\sim \bar{m}_K^2$, which is discussed here, was not established in Ref. [12].

For antiflavor (positive strangeness, etc.), the term proportional to $1/\Theta_K$ in Eq. (25) is large even for small m_K^2:

$$\Delta M_{HFS}^{\bar{S}}(\bar{m}_K = 0) = \frac{J(J+1)}{2\Theta_\pi} + \frac{1}{2\Theta_K} \times \times [-J(J+1) + I(I+1) + 3I_S(I_S+1)]. \quad (18')$$

This contribution to the position of the baryon mass agrees with the result of the RRM.

The case of exotic $S = +1$ Θ-hyperons is especially interesting. For $\Theta_0^+ \in$ "$\{\overline{10}\}$", we have $J = 1/2$, $I = 0$, and

$$M_{\Theta_0, J=1/2} = M_{cl} + \frac{2N_c+3}{4\Theta_K} + \frac{3}{8\Theta_\pi} + \bar{m}_K^2\Gamma\left(\frac{3}{N_c} - \frac{9}{N_c^2}\right).$$

For $\Theta_1^+ \in \{27\}$, we have $J = 3/2$, $I = 1$, and

$$M_{\Theta_1, J=3/2} = M_{cl} + \frac{2N_c+1}{4\Theta_K} + \frac{15}{8\Theta_\pi} + \bar{m}_K^2\Gamma\left(\frac{3}{N_c} - \frac{7}{N_c^2}\right).$$

For $\Theta_0^+ \in \{35\}$, $J = 5/2$ and $I = 2$, and the contribution to the mass is

$$M_{\Theta_2, J=5/2} = M_{cl} + \frac{2N_c - 1}{4\Theta_K} + \frac{35}{8\Theta_\pi} + + \bar{m}_K^2\Gamma\left(\frac{3}{N_c} - \frac{5}{N_c^2}\right). \quad (26)$$

The terms proportional to $1/\Theta_K$ agree with those obtained in the RRM for the antidecuplet, the $\{27\}$- and $\{35\}$-plets (the terms proportional to $K(p,q,J)$ in the RRM mass formula). This means that, indeed, we can interprete these positive-strangeness states as belonging to definite $SU(3)$ multiplets–the antidecuplet and the $\{27\}$- and $\{35\}$-plets[6)], at least when the expansion of μ made above is possible.

We also compare the contributions proportional to $\bar{m}_K^2\Gamma$ with the mass splitting correction from the RRM:

$$\delta M_{\Theta_0, J=1/2}^{RRM} = \bar{m}_K^2\Gamma\left(\frac{3}{N_c} - \frac{27}{N_c^2}\right),$$

$$\delta M_{\Theta_1, J=3/2}^{RRM} = \bar{m}_K^2\Gamma\left(\frac{3}{N_c} - \frac{25}{N_c^2}\right), \quad (27)$$

$$\delta M_{\Theta_2, J=5/2}^{RRM} = \bar{m}_K^2\Gamma\left(\frac{3}{N_c} - \frac{23}{N_c^2}\right),$$

and again, as for the "octet" and the "decuplet", considerable difference is observed between the RRM and BSM results.

The addition to the BSM result of a term allowed by the normal ordering ambiguity for the operators of (anti)strangeness production present in BSM (see discussion of this point in Ref. [6]),

$$\Delta M_{BSM-RRM} = -6\bar{m}_K^2\frac{\Gamma}{N_c^2}(2 + |S|), \quad (28)$$

brings the RRM and BSM results into agreement, for nonexotic as well as exotic $S = +1$ states. But this procedure does not look quite satisfactory: if we believe in the RRM, why should we need the BSM at all? Anyway, the RRM and BSM in their accepted form are different models.

The rotation–vibration approach [13] attempts to unify the RRM and BSM in some way, with Θ^+ having been confirmed with a somewhat higher energy and a considerable width ($\Gamma_\Theta \approx 50$ MeV)[7)].

5. THE ROLE OF CONFIGURATION MIXING

Configuration mixing due to the term proportional to $m_K^2\Gamma\sin^2\nu$ in the Lagrangian is important [14] because, for example, the $\Delta_{3/2}$ state from the decuplet of baryons is mixed with the $\Delta'_{3/2}$ state from the $\{27\}$-plet, and as a result, the splitting between these states becomes larger: the mass of $\Delta_{3/2}$ decreases, and the mass of $\Delta'_{3/2}$ increases (Fig. 2). Similar mixing occurs for other baryon states that have equal values of strangeness and isospin but belong to different $SU(3)$ multiplets.

