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QED CORRECTION TO ASYMMETRY FOR POLARIZED ep-SCATTERING FROM THE METHOD OF ELECTRON STRUCTURE FUNCTIONS

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The electron structure function method is applied to calculate model-independent QED radiative corrections to the asymmetry of electron–proton scattering. Representations for both spin-independent and spin-dependent parts of the cross section are derived. Master formulas include the leading corrections in all orders and the main contribution of the second order and provide accuracy of the QED corrections at the level of one per mill. Numerical calculations illustrate our analytic results for both elastic and deep inelastic events.

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1. INTRODUCTION

Precise polarization measurements in both inclusive [1, 2] and elastic [3, 4] scattering are crucial for understanding the structure and fundamental properties of a nucleon.

One important component of the precise data analysis is radiative effects, which always accompany the processes of electron scattering. The first calculation of radiative corrections to polarized deep inelastic scattering was done by Kukhto and Shumeiko [5], who applied the covariant method of extraction of the infrared divergence [6, 7] to this process. The polarization states were described by 4-vectors, which were kept in their general forms during the calculation. This required a tedious procedure of tensor integration over photonic phase space, and, as a result, led to a very complicated structure of the final formulas for the radiative corrections. The next step was taken in [8], where additional covariant expansion of polarization 4-vectors over a certain basis allowed simplifying the calculation and the final results. It resulted in producing the Fortran code POLRAD [9] and Monte Carlo generator RADGEN [10]. These tools are widely used in all current experiments in polarized deep inelastic scattering. Later, the calculation was applied to collider experiments on deep inelastic scattering [11, 12]. We also applied this method to elastic processes in [13, 14].

But the method of covariant extraction of the infrared divergence is essentially restricted by the lowest order radiative corrections. All attempts to go beyond the lowest order lead to very unwieldy formulas, which are difficult to cross check, or to a simple leading log approach [15]. The recent developments are reviewed in Ref. [16].

The resolution can be found in applying the formalism of electron structure functions (ESF). Within this approach, such processes as the electron–positron annihilation into hadrons and the deep inelastic electron–proton scattering in the one-photon exchange approximation can be considered as the Drell–Yan process [17] in the annihilation or scattering channel, respectively. Therefore, the QED radiative corrections to the corresponding cross sections can be written as a contraction of two electron structure functions and the hard part of the cross section, see [18, 19]. Traditionally, these radiative corrections include effects caused by loop corrections and soft and hard collinear radiation of pho-

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tons and e^+e^--pairs. But it was shown in Ref. [19] that this method can be improved by also including effects due to radiation of one noncollinear photon. The corresponding procedure results in a modification of the hard part of the cross section, which takes the lowest-order correction into account exactly and allows going beyond the leading approximation. We applied this approach to the recoil proton polarization in elastic electron scattering in Ref. [20]. In the present paper, we calculate radiative corrections to polarized deep inelastic and elastic scattering following Ref. [20].

Section 2 gives a short introduction to the structure function method. We there present two known forms of the electron structure functions, iterative and analytical, which resums singular infrared terms in all orders into the exponent. In this section, we also obtain master formulas for observed cross sections. Leading log results are presented in Sec. 3. These results are valid for both deep inelastic and elastic cases. We also use the iterative form of electron structure functions to extract the lowest-order correction, which can provide a cross-check via comparison with the known results. In Secs. 4 and 5, we describe the procedure of generalizing the results to the next-to-leading order in the deep inelastic and elastic cases. Numerical analysis is presented in Sec. 6. We consider kinematical conditions of current polarization experiments at fixed targets and collider kinematics. Some conclusions are given in Sec. 7.

2. ELECTRON STRUCTURE FUNCTIONS

A straightforward calculation based on the quasireal electron method [21] can be used to write the invariant cross section of the deep inelastic scattering process

$$e^-(k_1) + P(p_1) \to e^-(k_2) + X(p_x) \tag{1}$$

as

$$\frac{d\sigma(k_1,k_2)}{dQ^2\,dy} = \int\limits_{z_{1m}}^{1} dz_1 \int\limits_{z_{2m}}^{1} dz_2 D(z_1,L) \times \\ \times \frac{1}{z_2^2} D(z_2,L) \frac{d^2\sigma_{hard}(\tilde{k}_1,\tilde{k}_2)}{d\tilde{Q}^2\,d\tilde{y}}, \quad L = \ln\frac{Q^2}{m^2}, \tag{2}$$

where m is the electron mass and

$$Q^2 = -(k_1-k_2)^2, \quad y = \frac{2p_1(k_1-k_2)}{V}, \quad V = 2p_1k_1.$$

The reduced variables that define the hard cross section in the integrand are

$$\tilde{k}_1 = z_1 k_1, \quad \tilde{k}_2 = \frac{k_2}{z_2}, \\ \tilde{Q}^2 = \frac{z_1}{z_2} Q^2, \quad \tilde{y} = 1 - \frac{1-y}{z_1 z_2}. \tag{3}$$

The electron structure function $D(z,L)$ includes contributions due to the photon emission and pair production,

$$D = D^\gamma + D_N^{e^+e^-} + D_S^{e^+e^-}, \tag{4}$$

where D^γ is responsible for the photon radiation and $D_N^{e^+e^-}$ and $D_S^{e^+e^-}$ describe pair production in non-singlet (by single photon mechanism) and singlet (by double photon mechanism) channels, respectively.

The structure functions in the right-hand side of Eq. (4) satisfy the DGLAP equations [22] (see also [18]). The respective functions $D(z_1,L)$ and $D(z_2,L)$ are responsible for radiation of the initial and final electrons.

There exist different representations for the photon contribution to the structure function [18, 23, 24], but we here use the form given in [18] for D^γ, $D_N^{e^+e^-}$, and $D_S^{e^+e^-}$,

$$D^\gamma(z,Q^2) = \frac{1}{2}\beta(1-z)^{\beta/2-1} \times \\ \times \left[1 + \frac{3}{8}\beta - \frac{\beta^2}{48}\left(\frac{1}{3}L + \pi^2 - \frac{47}{8}\right)\right] - \frac{\beta}{4}(1+z) + \\ + \frac{\beta^2}{32}\left[-4(1+z)\ln(1-z) - \right. \\ \left. - \frac{1+3z^2}{1-z}\ln z - 5 - z\right], \quad \beta = \frac{2\alpha}{\pi}(L-1), \tag{5}$$

$$D_N^{e^+e^-}(z,Q^2) = \\ = \frac{\alpha^2}{\pi^2}\left[\frac{1}{12(1-z)}\left(1 - z - \frac{2m}{\varepsilon}\right)^{\beta/2}\left(L_1 - \frac{5}{3}\right)^2 \times \right. \\ \left. \times \left(1 + z^2 + \frac{\beta}{6}\left(L_1 - \frac{5}{3}\right)\right)\right]\theta\left(1 - z - \frac{2m}{\varepsilon}\right), \tag{6}$$

$$D_S^{e^+e^-} = \frac{\alpha^2}{4\pi^2}L^2\left[\frac{2(1-z^3)}{3z} + \right. \\ \left. + \frac{1}{2}(1-z) + (1+z)\ln z\right]\theta\left(1 - z - \frac{2m}{\varepsilon}\right), \tag{7}$$

where ε is the energy of the parent electron and

$$L_1 = L + 2\ln(1-z).$$

We note that the above form of the structure function $D_N^{e^+e^-}$ includes effects due to the real pair production only. The correction caused by the virtual pair is included in D^γ. Terms containing a contribution of the order $\alpha^2 L^3$ are canceled in the sum $D^\gamma + D_N^{e^+e^-}$.

