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RADIATIVE CORRECTIONS TO DEEP INELASTIC ELECTRON–DEUTERON SCATTERING. THE CASE OF TENSOR-POLARIZED DEUTERON

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Model-independent radiative corrections to deep inelastic scattering of an unpolarized electron beam off the tensor-polarized deuteron target are considered. The contribution to the radiative corrections due to the hard photon emission from the elastic electron–deuteron scattering (the so-called elastic radiative tail) is also investigated. The calculation is based on the covariant parameterization of the deuteron quadrupole polarization tensor. Radiative corrections to the polarization observables are estimated numerically for the kinematical conditions of the current experiment at HERA.

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1. INTRODUCTION

The flavor structure of nucleons is described in terms of parton distribution functions. Most of the information on these functions has up to now come from inclusive deep inelastic scattering processes: experiments where only the scattered lepton is detected. Investigation of the nucleon spin structure involves new types of reactions. For example, the HERMES experiment was specifically designed to perform accurate measurements of semi-inclusive reactions, where in addition to the scattered lepton, some of the produced hadrons are also detected [1].

The polarized nuclei of deuterium and helium-3 are used to extract information on the neutron spin-dependent structure function $g_1(x)$ [2]. In analyzing the experimental data on inclusive spin asymmetries for deuterium, a small effect due to a possible tensor polarization in this spin-one target must be taken into account in order to deduce the spin-dependent structure function g_1^d. This is connected with the presence of additional tensor-polarized structure functions in a deuteron target [1]. So far, the spin-structure studies have been focused on the spin-1/2 nucleon. Different spin physics, such as the tensor structure in the deuteron, exists for higher-spin hadrons. The measurement of these additional spin-dependent structure functions provides important information about nonnucleonic components in spin-one nuclei and tensor structures on the quark–parton level [3]. A general formalism of the deep inelastic electron–deuteron scattering was discussed in Ref. [4], where new four tensor structure functions $b_i(x)$, $i = 1, \dots, 4$, were introduced. They can be measured using a tensor-polarized target and an unpolarized electron beam. Among these new structure functions, only one, b_1, is the leading twist in QCD [4], and it was found that this function is small for a weakly coupled system of nucleons (for example, deuteron). Therefore, the measurement of b_1 for deuteron can give information about its possible exotic components.

From the theoretical standpoint, the spin-dependent structure function $b_1(x)$ was investigated in a number of papers. The available fixed targets with $J \geq 1$ are only nuclei (deuteron is the most commonly used nucleus). If the nucleons in the deuteron are in the S state, then $b_1(x) \equiv 0$. For nucleons in the D state, $b_1(x) \neq 0$ in general [4]. It was found [5] that in the quark–parton model, the sum rule

$$\int dx\, b_1(x) = 0$$

is generally true if the sea of quarks and antiquarks

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is unpolarized (and it was shown how this sum rule is modified in the presence of a polarized sea). Mankiewicz [6] has studied $b_1(x)$ for the ρ meson and noticed empirically that

$$\int dx\, b_1(x) = 0$$

in his model. It was shown in Ref. [7] that multiple scattering terms at low x can still lead to $b_1 \neq 0$ even in the case where only the S-wave component is present. Various twist-two structure functions of deuteron (in particular, b_1) have been calculated in a version of the convolution model that incorporates relativistic and binding energy corrections [8]. Simple parameterizations of these structure functions are given in terms of few deuteron wave function parameters and the free nucleon structure functions. The tensor structure functions were discussed in Ref. [9] in the case of lepton scattering and in hadron reactions such as the polarized proton – deuteron Drell – Yan process.

As is known, the HERMES experiment has been designed to measure the nucleon spin-dependent structure functions from deep inelastic scattering of longitudinally polarized positrons and electrons from polarized gaseous targets (H, D, ^{3}He). In 2000, HERMES collected a data set with a tensor-polarized deuterium target for the purpose of making the first measurement of the tensor structure function $b_1(x)$. The preliminary results on this structure function are presented in Ref. [10] for the kinematic range $0.002 < x < 0.85$ and $0.1\ \mathrm{GeV}^2 < Q^2 < 20\ \mathrm{GeV}^2$. The preliminary result for the tensor asymmetry is sufficiently small to produce an effect of more than 1 % on the measurement of g_1^d. The dependence of b_1 on the x variable is in qualitative agreement with the expectations based on coherent double-scattering models [11–13] and favors a sizeable value of b_1 in the low-x region. This suggests a significant tensor polarization of the sea quarks, violating the Close – Kumano sum rule [5].

The radiative corrections to deep inelastic scattering of unpolarized and longitudinally polarized electron beams on a polarized deuteron target were considered in Ref. [14] in a particular case of the deuteron polarization (which can be obtained from the general covariant spin-density matrix [15] when spin functions are eigenvectors of the spin projection operator). The leading-log model-independent radiative corrections in deep inelastic scattering of an unpolarized electron beam off the tensor-polarized deuteron target were considered in Ref. [16]. The calculation is based on the covariant parameterization of the deuteron quadrupole polarization tensor and use a Drell – Yan-like representation.

Current experiments at modern accelerators reached a new level of precision, and this circumstance requires a new approach to data analysis and inclusion of all possible systematic uncertainties. One of the important sources of such uncertainties is the electromagnetic radiative effects caused by physical processes occuring in higher orders of the perturbation theory with respect to the electromagnetic interaction. In the present paper, we give a covariant description of the deep inelastic scattering of an unpolarized electron beam off the tensor-polarized deuteron target (the polarization state of the target is described by the spin-density matrix of the general form) with the radiative corrections

$$e^-(k_1) + d(p) \to e^-(k_2) + X(p_x) \qquad (1)$$

taken into account.

The corresponding approach is based on the covariant parameterization of the deuteron quadrupole polarization tensor in terms of the 4-momenta of the particles in process (1) [16]. We also performed numerical calculations of the radiative corrections for the kinematical conditions of the experiment [10]. The contribution of the radiative tail from the elastic ed scattering is considered separately.

2. BORN APPROXIMATION

The standard set of variables used for the description of deep inelastic scattering processes is

$$x = \frac{-q^2}{2pq}, \quad y = \frac{2pq}{V}, \qquad V = 2pk_1, \quad q^2 = -Vxy, \quad q = k_1 - k_2, \qquad (2)$$

where q is the 4-momentum of the intermediate heavy photon that probes the deuteron structure. We first define the deep inelastic scattering cross section of process (1) in terms of the contraction of the leptonic $L_{\mu\nu}$ and hadronic $W_{\mu\nu}$ tensors (in the Born approximation, we can neglect the electron mass)

$$\frac{d\sigma}{dx dQ_B^2} = \frac{\pi\alpha^2}{VQ_B^4}\frac{y}{x}L_{\mu\nu}W_{\mu\nu}. \qquad (3)$$

We note that only in the Born approximation (without accounting for radiative corrections),

$$q = k_1 - k_2, \quad Q_B^2 = -q^2 = 2k_1k_2.$$

The Born leptonic tensor (in the unpolarized case) is

$$L_{\mu\nu}^B = q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}). \qquad (4)$$

The hadronic tensor is defined as

$$W_{\mu\nu} = (2\pi)^3 \sum_X \delta^{(4)}(k_1 + p - k_2 - p_x)\overline{J_\mu J_\nu^*},$$

where J_μ is the electromagnetic current for the $\gamma^* + d \to X$ transition (γ^* is the virtual photon). The sum means summation over the final states and the bar means averaging over the polarizations of the target and summation over the polarizations of the final particles. To write the hadron tensor in terms of the structure functions, we first define the deuteron spin-density matrix (we do not consider the effect caused by the vector polarization of the deuteron in what follows)

$$\rho_{\mu\nu} = -\frac{1}{3}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2}\right) - \frac{i}{2M}\varepsilon_{\mu\nu\lambda\rho}s_\lambda p_\rho + Q_{\mu\nu}, \quad Q_{\mu\nu} = Q_{\nu\mu}, \quad Q_{\mu\mu} = 0, \quad p_\mu Q_{\mu\nu} = 0, \tag{5}$$

where s_μ and $Q_{\mu\nu}$ are the target deuteron polarization 4-vector and the deuteron quadrupole polarization tensor. The corresponding hadron tensor has both the polarization-independent and polarization-dependent parts and in the general case can be written as

$$W_{\mu\nu} = W_{\mu\nu}(0) + W_{\mu\nu}(V) + W_{\mu\nu}(T), \tag{6}$$

where $W_{\mu\nu}(0)$ corresponds to the unpolarized case and $W_{\mu\nu}(V)(W_{\mu\nu}(T))$ corresponds to the case of the vector (tensor) polarization of the deuteron target. The $W_{\mu\nu}(0)$ term has the form

$$W_{\mu\nu}(0) = -W_1\tilde{g}_{\mu\nu} + \frac{W_2}{M^2}\tilde{p}_\mu\tilde{p}_\nu, \quad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad \tilde{p}_\mu = p_\mu - \frac{pq}{q^2}q_\mu, \tag{7}$$

where M is the deuteron mass and $W_{1,2}$ are the unpolarized structure functions depending on two independent variables x and q^2. The part of the hadron tensor that depends on the quadrupole polarization tensor can be represented as

$$W_{\mu\nu}(T) = \frac{M^2}{(pq)^2}\left\{Q_{\alpha\beta}q_\alpha q_\beta\left(B_1\tilde{g}_{\mu\nu} + \frac{B_2}{pq}\tilde{p}_\mu\tilde{p}_\nu\right) + B_3 q_\alpha(\tilde{p}_\mu Q_{\tilde{\nu}\alpha} + \tilde{p}_\nu Q_{\tilde{\mu}\alpha}) + pqB_4\widetilde{Q}_{\mu\nu}\right\}. \tag{8}$$

Here, B_i ($i = 1, 2, 3, 4$) are the spin-dependent structure functions (caused by the tensor polarization of the target). They are also functions of the two variables q^2 and x. Because the hadron tensor $W_{\mu\nu}(T)$ is symmetric under $\mu \leftrightarrow \nu$, measuring these new structure functions does not require the electron beam to be polarized.

