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HYPERNUCLEI AS CHIRAL SOLITONS

V. B. Kopeliovich[*]

Institute for Nuclear Research of the Russian Academy of Sciences
117312, Moscow, Russia

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Identification of flavored multiskyrmions with the ground states of known hypernuclei is successful for several of them, e.g., for the isodoublet ${}^{4}_{\Lambda}$H–${}^{4}_{\Lambda}$He and isoscalars ${}^{5}_{\Lambda}$He and ${}^{7}_{\Lambda}$Li. In other cases, agreement is not so good, but as the baryon number increases, the behavior of the binding energy qualitatively agrees with the data. Charmed or beautiful hypernuclei are predicted within this approach to be bound stronger than strange hypernuclei. This conclusion is stable with respect to a certain variation of poorly known heavy flavor decay constants.

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1. INTRODUCTION

One of the actual questions of nuclear and elementary particle physics is the possibility of the existence of nuclear matter fragments with unusual properties, e.g., with flavor being different from that of u and d quarks. This issue can have interesting consequences in astrophysics and cosmology. The stellar objects RXJ1856 and 3C58, recently observed at Chandra X-ray Observatory can be interpreted just as the strange quark matter stars. Experimental and theoretical studies of such nuclear fragments were first performed for strangeness (see, e.g., [1, 2] and references therein) and to some extent, also for charm and beauty quantum numbers [3–6]. Theoretical approaches vary from standard nuclear potential models to topological soliton models (the Skyrme model and its extensions). In the latter case, extension of the original $SU(2)$ model to the $SU(3)$ configuration space is necessary. It is known that several different local minima in the configuration space occur in $SU(3)$ extensions of the model [7]. Quantization of configurations near each of these minima is possible, leading to the prediction of the spectrum of quantum states with different flavor quantum numbers. Here, the quantization of $SU(2)$ bound skyrmions embedded in $SU(3)$ is considered following [8–10]. The physical interpretation of such quantum states seems to be simplest in comparison with the others because the lowest-energy states can be identified with the usual nuclei. In this way, we previously derived some spectrum of «flavored multiskyrmions» regardless of their interpretation [10]. Here, we make an attempt to identify some of these states with the known hypernuclei.

The chiral soliton models provide a picture of baryonic systems outside, at sufficiently large distances, based on several fundamental principles and ingredients incorporated in the model Lagrangian. The details of baryon–baryon interactions do not enter the calculations explicitly, although they certainly affect the results implicitly, via some integral characteristics of baryon systems, such as their masses, moments of inertia (Θ_F and Θ_T below), Σ-term (Γ), etc. The $SU(2)$ rational map ansatz [11], which well approximates the results of numerical calculations [12], was used as the starting point for the evaluation of static properties of bound states of skyrmions necessary for their quantization in the $SU(3)$ configuration space. The knowledge of the «flavor» moment of inertia and the Σ-term then allows estimating the flavor excitation energies [8, 10]. The masses of the lowest states with strangeness, charm or beauty are calculated within the rigid oscillator version of the bound state approach, and the binding energies of baryonic systems with different flavors, s, c or b, are estimated.

Within the rational map approximation, at sufficiently large B, the chiral field configuration has the form of a «bubble» with universal properties of the shell where the mass and baryon number of the baryon

[*]E-mail: kopelio@cpc.inr.ac.ru; kopelio@al20.inr.troitsk.ru

systems are concentrated. The width of the shell and its average mass density are independent of the baryon number [13]. This picture can be acceptable for not large B (where $B = A$ is the atomic number of the nucleus), e.g., up to $B \sim 16$, and therefore, we here discuss the hypernuclei not heavier than hyper-oxygen.