For the antidecuplet, the mixing slightly decreases the total splitting and pushes the N^* and Σ^* states toward higher energy. Mixing with components of the octet is important. An apparent contradiction with the simplest assumption of the equality of masses of strange

6) Obtaining other components of these multiplets within the BSM is an unresolved problem, however. Evaluations performed in the literature are not sufficient for this purpose. For example, the strange isospin, which is unique for the states with the strangeness $S = \pm 1$, is uncertain for the components of exotic multiplets different from the $S = 1$ states [7].

7) The RRM–BSM alternative is not properly resolved in the literature. In some cases involving an ambiguity, the priority is given to the RRM (see, e.g., [13, Sect. 3 and 4]). The HFS correction in [13] has the form different from ours. According to Eq. (3.21) in [13], it is

$$\Delta M_S = 1/2\Theta_\pi + [c_S J(J+1) + (1-c_S)I(I+1) + c_S(c_S-1)/4],$$

the last term being completely different from ours in Eq. (19). In view of this, the authors of [13] stated: "The comparison with the RR approximation suggests that these quartic terms contribute $9/8\Theta_K$ to the mass of the $S = 1$ baryons". According to our BSM formulas, we have $(\bar{c}_{\bar{S}} - c_{\bar{S}})I_S(I_S+1)/2\Theta_\pi|_{\bar{m}_K^2=0} = 9/8\Theta_K$ in agreement with the RRM, and there is no need to correct the BSM formulas "by hand".

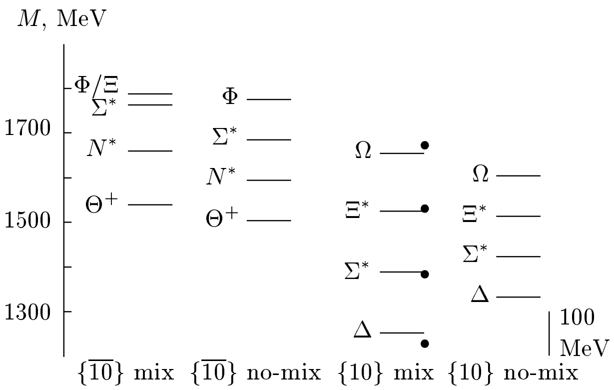


Fig. 2. Influence of the configuration mixing [14] on the mass splitting within the antidecuplet and decuplet of baryons, the RRM (the version described in [15]). For the decuplet, the data are shown by black dots

quarks and antiquarks, $m_s = m_{\bar{s}}$, then occurs (see the next section).

For the decuplet, the mixing increases the total splitting considerably, but an approximate equidistancy remains![8] Mixing with the components of the $\{27\}$-plet is important because, for example, $\Delta \in \{10\}$ after mixing with $\Delta^* \in \{27\}$ moves to a lower value of mass.

A note regarding the quark model should be made: states with different numbers of $q\bar{q}$ pairs can mix, and such mixing should be taken into account. In the diquark–diquark–antiquark picture proposed in Ref. [16], the mixing of pentaquark states with the ground-state baryon octet should be included because strong interactions do not preserve the number of quark–antiquark pairs present in a hadron. This mixing pushes the pentaquark states towards higher energy and changes the whole picture of relative positions of baryon states. Without this mixing, the diquark picture in [16] looks artificial, whereas within the CSA, this problem is resolved in a natural way.

We conclude this section with the following discussion of the case of large values of the mass $\bar{m}_F$, which, besides m_K, can also be $\bar{m}_D$ or $\bar{m}_B$. When this mass is large enough, the expansion of μ in (17) cannot be made, and we instead have $\mu \approx \bar{m}_F/M_0 = 4\bar{m}_F\sqrt{\Gamma\Theta_K}/N_c$. This linear dependence of μ and of the flavor excitation energies ω_F and $\bar{\omega}_F$ on the mass m_F, given by (17), is quite reasonable, but it is not possible to uniquely ascribe the quantized states to definite irreducible representations of $SU(3)$, as we did in Sec. 4. It is a challenging problem to obtain such a linear-in-m_F behavior of the flavored state energies within the rigid (RRM) or soft (SRM) rotator model. Probably, the strong configuration mixing that should occur in this case would be able to reduce the quadratic dependence on m_F (or linear in Γ) and to convert it to a linear dependence. Numerical calculations with the configuration mixing program arranged by H. Walliser and used in [15] confirm this point, but an analytic proof is desirable.

[8] Therefore, the statement made in several papers that the approximate equidistancy within the decuplet of baryons is an argument that the configuration mixing is negligible, is not correct.

