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RADIATION FROM COSMIC CHIRAL STRING LOOPS

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The gravitational and electromagnetic radiation from chiral superconducting cosmic string loops is calculated. The formulas for energy, momentum, and angular momentum losses due to gravitational and electromagnetic radiation from chiral loops of an arbitrary configuration are derived. After summation over all modes, expressions for the corresponding radiation rates averaged over the loop oscillation period have the form of four-dimensional integrals. These formulas are reduced to sums over the kinks for loops composed of piecewise linear strings. For three examples of string loops, the total radiation rates are calculated numerically in dependence on the chiral current along the string. In the limit of a nearly maximum current, which corresponds to a stationary loop (vorton) configuration, we determine the upper bounds on the gravitational and electromagnetic radiation. We also estimate the oscillation damping time of a nearly stationary loop.

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1. INTRODUCTION

We investigate the properties of the gravitational and electromagnetic radiation of energy, momentum, and angular momentum from superconducting closed cosmic strings with a chiral current. Formation of cosmic strings in the early universe phase transitions is predicted by many particle-physics models (see, e. g., reviews in [1, 2]). In 1985, Witten showed that cosmic strings can carry a superconducting electromagnetic current [3]. Exact solutions of the equations of motion for current-carrying cosmic strings were found by Carter and Peter [4], Davis et al. [5], and Blanco-Pillado et al. [6] in the case of a chiral (or null) current $J^a J_a = 0$, which does not couple to any gauge field.

Ordinary cosmic strings (without a current) radiate energy [7–12], momentum [7, 13, 14], and angular momentum [14] in the form of gravitational waves. If cosmic strings carry the electromagnetic current, cosmic string loops radiate both gravitational and electromagnetic waves. For a small current, the most intense radiation is generated by a cusp on the loop. The radiation from a single cusp of the chiral string loop with a small current was studied by Blanco-Pillado and Olum [15]. The radiation of loops in the opposite case of a nearly maximum current was considered in [16]. In this paper, we study the gravitational and electromagnetic radiation from closed chiral string loops in the entire range of the string current. The rates of the energy $\dot{E}$, momentum $\dot{P}$, and angular momentum $\dot{L}$ losses (averaged over the oscillation period) to the gravitational and electromagnetic waves can be expressed in the general form as

$$\begin{aligned} \dot{E}^{gr} &= \Gamma_E^{gr} G\mu^2, \quad \dot{P}^{gr} = \Gamma_P^{gr} G\mu^2, \\ \dot{L}^{gr} &= \Gamma_L^{gr} \mathcal{L} G\mu^2, \quad \dot{E}^{em} = \Gamma_E^{em} \mu q^2, \\ \dot{P}^{em} &= \Gamma_P^{em} \mu q^2, \quad \dot{L}^{em} = \Gamma_L^{em} \mathcal{L} \mu q^2, \end{aligned} \tag{1}$$

where the coefficients Γ_E^{gr}, Γ_P^{gr}, Γ_L^{gr}, Γ_E^{em}, Γ_P^{em}, and Γ_L^{em} depend on the particular string configuration and the current on the string, $\mathcal{L}$ is the string invariant length, μ is string mass per unit length, q is the electromagnetic charge, and we use units $\hbar = c = 1$. In what follows, we calculate the coefficients Γ_E^{gr}, Γ_P^{gr}, Γ_L^{gr}, Γ_E^{em}, Γ_P^{em}, and Γ_L^{em} as functions of the current on the string. It is known that for ordinary loops (without a current), the corresponding coefficients for the gravitational radiation are of the respective orders $\Gamma_E^{gr} \sim 100$, $\Gamma_P^{gr} \sim 10$, and $\Gamma_L^{gr} \sim 10$. We found that for loops with a chiral current, the same coefficients Γ_E^{gr}, Γ_P^{gr}, and Γ_L^{gr} behave as follows: they rapidly decrease with the current at small current values and slowly decrease at large current values. In general, the gravitational radiation

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rates are decreasing functions of the current on the string. For the electromagnetic radiation, the situation is quite different: the loss rates of the energy, momentum, and angular momentum to electromagnetic waves for all considered examples have a maximum near some rather small value of the current.