2. LAGRANGIAN AND THE MASS FORMULA

The Lagrangian of the Skyrme model, which in its well-known form depends on the meson decay constants F_π and F_D, the Skyrme constant e, etc., has been presented previously [9, 10], and we here give its density for completeness,

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \mathcal{L}^{SB}, \qquad (1)$$

which involves the term of the second order in the chiral derivative

$$\mathcal{L}^{(2)} = -\frac{F_\pi^2}{16} \operatorname{Tr} l_\mu l_\mu,$$

the antisymmetric 4th order, or Skyrme term

$$\mathcal{L}^{(4)} = \frac{1}{32e^2} \operatorname{Tr}[l_\mu l_\nu]^2,$$

the 6th order term

$$\mathcal{L}^{(6)} = c_6 \operatorname{Tr}([l_\mu l_\nu][l_\nu l_\gamma][l_\gamma l_\mu]),$$

and the symmetry (chiral and flavor) breaking terms

$$\mathcal{L}^{SB} = \frac{F_\pi^2 m_\pi^2}{16} \operatorname{Tr}(U + U^\dagger - 2) + \\ + \frac{F_D^2 m_D^2 - F_\pi^2 m_\pi^2}{24} \operatorname{Tr}(1 - \sqrt{3}\lambda_8)(U + U^\dagger - 2) + \\ + \frac{F_D^2 - F_\pi^2}{48} \operatorname{Tr}(1 - \sqrt{3}\lambda_8)(U l_\mu l_\mu + l_\mu l_\mu^\dagger U^\dagger). \qquad (2)$$

Here,

$$l_\mu = \partial_\mu U U^\dagger$$

is the left chiral derivative of the unitary matrix $U \in SU(3)$. The 6th order term $\mathcal{L}^{(6)}$, which can also be presented as a baryon (topological) number density squared, was not included in the original Skyrme model, and we omit it here as well. Recent calculations of flavor excitation energies performed by Shunderyuk provide the results that are close to those obtained in [10] and in the present paper. The Wess–Zumino term in the action, which can be written as a 5-dimensional differential form, plays a very important role in the quantization procedure, but it does not contribute to most of the static properties of skyrmions, see, e.g., [8, 10].

The physical values of these constants are as follows: $F_\pi = 186$ MeV and e is close to $e = 4$, and we here take the value $e = 4.12$ [14]. The chiral symmetry breaking part of the Lagrangian depends on meson masses, the pion mass m_π, and the mass of the K, D or B meson, which we call m_D. The flavor symmetry breaking part of the Lagrangian is of the usual form and is sufficient to describe the mass splittings of the octet and decuplet of baryons [14] within the collective coordinate quantization approach with configuration mixing. It is important that the flavor decay constant (pseudoscalar decay constant F_K, F_D or F_B) is different from the pion decay constant F_π. Experimentally, $F_K/F_\pi \approx 1.22$ and $F_D/F_\pi \approx 2.28^{+1.4}_{-1.1}$ [15]. The B-meson decay constant is not measured yet. In view of this uncertainty, we take two values of $r_c = F_D/F_\pi$ for our estimates, $r_c = 1.5$ and 2, and similarly for $r_b = F_B/F_\pi$, also following theoretical estimates [16].

We begin our calculations with a unitary matrix of chiral fields $U \in SU(2)$, as mentioned above. In the most general case, the classical mass of $SU(2)$ solitons and other static characteristics necessary for our purposes depend on 3 profile functions, f, α, and β. The general parameterization of U_0 for an $SU(2)$ soliton that we use here is given by

$$U_0 = c_f + s_f \boldsymbol{\tau} \cdot \mathbf{n}$$

with

$$n_z = c_\alpha, \quad n_x = s_\alpha c_\beta,$$

$$n_y = s_\alpha s_\beta, \quad s_f = \sin f, \quad c_f = \cos f, \quad \text{etc.}$$

For the rational map ansatz, $f = f(r)$, and the profile therefore depends on one variable only; the components of the vector $\mathbf{n}$ are some rational functions of two angular variables that define the direction of the radius vector $\mathbf{r}$ [11].

The quantization of solitons in the $SU(3)$ configuration space was done in the spirit of the bound state approach to the description of strangeness, proposed in [17] and used in [18, 19]. We here use a somewhat simplified and very transparent variant, the so-called rigid oscillator version proposed in [8]. The details of the quantization procedure can be found in [8–10], and we do not reproduce them here. We only note that the (u, d, c) and (u, d, b) $SU(3)$ groups are quite similar to the (u, d, s) one; a simple redefiniton of hypercharge must be made for the (u, d, c) group.

