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FERMI-LIQUID EFFECTS IN TRANSRESISTIVITY IN THE QUANTUM HALL DOUBLE LAYERS NEAR $\nu = 1/2$

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We present theoretical studies of the temperature and magnetic field dependences of the Coulomb drag transresistivity between two parallel layers of two-dimensional electron gases in the quantum Hall regime near half filling of the lowest Landau level. It is shown that Fermi-liquid interactions between the relevant quasiparticles can significantly affect the transresistivity, providing its independence from the interlayer spacing for spacings that take values reported in the experiments. The obtained results agree with the experimental evidence.

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During the last decade, double-layer two-dimensional ($2D$) electron gas systems were of significant interest due to many remarkable phenomena that they exhibit, including the so-called Coulomb drag. In Coulomb drag experiments, two $2D$ electron gases are arranged close to each other, such that they can interact via Coulomb forces. A current I is applied to one layer of the system, and the voltage V_D in the other nearby layer is measured, with no current allowed to flow in that layer. The ratio $-V_D/I$ gives the transresistivity ρ_D, which characterizes the strength of the effect. The physical interpretation of the Coulomb drag is that momentum is transferred from the current-carrying layer to the nearby one due to interlayer interactions [1–3].

It was shown theoretically [4, 5] and confirmed with experiments [5] that the transresistivity between two $2D$ electron gases in the quantum Hall regime at half filling of the lowest Landau level for both layers is proportional to $T^{4/3}$ (where T is the temperature of the system), which is quite different from the temperature dependence of ρ_D in the absence of the external magnetic field applied to $2D$ electron gases. This temperature dependence of the drag at $\nu = 1/2$ originates from the ballistic contribution to transresistivity. The latter reflects the response of the two-layer system to the driving disturbance of a finite wave vector $\mathbf{q}$ and finite frequency ω in the case where the relevant scales are smaller than the mean free path l of electrons ($ql \gg 1$), and times are shorter than their scattering time τ ($\omega\tau \gg 1$)[1)].

In further experiments [7], the Coulomb drag was measured between $2D$ electron gases where the layer filling factor was varied around $\nu = 1/2$. The transresistivity was reported to be enhanced quadratically with $\Delta\nu = \nu - 1/2$. It was also reported that the curvature of the enhancement depended on temperature but was insensitive to both the sign of $\Delta\nu$ and the distance d between the layers. The present work is motivated with these experiments of [7]. We calculate the transresistivity between two layers of $2D$ electron gases subject to a strong magnetic field that provides ν close to $1/2$ for both layers.

We start from the well-known expression [1, 3] that relates the Coulomb drag transresistivity to density–density components of the polarization in the layers $\Pi_{(1)}(\mathbf{q},\omega)$ and $\Pi_{(2)}(\mathbf{q},\omega)$,

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1) When the external driving disturbance applied to one of the layers is of small $\mathbf{q}$, ω ($ql \ll 1$, $\omega\tau \ll 1$), the transresisitivity is dominated by the diffusion contribution, and new effects could emerge (see. e.g., [6] and references therein).

$$\rho_D = \frac{1}{2(2\pi)^2}\frac{h}{e^2}\frac{1}{Tn^2}\int\frac{q^2 d\mathbf{q}}{(2\pi)^2}\int\frac{\hbar\, d\omega}{\mathrm{sh}^2(\hbar\omega/2T)}\times \times\left|U(\mathbf{q},\omega)\right|^2 \operatorname{Im}\Pi_{(1)}(\mathbf{q},\omega)\operatorname{Im}\Pi_{(2)}(\mathbf{q},\omega), \quad (1)$$

where $U(\mathbf{q},\omega)$ is the screened interlayer Coulomb interaction, and electron densities in the layers are supposed to be equal ($n_1 = n_2 = n$).