6. COMPARISON OF CSA RESULTS WITH THE SIMPLE QUARK MODEL

It is possible to compare the CSA results with expectations from the simple quark model in the pentaquark approximation (projection of the CSM on the quark model). The masses m_s, $m_{\bar{s}}$ and the mass $m_{s\bar{s}}$ of the $s\bar{s}$ pair come into play, as presented in Table 3 for pure states (without mixing). Examples of wave functions of pentaquarks in the diquark–diquark–antiquark picture given in [16] are as follows (see also [6, 17, 18]):

$$\Theta_0 \in \{\overline{10}\} \sim [ud][ud]\bar{s},$$

where $[ud]$ is a diquark with zero isospin (an antitriplet in $SU(3)$ flavors; see also the next section). Other states can be obtained, e.g., by acting with the operator U^- that transforms a d-quark into an s-quark ($U^-d = s$) and $\bar{s} \to \bar{d}$ ($U^-\bar{s} = -\bar{d}$) and with the well-known isotopic $I^\pm$ operators. For example,

$$N^{*+} \in \{\overline{10}\} \sim [\sqrt{2}\bar{s}\{[us][ud]\} - \bar{d}[ud][ud]]/\sqrt{3}, \ldots,$$

$$\Phi/\Xi^-_{3/2} \in \{\overline{10}\} \sim [sd][sd]\bar{u}, \ldots,$$

$$\Phi/\Xi^+_{3/2} \in \{\overline{10}\} \sim [su][su]\bar{d}.$$

For larger N_c, the number of diquarks, equal to $N_D = (N_c + 1)/2$, increases, and additional $s\bar{s}$ pairs appear in wave functions of some states[9].

For the antidecuplet at an arbitrary N_c, according

[9] The standard assumption is that the baryon number of the quark is equal to $1/N_c$. We also accept the relation between hypercharge and strangeness in the form $Y = S + N_cB/3$ (see, e.g., [12]). We note that the quantity Y' in Fig. 1 and Tables 1 and 2 is by definition $Y' = S + 1$. The wave function of the "pentaquark" in this case is $\Theta_0 \in$ "$\{\overline{10}\}$" $\sim [ud]\ldots[ud]\bar{s}$ with the number $(N_c + 1)/2$ of $[ud]$ diquarks, etc.

Table 3. Contributions of the strange quark (antiquark) masses (for $N_c = 3$) and calculation results within the RRM without and with the configuration mixing (respectively the first and the second lines of numbers [15]). For each value of strangeness, the states with the largest isospin value are considered here

$\|\{\overline{10}\},2,0\rangle$	$\left\|\{\overline{10}\},1,\frac{1}{2}\right\rangle$	$\|\{\overline{10}\},0,1\rangle$	$\left\|\{\overline{10}\},-1,\frac{3}{2}\right\rangle$		
$m_{\bar{s}}$	$2m_{s\bar{s}}/3$	$m_s + m_{s\bar{s}}/3$	$2m_s$		
564 600	655 722	745 825	836 847		
$\|\{27\},2,1\rangle$	$\left\|\{27\},1,\frac{3}{2}\right\rangle$	$\|\{27\},0,2\rangle$	$\left\|\{27\},-1,\frac{3}{2}\right\rangle$	$\|\{27\},-2,1\rangle$	
$m_{\bar{s}}$	$m_{s\bar{s}}/2$	m_s	$2m_s$	$3m_s$	
733 749	753 887	772 779	889 911	1005 1048	
$\|\{35\},2,2\rangle$	$\left\|\{35\},1,\frac{5}{2}\right\rangle$	$\|\{35\},0,2\rangle$	$\left\|\{35\},-1,\frac{3}{2}\right\rangle$	$\|\{35\},-2,1\rangle$	$\left\|\{35\},-3,\frac{1}{2}\right\rangle$
$m_{\bar{s}}$	0	m_s	$2m_s$	$3m_s$	$4m_s$
1152 1122	857 853	971 979	1084 1107	1197 1236	1311 1367

to Fig. 1 and Table 1, any state with strangeness S has the isospin $I = (1 - S)/2$ and its mass is

$$M(\{\overline{10}\}, S, I = (1-S)/2) = M(\{\overline{10}\}, S = 1, I = 0) + \\ + \bar{m}_K^2 \Gamma \frac{(1 - S)N_c}{(N_c + 3)(N_c + 9)}. \quad (29)$$

Interpretation of this relation in terms of the quark model is not straightforward. Simple relations can be obtained from Table 3 for the effective s-quark and antiquark masses m_s and $m_{\bar{s}}$ and from the total splitting of the antidecuplet ($N_c = 3$)

$$[2m_s - m_{\bar{s}}]_{\{\overline{10}\}} = \bar{m}_K^2 \Gamma/8, \quad (30)$$

which numerically equals 272 MeV for the parameters accepted in [15] ($\Gamma \approx 6.31$ GeV^{-1}). For an arbitrary number of colors, this relation should be rewritten as

$$[(N_c + 1)m_s - 2m_{\bar{s}}]_{\{\overline{10}\}} = \bar{m}_K^2 N_c \Gamma/(N_c + 9). \quad (30')$$