The following mass formula has been obtained for the masses of states with definite quantum numbers: the baryon (topological) number B, flavor F

(strangeness, charm or beauty), isospin I, and angular momentum J [8, 10],

$$E(B,F,I,J) = M_{B,cl} + |F|\omega_{F,B} + \\ + \frac{1}{2\Theta_{T,B}}\big[c_{F,B}T_r(T_r+1) + (1-c_{F,B})I(I+1) + \\ + (\bar{c}_{F,B} - c_{F,B})I_F(I_F+1)\big] + \frac{J(J+1)}{2\Theta_{J,B}}, \quad (3)$$

where $\omega_{F,B}$ or $\bar{\omega}_{F,B}$ are the frequences of flavor (antiflavor) excitations,

$$\omega_{F,B} = \frac{N_cB(\mu_{F,B}-1)}{8\Theta_{F,B}}, \quad \bar{\omega}_{F,B} = \frac{N_cB(\mu_{F,B}+1)}{8\Theta_{F,B}}, \quad (4)$$

with

$$\mu_{F,B} = \left[1 + \frac{16\Theta_{F,B}\big(\bar{m}_D^2\Gamma_B + (F_D^2 - F_\pi^2)\tilde{\Gamma}_B\big)}{(N_cB)^2}\right]^{1/2},$$

N_c is the number of colors of the underlying QCD ($N_c = 3$ in all numerical estimates), and

$$\bar{m}_D^2 = \frac{F_D^2 m_D^2}{F_\pi^2} - m_\pi^2.$$

The terms $\pm N_cB/8\Theta_{F,B}$ in (4) arise from the Wess–Zumino term in the action, which does not contribute to the masses and momenta of inertia of skyrmions [17, 8]. In terms of the quark models, the difference

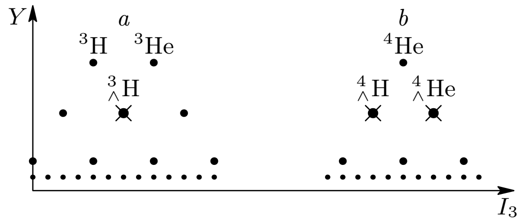
$$\bar{\omega} - \omega = \frac{N_cB}{4\Theta_{F,B}}$$

is the energy necessary for the production of an additional $q\bar{q}$ pair. The hyperfine structure constants $c_{F,B}$ and $\bar{c}_{F,B}$ are given by [8]

$$c_{F,B} = 1 - \frac{\Theta_{T,B}(\mu_{F,B}-1)}{2\Theta_{F,B}\,\mu_{F,B}}, \\ \bar{c}_{F,B} = 1 - \frac{\Theta_{T,B}(\mu_{F,B}-1)}{\Theta_{F,B}(\mu_{F,B})^2}. \quad (5)$$

Evidently, $\bar{c} \to 1$ as $\mu \to \infty$. The contributions of the order of $1/\Theta \sim N_c^{-1}$ that depend originally on angular velocities of rotations in the isospace and the usual space are taken into account in (3). This expression was obtained by quantizing the oscillator-type Hamiltonian describing the motion of the $SU(2)$ skyrmion in the $SU(3)$ collective coordinate space. The classical mass $M_{cl} \sim N_c$ and the energies $\omega_F \sim N_c^0 = 1$. The motion along the «flavor» direction s, c or b is described by the amplitude D [8, 10] that is small for the lowest quantum states (lowest $|F|$),

$$D \sim \big[16\Gamma_B\Theta_{F,B}\bar{m}_D^2 + N_c^2B^2\big]^{-1/4}(2|F|+1)^{1/2}.$$



The location of the isoscalar state with odd B and $|F| = 1$ in the upper part of the (I_3, Y) diagram (a). The same for isodoublet states with even B (b). The case of light hypernuclei $_\Lambda$H and $_\Lambda$He is considered as an example

The amplitude D therefore decreases as $1/\sqrt{m_D}$ with increasing the mass m_D and with increasing the number of colors N_c, and the method works for any value of m_D, also for charm and beauty quantum numbers.