The total rates of the energy, momentum, and angular momentum per unit time (averaged over the period) are usually calculated by summing the losses in different Fourier modes. As noted by Allen et al. [11], such calculations may not be accurate in practice because of a slow convergence of the corresponding sums over mode numbers. In this paper, we perform the summation over all radiation modes analytically and derive formulas for the energy, momentum, and angular momentum loss rates to the gravitational and electromagnetic radiation from the chiral string loops of a general configuration. As a result, the corresponding radiation rates into the unit solid angle averaged over the loop oscillation period are reduced to four-dimensional integrals. In general, these integrals can be calculated only numerically. For chiral loops composed of piecewise linear strings, these formulas lead to analytic expressions for the energy, momentum, and angular momentum radiation into the unit solid angle. For large currents (close to the maximum value), we determine the upper bounds on the gravitational and electromagnetic radiation. For weak radial oscillations of a chiral ring, we find the temporal behavior of the loop energy and current analytically. For some other less symmetric loop examples, we estimate the damping time of small-amplitude loop oscillations.

This paper is organized as follows. In Sec. 2, we review some general properties of chiral cosmic strings. In Sec. 3, we derive new expressions for the energy, momentum, and angular momentum gravitational radiation rates by chiral loops of a general configuration into the unit solid angle. These expressions are reduced to four-dimensional integrals where summation over all radiation modes is performed analytically. In Sec. 4, we derive similar formulas for the electromagnetic radiation rates. In Sec. 5, the radiation and oscillation damping to the vorton state of nearly stationary loops are described. In Sec. 6, we present numerical calculations of the electromagnetic and gravitational radiation rates for some illustrative examples of chiral loops and study the dependence of the chiral string radiation on the current. In Sec. 7, we describe the results obtained and discuss some qualitative features of the gravitational and electromagnetic radiation from chiral loops.

2. MOTION OF A CHIRAL STRING IN FLAT SPACE–TIME

In this section, for pedagogical reasons, we describe some general properties of chiral cosmic strings, i.e., strings with a chiral current $J^a J_a = 0$. The general solution of the equations of motion of the chiral string can be written as [4–6]

$$x^0 = t, \quad \mathbf{x}(t,\sigma) = \frac{\mathcal{L}}{4\pi}\left[\mathbf{a}(\xi) + \mathbf{b}(\eta)\right], \qquad (2)$$

where t is the Minkowski time, σ parameterizes the string total energy as

$$E = \mu \int d\sigma, \qquad (3)$$

$\mathcal{L}$ is the invariant length of the string, and $\mathbf{a}(\xi)$ and $\mathbf{b}(\eta)$ are arbitrary vector functions of $\xi = (2\pi/\mathcal{L})(\sigma - t)$ and $\eta = (2\pi/\mathcal{L})(\sigma + t)$ satisfying the conditions

$$\mathbf{a}'^2 = 1, \quad \mathbf{b}'^2 = k^2(\eta) \le 1. \qquad (4)$$

For closed chiral strings (loops), the vector functions $\mathbf{a}(\xi)$ and $\mathbf{b}(\eta)$ form closed loops, called a- and b-loops. The function $k(\eta)$ in (4) can be expressed as [6]

$$k^2(\eta) = 1 - \frac{4F'^2(\eta)}{\mu}, \qquad (5)$$

where the function $F(\eta)$ defines the auxiliary scalar field

$$\phi(\sigma,t) = \frac{\mathcal{L}}{2\pi}F(\eta). \qquad (6)$$

According to (6), the scalar field $\phi(\sigma,t)$ is an arbitrary function of the only parameter η. The four-dimensional current on the string is expressed through this scalar field $\phi(\sigma,t)$ as [21]

$$j^\mu(\mathbf{x},t) = q\int d\sigma \phi'(\sigma,t)(x'^\mu - \dot{x}^\mu)\delta^{(3)}\left(\mathbf{x}-\mathbf{x}(\sigma,t)\right), \quad (7)$$

where x' denotes $\partial x/\partial\sigma$ and $\dot{x}$ denotes $\partial x/\partial t$. The energy-momentum tensor of the string in this gauge is

$$T^{\mu\nu} = \mu \int d\sigma \left(\dot{x}^\mu \dot{x}^\nu - x'^\mu x'^\nu\right)\delta^{(3)}\left(\mathbf{x} - \mathbf{x}(\sigma,t)\right). \quad (8)$$

