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MAGNETOTRANSPORT IN A MODULATED TWO-DIMENSIONAL ELECTRON GAS

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We propose a semiclassical theory of dc magnetotransport in a two-dimensional electron gas modulated along one direction with weak electrostatic modulations. We show that oscillations of the magnetoresistivity $\rho_{||}$ corresponding to the current driven along the modulation lines observed at moderately low magnetic fields can be explained as commensurability oscillations.

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The theory of dc magnetotransport in modulated $2D$ electron gas is well developed at present and most of the effects observed in such systems at low magnetic fields have been explained by both quantum mechanical (in a semiclassical limit) [1–7] and classical [8–12] transport calculations, giving consistent results. One of a few exceptions is the effect of oscillations of the resistivity component $\rho_{||}$ that corresponds to the current driven parallel to the modulation lines. These oscillations were observed along with the commensurability oscillations of the other resistivity component $\rho_\perp$ corresponding to the current driven across the modulation lines. The $\rho_{\|}$ oscillations have the same period as the $\rho_\perp$ ones and the opposite phase. The oscillations of $\rho_{||}$ have been explained as an effect that originates in quantum oscillations of the electron density of states in the applied magnetic field [1, 3].

On the other hand, the observed coincidence of periods of the low-field commensurability (Weiss) oscillations of the resistivity component $\rho_\perp$ and the weaker anti-phase oscillations of $\rho_{||}$ provides grounds for the assumption that these oscillations have the same nature and origin for both resistivity components. The purpose of the present paper is to demonstrate that the most important characteristic features of the low-field oscillations of the resisitivity $\rho_{||}$ can be qualitatively reproduced within the semiclassical transport theory. To simplify the following calculations, the anisotropy effects in electron scattering are neglected and the relaxation time approximation is used. It is also assumed that the external magnetic field is moderately weak, such that the electron cyclotron radius R is considerably smaller than the electron mean free path l but larger than the period of modulations λ, and $R \gg \sqrt{l\lambda}$. This provides preferred conditions for observation of commensurability oscillations of transport coefficients of the $2D$ electron gas.

We consider electrostatic modulation with a single harmonic of the period $\lambda = 2\pi/g$ along the y direction given by

$$\Delta E(y) = -\frac{dV(y)}{dy}.$$

The screened modulation potential $V(y)$ is parameterized as

$$eV(y) = \epsilon E_F \sin gy,$$

where E_F is the Fermi energy of the $2D$ electron gas. We examine weak modulations, and hence $|\epsilon g l| \ll 1$. The electron current density in the $2D$ electron gas modulated along the y direction also depends on y and can be written as

$$\mathbf{j}(y) = Ne^2 \int_0^{2\pi} \frac{d\psi}{2\pi} \mathbf{v}(y,\psi)\Phi(y,\psi), \qquad (1)$$

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where $N = m/\pi\hbar^2$ is the electron density of states on the Fermi surface, and m and e are the effective mass and charge of the electron. The electron velocity vector $\mathbf{v}(y)$ has the direction $\mathbf{u}(\psi) = (\cos\psi, \sin\psi)$ and the magnitude

$$v(y) = v_F\sqrt{1+\epsilon\sin gy},$$

where v_F is the Fermi velocity in the unmodulated $2D$ electron gas. The distribution function $\Phi(y,\psi)$ satisfies the linearized Boltzmann transport equation

$$D[\Phi] + C[\Phi] = \mathbf{E}\cdot\mathbf{v}, \tag{2}$$

where $\mathbf{E}$ is the electric field. The collision term $C[\Phi]$ is written in the relaxation time approximation with the relaxation towards the local equilibrium distribution,

$$C[\Phi] = \frac{1}{\tau}\left(\Phi(y,\psi) - \frac{1}{2\pi}\int\limits_0^{2\pi}\Phi(y,\psi)\,d\psi\right), \tag{3}$$

and the drift term is given by

$$D[\Phi] = v(y)\sin\psi\frac{\partial\Phi}{\partial y} + (v'(y)\cos\psi + \Omega)\frac{\partial\Phi}{\partial\psi}, \tag{4}$$

where Ω is the electron cyclotron frequency. The linearized transport equation (2) with the collision and drift terms of form (3), (4) was used in [8] and agrees with the transport equations in [9–11].