We used the following notation in formula (8):

$$Q_{\mu\tilde{\nu}} = Q_{\mu\nu} - \frac{q_\nu q_\alpha}{q^2}Q_{\mu\alpha}, \quad Q_{\mu\tilde{\nu}}q_\nu = 0,$$

$$\widetilde{Q}_{\mu\nu} = Q_{\mu\nu} + \frac{q_\mu q_\nu}{q^4}Q_{\alpha\beta}q_\alpha q_\beta - \frac{q_\nu q_\alpha}{q^2}Q_{\mu\alpha} - \frac{q_\mu q_\alpha}{q^2}Q_{\nu\alpha}, \quad \widetilde{Q}_{\mu\nu}q_\nu = 0. \tag{9}$$

We note that the deuteron spin-dependent structure functions B_i are also related to the structure functions b_i introduced in Ref. [4] as

$$B_1 = -b_1, \quad B_2 = \frac{b_2}{3} + b_3 + b_4, \quad B_3 = \frac{b_2}{6} - \frac{b_4}{2}, \quad B_4 = \frac{b_2}{3} - b_3. \tag{10}$$

In calculating radiative corrections, it is convenient to parameterize the polarization state of the deuteron target in terms of the 4-momenta of the particles participating in the reaction under consideration. Therefore, first, we have to find the set of the axes and write them in a covariant form in terms of the 4-momenta. If we choose, in the laboratory system of reaction (1), the longitudinal direction $\mathbf{l}$ along the electron beam and the transverse one $\mathbf{t}$ in the plane $(\mathbf{k}_1, \mathbf{k}_2)$ and perpendicular to $\mathbf{l}$, then

$$S_\mu^{(l)} = \frac{2\tau k_{1\mu} - p_\mu}{M}, \quad S_\mu^{(t)} = \frac{k_{2\mu} - (1 - y - 2xy\tau)k_{1\mu} - xyp_\mu}{d}, \quad S_\mu^{(n)} = \frac{2\varepsilon_{\mu\lambda\rho\sigma}p_\lambda k_{1\rho}k_{2\sigma}}{Vd}, \tag{11}$$

$$d = \sqrt{Vxyb}, \quad b = 1 - y - xy\tau, \quad \tau = M^2/V.$$

We chose one of the axes along the direction $\mathbf{l}$ because in the experiment on measuring the b_1 structure function [10], the direction of the magnetic field used for polarization of the deuteron target is along the positron beam line. The direction of the magnetic field provides the quantization axis for the nuclear spin in the target.

It can be verified that the set of the 4-vectors $S_\mu^{(l,t,n)}$ has the properties

$$S_\mu^{(\alpha)}S_\mu^{(\beta)} = -\delta_{\alpha\beta}, \quad S_\mu^{(\alpha)}p_\mu = 0, \quad \alpha, \beta = l, t, n, \tag{12}$$

and that in the rest frame of the deuteron (the laboratory system),

$$S_\mu^{(l)} = (0, \mathbf{l}), \quad S_\mu^{(t)} = (0, \mathbf{t}), \quad S_\mu^{(n)} = (0, \mathbf{n}),$$

$$\mathbf{l}=\mathbf{n}_1,\quad \mathbf{t}=\frac{\mathbf{n}_2-(\mathbf{n}_1\cdot\mathbf{n}_2)\mathbf{n}_1}{\sqrt{1-(\mathbf{n}_1\cdot\mathbf{n}_2)^2}},$$
$$\mathbf{n}=\frac{\mathbf{n}_1\times\mathbf{n}_2}{\sqrt{1-(\mathbf{n}_1\cdot\mathbf{n}_2)^2}},\quad \mathbf{n}_{1,2}=\frac{\mathbf{k}_{1,2}}{|\mathbf{k}_{1,2}|}. \tag{13}$$

Adding one more 4-vector $S_\mu^{(0)}=p_\mu/M$ to set (11), we obtain a complete set of orthogonal 4-vectors with the properties

$$S_\mu^{(m)}S_\nu^{(m)}=g_{\mu\nu},$$
$$S_\mu^{(m)}S_\mu^{(n)}=g_{mn},\quad m,n=0,l,t,n. \tag{14}$$

This allows us to express the deuteron quadrupole polarization tensor, in the general case, as

$$Q_{\mu\nu}=S_\mu^{(m)}S_\nu^{(n)}R_{mn}\equiv S_\mu^{(\alpha)}S_\nu^{(\beta)}R_{\alpha\beta},$$
$$R_{\alpha\beta}=R_{\beta\alpha},\quad R_{\alpha\alpha}=0, \tag{15}$$

because the components R_{00}, $R_{0\alpha}$, and $R_{\alpha 0}$ are identically equal to zero due to the condition $Q_{\mu\nu}p_\nu=0$.

In the Born approximation, the components R_{ln} and R_{tn} do not contribute to the cross section (because the 4-momenta q_μ and $k_{1\mu}$ are orthogonal to the 4-vector $S_\mu^{(n)}$) and expansion (15) can be rewritten in the standard form

$$Q_{\mu\nu}=\left[S_\mu^{(l)}S_\nu^{(l)}-\frac{1}{2}S_\mu^{(t)}S_\nu^{(t)}\right]R_{ll}+$$
$$+\frac{1}{2}S_\mu^{(t)}S_\nu^{(t)}(R_{tt}-R_{nn})+\left(S_\mu^{(l)}S_\nu^{(t)}+S_\mu^{(t)}S_\nu^{(l)}\right)R_{lt}, \tag{16}$$

where we took into account that

$$R_{ll}+R_{tt}+R_{nn}=0.$$

In what follows, we consider the deep inelastic scattering of the unpolarized electron beam from the tensor-polarized deuteron target. Thus, we have to calculate only the contraction of the Born leptonic tensor $L^B_{\mu\nu}$ and the hadronic tensor $W_{\mu\nu}(T)$ caused by the tensor polarization of the target,

$$S^B(T)=L^B_{\mu\nu}W_{\mu\nu}(T)=$$
$$=8\frac{\tau}{y}\left\{-\frac{1}{y^2}[xy^2B_1+(a-1+y)B_2+yB_3]Q_0+\right.$$
$$\left.+\frac{1}{y}[(2-y)B_3-yB_4]Q_1+B_4Q_{11}\right\}, \tag{17}$$

where

$$a=xy\tau,\quad Q_0=Q_{\alpha\beta}q_\alpha q_\beta,$$

$$Q_1=Q_{\alpha\beta}q_\alpha k_{1\beta},\quad Q_{11}=Q_{\alpha\beta}k_{1\alpha}k_{1\beta}.$$

Using the formulas for the vectors $S_\mu^{(\alpha)}$, we can calculate the contractions. After simple calculation, we have

$$\frac{d\sigma_B(T)}{dxdQ_B^2}=$$
$$=\frac{2\pi\alpha^2}{xQ_B^4}\left[S_{ll}R_{ll}+S_{tt}(R_{tt}-R_{nn})+S_{lt}R_{lt}\right], \tag{18}$$

with

$$S_{ll}=\left[2xb\tau-y(1+2x\tau)^2\right]G+2b(1+3x\tau)B_3+(b-a)B_4,$$

$$S_{lt}=2\sqrt{\frac{xb\tau}{y}}\times$$
$$\times\left[2(y+2a)G+(2-y-4b)B_3+yB_4\right], \tag{19}$$

$$S_{tt}=-2xb\tau(G+B_3),\quad G=xyB_1-\frac{b}{y}B_2.$$

Therefore, in the general case, the cross section of deep inelastic scattering of an unpolarized electron beam from a tensor-polarized target is determined, in the Born approximation, by the components of the quadrupole polarization tensor R_{ll}, R_{lt}, and the combination $(R_{tt}-R_{nn})$.

We now consider just one more, commonly used, choice of the coordinate axes: components of the deuteron polarization tensor are defined in the coordinate system with the axes along the directions **L**, **T**, and **N** in the rest frame of the deuteron, where

$$\mathbf{L}=\frac{\mathbf{k}_1-\mathbf{k}_2}{|\mathbf{k}_1-\mathbf{k}_2|},\quad \mathbf{T}=\frac{\mathbf{n}_1-(\mathbf{n}_1\cdot\mathbf{L})\mathbf{L}}{\sqrt{1-(\mathbf{n}_1\cdot\mathbf{L})^2}},\quad \mathbf{N}=\mathbf{n}. \tag{20}$$

The corresponding covariant form of set (20) is given by

$$S_\mu^{(L)}=\frac{2\tau(k_1-k_2)_\mu-yp_\mu}{M\sqrt{yh}},$$
$$S_\mu^{(T)}=\frac{(1+2x\tau)k_{2\mu}-(1-y-2x\tau)k_{1\mu}-x(2-y)p_\mu}{\sqrt{Vxbh}}, \tag{21}$$

$$S_\mu^{(N)} = S_\mu^{(n)}, \quad h = y + 4x\tau,$$

and the expansion of the deuteron polarization tensor is defined in full analogy with (16),

$$Q_{\mu\nu} = \left[S_\mu^{(L)}S_\nu^{(L)} - \frac{1}{2}S_\mu^{(T)}S_\nu^{(T)}\right] R_{LL} + \\ + \frac{1}{2}S_\mu^{(T)}S_\nu^{(T)}\left(R_{TT} - R_{NN}\right) + \\ + \left(S_\mu^{(L)}S_\nu^{(T)} + S_\mu^{(T)}S_\nu^{(L)}\right)R_{LT}. \quad (22)$$

These two sets of orthogonal 4-vectors are connected by an orthogonal matrix that describes a rotation in the plane perpendicular to the direction $\mathbf{n} = \mathbf{N}$,

$$\begin{aligned} S_\mu^{(L)} &= \cos\theta S_\mu^{(l)} + \sin\theta S_\mu^{(t)}, \\ S_\mu^{(T)} &= -\sin\theta S_\mu^{(l)} + \cos\theta S_\mu^{(t)}, \end{aligned} \quad (23)$$

$$\cos\theta = \frac{y(1+2x\tau)}{\sqrt{yh}}, \quad \sin\theta = -2\sqrt{\frac{xb\tau}{h}}.$$

In this set of axes, the part of the differential cross section that depends on the tensor polarization can be written as

$$\frac{d\sigma_B(T)}{dxdQ_B^2} = \frac{2\pi\alpha^2}{xQ_B^4} \times \\ \times \left[S_{LL}R_{LL} + S_{TT}(R_{TT} - R_{NN}) + S_{LT}R_{LT}\right], \quad (24)$$

$$S_{LL} = -hG + 2bB_3 + \frac{B_4}{h}[(1-y)(y-2x\tau) - 2a(y+x\tau)],$$

$$\begin{aligned} S_{TT} &= \frac{2xb\tau}{h}B_4, \\ S_{LT} &= 2\sqrt{\frac{xb\tau}{y}}(2-y)\left(B_3 + \frac{y}{h}B_4\right). \end{aligned} \quad (25)$$

3. RADIATIVE CORRECTIONS

In this paper, we consider only the QED radiative corrections to the deep inelastic scattering process (1). We confine ourselves to calculation of the so-called model-independent radiative corrections, corresponding to photons radiated from a lepton line with the vacuum polarization taken into account. The reason is that it gives the leading contribution to radiative corrections due to the smallness of the electron mass, and can be calculated without any additional assumptions. Nevertheless, these radiative corrections depend on the shape of the deuteron structure functions (both spin-independent and spin-dependent) through their dependence on the x and Q^2 variables.