Correspondingly, the total momentum and angular momentum of the string are given by

$$\mathbf{P} = \mu \int d\sigma \dot{\mathbf{x}}(\sigma,t), \qquad (9)$$

$$\mathbf{L} = \mu \int d\sigma [\mathbf{x}(\sigma,t) \times \dot{\mathbf{x}}(\sigma,t)]. \qquad (10)$$

3. GRAVITATIONAL RADIATION FROM CHIRAL LOOPS

We consider a periodic system with the period T. In this system, the Fourier transform of the energy-momentum tensor $T^{\mu\nu}(\mathbf{x},t)$ is given by [14]

$$\hat{T}^{\mu\nu}(\omega_l,\mathbf{n}) = \frac{1}{T}\int_0^T dt \int d^3x T^{\mu\nu}(\mathbf{x},t) \times \times \exp\{i\omega_l(t-\mathbf{n}\cdot\mathbf{x})\}, \quad (11)$$

where $\omega_l = 2\pi l/T$ and $\mathbf{n}$ is an arbitrary unit vector. It is useful to also define the Fourier transform of the first moment,

$$\hat{T}^{\mu\nu p}(\omega_l,\mathbf{n}) = \frac{1}{T}\int dt \int d^3x T^{\mu\nu}(\mathbf{x},t)x^p \times \times \exp\{i\omega_l(t-\mathbf{n}\cdot\mathbf{x})\}. \quad (12)$$

For convenience, we define the four-dimensional symbol $n^\mu \equiv (1,\mathbf{n})$. For any periodic system, the corresponding gravitational energy, momentum, and angular momentum radiation rates per solid angle $d\Omega$ (averaged over the period T) are given by the series

$$\frac{d\dot{P}^\mu}{d\Omega} = \sum_{n=1}^{\infty}\frac{d\dot{P}^\mu(\omega_n)}{d\Omega}, \quad \frac{d\dot{\mathbf{L}}}{d\Omega} = \sum_{n=1}^{\infty}\frac{d\dot{\mathbf{L}}(\omega_n)}{d\Omega}, \quad (13)$$

where [17]

$$\frac{d\dot{P}^\mu(\omega)}{d\Omega} = -n^\mu\frac{G\omega^2}{\pi}P_{ij}P_{lm}\left[\hat{T}^*_{il}\hat{T}_{jm} - \frac{1}{2}\hat{T}^*_{ij}\hat{T}_{lm}\right] \quad (14)$$

and [14]

$$\frac{d\dot{L}_i(\omega)}{d\Omega} = -\frac{G}{2\pi}\epsilon^{ijk}n^j\left[i\omega n^l P^{pq}(3\hat{T}^*_{kl}\hat{T}_{qp} + 6\hat{T}^*_{kp}\hat{T}_{ql}) + \right. + \omega^2 P^{lm}P^{pq}\left(2\hat{T}^*_{kmq}\hat{T}_{lp} - 2\hat{T}^*_{km}\hat{T}_{lpq} - \right. \left.\left. - \hat{T}^*_{lpk}\hat{T}_{mq} + \frac{1}{2}\hat{T}^*_{lmk}\hat{T}_{pq}\right) + \text{c.c.}\right]. \quad (15)$$

Here, $P_{ij} = \delta_{ij} - n_i n_j$ is the projection operator to the plane perpendicular to the unit vector $\mathbf{n}$. It is possible to simplify (14) and (15) further by rewriting them in the corotating basis $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3) \equiv (\mathbf{n},\mathbf{v},\mathbf{w})$, where $\mathbf{v}$ and $\mathbf{w}$ are arbitrary unit vectors perpendicular to each other and to $\mathbf{n}$. In this corotating basis, Eqs. (14) and (15) become [14]

$$\frac{d\dot{P}^\mu(\omega)}{d\Omega} = n^\mu\frac{G\omega^2}{\pi}\left[\tau^*_{pq}\tau_{pq} - \frac{1}{2}\tau^*_{qq}\tau_{pp}\right], \quad (16)$$

$$\frac{d\dot{\mathbf{L}}(\omega)}{d\Omega} = \frac{d\dot{L}_2}{d\Omega}\mathbf{v} + \frac{d\dot{L}_3}{d\Omega}\mathbf{w}, \quad (17)$$