Within the usual composite fermion approach [8], a single layer polarizability describes the part of the density–current electromagnetic response that is irreducible with respect to the Coulomb interaction. Adopting the random-phase approximation (RPA) for simplicity, we obtain the following expression for the 2×2 polarizability matrix:

$$\Pi^{-1} = (K^0)^{-1} + C^{-1}. \quad (2)$$

Here, the matrix K^0 gives the response of noninteracting composite fermions and C is the Chern–Simons interaction matrix. Assuming the wave vector $\mathbf{q}$ to lie in the x direction for definiteness, we have

$$C = \begin{pmatrix} 0 & \dfrac{iq}{4\pi\hbar} \\ -\dfrac{iq}{4\pi\hbar} & 0 \end{pmatrix}. \quad (3)$$

Starting from expression (2), we arrive at the following results for the density–density response function $\Pi_{00(i)}(\mathbf{q},\omega)$:

$$\Pi_{00(i)}(\mathbf{q},\omega) = \Pi_{(i)}(\mathbf{q},\omega) = = \frac{K^0_{00(i)}(\mathbf{q},\omega)}{1 - \dfrac{8i\pi\hbar}{q}K^0_{01(i)}(\mathbf{q},\omega) - \left(\dfrac{4\pi\hbar}{q}\right)^2 \Delta_{(i)}(\mathbf{q},\omega)}. \quad (4)$$

Here,

$$\Delta_{(i)}(\mathbf{q},\omega) = = K^0_{00(i)}(\mathbf{q},\omega)K^0_{11(i)}(\mathbf{q},\omega) + \left(K^0_{01(i)}(\mathbf{q},\omega)\right)^2. \quad (5)$$

Within the RPA, the response functions included in Eqs. (4) and (5) are simply related to the components of the composite fermion conductivity tensor $\tilde\sigma$ [8],

$$\frac{1}{\tilde\sigma^{(i)}_{xx}(\mathbf{q},\omega)} = \frac{iq^2}{\omega e^2}\left[\frac{1}{K^0_{00(i)}(\mathbf{q},\omega)} - \frac{1}{K^0_{00(i)}(\mathbf{q},0)}\right],$$

$$\tilde\sigma^{(i)}_{yy}(\mathbf{q},\omega) = -\frac{ie^2}{\omega}\left[K^0_{11(i)}(\mathbf{q},\omega) - K^0_{11(i)}(\mathbf{q},0)\right], \quad (6)$$

$$\tilde\sigma^{(i)}_{xy} = -\tilde\sigma^{(i)}_{yx} = \frac{ie^2}{q}K^0_{01(i)}(\mathbf{q},\omega).$$

To proceed, we calculate the components of the composite fermion conductivity at ν slightly away from 1/2. In this case, composite fermions experience a nonzero effective magnetic field

$$B_{eff} = B - B_{1/2}.$$

We concentrate on the ballistic contribution to the transresistivity, and we therefore need asymptotics for the relevant conductivity components applicable in the nonlocal ($ql \gg 1$) and high-frequency ($\omega\tau \gg 1$) regime. The corresponding expressions for $\tilde\sigma_{ij}$ were obtained in earlier works [8]. But these results are not appropriate for our analysis because they do not provide a smooth passage to the $B_{eff} \to 0$ limit at finite $\mathbf{q}$. Therefore, we do not use them in further calculations. To obtain a suitable approximation for the composite fermion conductivity, we start from the standard solution of the Boltzmann transport equation for the composite fermion distribution function. This gives the following results for the composite fermion conductivity components for a single layer [9]:

$$\tilde\sigma_{\alpha\beta} = \frac{m^* e^2}{(2\pi\hbar)^2}\frac{1}{\Omega}\int_0^{2\pi} d\psi\, v_\alpha(\psi)\times \times\exp\left[-\frac{iq}{\Omega}\int_0^{\psi} v_x(\psi'')\,d\psi''\right]\times \times\int_{-\infty}^{\psi} v_\beta(\psi')\exp\left[\frac{iq}{\Omega}\int_0^{\psi'} v_x(\psi'')\,d\psi'' + +\frac{1}{\Omega\tau}(\psi'-\psi)(1-i\omega\tau)\right]d\psi'. \quad (7)$$