Configuration mixing decreases this quantity to 247 MeV (see Table 3). Relation (30) is the only relation that can be obtained, according to Table 3. If we assume that the strange quark mass in the antidecuplet is the same as in the decuplet, $m_s(\{\overline{10}\}) = m_s(\{10\})$, then the strange antiquark mass should be negative if the configuration mixing is not included: $m_{\bar{s}}(\{\overline{10}\}) = -m_s(\{\overline{10}\})$. This relation looks unrealistic. We note that if the mass of the s-antiquark within the antidecuplet were equal to that of the s-quark (we call this variant the simplistic model), then this splitting would be much smaller, just equal to $m_s \approx 130$ MeV. A natural resolution of this contradiction is to allow the masses of the strange quark/antiquark within the antidecuplet to be different from those within the decuplet and other multiplets.

It is remarkable that configuration mixing pushes the splitting towards the simplistic quark model, where the splitting of the antidecuplet should be about m_s, because $m_{\bar{s}} \approx m_s$. If we assume that the s-quark mass in $\{\overline{10}\}$ is about 150 MeV, as in the decuplet, then the strange antiquark within $\{\overline{10}\}$ should be very light, with the mass about 30–50 MeV.

For the components of the $\{27\}$-plet with strangeness $S \leq -1$, the relation

$$M(\{27\}, S, I = (5 + S)/2) = \\ = M(\{27\}, S = -1, I = 2) - \\ - \bar{m}_K^2 \Gamma \frac{(S + 1)\left(N_c^2 - N_c + 18\right)}{(N_c + 1)(N_c + 5)(N_c + 11)} \quad (31)$$

holds, and for $N_c = 3$, we obtain $m_s^{eff}(\{27\}) \approx \approx 3\bar{m}_K^2\Gamma/56 \approx 117$ MeV, which increases to 135 MeV when the configuration mixing is included.

From splittings within the $\{27\}$-plet between components with $Y' \geq 0$, we also obtain

$$[m_s - m_{\bar{s}}]_{\{27\}} = \bar{m}_K^2\Gamma/56, \tag{32}$$

which is numerically equal to 39 MeV [15] and reduces to 30 MeV when the configuration mixing is included.

It is interesting that when the configuration mixing is not included, then the mass of the strange quark–antiquark pair equals $m_{s\bar{s}} = (m_s + m_{\bar{s}})/2$ both for the antidecuplet and $\{27\}$-plet. This relation is in fact a consequence of the Gell-Mann–Okubo relation. For an arbitrary N_c, the interpretation of formula (31) in terms of effective quark/antiquark masses becomes more difficult, because additional $s\bar{s}$ pairs are present in simple wave functions.

We now consider the highest (in multiplicity) pentaquark. The remarkable property of the $\{35\}$-plet is that the lowest-mass state is not the state with the highest value of hypercharge, $Y' = 2$, but the state in the middle of the multiplet, which has $Y' = 1$, $S = 0$, and $I = 5/2$. In the pentaquark approximation ($N_c = 3$), this state contains neither a strange quark/antiquark, nor an $s\bar{s}$ pair, and has the numerically smallest strangeness content among all baryons considered here. As can be seen from Table 3, the mass of the $s\bar{s}$ pair does not enter the masses of all the $\{35\}$-plet components with the largest isospin values. The masses of these states with $S \leq 0$ are connected by the relation

$$M\left(\{35\}, S, I = \frac{5}{2} + \frac{S}{2}\right) = = M\left(\{35\}, S = 0, I = \frac{5}{2}\right) - - \bar{m}_K^2\Gamma \frac{S\left(N_c^2 + 12 - 11N_c/3\right)}{(N_c - 1)(N_c + 3)(N_c + 13)}, \tag{33}$$

and hence, for $N_c = 3$, the quantity

$$m_s^{eff}(\{35\}) = \bar{m}_K^2\Gamma\frac{5}{96} \approx 114 \text{ MeV} \tag{34}$$

can be considered the effective strange quark mass in this case. Configuration mixing increases this quantity to 130 MeV (see Table 3).

From the difference between the masses of the $S = 1$ and $S = 0$ states, we can extract the effective strange antiquark mass

$$[m_{\bar{s}}]_{\{35\}} = \bar{m}_K^2\Gamma\frac{13}{96} \approx 295 \text{ MeV}, \tag{35}$$

which is a remarkably large value. Configuration mixing slightly reduces this quantity to 270 MeV.