Following the standard approach [10], we write $\Phi(y,\psi)$ as

$$\Phi(y,\psi) = \Phi_0(\psi)\frac{v(y)}{v_F} + \rho_0\tau\chi(y,\psi), \tag{5}$$

where ρ_0 is the Drude resistivity and τ is the relaxation time. The homogeneous distribution function

$$\Phi_0(\psi) = \rho_0\tau\mathbf{v}_0\cdot\mathbf{j}_0$$

describes the linear response of the $2D$ electron gas to the field $\mathbf{E}$ in the absence of modulations, and the function $\chi(y,\psi)$ satisfies the transport equation

$$D[\chi] + C[\chi] = -v'(y)v(y)j_{0y}\,. \tag{6}$$

Here, as before, $\mathbf{j}_0$ is the current density for the unmodulated $2D$ electron gas.

To proceed, we expand $\chi(y,\psi)$ in a Fourier series in the spatial variable y, which leads to a system of differential equations for the Fourier components. Solving these equations and keeping the terms of the order of or larger than $(\epsilon gR)^2$, we obtain the approximation for the distribution function $\chi(y,\psi)$

$$\begin{aligned}\chi(y,\psi) = -\frac{v_F}{2}\epsilon g l j_0^y Q\left(\cos\psi - \frac{\sin\psi}{\Omega\tau}\right)\times\\ \times\Bigg\{\sin(gR\cos\psi + gy) -\\ -\frac{1}{2}\epsilon gR\Bigg[\cos(gR\cos\psi)\cos^2 gy -\\ -\frac{1}{2}\sin(gR\cos\psi)\sin 2gy\Bigg]\Bigg\},\end{aligned} \tag{7}$$

where

$$Q = \frac{J_0(gR)}{1-J_0^2(gR)} \tag{8}$$

and $J_0(gR)$ is the Bessel function. Using the obtained distribution function, we can easily calculate the electron current density given by (1).

Keeping only the leading terms in the expansion of $\chi(y,\psi)$ in powers of the small parameter $(\Omega\tau)^{-1}$, we obtain that only the j_x component receives a correction due to the modulations along the y direction, whereas the component j_y remains equal to j_y^0 and does not depend on spatial coordinates. This agrees with the continuity equation

$$\nabla\cdot\mathbf{j} = 0,$$

which is necessary in order to obtain correct results for electron transport coefficients in modulated $2D$ electron gas [10].

To proceed, we define the effective conductivity tensor σ_{eff} as

$$\mathbf{j} \equiv \langle\mathbf{j}(y)\rangle \equiv \frac{g}{2\pi}\int\limits_0^{2\pi/g}\mathbf{j}(y)dy \equiv \sigma_{eff}\mathbf{E}. \tag{9}$$

To justify the definition adopted in (9), we note that the expressions for transport coefficients obtained with either quantum mechanical or classical calculations must be consistent at low magnetic fields. Quantum mechanical calculations of magnetoconductivity [3, 6, 7] give an expression that passes to the classical conductivity tensor averaged over the period of modulations. The latter is therefore an accurate semiclassical analogue of the conductivity calculated within the proper quantum mechanical approach, and our definition of σ_{eff} agrees with this[1)]. The same definition was previously used in [11].

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1) For electrostatic modulations, definition (9) actually gives the same results for magnetoresistivity components as the alternative definition $\rho_{eff}\langle\mathbf{j}\rangle = \mathbf{E}$ used in [10]. But there is a significant difference in results based on these definitions when magnetic modulations are considered.

As a result, we obtain that only σ_{eff}^{xx} is affected by the modulations,

$$\sigma_{eff}^{xx} = \frac{\sigma_0}{1+(\Omega\tau)^2} + \frac{1}{4}(\epsilon gR)^2 \sigma_0 \frac{J_0^2(gR)}{1-J_0^2(gR)}, \qquad (10)$$

where $\sigma_0 = 1/\rho_0$ is the Drude conductivity of the electron system. The second term in (10) represents the electron diffusion along the x direction caused by the guiding center drift [8].