Here, m^* and Ω are the composite fermion effective mass and the cyclotron frequency at the effective magnetic field B_{eff}; ψ is the angular coordinate of the composite fermion cyclotron orbit. We now perform some formal transformations of this expression (7) following the way proposed before [9, 10]. First, we expand the composite fermion velocity components $v_\beta(\psi')$ in a Fourier series,

$$v_\beta(\psi') = \sum_k v_{k\beta}\exp(ik\psi'). \quad (8)$$

Substituting this expansion (8) in (7), we obtain

$$\tilde{\sigma} = \frac{m^* e^2}{(2\pi\hbar)^2} \sum_k v_{k\beta} \int_0^{2\pi} d\psi\, v_\alpha(\psi) \exp(ik\psi) \times \\ \times \int_{-\infty}^{0} \exp\left[\left(ik\Omega - i\omega + \frac{1}{\tau} + iqv_x(\psi) \right)\theta + \\ + iq \int_0^\theta \left(v_x(\psi + \Omega\theta') - v_x(\psi) \right) d\theta' \right] d\theta, \quad (9)$$

where
$$\theta = \frac{\psi' - \psi}{\Omega}.$$

Then we introduce a new variable η related to the variable θ as

$$\eta = \left(ik\Omega - i\omega + \frac{1}{\tau} + iqv_x(\psi) \right)\theta + \\ + iq \int_0^\theta \left[v_x(\psi + \Omega\theta') - v_x(\psi) \right] d\theta'. \quad (10)$$

The result is

$$\tilde{\sigma}_{\alpha\beta} = \frac{im^* e^2}{(2\pi\hbar)^2} \sum_k v_{k\beta} \int_{-\infty}^{0} e^\eta d\eta \times \\ \times \int_0^{2\pi} \frac{v_\alpha(\psi)\exp(ik\psi)}{\omega + i/\tau - k\Omega - qv_x(\psi + \Omega\theta)} d\psi. \quad (11)$$

Under the conditions of interest, $\omega\tau \gg 1$, $ql \gg 1$, and also assuming that the filling factor is close to $\nu = 1/2$, and hence $qv_F \gg \Omega$ (where v_F is the composite fermions Fermi velocity), the variable θ is approximately equal to

$$\eta\tau(1 + iql\cos\psi + ik\Omega\tau - i\omega\tau)^{-1}.$$

Taking this into account and expanding the last term in the denominator of (11) in powers of $\Omega\theta$, we obtain

$$qv_x(\psi + \Omega\theta) \approx qv_x(\psi) + \\ + \eta\Omega q\tau(1 + iql\cos\psi + ik\Omega\tau - i\omega\tau)^{-1} \frac{dv_x}{d\psi} + \\ + q\frac{\eta^2}{2}(\Omega\tau)^2(1 + iql\cos\psi + ik\Omega\tau - i\omega\tau)^{-2} \frac{d^2 v_x}{d\psi^2}. \quad (12)$$

Substituting this asymptotic expression in (9), we can calculate the first terms of the expansions of the relevant components of the composite fermion conductivity in powers of the small parameter $(qR)^{-1}$, where $R = v_F/\Omega$ is the composite fermion cyclotron radius. In the «collisionless» limit $1/\tau \to 0$, we have

$$\tilde{\sigma}_{xx} = -N\frac{i\omega}{q^2}e^2 \left\{ 1 + \frac{i\delta}{\sqrt{1-\delta^2}} + \\ + \frac{i\delta}{\sqrt{(1-\delta^2)^5}} \frac{1}{2(qR)^2} \left(1 - \frac{5}{4}\frac{1}{1-\delta^2} \right) \right\}, \quad (13)$$

$$\tilde{\sigma}_{yy} = N\frac{v_F e^2}{q} \left\{ \sqrt{1-\delta^2} + i\delta + \\ + \frac{1}{2(qR)^2} \left[\frac{7}{4}\frac{1}{\sqrt{(1-\delta^2)^5}} - \frac{1}{\sqrt{(1-\delta^2)^3}} \right] \right\}, \quad (14)$$