There exist two contributions to radiative corrections when we take the corrections of the order α into account. The first one is caused by virtual and soft photon emission that cannot affect the kinematics of process (1). The second one arises due to the radiation of a hard photon,

$$e^-(k_1) + d(p) \to e^-(k_2) + \gamma(k) + X(p_x). \quad (26)$$

The leptonic tensor corresponding to the hard-photon radiation is well-known [17, 18]. For an unpolarized electron beam, it can be written as

$$L_{\mu\nu}^\gamma = A_0\widetilde{g}_{\mu\nu} + A_1\widetilde{k}_{1\mu}\widetilde{k}_{1\nu} + A_2\widetilde{k}_{2\mu}\widetilde{k}_{2\nu}, \quad (27)$$

where

$$A_0 = -\frac{(q^2+\chi_1)^2 + (q^2-\chi_2)^2}{\chi_1\chi_2} - 2m^2q^2\left(\frac{1}{\chi_1^2} + \frac{1}{\chi_2^2}\right),$$

$$A_1 = -4\left(\frac{q^2}{\chi_1\chi_2} + \frac{2m^2}{\chi_2^2}\right),$$

$$A_2 = -4\left(\frac{q^2}{\chi_1\chi_2} + \frac{2m^2}{\chi_1^2}\right),$$

$$\tilde{k}_{i\mu} = k_{i\mu} - \frac{qk_i}{q^2}q_\mu, \quad i = 1, 2,$$

with

$$\chi_{1,2} = 2kk_{1,2},$$

m is the electron mass,

$$q^2 = \chi_2 - \chi_1 - Q_B^2,$$

and

$$q = k_1 - k_2 - k$$

in this section. The hadronic tensor in this case has the same form as the hadronic tensor in the Born approximation, but the momentum transfer q differs from the Born one and the structure functions B_i depend on the new momentum q. Here and in what follows, we neglect the terms vanishing as $m \to 0$.

We consider the hard photon (with the energy $\omega > \Delta\varepsilon$, where $\Delta \ll 1$) emission process using the approach in [19], where it was applied to the process of deep inelastic scattering on an unpolarized target. We introduce the variables suitable for this process,

$$z = \frac{M_x^2 - M^2}{V} = \frac{q^2 + 2pq}{V}, \quad r = -\frac{q^2}{Q_B^2},$$

$$x' = \frac{-q^2}{2pq} = \frac{xyr}{xyr + z}, \quad \chi_{1,2} = 2kk_{1,2},$$

where M_x is the invariant mass of the hadron system produced in scattering of the photon (with the virtuality q^2) by the target.

We note the physical meaning of the z variable: it shows the degree of deviation from the elastic process ($ed \to ed$). Therefore, the value $z = 0$ corresponds to the elastic ed scattering threshold and the value $z = \varepsilon_d/\varepsilon_1$ (where ε_d is the deuteron bound energy and ε_1 is the electron beam energy in the laboratory system) corresponds to the $ed \to enp$ reaction threshold (quasielastic ed scattering).

The contraction of the leptonic and hadronic tensors can be represented as

$$S^\gamma(T) = AA_0 + BA_1 + CA_2, \qquad (28)$$

$$A = NQ_0\left[3B_1 + \frac{2\tau}{c}B_2 + \frac{c}{2xyr}(B_2 + 2B_3 + B_4)\right],$$

$$B = N\left\{Q_0\left[\frac{V}{2c}B_2 - V\frac{Q_B^2+\chi_1}{2rQ_B^2}(B_2+B_3) + \frac{(Q_B^2+\chi_1)^2}{4rQ_B^2}\left(B_1 + \frac{Vc}{2rQ_B^2}(B_2+2B_3+B_4)\right)\right] + VQ_1\left[B_3 - \frac{Q_B^2+\chi_1}{2rQ_B^2}c(B_3+B_4)\right] + \frac{V}{2}cQ_{11}B_4\right\},$$

$$C = N\left\{Q_0\left[\frac{V}{2}\frac{(1-y)^2}{c}B_2 + V\frac{Q_B^2-\chi_2}{2rQ_B^2}(1-y)(B_2+B_3) + \frac{(Q_B^2-\chi_2)^2}{4rQ_B^2}\times \left(B_1 + \frac{Vc}{2rQ_B^2}(B_2+2B_3+B_4)\right)\right] + VQ_2\left[B_3(1-y) + \frac{Q_B^2-\chi_2}{2rQ_B^2}c(B_3+B_4)\right] + \frac{V}{2}cQ_{22}B_4\right\},$$

where

$$N = 4\tau/Vc^2, \quad c = z + xyr.$$

The quantities Q_0, Q_1, Q_2, Q_{11}, and Q_{22} are the contractions of the deuteron quadrupole polarization tensor and 4-momenta. They can be expressed in terms of the scalar products of the 4-momenta of the particles participating in the reaction and the set of 4-vectors $S_\mu^{(l,t,n)}$. Therefore, these contractions are given by

$$Q_0 = Q_{\alpha\beta}q_\alpha q_\beta = \left[(lq)^2 - \frac{1}{2}(tq)^2 - \frac{1}{2}(nq)^2\right]\times R_{ll} + 2lqtqR_{lt} + 2nqlqR_{ln} + 2nqtqR_{tn} + \frac{1}{2}[(tq)^2-(nq)^2](R_{tt}-R_{nn}),$$

$$Q_1 = Q_{\alpha\beta}q_\alpha k_{1\beta} = \left(lk_1lq - \frac{1}{2}tk_1tq\right)R_{ll} + (lk_1tq + tk_1lq)R_{lt} + lk_1nqR_{ln} + tk_1nqR_{tn} + \frac{1}{2}tk_1tq(R_{tt}-R_{nn}), \qquad (29)$$

$$Q_{11} = Q_{\alpha\beta}k_{1\alpha}k_{1\beta} = \left[(lk_1)^2 - \frac{1}{2}(tk_1)^2\right]R_{ll} + 2lk_1tk_1R_{lt} + \frac{1}{2}(tk_1)^2(R_{tt}-R_{nn}),$$

$$Q_2 = Q_1(k_1\to k_2), \quad Q_{22} = Q_{11}(k_1 \to k_2),$$

$$ia = S_\mu^{(i)}a_\mu, \quad i = l,t,n,$$

where we used the conditions

$$R_{ll} + R_{tt} + R_{nn} = 0, \quad nk_1 = nk_2 = 0.$$

For the set of the 4-vectors $S_\mu^{(l,t,n)}$, we also have $tk_1 = 0$.

It is convenient to separate the poles in the term $(\chi_1\chi_2)^{-1}$ using the relation

$$\frac{1}{\chi_1\chi_2} = \frac{1}{Q_B^2}\frac{1}{1-r}\left(\frac{1}{\chi_1} - \frac{1}{\chi_2}\right).$$

Then the radiative correction (caused by the hard-photon emission) to the differential cross section of deep inelastic scattering of an unpolarized electron beam by the tensor polarized target has the form

$$\frac{d\sigma^\gamma}{dxdQ_B^2} = \frac{\alpha y}{Vx}\int\frac{d^3k}{2\pi\omega}\Sigma(z,r), \qquad (30)$$

where ω is the energy of the hard photon and

$$\Sigma(z,r) = \frac{\alpha^2(q^2)}{Q_B^4}\left\{R_0(z,r) + \left(\frac{1}{\chi_1} - \frac{1}{\chi_2}\right)R_1(z,r) + \frac{m^2}{\chi_1^2}R_{1m}(z,r) + \frac{m^2}{\chi_2^2}R_{2m}(z,r)\right\}, \qquad (31)$$

$$R_0 = -\frac{2}{r^2}A,$$

$$R_1 = \frac{1}{r-1}\left[\left(1+\frac{1}{r^2}\right)Q_B^2A - \frac{4}{r}(B+C)\right],$$

$$R_{1m} = 2\left(\frac{Q_B^2}{r}A - \frac{4}{r^2}C\right),$$

$$R_{2m} = 2\left(\frac{Q_B^2}{r}A - \frac{4}{r^2}B\right).$$

It is convenient to write the integral in Eq. (30) as

$$I = \int\frac{d^3k}{2\pi\omega}\Sigma(z,r) = I_{1m} + I_{2m} + I_R, \qquad (32)$$

where we separate the contributions proportional to m^2,

$$I_{1m} = \int \frac{d^3k}{2\pi\omega} \frac{\alpha^2(q^2)}{Q_B^4} \frac{m^2}{\chi_1^2} R_{1m}(z,r),$$
$$I_{2m} = \int \frac{d^3k}{2\pi\omega} \frac{\alpha^2(q^2)}{Q_B^4} \frac{m^2}{\chi_2^2} R_{2m}(z,r). \qquad (33)$$