Instead of the photon structure function given by Eqs. (5)–(7), one can use their iterative form [23]

$$D^\gamma(z,L) = \delta(1-z) + \sum_{k=1}^{\infty} \frac{1}{k!}\left(\frac{\alpha L}{2\pi}\right)^k P_1(z)^{\otimes k}, \quad (8)$$

$$\underbrace{P_1(z)\otimes\ldots\otimes P_1(z)}_{k} = P_1(z)^{\otimes k},$$

$$P_1(z)\otimes P_1(z) = \int\limits_z^1 P_1(t)P_1\left(\frac{z}{t}\right)\frac{dt}{t},$$

$$P_1(z) = \frac{1+z^2}{1-z}\theta(1-z-\Delta) + \delta(1-z)\left(2\ln\Delta + \frac{3}{2}\right),$$
$$\Delta \ll 1.$$

The iterative form (8) of D^γ does not include any effects caused by pair production. The corresponding nonsinglet part of the structure due to the real and virtual pair production can be included into the iterative form of $D^\gamma(z,L)$ by replacing $\alpha L/2\pi$ in the right-hand side of Eq. (8) with the effective electromagnetic coupling

$$\frac{\alpha L}{2\pi} \to \frac{\alpha_{eff}}{2\pi} = -\frac{3}{2}\ln\left(1-\frac{\alpha L}{3\pi}\right), \quad (9)$$

which is (within the leading accuracy) the integral of the running electromagnetic constant.

The lower limits of integration with respect to z_1 and z_2 in the master equation (2) can be obtained from the condition for the existence of inelastic hadronic events,

$$(p_1+\tilde q)^2 > M_{th}^2, \quad \tilde q = \tilde k_1 - \tilde k_2, \quad M_{th} = M + m_\pi, \quad (10)$$

where m_π is the pion mass. This constraint can be rewritten in terms of dimensionless variables as

$$z_1 z_2 + y - 1 - xyz_1 \geq z_2 z_{th},$$
$$x = \frac{Q^2}{2p_1(k_1-k_2)}, \quad z_{th} = \frac{M_{th}^2 - M^2}{V}, \quad (11)$$

which leads to

$$z_{2m} = \frac{1-y+xyz_1}{z_1 - z_{th}}, \quad z_{1m} = \frac{1+z_{th}-y}{1-xy}.$$

The squared matrix element of the considered process in the one-photon exchange approximation is proportional to the contraction of the leptonic and hadronic tensors. Representation (2) reflects the properties of the leptonic tensor. Therefore, it has the universal nature (because of the universality of the leptonic tensor) and can be applied to processes with different final hadronic states. In particular, we can use the electron structure function method to compute radiative corrections to the elastic and deep inelastic (inclusive and semi-inclusive) electron–proton scattering cross sections.

On the other hand, straightforward calculations in the first order in α [5, 8, 21] and the recent calculations of the leptonic current tensor in the second order [25–28] for the longitudinally polarized initial electron demonstrate that in the leading approximation, spin-dependent and spin-independent parts of this tensor are the same for the nonsinglet channel contribution. The latter corresponds to photon radiation and e^+e^-–pair production through the single-photon mechanism. The difference appears in the second order due to the possibility of pair production in the singlet channel by the double-photon mechanism [28]. Therefore, representation (2), being slightly modified, can be used for the calculation of radiative corrections to cross sections of different processes with a longitudinally polarized electron beam.

In our recent paper [20], we applied the electron structure function method to compute radiative corrections to the ratio of the recoil proton polarizations measured at CEBAF by Jefferson Lab Hall A Collaboration [3]. The aim of this high-precision experiment is the measurement of the proton electric formfactor G_E. In the present work, we use this method for calculation of the model-independent part of the radiative corrections to the asymmetry in the scattering of longitudinally polarized electrons on polarized protons at the level of per mill accuracy for elastic and deep inelastic hadronic events.

The cross section of the scattering of the longitudinally polarized electron by the proton with given longitudinal ($\|$) or transverse ($\perp$) polarizations for both elastic and deep inelastic events can be written as a sum of the spin-independent and spin-dependent parts,

$$\frac{d\sigma(k_1,k_2,S)}{dQ^2dy} = \frac{d\sigma(k_1,k_2)}{dQ^2dy} + \eta\frac{d\sigma^{\|,\perp}(k_1,k_2,S)}{dQ^2dy}, \quad (12)$$

where S is the 4-vector of the target proton polarization and η is the product of the electron and proton polarization degrees. Hereafter, we assume $\eta = 1$.

Master equation (2) describes the radiative corrections to the spin-independent part of the cross section in the right-hand side of Eq. (12), and the corresponding equation for the spin-dependent part is given by

$$\frac{d\sigma^{\parallel,\perp}(k_1,k_2,S)}{dQ^2\,dy} = \int\limits_{z_{1m}}^{1} dz_1 \int\limits_{z_{2m}}^{1} dz_2 D^{(p)}(z_1,L)\times \\ \times\frac{1}{z_2^2}D(z_2,L)\frac{d^2\sigma^{\parallel,\perp}_{hard}(\tilde{k}_1,\tilde{k}_2,S)}{d\tilde{Q}^2\,d\tilde{y}}, \quad (13)$$

where

$$D^{(p)} = D^{\gamma} + D_N^{e^+e^-} + D_S^{e^+e^-(p)},$$

and [28]

$$D_S^{e^+e^-(p)} = \frac{\alpha^2}{4\pi^2}L^2\left(\frac{5(1-z)}{2} + (1+z)\ln z\right)\times \\ \times\theta\left(1-z-\frac{2m}{\varepsilon}\right) \quad (14)$$

describes radiation of the initial polarized electron.

This representation is valid if radiation of collinear particles does not change the polarizations $S^{\parallel}$ and $S^{\perp}$. Such stabilized 4-vectors of the proton polarization can be written as [8]

$$S_\mu^{\parallel} = \frac{2M^2k_{1\mu} - Vp_{1\mu}}{MV}, \\ S_\mu^{\perp} = \frac{up_{1\mu} + Vk_{2\mu} - [2u\tau + V(1-y)]k_{1\mu}}{\sqrt{-uV^2(1-y) - u^2M^2}}, \quad (15)$$

where

$$u = -Q^2, \quad \tau = M^2/V.$$

It can be verified that in the laboratory system, the 4-vector $S^{\parallel}$ has the components $(0,\mathbf{n})$, where the 3-vector $\mathbf{n}$ has the orientation of the initial electron 3-momentum $\mathbf{k}_1$. It can also be verified that $S^{\perp}S^{\parallel} = 0$ and that in the laboratory system,

$$S^{\perp} = (0,\mathbf{n}_{\perp}), \quad \mathbf{n}_{\perp}^2 = 1, \quad \mathbf{n}\cdot\mathbf{n}_{\perp} = 0,$$

where the 3-vector $\mathbf{n}_{\perp}$ belongs to the plane $(\mathbf{k}_1,\mathbf{k}_2)$.

If the longitudinal direction $\mathbf{L}$ is chosen along the 3-momentum $\mathbf{k}_1 - \mathbf{k}_2$ in the laboratory system, which coincides with the direction of the 3-vector $\mathbf{q}$ for nonradiative process, and the transverse direction $\mathbf{T}$ is chosen in the plane $(\mathbf{k}_1,\mathbf{k}_2)$, then we have the relations

$$\frac{d\sigma^L}{dQ^2dy} = \cos\theta\frac{d\sigma^{\parallel}}{dQ^2dy} + \sin\theta\frac{d\sigma^{\perp}}{dQ^2dy},$$

$$\frac{d\sigma^T}{dQ^2dy} = -\sin\theta\frac{d\sigma^{\parallel}}{dQ^2dy} + \cos\theta\frac{d\sigma^{\perp}}{dQ^2dy},$$

$$\cos\theta = \frac{y+2xy\tau}{\sqrt{y^2+4xy\tau}}, \quad \sin\theta = -2\sqrt{\frac{xy\tau(1-y-xy\tau)}{y^2+4xy\tau}},$$

and the master formula (13) for $d\sigma^{\parallel}$ and $d\sigma^{\perp}$.