where

$$\begin{aligned}\frac{d\dot{L}_2}{d\Omega} &= \frac{G}{2\pi}\Big[-i\omega(3\tau^*_{13}\tau_{pp} + 6\tau^*_{3p}\tau_{p1}) - \\ &- \omega^2\Big(2\tau^*_{3pq}\tau_{pq} - 2\tau^*_{3p}\tau_{pqq} - \\ &- \tau^*_{pq3}\tau_{pq} + \frac{1}{2}\tau^*_{qq3}\tau_{pp}\Big) + \text{c.c.}\Big], \\ \frac{d\dot{L}_3}{d\Omega} &= \frac{G}{2\pi}\Big[i\omega(3\tau^*_{12}\tau_{pp} + 6\tau^*_{2p}\tau_{p1}) + \\ &+ \omega^2\Big(2\tau^*_{2pq}\tau_{pq} - 2\tau^*_{2p}\tau_{pqq} - \\ &- \tau^*_{pq2}\tau_{pq} + \frac{1}{2}\tau^*_{qq2}\tau_{pp}\Big) + \text{c.c.}\Big].\end{aligned} \quad (18)$$

Here, τ_{pq} and τ_{pqr} are the respective Fourier transforms of the energy-momentum tensor and its first moment in the new corotating basis. We note that only the subscripts p and q with the values 2 and 3 appear in Eqs. (16) and (18). For chiral loops, the Fourier transforms τ_{pq} can be expressed as

$$\tau_{pq}(\omega_l,\mathbf{n}) = -\frac{\mathcal{L}\mu}{2}[I_p(l)Y_q(l) + Y_p(l)I_q(l)], \quad (19)$$

where the functions $I_p(l)$ and $Y_q(l)$ are expressed through the «fundamental integrals»,

$$\begin{aligned} I_i(l) &\equiv \frac{1}{2\pi}\int_0^{2\pi} d\xi \exp\{-il(\xi + \mathbf{n}\cdot\mathbf{a})\}\,\mathbf{a}'\cdot\mathbf{e}_i, \\ Y_j(l) &\equiv \frac{1}{2\pi}\int_0^{2\pi} d\eta \exp\{il(\eta - \mathbf{n}\cdot\mathbf{b})\}\,\mathbf{b}'\cdot\mathbf{e}_j. \end{aligned} \quad (20)$$

For the first moment (12), we can similarly find that

$$\tau_{ijk}(\omega_l,\mathbf{n}) = -\frac{\mathcal{L}^2\mu}{8\pi}[I_i(l)N_{jk}(l) + I_j(l)N_{ik}(l) + + Y_i(l)M_{jk}(l) + Y_j(l)M_{ik}(l)], \quad (21)$$

where

$$\begin{aligned} M_{ij}(l) &\equiv \frac{1}{2\pi} \times \\ &\times \int_0^{2\pi} d\xi \exp\{-il(\xi + \mathbf{n}\cdot\mathbf{a})\}(\mathbf{a}'\cdot\mathbf{e}_i)(\mathbf{a}\cdot\mathbf{e}_j), \\ N_{ij}(l) &\equiv \frac{1}{2\pi} \times \\ &\times \int_0^{2\pi} d\eta \exp\{il(\eta - \mathbf{n}\cdot\mathbf{b})\}(\mathbf{b}'\cdot\mathbf{e}_i)(\mathbf{b}\cdot\mathbf{e}_j). \end{aligned} \quad (22)$$

The crucial point of the calculation to follow is the summation over all mode numbers l in expressions (13) for the requested rates of the radiated gravitational energy, momentum, and angular momentum. For this summation, we first integrate expressions (20) and (22) by parts to obtain an additional l in the denominator. For example, the function I_i becomes