We first consider the integrals I_{im}, $i = 1,2$. The numerator of the integrands in $I_{1m}(I_{2m})$ is then calculated in the approximation $\chi_1 = 0$ ($\chi_2 = 0$) [19]. The integration measure over the hard-photon phase space is written as

$$\frac{d^3k}{2\pi\omega} = \frac{dz}{z_+ - z}\frac{\omega^2 d\Omega_k}{2\pi}, \quad z_+ = y(1-x). \qquad (34)$$

Using the invariance of $\omega^2 d\Omega_k$, we can integrate over the angular variables $d\Omega_k$ in the most suitable coordinate system, namely, in the coordinate frame where

$$\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{p} = 0$$

(the center-of-mass system of the scattered electron and the produced hadronic system). We obtain

$$\int \frac{\omega^2 d\Omega_k}{2\pi} \frac{m^2}{\chi_{1,2}^2} = \frac{1}{2}.$$

3.1. Integral I_{1m}

We calculate the integrand in the approximation where $\chi_1 = 0$ (except in the denominator). This approximation corresponds to the emission of a collinear photon along the initial-electron momentum. In this case, the variables take the values

$$r_1 = \frac{1-y+z}{1-xy}, \quad q_1^2 = -r_1 Q_B^2, \quad x_1' = \frac{xyr_1}{z+xyr_1}.$$

After integrating over the hard-photon angular variables, the integral I_{1m} can be represented as

$$I_{1m} = \frac{1}{Q_B^4}\int\limits_0^{z_m} \frac{dz}{z_+ - z}\alpha_1^2 N_1 \Sigma_1(z), \qquad (35)$$

$$\Sigma_1(z) = \Sigma_{1ll}R_{ll} + \Sigma_{1lt}R_{lt} + \Sigma_{1tt}(R_{tt} - R_{nn}),$$
$$\Sigma_{1tt} = b\frac{Q_B^2}{r_1}(G_t + B_{3t}),$$

$$\Sigma_{1lt} = -\frac{V}{r_1}\sqrt{\frac{xyb}{\tau}}\left[(y-1+r_1)B_{4t} + (a-3b+r_1)B_{3t} + {} + 2(a-b+r_1)G_t\right],$$

$$\Sigma_{1ll} = \frac{V}{2\tau r_1}\{(a-b)(y-1+r_1)B_{4t} + {} + 2b(b-2a-r_1)B_{3t} - [2ab - (a-b+r_1)^2]G_t\},$$

$$G_t = xyB_{1t} - \frac{b}{y-1+r_1}B_{2t}, \quad \alpha_1 = \alpha(q_1^2),$$
$$N_1 = \frac{4\tau}{(z+xyr_1)^2},$$
$$z_m = z_+ - \rho, \quad \rho = 2\Delta\varepsilon\sqrt{(\tau+z_+)/V},$$
$$B_{it} = B_i(q_1^2, x_1'), \quad i = 1\text{–}4.$$

It is convenient to explicitly extract the contribution containing the infrared divergence. For this, we add to the numerator of the integrand and subtract from it its value at $z = z_+$. At this value, we have

$$r_1 = 1, \quad \alpha_1 = \alpha, \quad N_1 = 4\tau/y^2, \quad x_1' = x.$$

The integral I_{1m} can thus be written as

$$I_{1m} = \frac{1}{Q_B^4}\int\limits_0^{z_+} \frac{dz}{z_+ - z} \times \left[\alpha_1^2 N_1 \Sigma_1(z) - \alpha^2\frac{4\tau}{y^2}\Sigma_1(z_+)\right] + {} + \frac{Vx}{\pi y}\ln\frac{\rho}{z_+}\frac{d\sigma_B}{dx\,dQ_B^2}. \qquad (36)$$

3.2. Integral I_{2m}

Calculation of the integrand is performed in the approximation $\chi_2 = 0$, which corresponds to the emission of a collinear photon along the final-electron momentum. In this case, the variables take the values

$$r_2 = \frac{1-z}{1-z_+}, \quad q_2^2 = -r_2 Q_B^2, \quad x_2' = \frac{xyr_2}{1-r_2(1-y)}.$$

After integrating over the hard-photon angular variables, the integral I_{2m} is represented as

$$I_{2m} = \frac{1}{Q_B^4}\int\limits_0^{z_m} \frac{dz}{z_+ - z}\alpha_2^2 N_2 \Sigma_2(z), \qquad (37)$$

$$\Sigma_2(z) = \Sigma_{2ll}R_{ll} + \Sigma_{2lt}R_{lt} + \Sigma_{2tt}(R_{tt} - R_{nn}),$$
$$\Sigma_{2tt} = bQ_B^2(r_2 G_s + B_{3s}),$$

$$\Sigma_{2lt} = -V\sqrt{\frac{xyb}{\tau}}\left\{\left(y-1+\frac{1}{r_2}\right)B_{4s} + {} + \left(a-3b+\frac{1}{r_2}\right)B_{3s} + 2[1+(a-b)r_2]G_s\right\},$$

$$\Sigma_{2ll} = \frac{V}{2\tau r_2}\{(a-b)[1-r_2(1-y)]B_{4s} - -2b[1+(2a-b)r_2]B_{3s} - [2abr_2^2 - (1+ar_2-br_2)^2]G_s\},$$

$$G_s = xyB_{1s} - \frac{b}{1-r_2(1-y)}B_{2s}, \quad \alpha_2 = \alpha(q_2^2),$$

$$N_2 = \frac{4\tau}{(z+xyr_2)^2}, \quad B_{is} = B_i(q_2^2, x_2'), \quad i = 1,2,3,4.$$

The contribution containing the infrared divergence is extracted explicitly in a similar manner as for the I_{1m} integral. At the value $z = z_+$, we have

$$r_2 = 1, \quad \alpha_2 = \alpha, \quad N_2 = 4\tau/y^2, \quad x_2' = x.$$

The integral I_{2m} is then rewritten as

$$I_{2m} = \frac{1}{Q_B^4}\int_0^{z_+}\frac{dz}{z_+-z} \times \times\left[\alpha_2^2 N_2\Sigma_2(z) - \alpha^2\frac{4\tau}{y^2}\Sigma_2(z_+)\right] + + \frac{Vx}{\pi y}\ln\frac{\rho}{z_+}\frac{d\sigma_B}{dx\,dQ_B^2}. \quad (38)$$

The radiative corrections due to the virtual photon exchange and real soft-photon emission (with energy $\omega < \Delta\varepsilon$) can be related to the Born cross section as[1)]

$$\frac{d\sigma^{(S+V)}}{dx\,dQ_B^2} = \delta^{SV}\frac{d\sigma_B}{dx\,dQ_B^2}, \quad (39)$$

where the factor δ^{SV} is [19]

$$\delta^{SV} = \frac{\alpha}{\pi}\left[(L-1)\ln\frac{(\Delta\varepsilon)^2}{\varepsilon_1\varepsilon_2} + \frac{3}{2}L - -\frac{1}{2}\ln^2\frac{\varepsilon_1}{\varepsilon_2} - \frac{\pi^2}{6} - 2 - f\left(\cos^2\frac{\theta}{2}\right)\right], \quad L = \ln\frac{Q_B^2}{m^2}, \quad (40)$$

$\varepsilon_1(\varepsilon_2)$ is the initial (final) electron energy, and θ is the electron scattering angle in the coordinate frame where

$$\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{p} = 0.$$

The function f is defined as

$$f(x) = \int_0^x \frac{dt}{t}\ln(1-t).$$

[1)] We note that the vacuum polarization effects are included in the Born cross section through the dependence of the coupling constant α on the virtual-photon momentum.

The quantities ε_1, ε_2, and θ can be expressed in terms of the invariant variables as

$$\varepsilon_1 = \frac{V(1-xy)}{2\sqrt{V(\tau+z_+)}}, \quad \varepsilon_2 = \frac{V(1-z_+)}{2\sqrt{V(\tau+z_+)}}, \quad \cos^2\frac{\theta}{2} = \frac{1-y-xy\tau}{(1-xy)(1-z_+)}. \quad (41)$$

The radiative correction δ^{SV} is finally rewritten as

$$\delta^{SV} = \frac{\alpha}{2\pi}\left\{-1-\frac{\pi^2}{3} - 2f\left[\frac{1-y-xy\tau}{(1-xy)(1-z_+)}\right] - -\ln^2\frac{1-xy}{1-z_+} + (L-1)\times \times\left(3 + 2\ln\frac{\rho^2}{(1-xy)(1-z_+)}\right)\right\}. \quad (42)$$

3.3. Integral I_R

To calculate this integral, we use the results in Ref. [19]. In addition to the integrals calculated in that paper, we need the integrals

$$\int\frac{d^3k}{2\pi\omega}F(z,r)\chi_1, \quad \int\frac{d^3k}{2\pi\omega}F(z,r)\chi_1^2. \quad (43)$$

To calculate these integrals, we write the hard-photon phase space measure as

$$\frac{d^3k}{2\pi\omega} = \frac{Q_B^2}{2\sqrt{y^2+4a}}\frac{d\varphi}{2\pi}dz\,dr. \quad (44)$$

Because the function F is independent of the φ variable in our case, we can integrate over this variable. We do this in the coordinate frame specified above. The results are

$$i_1 = \int\frac{d\varphi}{2\pi}\chi_1 = \frac{Q_B^2}{y^2+4a}\times \times\Big[(2-y)(y-c) - (1-r)(y+2a)\Big], \quad (45)$$

$$i_2 = \int\frac{d\varphi}{2\pi}\chi_1^2 = \frac{1}{2}\left[3i_1^2 - \frac{Q_B^4(1-xy)^2}{y^2+4a}(r-r_1)^2\right].$$