The asymmetry in elastic scattering and deep inelastic scattering processes is defined as the ratio

$$A^{\parallel,\perp} = \frac{d\sigma^{\parallel,\perp}(k_1,k_2,S)}{d\sigma(k_1,k_2)}, \quad (16)$$

and therefore calculating the radiative corrections to the asymmetry requires knowing radiative corrections to both spin-independent and spin-dependent parts of the cross section.

Radiative corrections to the spin-independent part were calculated (within the electron structure function approach) in [19]. In the present work, we compute the radiative corrections to the spin-dependent parts for longitudinal and transverse polarizations of the target proton and longitudinally polarized electron beam. For completeness, we briefly recall the result for the unpolarized case.

3. THE LEADING APPROXIMATION

Within the leading accuracy (with the terms of the order $(\alpha L)^n$ taken into account), the electron structure function can be computed, in principle, in all orders of the perturbation theory. In this approximation, we have to take the Born cross section as a hard part in the right-hand sides of Eqs. (2) and (13).

We express the Born cross section in terms of leptonic and hadronic tensors as

$$\frac{d\sigma}{dQ^2\,dy} = \frac{4\pi\alpha^2(Q^2)}{VQ^4}L^B_{\mu\nu}H_{\mu\nu}, \quad (17)$$

where $\alpha(Q^2)$ is the running electromagnetic constant, which accounts for the effects of vacuum polarization, and

$$H_{\mu\nu} = -F_1\tilde{g}_{\mu\nu} + \frac{F_2}{p_1q}\tilde{p}_{1\mu}\tilde{p}_{1\nu} - \\ - i\frac{M\epsilon_{\mu\nu\lambda\rho}q_\lambda}{p_1q}\left[(g_1+g_2)S_\rho - g_2\frac{Sq}{p_1q}p_{1\rho}\right], \quad (18)$$

$$L^B_{\mu\nu} = -\frac{Q^2}{2}g_{\mu\nu} + k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} + i\varepsilon_{\mu\nu\lambda\rho}q_\lambda k_{1\rho},$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad \tilde{p}_{1\mu} = p_{1\mu} - \frac{p_1q}{q^2}q_\mu.$$

In Eqs. (18), we assume the proton and electron polarization degrees equal to 1. The spin-independent $(F_1,\ F_2)$ and spin-dependent $(g_1,\ g_2)$ proton structure functions depend on the two variables

$$x' = \frac{-q^2}{2p_1q}, \quad q^2 = (p_x - p_1)^2.$$

In the Born approximation, $x' = x$, but these variables differ in the general case, when radiation of photons and electron–positron pairs is allowed.

Because the normalization is chosen, the elastic limit ($p_x^2 = M^2$) can be obtained by simply substituting

$$\begin{aligned} &F_1(x', q^2) \to \frac{1}{2}\delta(1-x')G_M^2(q^2), \\ &F_2(x', q^2) \to \delta(1-x')\frac{G_E^2(q^2) + \lambda G_M^2(q^2)}{1+\lambda}, \\ &g_1(x', q^2) \to \frac{1}{2}\delta(1-x')\{G_M(q^2)G_E(q^2) + \\ &+ \frac{\lambda}{1+\lambda}[G_M(q^2) - G_E(q^2)]G_M(q^2)\}, \\ &g_2(x', q^2) \to -\frac{1}{2}\delta(1-x')\frac{\lambda}{1+\lambda} \times \\ &\quad \times [G_M(q^2) - G_E(q^2)]G_M(q^2) , \\ &\lambda = -\frac{q^2}{4M^2} \end{aligned} \tag{19}$$

in the hadronic tensor, where G_M and G_E are the magnetic and electric proton form factors.

A simple calculation gives the spin-independent and spin-dependent parts of the well-known Born cross section in the form

$$\begin{aligned} \frac{d\sigma^B}{dQ^2 dy} &= \frac{4\pi\alpha^2(Q^2)}{Q^4 y} \times \\ &\times \left[(1 - y - xy\tau)F_2(x, Q^2) + xy^2 F_1(x, Q^2)\right], \end{aligned} \tag{20}$$

$$\begin{aligned} \frac{d\sigma_{\parallel}^B}{dQ^2 dy} &= \frac{8\pi\alpha^2(Q^2)}{V^2 y} \times \\ &\times \left[\left(\tau - \frac{2-y}{2xy}\right) g_1(x, Q^2) + \frac{2\tau}{y} g_2(x, Q^2)\right], \end{aligned} \tag{21}$$

$$\begin{aligned} \frac{d\sigma_{\perp}^B}{dQ^2 dy} &= -\frac{8\pi\alpha^2(Q^2)}{V^2 y}\sqrt{\frac{M^2}{Q^2}(1 - y - xy\tau)} \times \\ &\times \left[g_1(x, Q^2) + \frac{2}{y} g_2(x, Q^2)\right]. \end{aligned} \tag{22}$$

Thus, within the leading accuracy, the radiatively corrected cross section of process (1) is defined by Eq. (2) (for its spin-independent part) with (20) as the hard part of the cross section, and by Eq. (13) (for its spin-dependent part) with (21) or (22) as the hard part.

It is useful to extract the first-order correction to the Born approximation, as defined by master equation (2). For this purpose, we can use the iterative form of the photon structure function D^γ with $L \to L-1$ and

$$\Delta \to \Delta_1 = \frac{2(\Delta\varepsilon)}{\sqrt{V}(1-xy)}\sqrt{\tau + z_+},$$

$$z_+ = y(1-x), \quad \frac{2(\Delta\varepsilon)}{\sqrt{V}} \ll 1$$

for $D(z_1, L)$ and

$$\Delta \to \Delta_2 = \frac{2(\Delta\varepsilon)}{\sqrt{V}(1-z_+)}\sqrt{\tau + z_+}$$

for $D(z_2, L)$, where $(\Delta\varepsilon)$ is the minimal energy of a hard collinear photon in the special system ($\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{p}_1 = 0$). Straightforward calculations yield the expression

$$\begin{aligned} \frac{d\sigma^{(1)}(k_1, k_2)}{dQ^2 dy} &= \frac{\alpha(L-1)}{2\pi} \times \\ &\times \left\{ \frac{d\sigma^{(B)}(k_1, k_2)}{dQ^2 dy}\left[3 + 2\ln\frac{4(\Delta\varepsilon)^2(z_+ + \tau)}{V(1-z_+)(1-xy)}\right] + \right. \\ &+ \int\limits_{z_{th}}^{z_+ - \rho} dz \left[\frac{1+z_1^2}{(1-xy)(1-z_1)}\frac{d\sigma^{(B)}(z_1 k_1, k_2)}{dQ_t^2 dy_t} + \right. \\ &+ \left.\left. \frac{1+z_2^2}{(1-z_+)(1-z_2)}\frac{d\sigma^{(B)}(k_1, k_2/z_2)}{dQ_s^2 dy_s}\right]\right\}, \end{aligned} \tag{23}$$

where

$$z = \frac{M_x^2 - M^2}{V}, \quad z_1 = \frac{1-y+z}{1-xy},$$

$$z_2 = \frac{1-z_+}{1-z}, \quad \rho = \frac{2(\Delta\varepsilon)}{\sqrt{V}}\sqrt{\tau + z_+},$$

$$Q_t^2 = -q_t^2 = z_1 Q^2, \quad Q_s^2 = -q_s^2 = \frac{Q^2}{z_2},$$

$$y_{t,s} = 1 - \frac{1-y}{z_{1,2}}.$$

Similar equations can be derived for the first-order correction to the spin-dependent part of the cross section for both longitudinal and transverse polarizations of the target proton.

4. DEEP INELASTIC SCATTERING CROSS SECTION BEYOND THE LEADING ACCURACY

To go beyond the leading accuracy, we have to improve the expressions for hard parts of the cross sections in master equations (2) and (13) in order to include effects caused by radiation of a hard noncollinear photon. In principle, we can also improve the expression for the D function in order to take collinear next-to-leading effects in the second order of perturbation theory into account. The essential part of these effects is included in our D functions due to the replacement $L \to L - 1$. The rest can be written using the results of the corresponding calculations for the double photon emission [27,30], pair production [28, 31, 32], one-loop corrected Compton tensor [25, 26, 33], and virtual correction [34]. But we here restrict ourselves to the D functions given above in Eqs. (5), (6), (7), and (14).