$$\tilde{\sigma}_{xy} = iN\frac{v_F e^2}{q}\frac{\delta}{2qR} \left[\frac{1}{\sqrt{1-\delta^2}} + \frac{\delta^2}{\sqrt{(1-\delta^2)^3}} \right], \quad (15)$$

where $N = m^*/2\pi\hbar^2$ is the density of states at the composite fermion Fermi surface and $\delta = \omega/qv_F$. Using these results, we can easily obtain approximations for the functions $K^0_{\alpha\beta(i)}(\mathbf{q},\omega)$ $(\alpha, \beta = 0.1)$ and, subsequently, the desired density–density response function given by (4). It was shown in [3] that the integral over ω in the expression (1) for ρ_D is dominated by $\omega \sim T$, and the major contribution to the integral over q in this expression comes from

$$q \sim k_F(T/T_0)^{1/3},$$

where k_F is the Fermi wave vector and the scaling temperature T_0 is defined below. Therefore, we obtain an estimate for δ, namely

$$\delta \sim (T/\mu)(T_0/T)^{1/3},$$

where μ is the chemical potential of a single $2D$ electron gas included in the bilayer. For the parameter T_0 taking values of the order of room temperature, δ is small compared to unity at low temperatures ($T \sim 1$ K).

Here, we limit ourselves to the case of two identical layers ($\Pi_{(1)} = \Pi_{(2)} \equiv \Pi$). For $\delta \ll 1$, we obtain the approximation

$$\Pi_{00}(\mathbf{q},\omega) = \frac{q^3}{q^3\left(\frac{dn}{d\mu}\right)^{-1} - 8\pi i\hbar\omega k_F\left(1 + 2(k_F R)^{-1} + \frac{3}{8}(qR)^{-2}\right)}, \quad (16)$$

where $dn/d\mu$ is the compressibility of the $\nu = 1/2$ state, which is defined as [3]

$$\frac{dn}{d\mu} \equiv \Pi_{00}(\mathbf{q} \to 0;\ \omega \to 0) = \frac{3m^*}{8\pi\hbar^2}. \qquad (17)$$

This differs from the compressibility of the noninteracting $2D$ electron gas in the absence of the external magnetic field (the latter is equal to N). The difference in the compressibility values is a manifestation of the Chern–Simons interaction in strong magnetic fields.

In the following calculations, we adopt the expression used in [3] for the screened interlayer potential $U(\mathbf{q},\omega)$,

$$U(\mathbf{q},\omega) = \frac{1}{2}\frac{V_b + U_b}{1 + \Pi(\mathbf{q},\omega)(V_b + U_b)} - \frac{1}{2}\frac{V_b - U_b}{1 + \Pi(\mathbf{q},\omega)(V_b - U_b)}, \qquad (18)$$

where

$$V_b(\mathbf{q}) = \frac{2\pi e^2}{\epsilon q}, \quad U_b(\mathbf{q}) = \frac{2\pi e^2}{\epsilon q} e^{-qd}$$

are the respective Fourier components of the bare Coulomb potentials for intralayer and interlayer interactions and ϵ is the dielectric constant. Substitung (18) in (1) and using our result (16) for $\Pi(\mathbf{q},\omega)$, we can present the transresistvity in the «ballistic» regime as

$$\rho_D = \rho_{D0} + \delta\rho_D, \qquad (19)$$

where the first term ρ_{D0} is the transresistivity at $\nu = 1/2$ when the effective magnetic field is zero and the second term gives a correction arising in a nonzero effective magnetic field (away from $\nu = 1/2$). As was to be expected, our expression for ρ_{D0} coincides with the already known result [3],

$$\rho_{D0} = \frac{h}{e^2}\frac{\Gamma(7/3)\zeta(4/3)}{3\sqrt{3}}\left(\frac{T}{T_0}\right)^{4/3}, \qquad (20)$$