The effective magnetoresistivity tensor is here defined as the inverse of the effective conductivity,

$$\rho_{eff} = \sigma_{eff}^{-1}.$$

For the current driven across the modulation lines, the corresponding resistivity is

$$\rho_\perp = \rho_{yy} = \rho_0 \left\{ 1 + \frac{1}{4}(\epsilon gl)^2 \frac{J_0^2(gR)}{1-J_0^2(gR)} \right\}. \qquad (11)$$

Assuming that the current flows along the modulation lines, we obtain

$$\rho_{||} = \rho_{xx} = \rho_0 \left\{ 1 - \frac{1}{4}(\epsilon gR)^2 \frac{J_0^2(gR)}{1-J_0^2(gR)} \right\}. \qquad (12)$$

For moderately weak magnetic fields ($gR \gg 1$), the results in (12) and (13) describe oscillations of both magnetoresistivity components periodic in the inverse magnetic field magnitude. The oscillations of $\rho_\perp$ and $\rho_{||}$ have the same period in $1/B$ and the opposite phases, which corroborates the experimental results in [1]. The amplitude of the oscillations of $\rho_{||}$ is considerably smaller than that of $\rho_\perp$, and this also agrees with the experiments of [1] and with the results of numerical quantum mechanical calculations in the limit of a weak magnetic field [3]. The result for the resistivity $\rho_\perp$ also agrees with the corresponding results in [8–11] obtained within the classical magnetotransport theory.

But expression (12) for $\rho_{||}$ differs from the well-known result in the current semiclassical theory. To analyze this discrepancy, we now calculate the current density with the next terms in the expansion of distribution function (7) in powers of $(\Omega\tau)^{-1}$ taken into account. Keeping terms of the order not less than $(\epsilon gR/\Omega\tau)^2$, we obtain that the grating-induced correction to the Drude conductivity tensor $\hat{\sigma}_0$ is given by

$$\delta\hat{\sigma}(y) = \frac{\sigma_0}{1+(\Omega\tau)^2}\alpha(y) \begin{pmatrix} (\Omega\tau)^2 & \Omega\tau \\ -\Omega\tau & -1 \end{pmatrix}. \qquad (13)$$

Here, the correction $\alpha(y)$ is of the order $(\epsilon gR)^2$. With some formal transformations of transport equation (6), we can represent $\langle\alpha(y)\rangle$ in the form

$$\langle\alpha(y)\rangle = \frac{1}{2\pi}\int_0^{2\pi} \langle v(y)\sin\psi G(y,\psi)\rangle \, d\psi, \qquad (14)$$

where $G(y,\psi)$ satisfies the equation

$$D[G] + C[G] = -\frac{2v'(y)v(y)}{v_F^2}. \qquad (15)$$

This gives expressions for σ_{eff} and ρ_{eff} that are totally consistent with the existing semiclassical theory [8–12]. However, expression (13), which is the starting point in derivation of these results, is obviously incorrect because it violates the continuity equation for the current density. This gives grounds to seriously doubt the results of earlier works [8–12], especially those concerning $\rho_{||}$.

A detailed analysis shows that simplified transport equation (2) can be succesfully used in calculations of the leading terms in the expansions of transport coefficients in powers of $(\Omega\tau)^{-1}$, and expressions (12)–(14) are therefore valid. To obtain the next terms in these expansions, we must modify transport equation (2) in both the drift and collision terms. For that, we must consistently and systematically consider effects of the internal electrochemical field arising due to grating-induced inhomogeneity of the electron density. This is important because redistribution of the electron density at the presence of modulations provides the local equilibrium of the system[2)]. Considering magnetic modulations, we arrive at similar results [13].

Finally, the novel result in this paper is a qualitative explanation of the low-field oscillations of the magnetoresistivity component $\rho_{||}$ in the 2D electron gas modulated along one direction within a semiclassical approach. We have shown that these oscillations of $\rho_{||}$ at low magetic fields are commensurability oscillations. At low temperatures, with the quantum oscillations of the electron density of states at the Fermi surface resolved, Shubnikov–de Haas oscillations can be superimposed on the geometric

[2)] The method of calculations adopted in the current theory is mostly based on averaging the transport equation multiplied by a velocity component with respect to both ψ and y (see, e.g., [10, 11]). This procedure ignores the contribution from the second term of collision integral (3), which provides relaxation of the system towards the local equilibrium. As a result, one arrives at the expressions for transport coefficients corresponding to the relaxation of the system towards the total equilibrium that contradicts the continuity equation.

oscillations of the magnetoresistivity. However, this does not change the classical nature of the effect itself.

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