To compute the improved hard cross section, we must find the full first-order radiative corrections to the cross section of process (1) and subtract from it (to avoid double counting) its leading part defined by Eq. (23) (for the unpolarized case). Therefore, the improved hard part can be written as

$$\frac{d\sigma_{hard}}{dQ^2dy} = \frac{d\sigma^B}{dQ^2dy}+\frac{d\sigma^{(S+V)}}{dQ^2dy}+\frac{d\sigma^H}{dQ^2dy}-\frac{d\sigma^{(1)}}{dQ^2dy}, \quad (24)$$

where $d\sigma^{(S+V)}$ is the correction to the cross section of process (1) due to virtual and soft photon emission and $d\sigma^H$ is the cross section of the radiative process

$$e^-(k_1) + P(p_1) \to e^-(k_2) + \gamma(k) + X(p_x). \quad (25)$$

The virtual and soft corrections are factored in a similar way for both polarized and unpolarized cases [19] and can be written as

$$\frac{d\sigma^B}{dQ^2dy} + \frac{d\sigma^{(S+V)}}{dQ^2dy} = \frac{d\sigma^B}{dQ^2dy}\left[1+\frac{\alpha}{2\pi}\times \right.$$
$$\left.\times\left(\delta+(L-1)\left(3+2\ln\frac{\rho^2}{(1-xy)(1-z_+)}\right)\right)\right], \quad (26)$$

$$\delta = -1 - \frac{\pi^2}{3} - 2f\frac{1-y-xy\tau}{(1-xy)(1-z_+)} - \ln^2\frac{1-xy}{1-z_+},$$

$$f(x) = \int\limits_0^x \frac{dt}{t}\ln(1-t).$$

To calculate the cross section of radiative process (25), we use the corresponding leptonic tensor in the form

$$L^{\gamma}_{\mu\nu} = \frac{\alpha}{4\pi^2}(L^{H(un)}_{\mu\nu} + L^H_{\mu\nu})\frac{d^3k}{\omega},$$
$$L^H_{\mu\nu} = 2i\varepsilon_{\mu\nu\lambda\rho}q_\lambda(k_{1\rho}R_t + k_{2\rho}R_s), \quad (27)$$

$$R_t = \frac{u+t}{st} - 2m^2\left(\frac{1}{s^2}+\frac{1}{t^2}\right),$$

$$R_s = \frac{u+s}{st} - 2m^2\frac{s_t}{ut^2}, \quad s_t = \frac{-u(u+Vy-Vz)}{u+V},$$

where ω is the energy of the radiated photon, $L^{H(un)}_{\mu\nu}$ is the leptonic tensor for unpolarized particles, see Ref. [33], and we use the notation

$$s = 2kk_2, \quad t = -2kk_1, \quad q^2 = u+s+t$$

for kinematic invariants. The result for the unpolarized case was derived in [19], and we here rewrite it using standard notation as

$$\frac{d\sigma_{hard}}{dQ^2dy} = \frac{d\sigma^B}{dQ^2dy}\left(1+\frac{\alpha}{2\pi}\delta\right) + \frac{\alpha}{VQ^2}\times$$
$$\times\int\limits_{z_{th}}^{z_+} dz\left\{\frac{1-r_1}{1-xy}\hat{P}_tN - \frac{1-r_2}{1-z_+}\hat{P}_sN + \right.$$
$$+\int\limits_{r_-}^{r_+} dr\frac{2W}{\sqrt{y^2+4xy\tau}} +$$
$$+P\int\limits_{r_-}^{r_+}\frac{dr}{1-r}\left[\frac{1-\hat{P}_t}{|r-r_1|}\left(\frac{(1+r^2)N}{1-xy} + (r_1-r)T_t\right) - \right.$$
$$\left.\left.-\frac{1-\hat{P}_s}{|r-r_2|}\left(\frac{(1+r^2)N}{1-z_+}+(r_2-r)T_s\right)\right]\right\}\frac{\alpha^2(rQ^2)}{r^2}, \quad (28)$$

where $r = -q^2/Q^2$ and the limits of integration with respect to r are

$$r_\pm(z) = \frac{1}{2xy(\tau+z_+)}\times$$
$$\times\left[2xy(\tau+z) + (z_+-z)\left(y\pm\sqrt{y^2+4xy\tau}\right)\right].$$

Here, we used the notation

$$N = 2F_1(x',r) + \frac{2x'}{rxy}\left(\frac{1-y}{xy}-\tau\right)F_2(x',r),$$

$$W = 2F_1(x',r) - \frac{2x'\tau}{rxy}F_2(x',r),$$

$$T_t = -\frac{2x'[1-r(1-y)]}{x^2y^2r}F_2(x',r),$$
$$T_s = -\frac{2x'(1-y-r)}{x^2y^2r}F_2(x',r), \quad (29)$$

$$r_1 = z_1, \quad r_2 = \frac{1}{z_2}, \quad x' = \frac{xyr}{xyr+z}.$$

The action of the operators $\hat{P}_t$ and $\hat{P}_s$ is defined as

$$\hat{P}_tf(r,x') = f(r_1,x_t), \quad \hat{P}_sf(r,x') = f(r_2,x_s),$$

$$x_t = \frac{xyr_1}{xyr_1+z}, \quad x_s = \frac{xyr_2}{xyr_2+z}.$$

The hard cross section (29) has neither collinear nor infrared singularities. The different terms in the right-hand side of Eq. (29) have singularities at $r = r_1$, $r = r_2$, and $r = 1$. Singularities at first two points are collinear and the one at the third point is nonphysical, arising at integration. Collinear singularities vanish because of the action of the operators $\hat{P}_t$ and $\hat{P}_s$ on the terms containing N. The nonphysical singularity cancels because in the limiting case $r \to 1$, we have

$$\frac{r_2 - r}{|r_2 - r|} = 1, \quad \frac{r_1 - r}{|r_1 - r|} = -1, \quad T_t + T_s = 0.$$

To derive the hard cross section for the polarized case, we have to use the analogue of Eq. (24) for $d\sigma^{\|}$ and $d\sigma^{\perp}$. Taking into account that $d\sigma^{V+S}$ and $d\sigma^{(1)}$ are the same in the polarized and unpolarized cases and using expression (27) for the antisymmetric part of the leptonic tensor to compute $d\sigma^H$ in the polarized case, we arrive at

$$\frac{d\sigma^{\|,\perp}_{hard}}{dQ^2dy} = \frac{d\sigma^B_{\|,\perp}}{dQ^2dy}\left(1+\frac{\alpha}{2\pi}\delta\right) + \frac{\alpha}{Q^4}U^{\|,\perp} \times$$
$$\times \int\limits_{z_{th}}^{z_+} dz \left\{ \frac{1-r_1}{1-xy}\hat{P}_t N_t^{\|,\perp} + \frac{1-r_2}{1-z_+}\hat{P}_s N_s^{\|,\perp} + \right.$$
$$+ P\int\limits_{r_-}^{r_+}\frac{dr}{1-r}\left[\frac{1-\hat{P}_s}{|r-r_2|(1-z_+)} \times \right.$$
$$\times \left((1+r^2)N_s^{\|,\perp} + \frac{2(r_2-r)}{r_2}T_s^{\|,\perp}\right) - \frac{1-\hat{P}_t}{|r-r_1|} \times$$
$$\times \left.\left(\frac{(1+r^2)N_t^{\|,\perp}}{1-xy} + 2r(r_1-r)T_t^{\|,\perp}\right)\right] +$$
$$\left. + \int\limits_{r_-}^{r_+} dr \frac{2W^{\|,\perp}}{\sqrt{y^2+4xy\tau}}\right\} \frac{x'\alpha^2(Q^2r)}{r^3}, \quad (30)$$