$$I_i(l) = \frac{1}{2\pi}\int\limits_0^{2\pi} d\xi \left[\exp\left\{-il(\xi+\mathbf{n}\cdot\mathbf{a})\right\}(1+\mathbf{n}\cdot\mathbf{a}')\right] \times \times \frac{\mathbf{a}'\cdot\mathbf{e}_i}{1+\mathbf{n}\cdot\mathbf{a}'} = = -\frac{1}{2\pi il}\frac{\mathbf{a}'\cdot\mathbf{e}_i}{1+\mathbf{n}\cdot\mathbf{a}'}\exp\left\{-il(\xi+\mathbf{n}\cdot\mathbf{a})\right\}\Big|_0^{2\pi} + + \frac{1}{2\pi il}\int\limits_0^{2\pi} d\xi \left[\frac{\mathbf{a}'\cdot\mathbf{e}_j}{1+\mathbf{n}\cdot\mathbf{a}'}\right]'\exp\left\{-il(\xi+\mathbf{n}\cdot\mathbf{a})\right\}, \quad (23)$$

where the first term is equal to zero because of the periodicity of a- and b-loops. Expressions for the functions Y_j, M_{ij}, and N_{ij} can be integrated by parts similarly. We finally obtain

$$\begin{aligned} I_i &= \frac{1}{2\pi il}\int\limits_0^{2\pi} d\xi\, \mathcal{I}_i \exp\left\{-il(\xi+\mathbf{n}\cdot\mathbf{a})\right\}, \\ Y_j &= -\frac{1}{2\pi il}\int\limits_0^{2\pi} d\eta\, \mathcal{Y}_j \exp\left\{il(\eta-\mathbf{n}\cdot\mathbf{b})\right\}, \\ M_{ij} &= \int\limits_0^{2\pi} d\xi \left(\frac{1}{2\pi il}\mathcal{M}_{ij} - \frac{1}{2\pi l^2}\tilde{\mathcal{M}}_{ij}\right) \times \\ &\times \exp\left\{-il(\xi+\mathbf{n}\cdot\mathbf{a})\right\}, \\ N_{ij} &= -\int\limits_0^{2\pi} d\eta \left(\frac{1}{2\pi il}\mathcal{N}_{ij} + \frac{1}{2\pi l^2}\tilde{\mathcal{N}}_{ij}\right) \times \\ &\times \exp\left\{il(\eta-\mathbf{n}\cdot\mathbf{b})\right\}, \end{aligned} \quad (24)$$

where

$$\begin{aligned} \mathcal{I}_i &= \left[\frac{\mathbf{a}'\cdot\mathbf{e}_i}{1+\mathbf{n}\cdot\mathbf{a}'}\right]', \quad \mathcal{Y}_j = \left[\frac{\mathbf{b}'\cdot\mathbf{e}_j}{1-\mathbf{n}\cdot\mathbf{b}'}\right]', \\ \mathcal{M}_{ij} &= \left[\frac{\mathbf{a}'\cdot\mathbf{e}_i}{1+\mathbf{n}\cdot\mathbf{a}'}\right]'(\mathbf{a}\cdot\mathbf{e}_j), \\ \tilde{\mathcal{M}}_{ij} &= \left[\frac{(\mathbf{a}'\cdot\mathbf{e}_i)(\mathbf{a}'\cdot\mathbf{e}_j)}{(1+\mathbf{n}\cdot\mathbf{a}')^2}\right]', \\ \mathcal{N}_{ij} &= \left[\frac{\mathbf{b}'\cdot\mathbf{e}_i}{1-\mathbf{n}\cdot\mathbf{b}'}\right]'(\mathbf{b}\cdot\mathbf{e}_j), \\ \tilde{\mathcal{N}}_{ij} &= \left[\frac{(\mathbf{b}'\cdot\mathbf{e}_i)(\mathbf{b}'\cdot\mathbf{e}_j)}{(1-\mathbf{n}\cdot\mathbf{b}')^2}\right]'. \end{aligned} \quad (25)$$

Substituting (24) in (19) and (21), we find

$$\begin{aligned} \tau_{ij} &= -\frac{\mathcal{L}\mu}{8\pi^2 l^2}\int\limits_0^{2\pi}\int\limits_0^{2\pi} d\xi d\eta\, \mathcal{T}_{ij} \times \\ &\times \exp\left\{-il[\xi-\eta+\mathbf{n}\cdot(\mathbf{a}+\mathbf{b})]\right\}, \\ \tau_{ijk} &= -\frac{\mathcal{L}^2\mu}{32\pi^3 l^2}\int\limits_0^{2\pi}\int\limits_0^{2\pi} d\xi d\eta \left(\mathcal{T}_{ijk} + \frac{1}{il}\tilde{\mathcal{T}}_{ijk}\right) \times \\ &\times \exp\left\{-il[\xi-\eta+\mathbf{n}\cdot(\mathbf{a}+\mathbf{b})]\right\}, \end{aligned} \quad (26)$$