In (3), I is the isospin of the multiplet with a flavor F, $T_r = p/2$ is the so-called «right» isospin, the isospin of the nonflavored component of the $SU(3)$ multiplet under consideration, with (p, q) being the numbers of the upper and lower indices in the spinor that describes it. I_F is the isospin carried by flavored mesons that are bound by the $SU(2)$ skyrmion,

$$\mathbf{I} = \mathbf{T}_r + \mathbf{I}_F.$$

Evidently, $I_F \le |F|/2$. The states predicted in the rigid oscillator model do not correspond to definite $SU(3)$ or $SU(4)$ representations. How they can be ascribed to them was shown in [8, 10]. For example, the state with $B = 1$, $|F| = 1$, and $I = 0$ must belong to the octet of the (u, d, s) or (u, d, c) $SU(3)$ group if $N_c = 3$. Here, we consider quantized states of the baryon system that belong to the lowest possible $SU(3)$ irreps (p, q), $p + 2q = 3B$,

$$p = 0, \quad q = 3B/2 \quad \text{for even} \quad B$$

and

$$p = 1, \quad q = (3B-1)/2 \quad \text{for odd} \quad B.$$

These are $\bar{35}$, $\bar{80}$, and $\bar{143}$-plets for $B = 3, 5$, and 7; $\bar{28}$, $\bar{55}$, and $\bar{91}$-plets for $B = 4, 6$, and 8, etc. For even B, $T_r = 0$ and for odd B, $T_r = 1/2$ for the lowest $SU(3)$ irreps (see the Figure).

The flavor moment of inertia that enters the above formulas [8, 10, 17] for arbitrary $SU(2)$ skyrmions is given by [10]

$$\Theta_F = \frac{1}{8}\int (1-c_f) \times \times \left\{ F_D^2 + \frac{1}{e^2}\left[(\boldsymbol{\partial} f)^2 + s_f^2(\boldsymbol{\partial} n_i)^2\right]\right\} d^3\mathbf{r}, \quad (6)$$

where

$$(\boldsymbol{\partial} n_i)^2 = (\boldsymbol{\partial}\alpha)^2 + s_\alpha^2(\boldsymbol{\partial}\beta)^2.$$

It is simply related to $\Theta_F^{(0)}$ for the flavor symmetric case,

$$\Theta_F = \Theta_F^{(0)} + (F_D^2/F_\pi^2 - 1)\Gamma/4,$$

with Γ defined in (7) below. The flavor inertia increases with B almost proportionally to B. The isotopic moments of inertia are the diagonal components of the corresponding tensor of inertia; in our case, this tensor of inertia is close to the unit matrix multiplied by Θ_T.

The quantities Γ (or the Σ-term), which define the contribution of the mass term to the classical mass of solitons, and $\tilde{\Gamma}$ in $\mu_{F,B}$ are given by

$$\Gamma = \frac{F_\pi^2}{2}\int (1-c_f) d^3\mathbf{r}, \qquad \tilde{\Gamma} = \frac{1}{4}\int c_f\left[(\boldsymbol{\partial} f)^2 + s_f^2(\boldsymbol{\partial} n_i)^2\right] d^3\mathbf{r}. \quad (7)$$

For the rational map ansatz, formulas (6) and (7) can be slightly modified [10], but they already look sufficiently simple in such a general form. The masses of solitons were calculated in [12] and [10], the moments of inertia Γ and $\tilde{\Gamma}$ were calculated in [10] for several values of B, and the missing quantities are calculated here.

The contribution to $\mu_{F,B}$ proportional to $\tilde{\Gamma}_B$ is suppressed in comparison with the term of the order of Γ by the small factor of the order of F_D^2/m_D^2, and is more important for strangeness.