where

$$U^{\|} = 1, \quad U^{\perp} = \sqrt{\frac{M^2}{Q^2}(1-y-xy\tau)^{-1}},$$

$$W^{\|} = 4y\tau W, \quad W^{\perp} = 2y^2(1+2x\tau)W,$$
$$W = (1+r)xg_1 + x'g_2,$$
$$N_t^{\|} = 2[2r - z - xy(r+2\tau)]g_1 - 8x'\tau g_2,$$
$$N_s^{\|} = 2[2 - z - xyr(1+2\tau)]g_1 - 8x'\tau g_2,$$
$$N_t^{\perp} = 2[1 - y - z + r - xy(r+2\tau)](xyg_1 + 2x'g_2),$$
$$N_s^{\perp} = 2\left[1 - y + \frac{1-z}{r} - xy(1+2\tau)\right](xyrg_1 + 2x'g_2),$$
$$T_t^{\|} = 2rg_1 - 4x'\tau g_2, \quad T_s^{\|} = 2(z-1)(g_1 - 2x'\tau g_2),$$
$$T_t^{\perp} = 2xyrg_1 + 2x'(1 - y + r - 2xy\tau)g_2,$$
$$T_s^{\perp} = 2(z-1)\left[xyg_1 + x'(1 - y + 1/r - 2xy\tau)g_2\right].$$

The polarized hard cross section defined by Eq. (30) is also free from collinear singularities due to the action of the operators $1-\hat{P}_t$ and $1-\hat{P}_s$. The nonphysical singularity at $r=1$ in the right-hand side of Eq. (30) cancels because

$$T_t{}^{\|,\perp} = \frac{1}{z-1}T_s{}^{\|,\perp}$$

in this limit. We note that radiation of a photon at large angles by the initial and final electrons increases the range of r in (28) and (30), because $r_1 < r < r_2$ for collinear radiation, and now $r_- < r_1$ and $r_+ > r_2$. This may be important if the hadron structure functions are large in these additional regions.

5. HARD CROSS SECTION FOR ELASTIC HADRONIC EVENTS

To describe the hard cross section for elastic hadronic events, we use the replacement defined by (19) in Eqs. (28) and (30). We refer to Eqs. (21)–(23) for the Born cross sections that enter these equations. The function $\delta(1-x')$ is used to integrate with respect to the inelasticity z,

$$\int dz\,\delta(1-x') = xyr. \quad (31)$$

The final result for the unpolarized case is given by (we do not introduce special notation for the elastic cross section)

$$\frac{d\sigma_{hard}}{dQ^2dy} = \frac{d\sigma^B}{dQ^2dy}\left(1+\frac{\alpha}{2\pi}\delta\right) + \frac{\alpha}{V^2} \times$$
$$\times \left\{ \frac{1-r_1}{1-xy}\hat{P}_t N - \frac{1-r_2}{1-z_+}\hat{P}_s N + \int\limits_{r_-}^{r_+} dr \frac{2W}{\sqrt{y^2+4xy\tau}} + \right.$$
$$+ P\int\limits_{r_-}^{r_+}\frac{dr}{1-r}\left[\frac{1-\hat{P}_t}{|r-r_1|}\left(\frac{1+r^2}{1-xy}N + (r_1-r)T_t\right) - \right.$$
$$\left.\left. - \frac{1-\hat{P}_s}{|r-r_2|}\left(\frac{1+r^2}{1-z_+}N + (r_2-r)T_s\right)\right]\right\}\frac{\alpha^2(Q^2r)}{r}, \quad (32)$$

where

$$N = G_M^2 + \frac{2}{xyr}\left(\frac{1-y}{xy} - \tau\right)\frac{G_E^2 + \lambda G_M^2}{1+\lambda},$$

$$W = G_M^2 - \frac{2\tau}{xyr}\frac{G_E^2 + \lambda G_M^2}{1+\lambda},$$

$$T_t = -\frac{2}{x^2y^2r}[1 - r(1-y)]\frac{G_E^2 + \lambda G_M^2}{1+\lambda},$$

$$T_s = -\frac{2}{x^2y^2r}(1-r-y)\frac{G_E^2+\lambda G_M^2}{1+\lambda}.$$

The Born cross section in the right-hand side of Eq. (32) is defined as

$$\frac{d\sigma^B}{dQ^2dy} = \frac{4\pi\alpha^2(Q^2)}{V^2}\times \times\left[\frac{1}{2}G_M^2+[1-y(1+\tau)]\frac{G_E^2+\lambda G_M^2}{y^2(1+\lambda)}\right]\delta\left(y-\frac{Q^2}{V}\right). \quad (33)$$

In writing this last equation, we took into account that

$$\delta(1-x) = y\delta\left(y-\frac{Q^2}{V}\right).$$

The spin-dependent hard cross section for elastic hadronic events can be written in the form very similar to (32),

$$\frac{d\sigma^{\parallel,\perp}_{hard}}{dQ^2dy} = \frac{d\sigma^B_{\parallel,\perp}}{dQ^2dy}\left(1+\frac{\alpha}{2\pi}\delta\right)+\frac{\alpha}{V}U^{\parallel,\perp}\times \times\left\{\frac{1-r_1}{1-xy}\hat{P}_tN_t^{\parallel,\perp}+\frac{1-r_2}{1-z_+}\hat{P}_sN_s^{\parallel,\perp}+ +\int\limits_{r_-}^{r_+}dr\frac{W^{\parallel,\perp}}{\sqrt{y^2+4xy\tau}}+P\int\limits_{r_-}^{r_+}\frac{dr}{1-r}\left[-\frac{1-\hat{P}_t}{|r-r_1|}\times \times\left(\frac{1+r^2}{1-xy}N_t^{\parallel,\perp}+2r(r_1-r)T_t^{\parallel,\perp}\right)+ +\frac{1-\hat{P}_s}{|r-r_2|(1-z_+)}\times \times\left((1+r^2)N_s^{\parallel,\perp}+\frac{2(r_2-r)}{r_2}T_s^{\parallel,\perp}\right)\right]\right\}\times \times\frac{\alpha^2(Q^2r)}{(4M^2+Q^2r)r^2}, \quad (34)$$

where

$$W^{\parallel} = 4y\tau W, \quad W^{\perp} = 2y^2(1+2x\tau)W,$$

$$W = r[x(1+r)-1]G_M^2+\left[r+\frac{4\tau}{y}(1+r)\right]G_MG_E,$$

$$N_t^{\parallel} = r(2\tau+r)(2-xy)G_M^2+8\tau\left[r\left(\frac{1}{xy}-1\right)-\tau\right]G_MG_E,$$

$$N_s^{\parallel} = r(2\tau+1)(2-xyr)G_M^2+8\tau\left[\frac{1}{xy}-r(1+\tau)\right]G_MG_E,$$

$$N_t^{\perp} = [1-y+r-xy(r+2\tau)]\times \times[-r(2-xy)G_M^2+2(r+2\tau)G_MG_E],$$

$$N_s^{\perp} = \left[1-y+\frac{1}{r}-xy(1+2\tau)\right]\times \times[-r(2-xyr)G_M^2+2r(1+2\tau)G_MG_E],$$

$$T_t^{\parallel} = r\left[(r+2\tau)G_M^2+2\tau\left(\frac{2}{xy}-1\right)G_MG_E\right],$$

$$T_s^{\parallel} = -r(1+2\tau)G_M^2-2\tau\left(\frac{2}{xy}-r\right)G_MG_E,$$

$$T_t^{\perp} = r\{-[r(1-xy)+1-y-2xy\tau)]G_M^2+ +[1-y-2xy\tau+r+4\tau]G_MG_E\},$$

$$T_s^{\perp} = r\left[\frac{1}{r}-xy(1+2\tau)+1-y\right]G_M^2- -\left[2\tau(2-xyr)+1+r(1-y)\right]G_MG_E.$$

We note that the argument of the electromagnetic form factors in Eqs. (32) and (34) is $-Q^2r$.