where

$$T_0 = \frac{\pi e^2 nd}{\epsilon}(1+\alpha),$$

and

$$\frac{1}{\alpha} = \frac{2\pi e^2 d}{\epsilon}\frac{dn}{d\mu}. \qquad (21)$$

The leading term of the correction $\delta\rho_D$ at low temperatures,

$$T/T_0 \ll 1,$$

can be writen as

$$\delta\rho_D = \frac{2}{3}\rho_{D0}\frac{1}{k_F R}\left(1 + \frac{3}{8}\frac{1}{k_F R}\right) + a^2\frac{h}{e^2}\left(\frac{2T}{T_0}\right)^{2/3}\frac{1}{(k_F R)^2} \approx \frac{4}{3}\rho_{D0}\Delta\nu\left(1 + \frac{3}{4}\Delta\nu\right) + 4a^2\frac{h}{e^2}\left(\frac{2T}{T_0}\right)^{2/3}(\Delta\nu)^2, \qquad (22)$$

where the dimensionless positive constant a^2 can be approximated as

$$a^2 = \frac{7}{24\sqrt{3}}\int\limits_0^\infty\left(\frac{y^{2/3}}{\operatorname{sh}^2 y} - \frac{1}{y^{4/3}\operatorname{ch}^2 y}\right)dy. \qquad (23)$$

We have to remark that our result (23) cannot be used in the limit as $T \to 0$. Actually, this expression provides a good asymptotic form for the coefficient a^2 when $(Tk_F l/\mu)^{1/3} \geq 1.5$. Assuming that the mean free path is of the order 1.0μm as in the experiments [11] on dc magnetotransport in a single modulated $2D$ electron gas at ν close to $1/2$, and using the estimate in [7] for the electron density $n = 1.4 \cdot 10^{15}$ m^{-2}, we obtain that expression (23) gives good approximation for a^2 when T/μ is not less than 10^{-2}.

It follows from our results (19) and (22) that the transresistivity ρ_D enhances nearly quadratically with $\Delta\nu$ when the filling factor deviates from $\nu = 1/2$. The term linear in $\Delta\nu$ is also present in the expression for $\delta\rho_D$. This causes an asymmetric shape of the plot of Eq. (22) with respect to $\Delta\nu = 0$. But this asymmetry is not very significant because the linear term is smaller than the last term in the right-hand side of (22). This difference in magnitudes is due to different temperature dependences of the terms considered. The first term, including the correction linear in $(k_F R)^{-1}$, is proportional to $(T/T_0)^{4/3}$, whereas the second one is proportional to $(T/T_0)^{2/3}$ and predominates at low temperatures. Therefore, the magnetic field dependence of the transresistivity near $\nu = 1/2$ matches that observed in the experiments (see Fig. 1).

Keeping only the greatest term in (22), we can represent the ratio ρ_D/ρ_{D0} as

$$\frac{\rho_D}{\rho_{D0}} = 4\beta(\Delta\nu)^2 + 1 \qquad (24)$$

with the coefficient

$$\beta = \frac{3\sqrt{3}a^2}{\Gamma(7/3)\zeta(4/3)}\left(\frac{2T_0}{T}\right)^{2/3}. \qquad (25)$$

This coefficient is proportional to the curvature of the plot of Eq. (22) assuming that the first term is neglected. The curvature reveales a strong dependence