3. STRANGE AND BEAUTIFUL HYPERNUCLEI

It is convenient to calculate the energy difference between the state with a flavor F belonging to the (p,q) irrep and the ground state with $F=0$ and the same B, J, and (p,q) [10],

$$\Delta E_{B,F} = |F|\omega_{F,B} + \frac{\mu_{F,B}-1}{4\mu_{F,B}\Theta_{F,B}} \times \times \left[I(I+1) - T_r(T_r+1)\right] + + \frac{(\mu_{F,B}-1)(\mu_{F,B}-2)}{4\mu_{F,B}^2\Theta_{F,B}} I_F(I_F+1). \quad (8)$$

In deriving (3) and (8), we used that the so-called «interference» moment of inertia, whose contribution to the Lagrangian is proportional to the product of angular rotation velocities in the isotopic and ordinary $3D$ spaces, is negligible compared with the isotopic and orbital tensors of inertia [20] for all multiskyrmions except those with $B=1,2$. We also note that (8) is independent of Θ_T and depends only on Θ_F when the formulas for hyperfine splitting constants are used.

For the state with the isospin $I=0$ and the unit flavor number $|F|=1$, the binding energy difference in comparision with the ground state of the nucleus with the same B, (p,q) and $|F|=0$ is

$$\Delta\epsilon_{B,F} = \omega_{F,1} - \omega_{F,B} - \frac{3(\mu_{F,1}-1)}{8\mu_{F,1}^2\Theta_{F,1}} + \frac{3(\mu_{F,B}-1)}{8\mu_{F,B}^2\Theta_{F,B}}. \quad (9)$$

Such states can exist for odd B, with $I_F = T_r = 1/2$, see Fig. *a*. For antiflavor excitations, we have similar formulas with the substitution $\mu \to -\mu$.

For states with the maximal isospin

$$I = T_r + \frac{|F|}{2},$$

the energy difference can be simplified to [10]

$$\Delta E_{B,F} = |F| \times \times \left[\omega_{F,B} + T_r\frac{\mu_{F,B}-1}{4\mu_{F,B}\Theta_{F,B}} + \frac{(|F|+2)}{8\Theta_{F,B}}\frac{(\mu_{F,B}-1)^2}{\mu_{F,B}^2}\right]. \quad (10)$$

The case of isodoublets, even B, is described by (8) with $T_r=0$, see Table 2 and Fig. *b*. It follows from (10) that when a nucleon is replaced by a flavored hyperon in a baryon system, the binding energy of the system with $|F|=1$ and $T_r=0$ changes by

$$\Delta\epsilon_{B,F} = \omega_{F,1} - \omega_{F,B} - \frac{3(\mu_{F,1}-1)}{8\mu_{F,1}^2\Theta_{F,1}} - \frac{3(\mu_{F,B}-1)^2}{8\mu_{F,B}^2\Theta_{F,B}}. \quad (11)$$

For strangeness, Eq. (11) is negative, indicating that stranglets should have binding energies smaller than those of nuclei with the same B.

To obtain the values of the total binding energy of hypernuclei shown in the Tables, we add the calculated difference of binding energies given by (9) or (11) to the known binding energy value of the usual (u,d) nucleus. For example, for $B=3$, it is the average of binding energies of ^{3}H and ^{3}He, for $B=4$ it is the binding energy of ^{4}He (5.3 MeV = (28.3—23) MeV), etc., see the Figure. A special care should be taken about the spin

Table 1. The collective motion contributions to the binding energies of isoscalar hypernuclei with the unit flavor, strangeness or beauty, $S=-1$ or $b=-1$, in MeV