The Born cross sections in the right-hand side of Eq. (34) are given by

$$\frac{d\sigma^B_{\parallel}}{dQ^2dy} = \frac{4\pi\alpha^2(Q^2)}{V(4M^2+Q^2)}\left[4\tau\left(1+\tau-\frac{1}{y}\right)G_MG_E- -(1+2\tau)\left(1-\frac{y}{2}\right)G_M^2\right]\delta\left(y-\frac{Q^2}{V}\right), \quad (35)$$

for the longitudinal polarization of the target proton and by

$$\frac{d\sigma^B_{\perp}}{dQ^2dy} = \frac{8\pi\alpha^2(Q^2)}{V(4M^2+Q^2)}\sqrt{\frac{M^2}{Q^2}[1-y(1+\tau)]}\times \times\left[\left(1-\frac{y}{2}\right)G_M^2-(1+2\tau)G_MG_E\right]\delta\left(y-\frac{Q^2}{V}\right), \quad (36)$$

for the transverse one. The argument of the form factors in (35) and (36) is $-Q^2$.

The results in this section can be generalized to elastic electron–deuteron scattering in both polarized and unpolarized cases in a very simple way because the respective deuteron tensors $H^d_{\mu\nu}$ are connected with the proton ones $H^p_{\mu\nu}$ by the relations

$$H^{d(un)}_{\mu\nu} = \frac{4\tau+xyr}{4\tau}H^{p(un)}_{\mu\nu}\times \times\left(G_M^2\to\frac{2}{3}{G_M^{(d)}}^2,\ G_E^2\to G_C^2+\frac{8x^2y^2r^2}{9(4\tau)^2}G_Q^2\right),$$

$$H^{d(\parallel,\perp)}_{\mu\nu} = -\frac{4\tau+xyr}{8\tau}H^{p(\parallel,\perp)}_{\mu\nu}\times \times\left(G_M\to G_M^{(d)},\ G_E\to 2G_C+\frac{xyr}{6\tau}G_Q\right),$$

where $G_M^{(d)}$, G_C, and G_Q are the magnetic, charged, and quadrupole deuteron form factors respectively.

6. NUMERICAL ESTIMATIONS

The formulas obtained in the last section include some operators that emphasize the physical meaning of the transformations performed. But they are not convenient in numerical analysis. Here, we present a unified version of the formulas without any operators. For example, the symbol P is explicitly treated as

$$P\int\limits_{r_-}^{r_+}\frac{dr}{1-r}F(r) = \int\limits_{r_-}^{r_+}\frac{dr}{1-r}\big(F(r)-F(1)\big)+F(1)\ln\frac{1-r_-}{r_+-1}.$$

Therefore, all cross sections given by Eqs. (28), (30), (32), and (34) can be written by means of the unified formula

$$\frac{d\sigma^i_{hard}}{dQ^2dy} = \frac{d\sigma^B_i}{dQ^2dy}\left(1+\frac{\alpha}{2\pi}\delta\right)+\alpha U_i\times \times\int\limits_{z_{th}}^{z_+}dz\left(\left\{L^i_1N_i(r_1)+L^i_2N_i(r_2)\right\}+ +\int\limits_{r_-}^{r_+}dr\left\{W_i+T_i+\frac{1}{1-r}\left[N_i(r_1)-N_i(r_2)+ +\frac{1-r_1}{|r-r_1|}\Big[N_i(r)-N_i(r_1)\Big]+ +\frac{1-r_2}{|r-r_2|}\Big[N_i(r)-N_i(r_2)\Big]\right]\right\}\right), \qquad (37)$$

where

$$L^i_{1,2} = \mp b_i\frac{(1-r_{1,2})^2}{1+r^2_{1,2}}\mp\ln\frac{1-r_-}{r_+-1}, \quad b_u=-1, \quad b_{l,t}=1.$$

The index i runs over all polarization states ($i=u,l,t$). The functions $N_i(r)$ and T_i are given by

$$N_i(r) = \frac{1+r^2}{z_+-z}N_i\frac{x'\alpha^2}{r^3},$$

$$T_i = \begin{cases} \pm\dfrac{T_{i1}}{1-r}\dfrac{x'\alpha^2}{r^3}, & r<r_1, \quad r>r_2, \\ T_{i2}\dfrac{x'\alpha^2}{r^3}, & r_1<r<r_2. \end{cases}$$

The pole at $r=1$ can be reached only in the region $r_1<r<r_2$, and hence there is no singularity in the terms involving T_{i1}. For T_{i2}, this pole cancels explicitly,

$$T_{u2} = \frac{2(2-y)F_2}{x^2y^2}, \quad T_{l2} = -4(1+r)g_1+8x'\tau g_2,$$

$$T_{t2} = -4(1+r)xyg_1-4x'\left(r+\frac{1}{r}+2-y-2xy\tau\right)g_2.$$

In the unpolarized case, $N_u = rN/x'$ with N from (28). In other cases, they are

$$N_l = 2\left[-1-r+\frac{y(1+2x\tau)[1-z+r(1-xy)]}{2-y}\right]g_1+ +8x'\tau g_2,$$

$$N_t = -\frac{4[1-z+r(1-xy)]}{r(2-y)}\,[xyr(1-y-xy\tau)g_1+ +x'(1-y+z+r(1-y+xy))g_2]+4x'y(1+2x\tau)g_2,$$

$$T_{u1} = -\frac{2(1+r)F_2}{x^2y},$$

$$T_{l1} = 4\frac{y(1+r^2)(1+2x\tau)}{2-y}g_1+8x'(1+r)\tau g_2,$$

$$T_{t1} = 4\left\{\frac{1+r^2}{2-y}\left[-2xy(1-y-xy\tau)g_1+ +(y-2z+yr(1-2x))\frac{x'}{r}g_2\right]+ +x'y(1+2x\tau)(1+r)g_2\right\},$$

and

$$W_u = \frac{2W}{\sqrt{y^2+4xy\tau}}\frac{\alpha^2}{r^2}, \quad W_{l,t} = \frac{2W^{\parallel,\perp}}{\sqrt{y^2+4xy\tau}}\frac{x\alpha^2}{r^3},$$

$$U_u = \frac{1}{VQ^2} \quad U_{l,t} = \frac{\widetilde{U}^{\parallel,\perp}}{Q^4}.$$

The same formulas can be used in the elastic case. Only Eqs. (19) and (31) are needed here. In the elastic case, we must therefore substitute

$$\int dz \to xyr,$$

set $x'=1$ and $z=0$, and replace the proton structure functions in accordance with (19).

It is believed that the formulas obtained within the presented formalism are not convenient for numerical analysis. There are two reasons for such an opinion. First, the electron structure function in form (5), (6) has a very sharp peak as z tends to unity. Second, because absolute values appear in denominators, the integrand cannot be a continuous function of the integration variables. This produces obstacles for numerical analysis if it is carried out in the traditional style based on adaptive methods of numerical integration, which

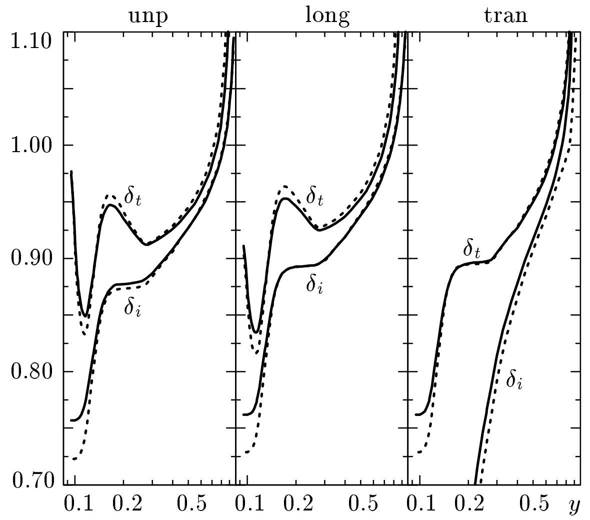

Fig. 1. Radiative correction to unpolarized and polarized (both longitudinal and transverse) parts of the cross section for kinematics close to JLab experiments, $V = 10$ GeV^2, $x = 0.5$