For an arbitrary N_c, we can obtain some information about the behavior of the strange antiquark mass for the "antidecuplet", the "$\{27\}$"-plet, and the "$\{35\}$"-plet if we make some assumption about the contribution of the strange quark sea, in particular, that it is the same as for the "nucleon" and the "Δ"-isobar (coinciding in the leading and next-to-leading orders of the $1/N_c$ expansion; see Table 1). In this way, from the RRM, we obtain (with the contribution of the sea of $s\bar{s}$ pairs subtracted)

$$\begin{aligned} [m_{\bar{s}}]_{\{\overline{10}\}} &\sim \frac{\bar{m}_K^2\Gamma}{N_c}\left(1 - \frac{15}{N_c}\right), \\ [m_{\bar{s}}]_{\{27\}} &\sim \frac{\bar{m}_K^2\Gamma}{N_c}\left(1 - \frac{13}{N_c}\right), \\ [m_{\bar{s}}]_{\{35\}} &\sim \frac{\bar{m}_K^2\Gamma}{N_c}\left(1 - \frac{11}{N_c}\right), \end{aligned} \tag{36}$$

and within the BSM

$$\begin{aligned} [m_{\bar{s}}]_{\{\overline{10}\}} &\sim \frac{\bar{m}_K^2\Gamma}{N_c}\left(1 - \frac{9}{N_c}\right), \\ [m_{\bar{s}}]_{\{27\}} &\sim \frac{\bar{m}_K^2\Gamma}{N_c}\left(1 - \frac{7}{N_c}\right), \\ [m_{\bar{s}}]_{\{35\}} &\sim \frac{\bar{m}_K^2\Gamma}{N_c}\left(1 - \frac{5}{N_c}\right). \end{aligned} \tag{37}$$

It thus follows that numerical results shown in Table 3 can be understood qualitatively from this expansion, although the extrapolation back to the real $N_c = 3$ world is not possible. It is also worth noting that the changes of the effective s-antiquark mass from the antidecuplet to the $\{35\}$-plet are equal within the RRM and the BSM, although the mass itself is smaller in the RRM, in the next-to-leading order of the $1/N_c$ expansion.

We summarize our results for the first two terms of the $1/N_c$ expansion of the effective strange quark and antiquark masses in Table 4. The "octet" and "decuplet" of baryons do not contain valent $s\bar{s}$ pairs, and the mass difference between the components is defined entirely by the valent strange quarks. The mass m_s is defined as half the total splitting for the "octet" and $1/3$ of the total splitting for the "decuplet".

A strong dependence of the s-antiquark mass on the multiplet is required when we project the CSA results on simple quark model: it is presently unclear whether it is an artefact of the CSA or is physically significant.

Table 4. First terms of the $1/N_c$ expansion for the effective strange quark and antiquark masses within different $SU(3)$ multiplets, in units $\bar{m}_K^2\Gamma/N_c$. Empty spaces are left in the cases of theoretical uncertainty. The assumption concerning the sea of strange quarks/antiquarks, described in the text, should be kept in mind

	$\{8\}$	$\{10\}$	$\{\overline{10}\}$	$\{27\}$	$\{35\}$
m_s^{RRM}	$1-8/N_c$	$1-11/N_c$	–	–	–
m_s^{BSM}	$1-2/N_c$	$1-5/N_c$	–	–	–
$m_{\bar{s}}^{RRM}$	–	–	$1-15/N_c$	$1-13/N_c$	$1-11/N_c$
$m_{\bar{s}}^{BSM}$	–	–	$1-9/N_c$	$1-7/N_c$	$1-5/N_c$

The effect of the configuration mixing on the contribution of m_s, $m_{\bar{s}}$, and $m_{s\bar{s}}$ to baryon states should be included in a more detailed consideration.

7. DIQUARKS MASS DIFFERENCE ESTIMATES

The diquark mass differences can be roughly estimated using results obtained from the CSA. As was suggested by Wilczek [17], the singlet in the spin diquark $[q_1q_2]$, which is an antitriplet $\bar{3}_F$ in flavor, is called the "good" diquark d_0, and the triplet in the spin diquark (q_1q_2), which is 6_F in flavor, is called the "bad" diquark d_1. Both good and bad diquarks are antitriplets in color. As was shown in the preceding section, the wave function for pentaquarks from the antidecuplet can be written in terms of diquark wave functions [16, 18] as

$$\Theta_0 \in \{\overline{10}\} \sim [ud][ud]\bar{s}, \ldots,$$

$$\Phi/\Xi^{--}_{3/2} \in \{\overline{10}\} \sim [sd][sd]\bar{u} \ldots$$

It is not possible to build the $\{27\}$- and $\{35\}$-plets from good diquarks only; that the bad diquarks are needed is well illustrated by these examples of wave functions of positive-strangeness baryons:

$$\Theta_1^0 \in \{27\} \sim (dd)[ud]\bar{s}, \quad \Theta_1^+ \in \{27\} \sim (ud)[ud]\bar{s},$$

$$\Theta_1^{++} \in \{27\} \sim (uu)[ud]\bar{s}, \quad \Theta_2^- \in \{35\} \sim (dd)(dd)\bar{s},$$

$$\Theta_2^0 \in \{35\} \sim (ud)(dd)\bar{s}, \ldots,$$

$$\Theta_2^{+++} \in \{35\} \sim (uu)(uu)\bar{s}.$$

It seems to be natural to ascribe the difference of the rotation energies for different multiplets, given by the term proportional to $K(p,q,I_R)$ in expression (5), to the difference of masses of bad and good diquarks. Because the bad diquark is heavier, this is an obvious reason why Θ_1 is heavier than Θ_0, and Θ_2 is even heavier.

From the difference of the $\{27\}$-plet and antidecuplet masses, it follows

$$M(d_1) - M(d_0) \approx \frac{3}{2\Theta_\pi} - \frac{1}{2\Theta_K} \approx 100 \text{ MeV}. \quad (38)$$

From the $\{35\}$-plet and $\{27\}$-plet mass difference,

$$M(d_1) - M(d_0) \approx \frac{5}{2\Theta_\pi} - \frac{1}{2\Theta_K} \approx 250 \text{ MeV}. \quad (39)$$

This result seems to be qualitatively acceptable, in agreement with previous estimates [17] and, e.g., lattice calculations [19], but this picture is too naive. In particular, the interaction between diquarks may be important, which makes the Θ_2 $(J = 5/2)$ even heavier.

8. THE RIGID ROTATOR – SOFT ROTATOR DILEMMA

The RRM is a limit case of the rotator model when deformations of skyrmions under rotation in the $SU(3)$ configuration space are totally neglected. In the SRM, opposite to the RRM, it is supposed that the soliton is deformed under the influence of flavor symmetry breaking forces: the static energy minimization is made at a fixed value of ν. The dependence of static characterstics of skyrmions on ν is taken into account in the quantization procedure.

Static characteristics of skyrmions depend on ν, the angle of rotation into the "strange" direction. It is most important for "strange", or kaonic inertia moments:

$$\Theta_K = \frac{1}{8}\int (1-\cos f)\left[F_K^2 - \frac{1}{2}(F_K^2 - F_\pi^2) \times \right.$$
$$\left. \times\, (2-\cos f)\sin^2\nu + \frac{1}{e^2}\left(f'^2 + \frac{2\sin^2 f}{r^2}\right)\right] d^3r. \quad (40)$$

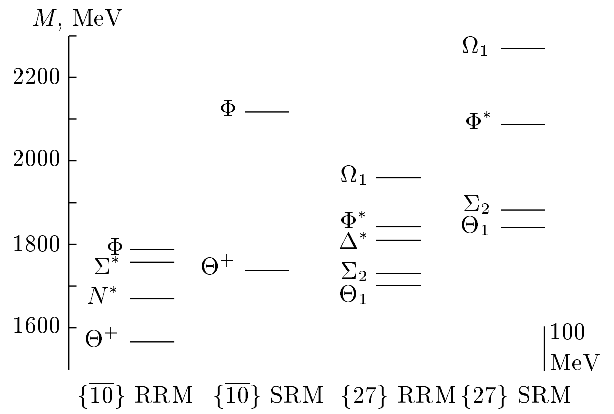


Fig. 3. Comparison of the RRM and SRM predictions for the masses of exotic baryons, the antidecuplet and the {27}-plets. Not all states are shown for the SRM. The code for the SRM used here was provided in [20]

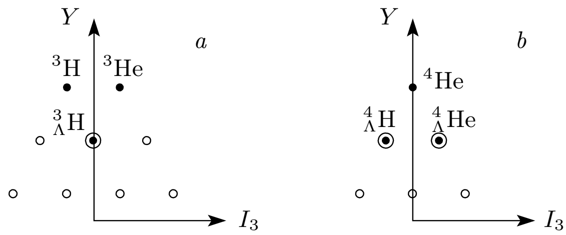


Fig. 4. a) The location of the isoscalar state (shown by a double circle) with an odd B-number, $J = 1/2, 3/2$ and $|S| = 1$ in the upper part of the I_3–Y diagram. b) The same for isodoublet states with even B ($J = 0$). The case of light hypernuclei $_\Lambda$H and $_\Lambda$He is presented as an example. The lower parts of diagrams with $Y \le B - 3$ are not shown here