${}_{\Lambda}A$	ω_s	$\Delta\epsilon_s$	ϵ_s^{tot}	$\epsilon_{exp,s}^{tot}$	$\omega_b^{r_b=1.5}$	$\Delta\epsilon_b$	ϵ_b^{tot}	$\omega_b^{r_b=2}$	$\Delta\epsilon_b$	ϵ_b^{tot}
1	306	–	–	–	4501	–	–	4805	–	–
${}^{3}_{\Lambda}$H	289	−3	5	2.35	4424	75	83	4751	53	61
${}^{5}_{\Lambda}$He	287	−6	33	31.4	4422	76	103	4749	54	81
${}^{7}_{\Lambda}$Li	282	−3	29	37.6	4429	81	119	4744	59	97
${}^{9}_{\Lambda}$Be	291	−13	40	63.2	4459	40	97	4773	31	88
${}^{11}_{\Lambda}$B	294	−16	59	–	4478	21	96	4786	18	93
${}^{13}_{\Lambda}$C	295	−18	78	104	4488	10	106	4793	11	107
${}^{15}_{\Lambda}$N	300	−23	91	118	4515	−17	97	4810	−7	108

ω_s and ω_b are the strangeness and beauty excitation energies, $\Delta\epsilon_{s,b}$, in MeV, are the changes of binding energies of the lowest baryon system with flavor s or b, $|F|=1$, in comparison with the usual (u,d) nuclei with the same B. ϵ^{tot} is the total binding energy of the hypernucleus. Experimental values ϵ_{exp}^{tot} are taken from [1, 2]. The energies ω for $B=1$ are given for comparison. For beauty, the first 3 columns correspond to $r_b=F_B/F_\pi=1.5$, and the last 3 ones to $r_b=2$.

Table 2. The binding energies of isodoublets of hypernuclei with the unit flavor, $S=-1$ or $b=-1$, in MeV

${}_{\Lambda}A$	ω_s	$\Delta\epsilon_s$	ϵ_s^{tot}	ϵ_{exp}^{tot}	$\omega_b^{r_b=1.5}$	$\Delta\epsilon_b$	ϵ_b^{tot}	$\omega_b^{r_b=2}$	$\Delta\epsilon_b$	ϵ_b^{tot}
${}^{4}_{\Lambda}$H–${}^{4}_{\Lambda}$He	283	−23	5.3	10.52; 10.11	4402	71	99	4735	52	80
${}^{6}_{\Lambda}$He–${}^{6}_{\Lambda}$Li	287	−22	10.3	31.7; 30.8	4430	52	84	4752	40	72
${}^{8}_{\Lambda}$Li–${}^{8}_{\Lambda}$Be	288	−20	36.5	46.05; 44.4	4443	43	99	4765	33	89
${}^{10}_{\Lambda}$Be–${}^{10}_{\Lambda}$B	292	−23	42	67.3; 65.4	4465	24	89	4778	20	85
${}^{12}_{\Lambda}$B–${}^{12}_{\Lambda}$C	294	−24	67	87.6; 84.2	4481	10	102	4788	11	103
${}^{14}_{\Lambda}$C–${}^{14}_{\Lambda}$N	299	−28	77	109.3; 106.3	4506	−14	91	4805	−5	100
${}^{16}_{\Lambda}$N–${}^{16}_{\Lambda}$O	301	−30	97	–	4521	−28	100	4815	−14	114

The rest of the notation and other detailes are as in Table 1.

of the nucleus. For ${}^{3}_{\Lambda}$H and ^{3}H, ${}^{4}_{\Lambda}$He and ^{4}He, ${}^{6}_{\Lambda}$Li and ^{6}Li, ${}^{13}_{\Lambda}$C and ^{13}C, and in several other cases, the spins of the ground states of the hypernucleus and the nucleus coincide. For ${}^{5}_{\Lambda}$He ($J=1/2$) and ^{5}He ($J=3/2$), ${}^{9}_{\Lambda}$Be ($J=1/2$) and ^{9}Be ($J=3/2$), ${}^{12}_{\Lambda}$C ($J=1$) and ^{12}C ($J=0$) and in some other cases, the difference in the rotation energies

$$E_J=\frac{J(J+1)}{2\Theta_J}$$

must be taken into account. For example, this difference decreases the theoretical value of the binding energy for ${}^{7}_{\Lambda}$Li by about 7 MeV, we have 29 MeV instead of 36 MeV. In those cases where the spin of the hypernucleus is not known, this correction was not included in Tables 1 and 2. Beginning with $B\sim 10$, the correction to the energy of quantized states due to nonzero angular momentum is small and decreases with increasing B because the corresponding moment of inertia increases proportionally to B^2.