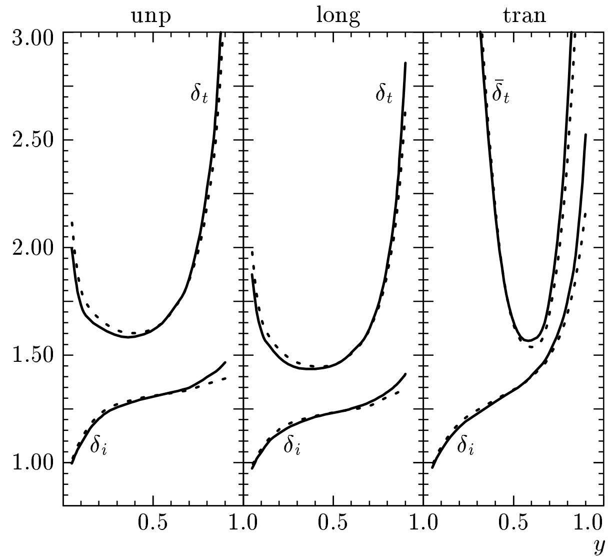

Fig. 2. Radiative correction to unpolarized and polarized (both longitudinal and transverse) parts of the cross section for kinematics close to HERMES experiment, $V = 50$ GeV^2, $x = 0.1$, $\bar{\delta}_t = -\delta_t$

is used in such programs as TERAD/HECTOR [36] or POLRAD [9]. But it is possible to perform numerical analysis if Monte Carlo integration is used instead of adaptive integration and the regions with sharp peaks are extracted into separate integration subregions. Based on these ideas, we developed Fortran code ESFRAD[1)] that allows performing the numerical analysis without any serious difficulties.

We considered two radiative processes. In the first case, the continuum of hadrons is produced, while in the second case, the proton remains in the ground state. Both of the effects considered contribute to the experimentally observed cross section[2)] of deep inelastic scattering. They are usually called the radiative tails from the continuous spectrum and the elastic peak, or simply the inelastic and elastic radiative tails. Below, we study the contributions of the tails numerically within kinematical conditions of the current experiments on deep inelastic scattering.

We take three typical values of V equal to 10, 50 and 10^5 GeV^2. They correspond to JLab, HERMES, and HERA measurements. Figures 1, 2, and 3 give the radiative correction factor for all polarization states (unpolarized, longitudinal or transverse)

$$\delta_{i,t} = \frac{\sigma^{obs}}{\sigma^B}. \tag{38}$$

The observed double differential cross section is given by the master formulas (2) and (13), and the Born cross section is calculated via (20), (21), and (22). Both elastic and inelastic contributions must be taken for σ_{hard}. In this case, we obtain the total radiative correction factor (δ_t). The subscripts i and t correspond to the cases where the elastic radiative tail is included into the total correction (δ_t) or the inelastic radiative tail contributes only (δ_i). The elastic radiative tail may optionally not be included because there sometimes exist experimental methods to separate this contribution. We note that for the HERA kinematics, we do not include it because it is usually separated experimentally. Also we can extract a one-loop contribution in order to study the effect of higher-order corrections. The observed cross section in this case is given by the sum of the cross sections in Eqs. (23) and (37). We note that this can provide an additional cross check by comparison with POLRAD.

We use rather simple models for spin-averaged and spin-dependent structure functions. It allows us not to mix the pure radiative effects, which are of interest, with the effects due to hadron structure functions. Specifically, we use the so-called D8 model for the spin-average structure function [35] (see also discussion in [9]), and $A_1(x) = x^{0.725}$ suggested in [37]; we set $g_2 = 0$ (the definition of $A_1(x)$ is given below).

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1) Electron Structure Function method for RADiative corrections.

2) Here and below, we mean double differential cross section $\sigma = d\sigma/dy\,dQ^2$.

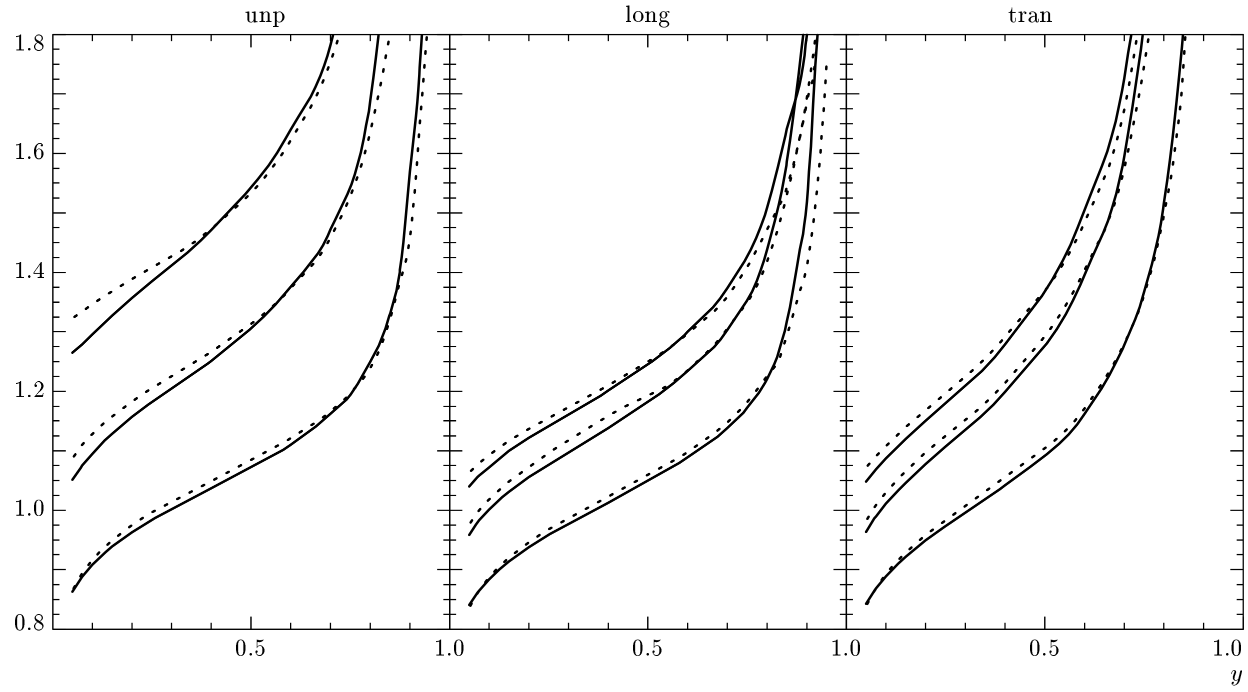

Fig. 3. One-loop and total radiative corrections (dashed and solid lines) for collider kinematics (HERA); $V = 10^5$ GeV^2. Lines from top to bottom correspond to different values of $x = 0.001$, 0.01, and 0.1

From these plots, we can see that the total radiative correction is basically determined by the one-loop correction with some important effect around kinematical boundaries. The sign and value of the higher-order effects are in agreement with the leading log estimations and calculations of the correction to the elastic radiative tail in Refs. [38, 39]. Two regions require special consideration, the region of higher y for the HERMES and JLab kinematics and the region near the pion threshold at JLab.