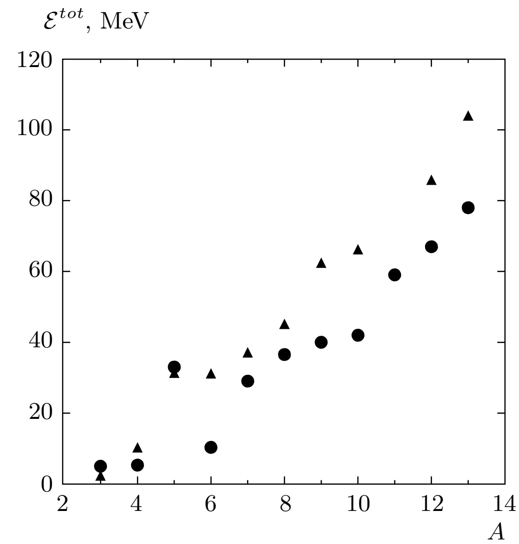


Fig. 5. Total binding energies of light hypernuclei. Triangles, correspond to experimental data; circles, are the theoretical results in a version of the BSM applied to multiskyrmions [23]. The figure is taken from the second reference in [4]

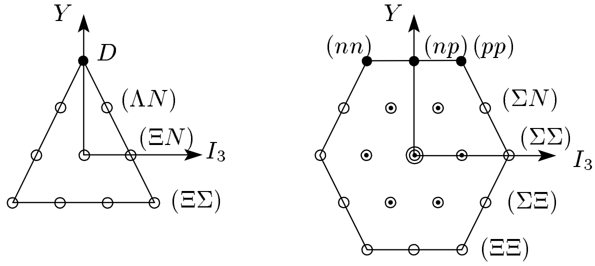
It is a decreasing function of $\sin^2 \nu$. The RRM corresponds to $\nu = 0$, the maximal value of the kaonic inertia moment Θ_K and relatively low values of masses of exotic baryons (Θ, $\Phi/\Xi_{3/2}$, etc.). Within the SRM, the masses of baryons from the antidecuplet and the {27}-plet are considerably greater than in the RRM, mostly due to the smaller value of Θ_K (see Fig. 3). The truth is somewhere between the RRM and SRM, but making reasonable calculations seems to be unrealistic presently because the properties of baryonic matter are not known, in particular, the response of matter to the flavor symmetry breaking forces.

9. STRANGE MULTIBARYONS OR HYPERNUCLEI

The great advantage of the CSA is that multibaryon states (nuclei, hypernuclei, etc.) can be considered on equal footing with the $B = 1$ case. The rational map approximation proposed in Ref. [21] simplifies this work considerably and allows easily calculating all static characteristics of multiskyrmions necessary for the spectrum evaluation. In particular, the B-number dependence of the quantities of interest has been established, $\Theta_I \sim B$ and $\Theta_J \sim B^2$ for $B \le 20$–30. Some kind of the "bag model" for multibaryons can be obtained with the help of this ansatz, starting with an effective Lagrangian [22].

Ordinary nuclei and hypernuclei (ground states) can be assigned to definite $SU(3)$ multiplets, as shown in Fig. 4 for baryon numbers 3 and 4. In a version of the BSM, it is possible to describe the total binding energies of light hypernuclei in a qualitative, even semi-quantitative agreement with data [23]. The collective motion of the multiskyrmion in the $SU(3)$ collective coordinates space is taken into account. The results of such estimates within the rigid oscillator model (a variant of the bound state model) are presented in Fig. 5, and a quite satisfactory qualitative agreement with the existing data on total binding energies is observed.

For $B = 2$, more detailed investigations have been performed. The lowest multiplets of dibaryons are shown in Fig. 6: the left figure shows the antidecuplet of the $J = 1$ dibaryons, the $I = 0$ deuteron being the nonstrange state; the right figure shows the $J = 0$ {27}-plet, the $I = 1$ nucleon–nucleon scattering state



$B=2,\ \{\overline{10}\},\ J=1\,(2,\ldots)$ $B=2,\ \{27\},\ J=0\,(1,\ldots)$

Fig. 6. The I_3–Y diagram of multiplets of dibaryons, $B=2$: the $J=1$ antidecuplet (not to be mixed with the antidecuplet of pentaquarks, $B=1$) and the $J=0$ $\{27\}$-plet. Virtual levels (scattering states) are shown in brackets, e.g., (ΛN) is a scattering state that appears as a near-threshold enhancement

being the upper (nonstrange) component. There is also a $\{35\}$-plet with the $N\Delta$-like nonstrange upper component (isospin $I=2$) and a $\{28\}$-plet with a $\Delta\Delta$-like upper component (isospin $I=3$). The $\{28\}$-plet contains the state with $S=-6$ (di-Omega). The $\{35\}$-plet and the $\{28\}$-plet are not shown in Fig. 6.