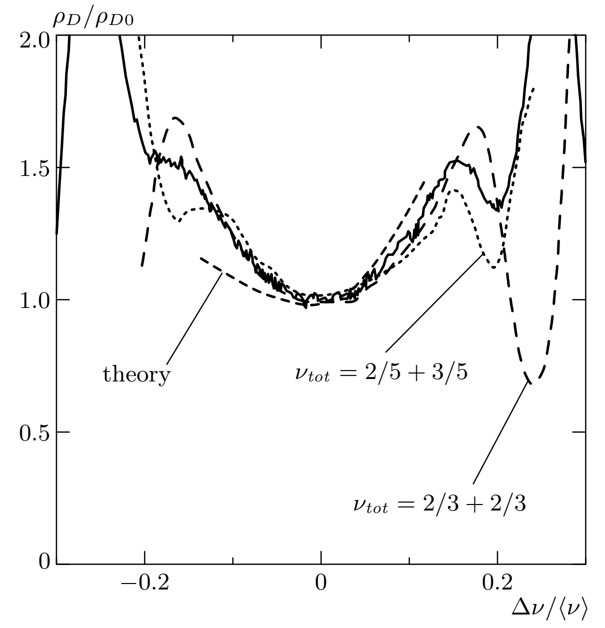

Fig. 1. Scaled drag resistivity versus $\Delta\nu$ at $T = 0.6$ K; lowest dashed curve is the plot of Eq. (22) at $m^* = 4m_b$; $A_0 = 15$, and remaining curves represent the experimental data in [7]

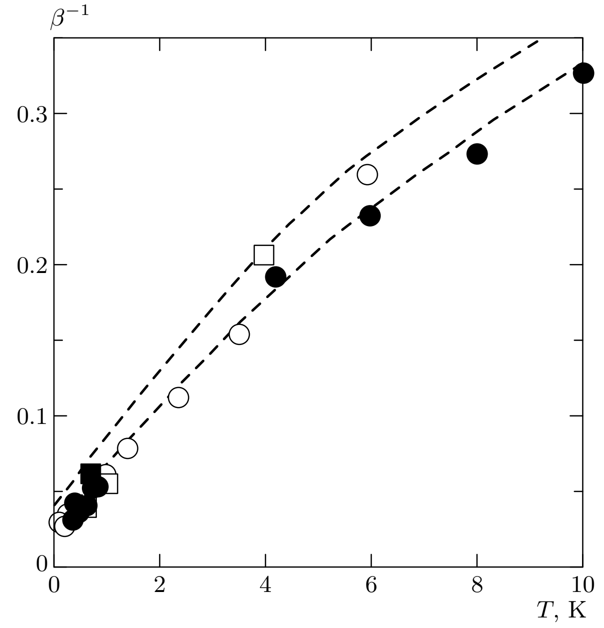

Fig. 2. Temperature dependence of the coefficient β^{-1} for interlayer distances $d = 10$ nm (upper curve) and $d = 22.5$ nm (lower curve) compared to the summary of experimental curvature at both spacings [7]

on temperature; its character also agrees with experiments of [7], as shown in Fig. 2.

A striking feature of the experimental results is that they appear to be insensitive to the distance between the $2D$ electron gases. Sets of data corresponding to samples with different interlayer spacings $d_A = 10$ nm and $d_B = 22.5$ nm fall on the same curve. This concerns both magnetic field dependence of the transresistivity and temperature dependence of the parameter β. The results of the present analysis provide a possible explanation for this feature. It follows from (20)–(25) that the dependence of ρ_D on the interlayer spacing is completely included in the characteristic temperature T_0, which is defined with Eq. (21). The above quantity is nearly independent of the interlayer separation d when the parameter α takes values larger that unity. Estimating the parameter α given by Eq. (21), we obtain that the condition $\alpha > 1$ could be satisfied for small values of the compressibility of the $\nu = 1/2$ state. But within the RPA, the effective mass of composite fermions coincides with the single electron band mass m_b, which takes the value $m_b \approx 0.07 m_e$ for GaAs wells (m_e is the mass of a free electron). Using this value to estimate the compressibility introduced by Eq. (17), we obtain $\alpha \approx 0.44$. This is too small to provide insensitivity of the coefficient β determined by Eq. (25) to the interlayer distance for interlayer spacings reported in the experiments [3]. The above discrepancy could be removed by taking Fermi liquid interactions among quasiparticles (composite fermions) into account. To include Fermi liquid effects into consideration, we write the renormalized polarizability Π^* as [8]

$$\Pi^{*-1} = \Pi^{-1} + F_{(0)} + F_{(1)}, \qquad (26)$$

where Π is the polarizability of noninteracting composite fermions defined with Eq. (2), and the remaining terms represent contributions arising due to the Fermi liquid interaction in the composite fermion system. Only contributions from the first and greatest two terms in the expansion of the Fermi liquid interaction function in Legendre polynomials (f_0 and f_1, respectively) are kept in Eq. (26) to avoid too lengthy calculations. Matrix elements of the 2×2 matrices $F_{(0)}$ and $F_{(1)}$ are equal to

$$F_{(0)} = \begin{pmatrix} f_0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad F_{(1)} = \begin{pmatrix} \dfrac{m^* - m_b}{ne^2}\dfrac{\omega^2}{q^2} & 0 \\ 0 & -\dfrac{m^* - m_b}{ne^2} \end{pmatrix}. \qquad (27)$$