We define the polarization asymmetries in the standard way,

$$A_L = \frac{\sigma_\parallel}{\sigma}, \quad A_T = \frac{\sigma_\perp}{\sigma}. \tag{39}$$

We can also define the spin asymmetry A_1 as $A_L = DA_1$ (for the chosen model where $g_2 = 0$), where D is the kinematical depolarization factor depending on the ratio R of the longitudinal and transverse photoabsorption cross sections,

$$D = \frac{y(2-y)(1+\gamma^2 y/2)}{y^2(1+\gamma^2) + 2(1-y-\gamma^2 y^2/4)(1+R)},$$

$$R = \frac{\sigma_L}{\sigma_T} = \frac{M(Q^2+\nu^2)}{Q^2\nu}\frac{F_2}{F_1} - 1,$$

where $\nu = yV/2M$ and $\gamma^2 = Q^2/\nu^2$. For fixed x, A_1 is a constant within our model, and it is therefore very convenient for graphical presentation and analysis of different radiative effects. Figure 4 gives the asymmetries A_1 and A_T for the kinematics of HERMES and JLab up to $y = 0.95$. Influence of higher-order and elastic radiative effects can be seen. Figure 5 gives the total corrections to the cross sections and asymmetries for the threshold region of JLab.

7. CONCLUSION

We have considered model-independent QED radiative correction to the polarized deep inelastic and elastic electron–proton scattering. Together with the analytic expression for the radiative corrections, we give its numerical values for different experimental situations.

Our analytic calculations are based on the electron structure function method, which allows us to write both the spin-independent and spin-dependent parts of the cross section with the radiative corrections to the leptonic part of interaction taken into account in the form of the well-known Drell–Yan representation. The corresponding radiative corrections explicitly includes the first-order correction as well as the leading-log contribution in all orders of the perturbation theory and the main part of the second-order next-to-leading-log contribution. Moreover, any model-dependent radia-

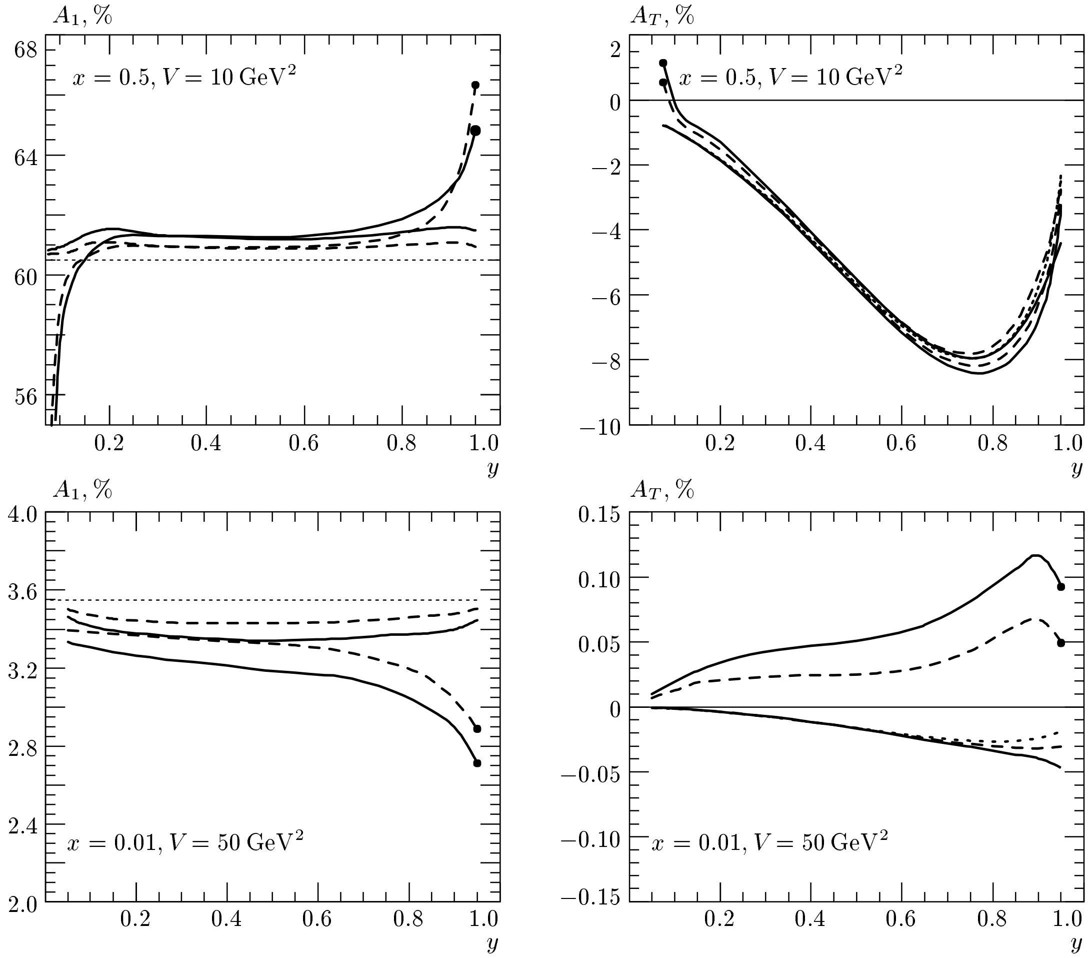

Fig. 4. Radiative correction to asymmetries for the HERMES (lower plots) and JLab (upper plots) kinematics. The dotted line shows the Born asymmetry. The full and dashed lines correspond to the total and one-loop contributions. Asymmetries with the elastic contribution taken into account are marked by dots at the end

tive correction to the hadronic part of the interaction can be included in our analytic result by inserting it as an additive part of the hard cross section in the integrand sign in master formulas (2) and (13).

To derive the radiative corrections, we take into account the radiation of photons and e^+e^- pairs in collinear kinematics, which produces a large logarithm L in the radiation probability (in D-functions), and the radiation of one noncollinear photon, which enlarges the range of the hadron structure function arguments. It may be important that these functions are sufficiently sharp. In this case, the loss in the radiation probability (the loss of L) can be compensated by the increase in the value of the hard cross section.

We note that we extracted the explicit formulas for the first-order contribution at both leading and next-to-leading order levels. We found analytic agreement between these results for the one-loop correction with the previous results in [8], which provides the most important test of the total correction.

On the basis of the analytic results, we constructed the Fortran code ESFRAD[3)]. Because of several known reasons discussed in Sec. 6, results obtained by the electron structure function method are usually not very convenient for precise numerical analysis. But we believe that our numerical procedure based on Monte Carlo integration allows us to overcome the obstacles.

Using the ESFRAD code, we performed numerical analysis for kinematical conditions of the current and future polarization experiments. We found two kinematical regions where the higher-order radiative

3) Fortran code ESFRAD is available at http://www.jlab.org/~aku/RC.

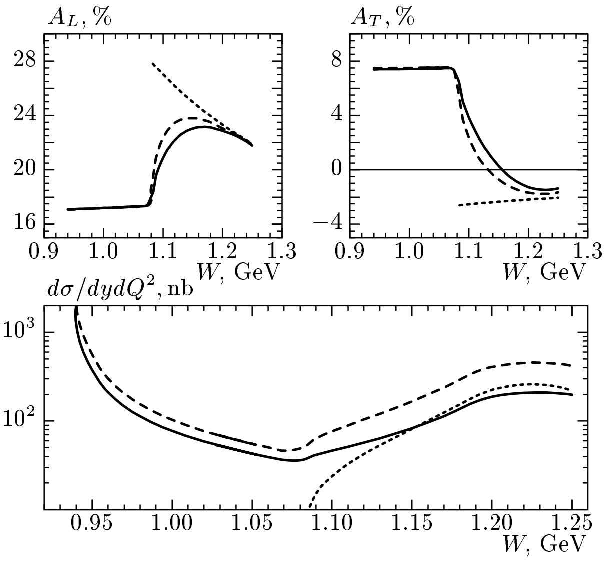

Fig. 5. The cross section (lower plot) and polarization asymmetries (both longitudinal and transverse) for the JLab kinematics ($Q^2 = 1$ GeV^2) near the pion threshold. The dotted line shows the Born cross section and asymmetry. Full and dashed lines correspond to the total and one-loop contributions

correction can be important. These are the traditional region of high y and the region around the pion threshold. We gave a detailed analysis of the effects within these regions and presented numerical results within one of the simplest possibilities for modeling the deep inelastic scattering structure functions. Model dependence of the result is certainly an important issue requiring separate investigation for specific applications within the experimental data analysis.

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