After simple calculations, the integral I_R is (with the contributions proportional to the R_{ln} and R_{tn} components omitted)

$$I_R = \frac{1}{2Q_B^4} \times \sum_{i=1}^{4}\sum_{m,n} R_{mn}\left\{ \frac{L_1}{1-xy}\int\limits_0^{z_m} \frac{dz}{1-r_1} G_i^{mn}(z,r_1) + \frac{L_2}{1-z_+}\int\limits_0^{z_m}\frac{dz}{1-r_2}\tilde{G}_i^{mn}(z,r_2) + \frac{1}{1-xy}\int\limits_0^{z_m} dz \int\limits_{r_-}^{r_+}\frac{dr}{|r-r_1|}\left[\frac{G_i^{mn}(z,r)}{1-r} - \frac{G_i^{mn}(z,r_1)}{1-r_1}\right] + \frac{1}{1-z_+}\int\limits_0^{z_m} dz \int\limits_{r_-}^{r_+}\frac{dr}{|r-r_2|}\left[\frac{\tilde{G}_i^{mn}(z,r)}{1-r} - \frac{\tilde{G}_i^{mn}(z,r_2)}{1-r_2}\right] + \frac{Q_B^2}{\sqrt{y^2+4a}}\int\limits_0^{z_m} dz \int\limits_{r_-}^{r_+} dr \frac{\alpha^2}{r^2} \times B_i\left[C_{0i}^{mn}(z,r) + i_1 C_{1i}^{mn}(z,r) + i_2 C_{2i}^{mn}(z,r)\right]\right\}, \quad (46)$$

where

$$L_1 = \ln\frac{Q_B^2(1-xy)^2}{m^2 xy(\tau+z_+)}, \quad L_2 = \ln\frac{Q_B^2(1-z_+)^2}{m^2 xy(\tau+z_+)}, \quad (47)$$

$$r_\pm(z) = \frac{1}{2xy(\tau+z_+)} \times \left[2xy(\tau+z) + (z_+ - z)(y \pm \sqrt{y^2+4a})\right],$$

$$G_i^{mn}(z,r) = \frac{\alpha^2}{r^2}(1-r)B_i A_i^{mn}(z,r),$$

$$\tilde{G}_i^{mn}(z,r) = \frac{\alpha^2}{r^2}(1-r)B_i B_i^{mn}(z,r),$$

with $m,n = l,t,n$. We note that the structure functions B_i are functions of two independent variables

$$q^2 = -rQ_B^2, \quad x' = \frac{xyr}{z+xyr}.$$

The expressions for the coefficients A_i^{mn}, B_i^{mn}, C_{ki}^{mn}, $k = 0,1,2$, are given in Appendix A. The contributions proportional to the R_{ln} and R_{tn} components are considered in more detail in Appendix B.

We now briefly discuss the singularities in the I_R integral. The value $r = 1$ corresponds to the real soft-photon emission (there is an infrared divergence at this point), and the value $r = r_1(r_2)$ corresponds to the emission of a collinear photon along the initial- (final-) electron momentum (the so-called collinear divergence). The singularity at the point $z = z_+$ is the infrared one. The divergence at $r = 1$ is unphysical. It arises during the integration procedure due to the separation of the poles in the expression $(\chi_1\chi_2)^{-1}$. It is necessary to explicitly extract the collinear and infrared divergences in the above formula.

The integrand in the above expression can be written in the form that does not explicitly contain the infrared divergences if we add term (39) to it. For this, we use the transformations

$$\frac{G(z,r_i)}{1-r_i}\ln\frac{\varphi_i(x,y)}{xy(\tau+z_+)} + \int\limits_{r_-}^{r_+}\frac{dr}{|r-r_i|} \times \left[\frac{G(z,r)}{1-r} - \frac{G(z,r_i)}{1-r_i}\right] = P\int\limits_{r_-}^{r_+}\frac{dr}{(1-r)|r-r_i|} \times \left[G(z,r) - G(z,r_i)\right], \quad i = 1,2, \quad (48)$$

where

$$\varphi_1(x,y) = (1-xy)^2, \quad \varphi_2(x,y) = (1-z_+)^2,$$

and the symbol P denotes the principal value of the integral. The total radiative correction (which is the sum of the contribution due to the hard-photon emission and the contribution due to the real soft-photon emission and virtual-photon contribution) to the part of the differential cross section caused by the tensor polarization of the target is written as

$$\frac{d\sigma}{dx\,dQ_B^2} = \frac{d\sigma_B}{dx\,dQ_B^2} + \delta^{tot}, \quad (49)$$

where

$$\delta^{tot} = \frac{\alpha}{2\pi}\left\{3L + 2(L-1)\ln\frac{z_+^2}{(1-xy)(1-z_+)} - \ln^2\frac{1-xy}{1-z_+} - 4 - \frac{\pi^2}{3} - 2f\left[\frac{b}{(1-xy)(1-z_+)}\right]\right\}\frac{d\sigma_B}{dx\,dQ_B^2} + \frac{\alpha y}{xQ_B^4} \times \int\limits_0^{z_+}\frac{dz}{z_+ - z}\left[\alpha_1^2 N_1 \Sigma_1(z) + \alpha_2^2 N_2\Sigma_2(z) - \alpha^2\frac{8\tau}{Vy^2}\Sigma_1(z_+)\right] + \frac{\alpha y}{2xVQ_B^4} \times \sum_{i=1}^{4}\sum_{m,n} R_{mn}\left\{L\int\limits_0^{z_+}\frac{dz}{z_+ - z}\Big[G_i^{mn}(z,r_1) - G_i^{mn}(z_+,1) - \tilde{G}_i^{mn}(z,r_2) + \tilde{G}_i^{mn}(z_+,1)\Big] + \frac{Q_B^2}{\sqrt{y^2+4a}} \times \int\limits_0^{z_+} dz\int\limits_{r_-}^{r_+} dr\frac{\alpha^2}{r^2}B_i\Big[C_{0i}^{mn}(z,r) + i_1 C_{1i}^{mn}(z,r) + i_2 C_{2i}^{mn}(z,r)\Big] + R_i^{mn}\right\}. \quad (50)$$

The term R_i^{mn} has different forms depending on the integration region of the variable r. In the regions $r_- \le r \le r_1$ and $r_2 \le r \le r_+$ (where $r \ne 1$, and therefore the divergence at the point $r = 1$ is absent), the function R_i^{mn} has the form

$$R_i^{mn} = \frac{1}{1-xy}\int_0^{z_+} dz \int_{r_-}^{r_+} \frac{dr}{(1-r)|r-r_1|} \times \Big[G_i^{mn}(z,r) - G_i^{mn}(z,r_1)\Big] + + \frac{1}{1-z_+}\int_0^{z_+} dz \int_{r_-}^{r_+} \frac{dr}{(1-r)|r-r_2|} \times \Big[\tilde{G}_i^{mn}(z,r) - \tilde{G}_i^{mn}(z,r_2)\Big]. \quad (51)$$

In the region $r_1 < r < r_2$, we have

$$R_i^{mn} = \int_0^{z_+} dz \ln\frac{1-r_-}{r_+-1} \times \Big\{ g_{i1}^{mn}(z,1) - f_{i1}^{mn}(z,1) + \frac{1}{z_+-z} \times \Big[g_{i0}^{mn}(z,1) - g_{i0}^{mn}(z,r_1) + + f_{i0}^{mn}(z,1) - f_{i0}^{mn}(z,r_2)\Big]\Big\} + + \int_0^{z_+} dz \int_{r_-}^{r_+} \frac{dr}{1-r}\Big\{ g_{i1}^{mn}(z,r) - g_{i1}^{mn}(z,1) - - f_{i1}^{mn}(z,r) + f_{i1}^{mn}(z,1) + + \frac{1}{1-xy}\Big[F^{mn}(z,r) - F^{mn}(z,1)\Big] - \frac{1}{1-z_+} \times \Big[\tilde{F}^{mn}(z,r) - \tilde{F}^{mn}(z,1)\Big]\Big\}, \quad (52)$$

where we introduce the notation

$$G_i^{mn}(z,r) = g_{i0}^{mn}(z,r) + \Delta_1 g_{i1}^{mn}(z,r),$$

$$\tilde{G}_i^{mn}(z,r) = f_{i0}^{mn}(z,r) + \Delta_2 f_{i1}^{mn}(z,r),$$

$$F^{mn}(z,r) = \frac{1}{r-r_1}\Big[g_{i0}^{mn}(z,r) - g_{i0}^{mn}(z,r_1)\Big],$$
$$\tilde{F}^{mn}(z,r) = \frac{1}{r-r_2}\Big[f_{i0}^{mn}(z,r) - f_{i0}^{mn}(z,r_2)\Big], \quad (53)$$

$\Delta_1 = (1-xy)r - a - b - z, \quad \Delta_2 = (1-y+xy)r + z - 1.$

In obtaining the above formula, we use the relation

$$P\int_{r_-}^{r_+} \frac{dr}{1-r}\Psi(r) = \int_{r_-}^{r_+} \frac{dr}{1-r}\Big[\Psi(r) - \Psi(1)\Big] + + \Psi(1)\ln\frac{1-r_-}{r_+-1}. \quad (54)$$

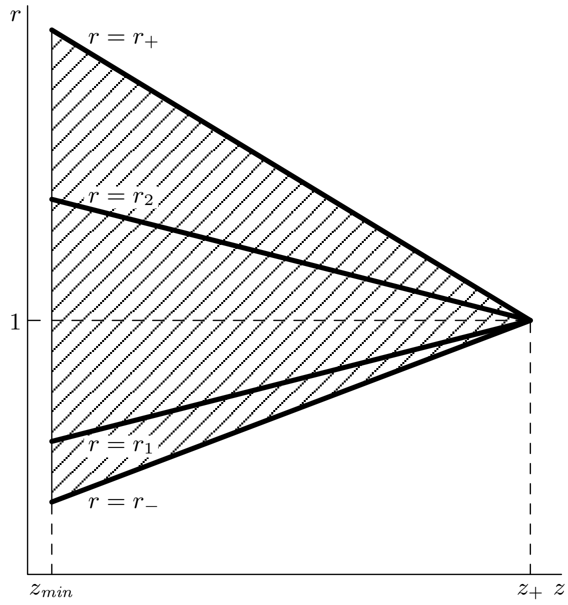

Fig. 1. The integration domain in the r and z variables

We finally consider the part of the integral I caused by the R_{ln} and R_{tn} components of the deuteron quadrupole polarization tensor. As stated above, these components do not contribute to the cross section treated in the Born approximation. If these terms are integrated over the whole region of the φ variable, then these integrals are equal to zero as well (because only one plane remains after such integration). We discuss this problem in more detail in Appendix B.