Calculations of the spectrum of strange dibaryons were performed [24] in the SRM, which is more relevant in the $B=2$ case than in the $B=1$ case. When the NN-scattering state was fitted to be in the right place (the deuteron binding energy is then about 30 MeV), all strange and multistrange dibaryons are above the threshold by few tens of megaelectronvolt, and hence can appear as near-threshold enhancements in scattering cross sections of baryons with appropriate quantum numbers. These results are in qualitative agreement with quark model calculations [25].

Multibaryons with positive strangeness or beauty (or negative charm) have also been predicted within a similar approach [26].

Rotational excitations of any state have the additional energy

$$\Delta E = \frac{J(J+1)}{2\Theta_J}. \qquad (41)$$

Excited states with $J=2^+$ have the energy by $2/\Theta_J$ greater than the energy of the $J=1\,\{\overline{10}\}$. The state with $S=-1$, $I=1/2$, $J^P=2^+$ can be interpreted as an $NN\bar{K}$ state with the binding energy about 100 MeV. For the $B=2$ $\{27\}$-plet, $J=1$ states have the energy by $1/\Theta_J \approx 60$ MeV greater than $J=0$ ground states.

The orbital inertia rapidly increases with increasing the baryon (atomic) number, $\Theta_J \sim B^p$, where p is between 1 and 2. Therefore, the number of rotational states becomes larger for large baryon numbers. Some of them can be interpreted as deeply bound anti-kaon states intensively discussed in [27] and other papers. More detailed investigations of this issue are necessary.

10. SUMMARY AND CONCLUSIONS

We can summarize our discussion as follows.

The parameter of the $1/N_c$ expansion is large in the case of the baryon spectrum, the extrapolation to real world is not possible in this way, and conclusions made in the limit $N_c \to \infty$ may not be valid in the real world. Rigid (as well as soft) rotator and bound state models coincide in the first order of the $1/N_c$ expansion, but differ in the next orders.

Configuration mixing is important, according to the RRM, and substantially affects the effective quark masses within the simple quark model.

Transition to the SRM from the RRM may be crucial, leading to an increase in the masses, especially for exotic states.

There is a correspondence between the chiral soliton RRM and quark model predictions for pentaquark spectra in the negative-S sector of $\{27\}$- and $\{35\}$-plets: the effective mass of the strange quark is about 135–130 MeV and slightly smaller for $\{35\}$-plet.

For positive strangeness components, the link between the CSA and the quark model requires a strong dependence of the effective $\bar{s}$ mass on a particular $SU(3)$ multiplet. The $1/N_c$ expansion for the effective strange antiquark mass provides different results in the rotator and bound state models in the next-to-leading order, but the changes in the effective mass $m_{\bar{s}}$ in passing from one multiplet to another are the same for the RRM and BSM. Configuration mixing pushes spectra towards the simplistic model, which is a nice property, but the reasons for this are not clear presently. Diquarks mass difference estimates from the CSA seem to be reasonable.

To conclude, we state that chiral soliton models, based on few principles and ingredients incorporated in an effective Lagrangian, allow a qualitative, in some cases even a quantitative description of various chracteristics of baryons and nuclei, from the ordinary $(S=0)$ nuclei to the known hypernuclei. This suggests that predictions of pentaquark states, as well as multibaryons with strangeness, are of interest. The existence of pentaquarks themself is without any doubt, although very narrow pentaquarks may not exist. Wide, even very wide pentaquarks should exist, and searches for pentaquarks remain a topical problem.

However, problems are encountered in trying to project the CSA results on quark models: a strong dependence of the strange antiquark mass on the $SU(3)$

multiplet and a difference in the masses of "bad" and "good" diquarks, which is not unique in naive picture, at least.

In view of theoretical uncertainties, experimental investigations are of crucial importance. In particular, experiments at the J-PARC accelerator (50 GeV) can provide a great chance to shed more light on the puzzles of baryon spectroscopy.

The author is indebted to H. Walliser and A. Shunderuk for the fruitful collaboration. Helpful discussions with T. Cohen, R. Jaffe, I. Klebanov, N. Manton, C. Rebbi, H. Walliser, and H. Weigel at different stages of the work, as well as the useful remarks by R. Faustov are thankfully acknowledged.

This paper is partly based on the talks presented at the Workshop on Nuclear and Particle Physics at J-PARC (NP08), Mito, Ibaraki, Japan, March 05-07, 2008; the 15th International Seminar on High Energy Physics Quarks-2008, Sergiev Posad, Russia, May 23–29, and the International Workshop on Hadron Structure and QCD (HSQCD'2008), Gatchina, Russia, June 30–July 4, 2008.

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