where

$$\begin{aligned} \mathcal{T}_{ij} &= \mathcal{I}_i\mathcal{Y}_j + \mathcal{I}_j\mathcal{Y}_i, \\ \mathcal{T}_{ijk} &= \mathcal{I}_i\mathcal{N}_{jk} + \mathcal{I}_j\mathcal{N}_{ik} + \mathcal{Y}_i\mathcal{M}_{jk} + \mathcal{Y}_j\mathcal{M}_{ik}, \\ \tilde{\mathcal{T}}_{ijk} &= -\mathcal{I}_i\tilde{\mathcal{N}}_{jk} - \mathcal{I}_j\tilde{\mathcal{N}}_{ik} + \mathcal{Y}_i\tilde{\mathcal{M}}_{jk} + \tilde{\mathcal{Y}}_j\mathcal{M}_{ik}. \end{aligned} \quad (27)$$

Next, substituting (26) in (16) and (18), we find the radiation rates of E, $\mathbf{P}$, and $\mathbf{L}$ on the particular eigenfrequency $\omega_l = 2\pi l/T$,

$$\frac{d\dot{P}^\mu(\omega)}{d\Omega} = n^\mu \frac{G\mu^2}{4\pi^3 l^2}\int d^4\xi\, \mathcal{P}\cos(l\Delta x), \quad (28)$$

$$\begin{aligned} \frac{d\dot{L}_v}{d\Omega} &= -\frac{G\mathcal{L}\mu^2}{16\pi^4} \times \\ &\times \int d^4\xi \left[\frac{\sin(l\Delta x)}{l^3}(3\lambda_2+\tilde{\Lambda}_2) + \frac{\cos(l\Delta x)}{l^2}\Lambda_2\right], \\ \frac{d\dot{L}_w}{d\Omega} &= \frac{G\mathcal{L}\mu^2}{16\pi^4} \times \\ &\times \int d^4\xi \left[\frac{\sin(l\Delta x)}{l^3}(3\lambda_3+\tilde{\Lambda}_3) + \frac{\cos(l\Delta x)}{l^2}\Lambda_3\right], \end{aligned} \quad (29)$$

where we use the notation

$$\begin{aligned} \Delta x &= \xi - \xi' - (\eta - \eta') + \\ &+ \mathbf{n}[\mathbf{a}(\xi) - \mathbf{a}(\xi') + \mathbf{b}(\eta) - \mathbf{b}(\eta')], \\ \mathcal{P} &= \mathcal{T}'_{pq}\mathcal{T}_{pq} - \frac{1}{2}\mathcal{T}'_{qq}\mathcal{T}_{pp}, \\ \lambda_2 &= \mathcal{T}'_{13}\mathcal{T}_{pp} + 2\mathcal{T}'_{3p}\mathcal{T}_{p1}, \\ \lambda_3 &= \mathcal{T}'_{12}\mathcal{T}_{pp} + 2\mathcal{T}'_{2p}\mathcal{T}_{p1}, \\ \Lambda_2 &= 2\mathcal{T}'_{3pq}\mathcal{T}_{pq} - 2\mathcal{T}'_{3p}\mathcal{T}_{pqq} - \mathcal{T}'_{pq3}\mathcal{T}_{pq} + \\ &+ \frac{1}{2}\mathcal{T}'_{qq3}\mathcal{T}_{pp}, \\ \tilde{\Lambda}_2 &= 2\tilde{\mathcal{T}}'_{3pq}\mathcal{T}_{pq} + 2\mathcal{T}'_{3p}\tilde{\mathcal{T}}_{pqq} - \tilde{\mathcal{T}}'_{pq3}\mathcal{T}_{pq} + \\ &+ \frac{1}{2}\tilde{\mathcal{T}}'_{qq3}\mathcal{T}_{pp}, \\ \Lambda_3 &= 2\mathcal{T}'_{2pq}\mathcal{T}_{pq} - 2\mathcal{T}'_{2p}\mathcal{T}_{pqq} - \mathcal{T}'_{pq2}\mathcal{T}_{pq} + \\ &+ \frac{1}{2}\mathcal{T}'_{qq2}\mathcal{T}_{pp}, \\ \tilde{\Lambda}_3 &= 2\tilde{\mathcal{T}}'_{2pq}\mathcal{T}_{pq} + 2\mathcal{T}'_{2p}\tilde{\mathcal{T}}_{pqq} - \tilde{\mathcal{T}}'_{pq2}\mathcal{T}_{pq} + \\ &+ \frac{1}{2}\tilde{\mathcal{T}}'_{qq2}\mathcal{T}_{pp}. \end{aligned} \quad (30)$$