We note that the integration limits for the variable z in formula (50) are given somewhat schematically. This integral contains two contributions (we neglect here the contribution of the radiative tail from the quasielastic scattering). One of them is the so-called inelastic contribution; the integration region for it in the variables r and z is presented in Fig. 1 by the dashed triangle. The integration over z for this contribution must be carried out from

$$z_{min} = \frac{M_{th}^2 - M}{V}$$

to z_+, where M_{th} is the inelastic threshold ($M_{th} = M + m_\pi$). The second contribution, related to the radiative tail of the elastic peak, is given by the interval

$$z = 0, \quad r_-(0) \le r \le r_+(0).$$

The contribution of the elastic radiative tail to the total radiative correction δ^{tot} (i.e., inclusion of radiative corrections to the elastic ed scattering) can be obtained from formula (30) by a simple substitution in the hadronic tensor,

$$B_i(q^2, x') \to -\frac{1}{q^2}\delta(1-x')B_i^{(el)}, \quad i = 1,2,3,4, \quad (55)$$

where $B_i^{(el)}$ are expressed in terms of the deuteron electromagnetic form factors as

$$B_1^{(el)} = \eta q^2 G_M^2, \quad B_2^{(el)} = -2\eta^2 q^2 \times \times \left[G_M^2 + \frac{4G_Q}{1+\eta}(G_C + \frac{\eta}{3}G_Q + \eta G_M)\right], \quad (56)$$

$$B_3^{(el)} = 2\eta^2 q^2 G_M(G_M + 2G_Q),$$

$$B_4^{(el)} = -2\eta q^2(1+\eta)G_M^2, \quad \eta = -q^2/4M^2.$$

Here, G_C, G_M, and G_Q are the deuteron charge monopole, magnetic dipole, and quadrupole form factors, respectively. These form factors have the normalizations

$$G_C(0) = 1, \quad G_M(0) = (M/m_n)\mu_d,$$

$$G_Q(0) = M^2 Q_d,$$

where m_n is the nucleon mass and $\mu_d(Q_d)$ is the deuteron magnetic (quadrupole) moment, with the values

$$\mu_d = 0.857, \quad Q_d = 0.2859 \text{ fm}^2.$$

After substitution of $B_i^{(el)}$ in formula (30), we have to do a trivial integration over the z variable using the δ-function

$$\delta(1-x') = xyr\delta(z).$$

4. NUMERICAL ESTIMATE

We calculate the radiative corrections for the kinematical conditions of the HERMES experiment [10]. The energy of the positron beam is 27.6 GeV. The HERMES installation has provided the first direct measurement of the structure function b_1 in the kinematic range $0.002 < x < 0.85$ and $0.1\ \text{GeV}^2 < Q^2 < 20\ \text{GeV}^2$. A cylindrical target cell confines the polarized gas along the positron beam line, where a longitudinal magnetic field provides the quantization axis for the nuclear spin. The corresponding tensor atomic polarization is $T = 0.83$ (see Appendix C for the definition of this quantity).

The analysis of the experimental data was performed in the approximation $b_3 = b_4 = 0$. In the numerical estimate below, we also neglect these functions.

The deuteron spin-dependent structure function b_1 is extracted from the measured tensor asymmetry A_{zz} via the relation [10]

$$b_1 = -\frac{3}{2}A_{zz}\frac{(1+\gamma^2)F_2^d}{2x(1+R)}, \quad (57)$$

where the deuteron spin-independent structure function F_1^d is expressed in terms of the ratio

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_2^d(1+4M^2x^2/Q^2)}{2xF_1^d} - 1$$

(see [20]) and

$$\gamma^2 = \frac{4M^2x^2}{Q^2}$$

is a kinematic factor. Here, $\sigma_T(\sigma_L)$ is the cross section for the absorption of transversely (longitudinally) polarized virtual photons by the unpolarized target. The Born cross section of the deep inelastic scattering of the unpolarized electron beam by the unpolarized target has the form

$$\frac{d\sigma_B^{un}}{dxdQ_B^2} = \frac{4\pi\alpha^2}{xQ_B^4} \times \times [(1-y-xy\tau)F_2^d(x,Q^2) + xy^2F_1^d(x,Q^2)]. \quad (58)$$

The structure functions $F_{1,2}^d$ are related to the structure functions $W_{1,2}$ (introduced in formula (7)) as

$$W_1 = 2F_1^d, \quad W_2 = 4(\tau/y)F_2^d.$$

The deuteron spin-independent structure function

$$F_2^d = \frac{F_2^p(1 + F_2^n/F_2^p)}{2}$$

is calculated using parameterizations for the proton structure functions F_2^p [21] and the ratio F_2^n/F_2^p [22]. The deuteron spin-dependent structure function b_2 is also extracted from the experiment using the Callan–Gross relation

$$b_2 = 2x\frac{1+R}{1+\gamma^2}b_1. \quad (59)$$

According to the preliminary results of the HERMES experiment, the tensor asymmetry can be parameterized as [23]

$$A_{zz} = -1.56 \cdot 10^{-2}\left(1 - 1.74x - 1.45\sqrt{x}\right). \quad (60)$$

The influence of the radiative correction on the spin-dependent part of the Born cross section is shown in Fig. 2 as a function of the variable x for various Q^2 values. Inclusion of the radiative correction shifts the zero value of b_1 and b_2 to the region of smaller x (see

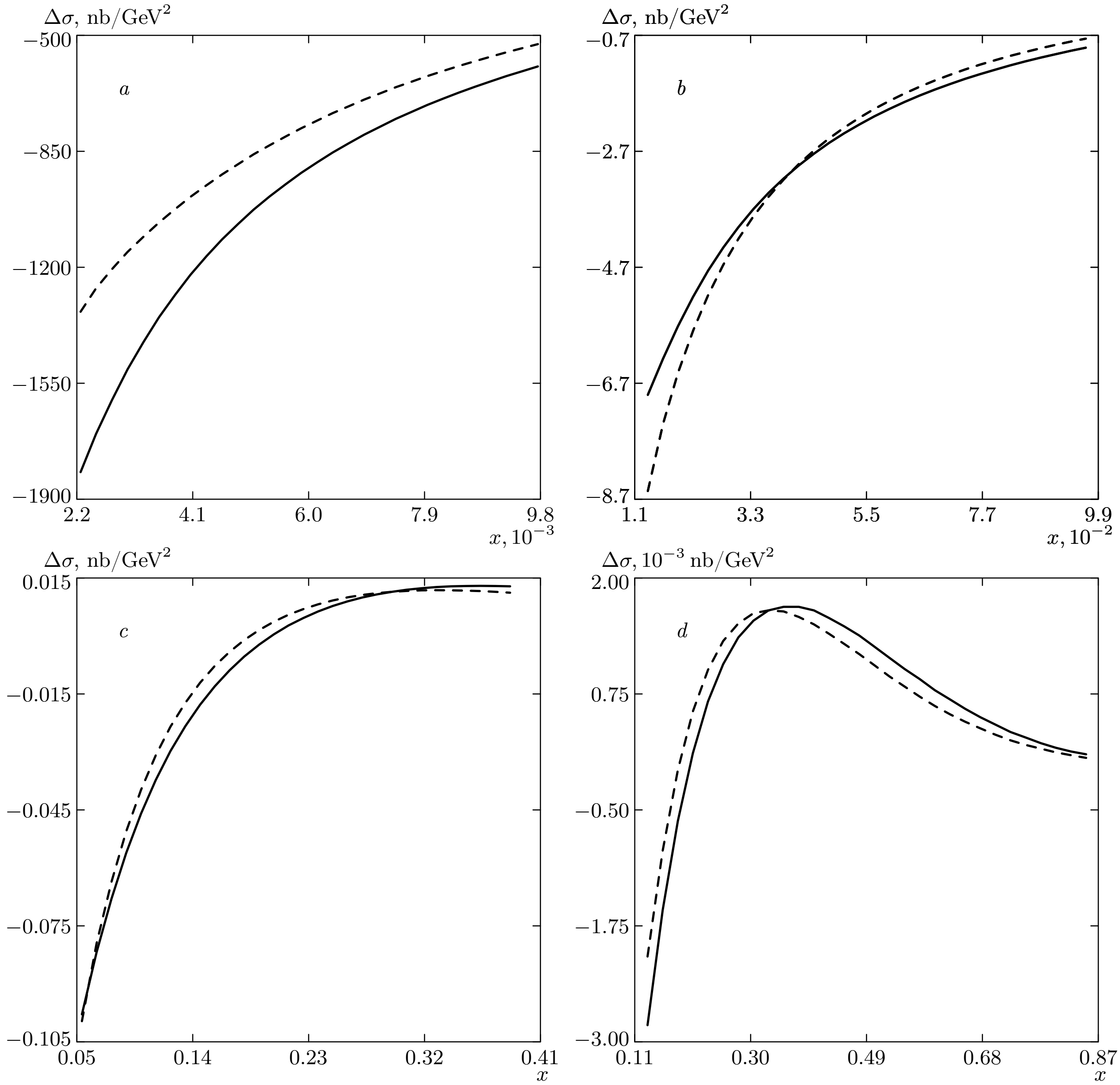

Fig. 2. The spin-dependent part of the cross section calculated for the kinematical conditions of the HERMES experiment [10]. The solid line is the Born approximation, the dotted line corresponds to the inclusion of the radiative corrections. The Q^2 values are as follows: a — 0.1 GeV^2, b — 1 GeV^2, c — 4 GeV^2, d — 10 GeV^2

Fig. 2c and d). In the range of low x ($x \sim 10^{-3}$–10^{-2}), the value of the radiative correction changes from 10 % to 30 % compared with the Born contribution. This region is of the utmost importance for b_1 measurements. According to the theoretical predictions in [11–13], the structure function b_1 increases very rapidly in this region, and this fact was confirmed in the HERMES experiment [10].

From our estimate, we conclude that the radiative corrections to process (1) are not small, especially for the low-x region, and they have to be taken into account in the data analysis.

We wish to thank N. P. Merenkov for useful discussions and comments. We warmly acknowledge M. Contalbrigo for useful discussions on the HERA experimental conditions, as well as for sending us preliminary results on the A_{zz} parameterization.