Because $\Theta_{F,B}$ increases with increasing B and F_D (m_D), this leads to the increase of binding with increasing B and the mass of the «flavor», in agreement with [9, 10]. For beauty (and charm, see below), Eq. (11) is positive for $3 \le B \le 12$. As follows from Tables 1 and 2, our method underestimates the binding energy of strangeness in nuclei beginning with $B = A \sim 9$. This means that other sources of binding should be taken into account, in addition to the collective motion of baryon system in the $SU(3)$ configuration space.

4. CHARMED HYPERNUCLEI

In this section, binding energies of charmed hypernuclei are presented for two values of the charm decay constant that correspond to the ratio $r_c = F_D/F_\pi = 1.5$ and $r_c = 2$. Although the measurement of this constant has been performed in [15], its variation in some interval seems to be reasonable in view of its big uncertainty. As follows from Tables 3 and 4, the predicted binding energies of charmed hypernuclei do not essentially differ for the values $r_c = 1.5$ and $r_c = 2$. This difference increases with increasing the atomic number. For light hypernuclei, this difference is considerably smaller than the difference between binding energies for $r_b = 1.5$ and $r_b = 2$ (see Sec. 3).

For charmed nuclei, the repulsive Coulomb interaction is greater than for ordinary nuclei with the same atomic number. Moreover, because a charmed nucleus has somewhat smaller dimensions than the ordinary nuclei (the effect that has not been taken into account in the present analysis), this repulsion can decrease the binding energies for charm by several MeV. This does not change our qualitative conclusions, however. For $B = A = 5$ and 13, our results shown in Tables 3 and 4 agree, within 15–20 MeV, with the early result by Dover and Kahana [4], where binding of the charm by several nuclei was studied within the potential approach. In general, we can speak about a qualitative agreement with the results of this approach for $B \sim 5$–10 [5, 6] (the results of the potential approach have been reviewed in [6]).

Table 3. The binding energies of the charmed hypernuclei (isoscalars) with unit charm, $c = 1$, in MeV

${}_{\Lambda}$A	$\omega_c^{r_c=1.5}$	$\Delta\epsilon_c$	ϵ_c^{tot}	$\omega_c^{r_c=2}$	$\Delta\epsilon_c$	ϵ_c^{tot}
1	1535	–	–	1673	–	–
${}^3_\Lambda$He	1504	27	35	1647	24	32
${}^5_\Lambda$Li	1505	25	52	1646	25	52
${}^7_\Lambda$Be	1497	32	70	1641	30	68
${}^9_\Lambda$B	1518	11	68	1654	17	74
${}^{11}_\Lambda$C	1525	4	79	1658	13	87
${}^{13}_\Lambda$N	1529	0	96	1660	10	106
${}^{15}_\Lambda$O	1540	−11	103	1668	3	117

$\Delta\epsilon_c$, in MeV, and ϵ^{tot} are the same as in Tables 1, 2, for the charm quantum number. The results are shown for two values of the charm decay constant corresponding to $r_c = 1.5$ and $r_c = 2$. The chemical symbol is assigned to each nucleus in accordance with its total electric charge.

Table 4. The binding energies of the charmed hypernuclei (isodoublets), with unit charm, $c = 1$, in MeV

${}_{\Lambda}$A	$\omega_c^{r_c=1.5}$	$\Delta\epsilon_c$	ϵ_c^{tot}	$\omega_c^{r_c=2}$	$\Delta\epsilon_c$	ϵ_c^{tot}
${}^4_\Lambda$He–${}^4_\Lambda$Li	1493	12	40	1639	16	44
${}^6_\Lambda$Li–${}^6_\Lambda$Be	1504	9	41	1646	14	46
${}^8_\Lambda$Be–${}^8_\Lambda$B	1510	7	63	1648	15	71
${}^{10}_\Lambda$B–${}^{10}_\Lambda$C	1520	0	65	1655	10	75
${}^{12}_\Lambda$C–${}^{12}_\Lambda$N	1526	−4	88	1659	7	99
${}^{14}_\Lambda$N–${}^{14}_\Lambda$O	1536	−14	91	1666	1	106
${}^{16}_\Lambda$O–${}^{16}_\Lambda$F	1543	−19	109	1670	−2	126

The rest of the notation and other details are as in Table 3.