It is assumed that integration in (28) and (29) is over the four-dimensional cube with the side $(0, 2\pi)$; we also use the notation $d^4\xi = d\xi\, d\xi'\, d\eta\, d\eta'$.

We now find the form of expressions (28) and (29) suitable for summing over the modes l. Using the known values for infinite series [18]

$$\sum_{l=1}^{\infty}\frac{\cos(lx)}{l^2} = \frac{1}{4}(x-\pi)^2 - \frac{\pi^2}{12}, \quad 0 \le x \le 2\pi,$$
$$\sum_{l=1}^{\infty}\frac{\sin(lx)}{l^3} = \frac{1}{12}[(x-\pi)^3 - \pi^2 x] + \frac{\pi^3}{12}, \quad 0 \le x \le 2\pi, \tag{31}$$

we obtain the final expressions for the gravitational radiation of energy, momentum, and angular momentum rates [19] from (28) and (29) as

$$\frac{d\dot{P}^\mu}{d\Omega} = n^\mu \frac{G\mu^2}{16\pi^3}\int d^4\xi \mathcal{P}(\Delta x \bmod 2\pi - \pi)^2, \tag{32}$$

$$\frac{d\dot{L}_v}{d\Omega} = -\frac{G\mathcal{L}\mu^2}{64\pi^4}\int d^4\xi \left\{ [(\Delta x \bmod 2\pi - \pi)^3 - \pi^2 \Delta x \bmod 2\pi]\left(\lambda_2 + \frac{1}{3}\tilde{\Lambda}_2\right) + (\Delta x \bmod 2\pi - \pi)^2 \Lambda_2 \right\},$$
$$\frac{d\dot{L}_w}{d\Omega} = \frac{G\mathcal{L}\mu^2}{64\pi^4}\int d^4\xi \left\{ [(\Delta x \bmod 2\pi - \pi)^3 - \pi^2 \Delta x \bmod 2\pi]\left(\lambda_3 + \frac{1}{3}\tilde{\Lambda}_3\right) + (\Delta x \bmod 2\pi - \pi)^2 \Lambda_3 \right\}. \tag{33}$$

We note that the integrals in (32) and (33) do not contain the terms $\pi^2/12$ and $\pi^3/12$ originating in (31) because the corresponding contributions vanish in the integrals. The advantage of formulas (32) and (33) with respect to the corresponding formulas (16) and (17) is that there are no summations over modes. But because of the presence of the function $\Delta x(\mathrm{mod})2\pi$, the four-dimensional integrals in (32) and (33) cannot be reduced to products of lower-dimensional integrals, and therefore numerical calculations of the four-dimensional integrals become more complicated.

4. ELECTROMAGNETIC RADIATION FROM CHIRAL LOOPS

We now consider the electromagnetic radiation from an arbitrary relativistic periodic system in a similar way. We calculate the electromagnetic radiation by analogy with Durrer's calculations of the gravitational radiation [14]. In the Lorentz gauge, a retarded solution for the electromagnetic potential A_μ in such a system is given by

$$A_\mu(\mathbf{x}, t) = -\int \frac{j_\mu(\mathbf{x}', t_{ret})}{|\mathbf{x}-\mathbf{x}'|}d\mathbf{x}', \tag{34}$$

where j_μ is the four-dimensional current and we set $t_{ret} = t - |\mathbf{x}-\mathbf{x}'|$. We consider formula (34) in the limit $r = |\mathbf{x}| \gg |\mathbf{x}'|$. Expanding (34) in a series in $1/r$ and taking the first two terms into account, we obtain