APPENDIX A

In this Appendix, we present the formulas for the coefficients A_i^{mn}, B_i^{mn}, and C_{ji}^{mn}, $(m,n = l,t$, $i = 1,2,3,4$, $j = 0,1,2)$ determining the cross section of the hard-photon emission process (see formula (50)).

The coefficients determining the contribution proportional to the B_1 structure function are

$$A_1^{ll} = -\frac{n_1}{\tau}[(\bar r-\Delta_1)^2 - 2a(b+\Delta_1)],$$

$$B_1^{ll} = \frac{n_1}{\tau}\Big\{[(2a-b)r+1+\Delta_2]^2 - ar(2+3ar)\Big\},$$

$$C_{01}^{ll} = -\frac{VN}{\tau}\Big\{(\bar r-\Delta_1)^2 + a[3a(1+r^2)-2(b+\Delta_1)]\Big\},$$

$$C_{11}^{ll} = -6N(c+2a), \quad C_{21}^{ll} = -6N\frac{\tau}{V},$$

$$A_1^{lt} = \frac{2n_1}{Md}Q_B^2(2b+\Delta_1)(\bar r-\Delta_1),$$

$$B_1^{lt} = \frac{2n_1}{Md}Q_B^2(\Delta_2-2br)[(a-b)r+1+\Delta_2],$$

$$C_{01}^{lt} = -4n_2Q_B^2\Big[a(1+r^2)(y+2a) - 2b\bar r - \Delta_1(c+2a-2b)\Big],$$

$$C_{11}^{lt} = -4n_2\Big[(y+4a)(c+2a) - 2a(\bar r+2b)\Big],$$

$$C_{21}^{lt} = -8n_2\frac{\tau}{V}(y+2a),$$

$$A_1^{tt} = -\frac{n_1}{b}\,\frac{Q_B^2}{V}[b^2+(b+\Delta_1)^2], \tag{A.1}$$

$$B_1^{tt} = \frac{n_1}{b}\,\frac{Q_B^2}{V}(2b^2r^2 - 2br\Delta_2 + \Delta_2^2),$$

$$C_{01}^{tt} = -\frac{NQ_B^2}{2b}\Big[(1+r^2)(y^2+4a-2ab) + (2b+\Delta_1)^2 + \Delta_1^2\Big],$$

$$C_{11}^{tt} = \frac{2N}{b}\Big[b(1+y+2a-r) + (1+a)\Delta_1\Big],$$

$$C_{21}^{tt} = -\frac{N}{d^2}\Big[y^2 + 2a(2-b)\Big].$$

The coefficients determining the contribution proportional to the B_2 structure function are

$$A_2^{ll} = \frac{n_3}{\tau}\Big[b(1+r^2) + (1-r+ry)\Delta_1\Big]\Big[(\bar r-\Delta_1)^2 - 2a(b+\Delta_1)\Big],$$

$$B_2^{ll} = -\frac{n_3}{\tau}\Big[b(1+r^2) - \Delta_2(\bar r-2a)\Big]\Big\{[(2a-b)r+1+\Delta_2]^2 - ar(2+3ar)\Big\},$$

$$C_{02}^{ll} = -\frac{NV}{c}\Big\{(7-3y)c^2 + 3a(5-y+r)c + 3a^2(3+r^2) - ar[5+3(a+b)^2]\Big\},$$

$$C_{12}^{ll} = -3\tau\frac{N}{c}\Big[4(a+c) - cy\Big], \ C_{22}^{ll} = -6N\frac{\tau^2}{cV},$$

$$A_2^{lt} = -2\frac{n_3Q_B^2}{Md}(2b+\Delta_1)(\bar r-\Delta_1)\Big[b(1+r^2) + (1-r+ry)\Delta_1\Big],$$

$$B_2^{lt} = -2\frac{n_3Q_B^2}{Md}(\Delta_2-2br)[1+\Delta_2+(a-b)r]\Big[b(1+r^2) + (a+b-r)\Delta_2\Big], \tag{A.2}$$

$$C_{02}^{lt} = \frac{NV^2}{Mdc}\Big\{2ac\Big[y\bar r + (3b+a)(1+r) - y - 8a\Big] - c^2\Big[2a + (2-y)(y+4a)\Big] +$$
$$2a\Big[2a(b-a+r) + (y+2a)(r - a(1+r^2) + r(a+b)^2)\Big]\Big\},$$

$$C_{12}^{lt} = -4n_2\frac{\tau}{c}\Big[2a(1-3r+2\bar{r}+4c)+cy(2+b-a)\Big], \quad C_{22}^{lt} = -8n_2\frac{\tau^2}{cV}(y+2a),$$

$$A_2^{tt} = \frac{xy}{b}n_3Z_1[2b^2+\Delta_1(2b+\Delta_1)],$$

$$B_2^{tt} = -\frac{xy}{b}n_3Z_2[2b^2r^2+\Delta_2(\Delta_2-2br)],$$

$$C_{02}^{tt} = \frac{VN}{2bc}\Big\{-2c^2\Big[a+(1+a)(2-y)\Big]+c\Big[(3-2y+a^2+b^2)(\bar{r}-2a)+4(ab+b-a^2)+4r(a-b^2)\Big]-$$
$$-2a\Big[(r-a)^2+b^2\Big]+(1+2a-2b+a^2+b^2)\Big[r-a(1+r^2)+(a+b)^2r\Big]\Big\},$$

$$C_{12}^{tt} = -\frac{VN}{2cd^2}\Big\{c\Big[1+7a(1+a)-b(1+b)+(a+b)(a^2+b^2)\Big]+4a\Big[(a-b)(1-r)+a^2+b^2-r\Big]\Big\},$$

$$C_{22}^{tt} = -\frac{N\tau}{cd^2}\Big[y^2+2a(2-b)\Big].$$

The coefficients determining the contribution proportional to the B_3 structure function are

$$A_3^{ll} = n_3\frac{c}{\tau}\Big\{(a+\bar{r})\Big[2Z_1+r\Delta_1(2a+r-\Delta_1)\Big]-\Delta_1\Big[r^2(r-\Delta_1)+2(b+\Delta_1)+r(a+b)(a+r-\Delta_1)\Big]\Big\},$$

$$B_3^{ll} = -n_3\frac{c}{\tau}\Big\{2Z_2\Big[1+(2a-b)r+\Delta_2\Big]+3a\Delta_2\Big[(b-a)r-1-\Delta_2\Big]\Big\},$$

$$C_{03}^{ll} = \frac{VN}{2}\Big[c(6a-16+9y)+6a(y-3-r)\Big], \quad C_{13}^{ll} = -3\tau N(2-y), \quad C_{23}^{ll} = 0,$$

$$A_3^{lt} = -n_3\frac{cQ_B^2}{Md}\Big\{2Z_1(3b-a-r)+\Delta_1\Big[4r(1+b^2+3ab)-2a(1+r^2)+c(ar-3+5br)\Big]\Big\},$$

$$B_3^{lt} = n_3\frac{cQ_B^2}{Md}\Big\{2Z_2\Big[(3b-a)r-1\Big]+\Delta_2\Big[(1+ar)(a-6b)-(a+3b)(r^2+\Delta_2)+r(b^2-1)+b+\Delta_2(3r-2b)\Big]\Big\}, \tag{A.3}$$

$$C_{03}^{lt} = n_2V\Big\{4a\Big[2b\bar{r}-y^2+4(b^2-a)\Big]-c\Big[3y(2-y)+8a(1+a+2b)\Big]\Big\},$$

$$C_{13}^{lt} = -2N\frac{M}{d}(2-y)(y+2a), \; C_{23}^{lt} = 0,$$

$$A_3^{tt} = n_3xy\frac{c}{b}\Big\{(\Delta_1-2b)[b(1+r^2)+(1-r+ry)\Delta_1]+b\Delta_1[1+r(b-a+\Delta_1)]\Big\},$$

$$B_3^{tt} = n_3xy\frac{c}{b}\Big\{(2br-\Delta_2)[b(1+r^2)+(1-r-y)\Delta_2]+b\Delta_2[r(b-a+r)-\Delta_2)]\Big\},$$

$$C_{03}^{tt} = \frac{VN}{2b}\Big\{3b-a-(a^2+b^2)(2+a+b)+r[y^2+2y(2b-a)+2a(3-a)]+c[y(1+y+3a)-4(1+a)-2ab]\Big\},$$

$$C_{13}^{tt} = -\frac{VN}{2d^2}(2-y)[y^2+2a(2-b)], \; C_{23}^{tt} = 0.$$

The coefficients determining the contribution proportional to the B_4 structure function are

$$A_4^{ll} = n_3\frac{c^2}{\tau}\Big\{(b-a)(1+r^2)+\Delta_1\Big[1+r(2a-b)\Big]\Big\}, \quad B_4^{ll} = n_3\frac{c^2}{\tau}\Big[(a-b)(1+r^2)+\Delta_2(a+\bar{r})\Big],$$

$$C_{04}^{ll} = -\frac{cNV}{2}\Big[1+3(b-a)\Big], \quad C_{14}^{ll} = C_{24}^{ll} = 0,$$

$$A_4^{lt} = n_3\frac{c^2Q_B^2}{Md}\Big\{2b(1+r^2)+\Delta_1\Big[1-r(3b-a)\Big]\Big\}, \; B_4^{lt} = n_3\frac{c^2Q_B^2}{Md}\Big[-2b(1+r^2)+\Delta_2(\bar{r}-2b)\Big],$$

$$C_{04}^{lt} = -c\frac{NV^2}{2Md}\Big[1+4ab-(a-b)^2\Big], \quad C_{14}^{lt} = C_{24}^{lt} = 0, \tag{A.4}$$

$$A_4^{tt} = -xyrc^2\Delta_1 n_3, \quad B_4^{tt} = -xyc^2\Delta_2 n_3, \quad C_{04}^{tt} = -\frac{cNV}{2}(y+2a), \quad C_{14}^{tt} = C_{24}^{tt} = 0.$$