Within the Fermi liquid theory, the effective mass m^* is related to the «bare» mass m_b as

$$\frac{1}{m_b} = \frac{1}{m^*} + \frac{f_1}{2\pi\hbar^2} \equiv \frac{1 + A_1}{m^*}. \qquad (28)$$

Using expressions (26)–(28) and performing calculations within the relevant limit $\delta \ll 1$, we obtain that the expression for the density–density response function for a single layer preserves the form given by Eq. (16), where the compressibility $dn/d\mu$ is replaced with the quantity $dn^*/d\mu$ renormalized due to the Fermi liquid interaction,

$$\frac{dn^*}{d\mu} = \frac{3m^*}{8\pi\hbar^2}\left(1 + \frac{3m^*}{8\pi\hbar^2} f_0\right)^{-1} \equiv \equiv \frac{dn}{d\mu}\left(1 + \frac{dn}{d\mu} f_0\right)^{-1}. \qquad (29)$$

For strongly correlated quasiparticles, this renormalization may significantly reduce the compressibility of the composite fermion liquid, and, consequently, increase the value of the parameter α. It is usually assumed [3, 8] that the Fermi liquid renormalization of the effective mass significantly changes its value: $m^* \sim (5\text{–}10) m_b$. This gives the values of the order 10 for the Fermi liguid coefficient A_1. Using this estimate and substituting our renormalized compressibility (29) in expression (21), we arrive at the conclusion that $dn^*/d\mu$ is low enough for the condition $\alpha > 1$ to be satisfied when the Fermi liquid parameter $A_0 \equiv f_0/2\pi\hbar^2$ takes values of the order 10–100. This conclusion does not seem unrealistic because it is reasonable to expect A_0 to be of the order of or greater than the next Fermi liquid parameter A_1. We obtain a reasonably good agreement between the plot of our Eq. (22) and the experimental results using $A_0 = 15$ and $A_1 = 3$ ($m^* = 4m_b$) (Fig. 1).

Our results for the temperature dependence of β^{-1} also agree with the results of experiments [7]. The upper curve in Fig. 2 corresponds to the double-layer system with smaller interlayer spacing $d_A = 10$ nm, which gives $T_0 = 487$ K, and the lower curve exhibits the temperature dependence of β^{-1} for greater spacing $d_B = 22.5$ nm ($T_0 = 587$ K). The curves do not coincide, but they are arranged rather close to each other.

Finally, the results of the present analysis enable us to qualitatively describe all important features observed in the experiments in [7] on the Coulomb drag slightly away from half filling of the lowest Landau levels of both interacting $2D$ electron gases. They also give us grounds to treat these experimental results as one more evidence of a strong Fermi liquid interaction in the composite fermion system near half filling of the lowest Landau level. The above interaction provides a significant reduction of the compressibility of the composite fermion liquid and a consequent enhancement in the screening length in single layers. Essentially, the parameter α characterizes the ratio of the Thomas–Fermi screening length in a single $2D$ electron gas at $\nu = 1/2$ and the separation between the layers [3]. When $\alpha > 1$, intralayer interactions predominate those between the layers, which could be the reason for low sensitivity of the bilayer to changes in the interlayer spacing. It is likely that here is an explanation for the reported nearly-independence of the drag from the interlayer separation [7]. We believe that at larger distances between the layers, the dependence of the transresistivity on d could be revealed in the experiments. At the same time, the results in [7] give us a valuable opportunity to estimate the strength of Fermi-liquid interactions between quasiparticles at the $\nu = 1/2$ state, which is important for further studies of such systems.

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