As in the case where $B = 1$, the absolute values of masses of multiskyrmions are controlled by poorly known loop corrections to the classic masses, or the Casimir energy [21]. As was done for the $B = 2$ states, the renormalization procedure is necessary to obtain physically reasonable values of the masses of multibaryons. This generates an uncertainty of about few tens of MeV; because the binding energy of the deuteron is 30 MeV instead of the measured value 2.225 MeV, approximately 30 MeV characterizes the uncertainty of our approach [10]. This uncertainty is mainly canceled in the differences of binding energies $\Delta\epsilon$ shown in Tables 1–4.

5. COMMENTS AND CONCLUSIONS

The version of the bound state soliton model proposed in [8] and modified in [9, 10] for the flavor symmetry breaking case ($F_D > F_\pi$) allows calculating the binding energy differences of ground states of flavored and unflavored nuclei. Combined with several phenomenological arguments, this model is very successful in some cases of light hypernuclei, e.g., isoscalars ${}^5_\Lambda$He and ${}^7_\Lambda$Li. In other cases, the accuracy of describing the binding energies is at the level 10–30 MeV, expected for the whole method that takes only the collective motion of the baryonic systems into account. There is also a general qualitative agreement with the data in the behavior of binding energy with increasing the atomic number. It should be stressed that it is possibly one of interesting examples where a field theoretical model provides results that can be directly compared with observation data. This can be considered as an additional argument in favor of the applicability of the chiral soliton approach to the description of realistic properties of nuclei. For the charm and beauty quantum numbers, the results only slightly depend on the poorly known values of the decay constants F_D or F_B.

The tendency of the binding energies to decrease with increasing the B number beginning with $B \sim 10$ is related to the fact that the rational map approximation, leading to the one-shell bubble structure of the classical configuration [11–13], is not good for such values of B. At large values of the flavor symmetry breaking mass, we have approximately

$$\omega_F \approx \frac{m_D\sqrt{\Gamma/\Theta_F}\, F_D}{2F_\pi}.$$

For rational map configurations at large B, the Σ-term Γ grows faster than the inertia Θ_F because the contribution of the volume occupied by the chiral field configuration is more important for Γ [13]. For larger $B = A$, beginning with several tens, configurations of the type of skyrmion crystals seem to be more realistic than configurations of the rational map type.

Hypernuclei with $|F| \geq 2$ can be studied using similar methods [10]. The analysis of hypernuclei with «mixed» flavors is possible in principle, but is more involved technically. For example, the isodoublet ${}^3_{s,c}$H–${}^3_{s,c}$He consisting of (n, Λ, Λ_c) and (p, Λ, Λ_c) is expected.

There is a rough agreement of our results with the results in [19, 20], where the flavor excitation frequences were calculated within another version of the bound state approach and the collective coordinate quantization method was used for strangeness. Some details are different, however, and it would be interesting to reproduce our results within other variants of the chiral soliton model. The model that we used overestimates the strangeness excitation energies, but is more reliable for differences of energies entering (9) and (11) and for charm and beauty quantum numbers. Further theoretical studies and experimental search for the baryonic systems with flavor different from u and d could shed more light on the dynamics of heavy flavors in baryonic systems.

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Note added in proof (05.03.03). The variation of the only model parameter, Skyrme constant e, has small influence on the results presented here, negligible for charm or beauty quantum numbers. Both quantities Γ and inertia Θ_F scale as $1/F_\pi e^3$, and the flavor excitation energies given by (4) at large m_D depend on their ratio, and are therefore scale-invariant.

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