We here use the notation

$$c = z + xyr, \quad \bar{r} = a - b + r, \quad n_1 = \frac{N}{2}\frac{1+r^2}{1-r}VQ_B^2, \quad n_2 = \frac{NV}{2Md}, \quad n_3 = \frac{N}{2c}\frac{V^2}{1-r},$$

$$d^2 = bQ_B^2, \quad \Delta_1 = (1-xy)r - a - b - z, \quad \Delta_2 = (1-y+xy)r + z - 1, \quad N = \frac{4\tau}{Vc^2},$$

$$Z_1 = b(1+r^2) + \Delta_1(1-r+yr), \quad Z_2 = b(1+r^2) + \Delta_2(1-y-r).$$

APPENDIX B

In this Appendix, we consider the part of the integral I that is caused by the R_{ln} and R_{tn} components of the deuteron quadrupole polarization tensor (these components do not contribute to the differential cross section treated in the Born approximation). We define the integral caused by the R_{ln} component as

$$I_{ln} = \int \frac{d^3k}{2\pi\omega}\Sigma_{ln}(z,r,\varphi)R_{ln}, \tag{B.1}$$

with

$$\Sigma_{ln}(z,r,\varphi) = \frac{\alpha^2(q^2)}{Q_B^4}\frac{2VN}{Mr^2}nq\left(\frac{P_{1ln}}{\chi_1} - \frac{P_{2ln}}{\chi_2} + U_{0ln} + U_{1ln}\chi_1\right),$$

$$P_{1ln} = \frac{V}{1-r}\Big\{cgxy(1+r^2)B_1 + g\Big[c(1-r(1-y)) + a(1+r^2) - 4fr\Big]B_2 + \\ + \Big[2a - fr + \frac{1}{2}(3(1-r+yr)+2ar)\Big]B_3 + \frac{1}{2}c[1+(a-b)r]B_4\Big\}, \tag{B.2}$$

$$P_{2ln} = -\frac{V}{1-r}\Big\{-xy(1+r^2)(c+2ar)B_1 + \frac{2}{c}[-ar(a(1+r^2)-4fr) + \frac{c^2}{2}(1-y-r) + \\ + \frac{c}{2}(4fr - a(1+3r^2+2yr-2r))]B_2 + [r(f-2ar) - \frac{c}{2}(2a+3(r+y-1))]B_3 - \frac{c}{2}(a-b+r)B_4\Big\},$$

$$U_{0ln} = 2g(cB_1 + \tau B_2) + 2\tau(2-y)(B_2+B_3),$$

$$U_{1ln} = \frac{4\tau}{V}(B_1 + \frac{\tau}{c}B_2),$$

and

$$c = z + xyr, \quad f = 1 + (1-y)^2, \quad g = 1 + 2a/c, \quad nq = S_\mu^{(n)}q_\mu.$$

The second integral, caused by the R_{tn} component, is defined as

$$I_{tn} = \int \frac{d^3k}{2\pi\omega}\Sigma_{tn}(z,r,\varphi)R_{tn}, \tag{B.3}$$

where the integrand is

$$\Sigma_{tn}(z,r,\varphi) = \frac{\alpha^2(q^2)}{Q_B^4}\frac{2VN}{dr^2(r-1)}nq\left(\frac{P_{1tn}}{\chi_1} - \frac{P_{2tn}}{\chi_2} + U_{0tn} + U_{1tn}\chi_1\right),$$

$$P_{1tn} = Q_B^2\Big\{\bar{f}xy(1+r^2)B_1 - \frac{\bar{f}}{c}\Big[a(1+r^2) - 4r(f+4y)\Big]B_2 + \\ + \bar{f}[1+r(y-1)](B_2+B_3) + brc(B_3+B_4) + 2br(1-y)B_3\Big\},$$

$$P_{2tn} = -Q_B^2\Big\{xy\bar g(1+r^2)B_1 + \frac{\bar g}{c}[a(1+r^2)-4fr]B_2 + \\ + \bar g(r-1+y)(B_2+B_3) - bc(B_3+B_4) - 2br(1-y)B_3\Big\}, \quad \text{(B.4)}$$

$$U_{0tn} = (r-1)\Big[-2xy\bar f(B_1+\frac{\tau}{c}B_2) + (2-y)(2a+y)(B_2+B_3)\Big],$$

$$U_{1tn} = \frac{1}{V}\Big\{(r-1)(2a+y)G_1 - \bar f G_2 - \frac{y-2}{Mxy}[M(2a+y)-2\tau d](B_2+B_3) - \frac{d}{Mxy}[c-2a(r-2)]G_2\Big\},$$

$$G_1 = 3B_1 + \frac{2\tau}{c}B_2 + \frac{c}{2xyr}(B_2+2B_3+B_4), \quad G_2 = -B_1 - \frac{c}{2xyr}(B_2+2B_3+B_4),$$

and

$$d^2 = bQ_B^2, \quad \bar f = b-a-z+r(1-xy), \quad \bar g = z-1+r(a-b+xy).$$

As before, we calculate the above integrals in the center-of-mass system of the hard photon and the undetected hadron system:

$$\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{p} = 0.$$

The electron momenta $\mathbf{k}_1$ and $\mathbf{k}_2$ define the xz plane, the z axis is directed along the deuteron momentum $\mathbf{p}$. Then the hard-photon momentum $\mathbf{k}$ is determined by the azimuthal (φ) and polar (θ) angles, and the phase space of the hard photon can be written as

$$\frac{d^3k}{2\pi\omega} = \frac{Q_B^2}{2\sqrt{y^2+4a}}\frac{d\varphi}{2\pi}dzdr, \quad \text{(B.5)}$$

where ω is the hard-photon energy.

The quantity nq can be written in this coordinate system as $nq = \bar n \sin\varphi$, where $\bar n$ is a factor independent of φ. Then the integration over the φ variable in the region $(0, 2\pi)$ leads to the result

$$I_{ln} = I_{tn} = 0.$$

Therefore, the R_{ln} and R_{tn} components of the deuteron quadrupole polarization tensor do not contribute to the differential cross section of deep inelastic scattering of the unpolarized electron beam by the tensor polarized target. This is because only the scattered-electron variables are measured (this corresponds to the HERA experimental conditions, for example).

If the hard photon is detected, then I_{ln} and I_{tn} survive and the expressions for Σ_{ln} and Σ_{tn} have to be taken into account.

APPENDIX C

In this Appendix, we give some formulas describing the polarization state of the deuteron target in different cases. In the case of an arbitrary polarization of the target, it is described by the general spin-density matrix (defined by 8 parameters in general), which in the coordinate representation has the form

$$\rho_{\mu\nu} = -\frac{1}{3}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{M^2}\right) - \frac{i}{2M}\varepsilon_{\mu\nu\lambda\rho}s_\lambda p_\rho + Q_{\mu\nu}, \\ Q_{\mu\nu} = Q_{\nu\mu}, \quad Q_{\mu\mu} = 0, \quad p_\mu Q_{\mu\nu} = 0, \quad \text{(C.1)}$$

where p_μ is the deuteron 4-momentum, and s_μ and $Q_{\mu\nu}$ are the deuteron polarization 4-vector and the deuteron quadrupole polarization tensor.

In the deuteron rest frame, the above formula is written as

$$\rho_{ij} = \frac{1}{3}\delta_{ij} + \frac{i}{2}\varepsilon_{ijk}s_k + Q_{ij}, \quad ij = x, y, z. \quad \text{(C.2)}$$

This spin-density matrix can be written in the helicity representation using the relation

$$\rho_{\lambda\lambda'} = \rho_{ij}e_i^{(\lambda)*}e_j^{(\lambda')}, \quad \lambda, \lambda' = +, -, 0, \quad \text{(C.3)}$$

where $e_i^{(\lambda)}$ are the deuteron spin functions that have the deuteron spin projection λ on the quantization axis (the z axis). They are

$$e^{(\pm)} = \mp\frac{1}{\sqrt2}(1, \pm i, 0), \quad e^{(0)} = (0, 0, 1). \quad \text{(C.4)}$$

The elements of the spin-density matrix in the helicity representation are related to those in the coordinate representation by

$$\rho_{\pm\pm} = \frac{1}{3} \mp \frac{1}{2}s_z - \frac{1}{2}Q_{zz}, \quad \rho_{00} = \frac{1}{3} + Q_{zz}, \\ \rho_{+-} = -\frac{1}{2}(Q_{xx} - Q_{yy}) + iQ_{xy}, \quad \text{(C.5)}$$

$$\rho_{+0} = -\frac{1}{2\sqrt2}(s_x - is_y) - \frac{1}{\sqrt2}(Q_{xz} - iQ_{yz}),$$

$$\rho_{-0} = -\frac{1}{2\sqrt2}(s_x + is_y) + \frac{1}{\sqrt2}(Q_{xz} + iQ_{yz}),$$

$$\rho_{\lambda\lambda'} = (\rho_{\lambda'\lambda})^*.$$

To obtain these relations, we use that $Q_{xx} + Q_{yy} + Q_{zz} = 0$.

The polarized deuteron target described by the population numbers n_+, n_-, and n_0 is often used in spin experiments. Here, n_+, n_-, and n_0 are the fractions of atoms with the respective nuclear spin projection on the quantization axis $m = +1$, $m = -1$, and $m = 0$. If the spin-density matrix is normalized to 1, i.e.,

$$\mathrm{Sp}\,\rho = 1,$$

then we have

$$n_+ + n_- + n_0 = 1.$$

Thus, the polarization state of the deuteron target is defined in this case by two parameters: the so-called V (vector) and T (tensor) polarizations,

$$V = n_+ - n_-, \quad T = 1 - 3n_0. \tag{C.6}$$

Using the definitions of the quantities $n_{\pm,0}$,

$$n_\pm = \rho_{ij} e_i^{(\pm)*} e_j^{(\pm)}, \quad n_0 = \rho_{ij} e_i^{(0)*} e_j^{(0)}, \tag{C.7}$$

we have the following relation between V and T and the parameters of the spin-density matrix in the coordinate representation (in the case where the quantization axis is directed along the z axis):

$$n_0 = \frac{1}{3} + Q_{zz}, \quad n_\pm = \frac{1}{3} \mp \frac{1}{2} s_z - \frac{1}{2} Q_{zz}, \tag{C.8}$$

or

$$T = -3Q_{zz}, \quad V = -s_z. \tag{C.9}$$

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