$$A_\mu(\mathbf{x}, t) = \frac{1}{r}\int j_\mu(\mathbf{x}', t_{ret})d\mathbf{x}' - \frac{1}{r^2}\int j_\mu(\mathbf{x}', t_{ret})x'^i d\mathbf{x}' + O(r^{-3}), \tag{35}$$

where $\mathbf{n} = \mathbf{x}/r$. Expanding t_{ret} in a series in $|\mathbf{x}'|/r$, we then find

$$t_{ret} = t - r + \mathbf{n}\cdot\mathbf{x}' - \frac{1}{2r}P_{ij}x'^i x'^j + O(|\mathbf{x}'|^2/r^2)|\mathbf{x}'|. \tag{36}$$

Equation (36) implies the useful relation

$$A_{\mu,j} = -A_{\mu,0}n_j + O(A_\mu/r). \tag{37}$$

Similarly to the case of the gravitational field ($T^{\mu\nu} \leftrightarrow j^\mu, h^{\mu\nu} \leftrightarrow j^\mu$, etc.), we have the Fourier transforms of the current $\tilde{j}^\mu$ and its first and second moments $\tilde{j}^{\mu p}$ and $\tilde{j}^{\mu pq}$,

$$\tilde{j}^\mu(\omega_l, \mathbf{n}) = \frac{1}{T}\int_0^T dt \int d^3x j^\mu(\omega_l, \mathbf{x}) \times \exp\{i\omega_l(t - \mathbf{n}\cdot\mathbf{x})\},$$
$$\tilde{j}^{\mu p}(\omega_l, \mathbf{n}) = \frac{1}{T}\int_0^T dt \int d^3x j^\mu(\omega_l, \mathbf{x})x^p \times \exp\{i\omega_l(t - \mathbf{n}\cdot\mathbf{x})\}, \tag{38}$$
$$\tilde{j}^{\mu pq}(\omega_l, \mathbf{n}) = \frac{1}{T}\int_0^T dt \int d^3x j^\mu(\omega_l, \mathbf{x})x^p x^q \times \exp\{i\omega_l(t - \mathbf{n}\cdot\mathbf{x})\}.$$

These quantities satisfy the conditions

$$\tilde{j}^0 - n^k \tilde{j}^k = 0,$$
$$-i\omega \tilde{j}^{0p} - \tilde{j}^p + i\omega n_k \tilde{j}^{kp} = 0, \tag{39}$$
$$i\omega P_{mn}(\tilde{j}^{0mn} - n_p \tilde{j}^{pmn}) + 2P_{pq}\tilde{j}^{pq} = 0,$$

which follow from the relations

$$\begin{aligned} &j^\mu_{,\mu}=0,\\ &\int j^\mu(t,\mathbf{x}')_{,\mu}[x'^p\exp\{i\omega(t-\mathbf{n}\cdot\mathbf{x}')\}]\,dt\,d^3x=0,\\ &\int j^\mu(t,\mathbf{x}')_{,\mu}\left\{[x'^2-(\mathbf{n}\cdot\mathbf{x}')]\times\right.\\ &\times\exp\{i\omega(t-\mathbf{n}\cdot\mathbf{x}')\}\}\,dt\,d^3x=0. \end{aligned} \tag{40}$$

Using (38) and (40), we obtain from (35) that

$$A_\mu(\mathbf{x},t)=\frac{1}{r}\sum_{l=1}^{\infty}e^{-i\omega_l(t-r)}\left[\tilde{j}_\mu(\omega_l,\mathbf{n})+\right.$$
$$\left.+\frac{n^p}{r}\tilde{j}_{\mu p}(\omega_l,\mathbf{n})+\frac{i\omega_l}{2r}P^{pq}\tilde{j}_{\mu pq}(\omega_l,\mathbf{n})\right]+$$
$$+\,\text{c.c.}+O(r^{-3}). \tag{41}$$

To calculate the energy and momentum radiation losses, we keep only terms of the order of $1/r$ in (41). The radiation of energy from the system is determined by the Poynting vector as [20]

$$\frac{d\dot{E}^{em}}{d\Omega}=\frac{|\mathbf{E}\times\mathbf{H}|}{4\pi}, \tag{42}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields. Using (41), we obtain from (42) that

$$\frac{d\dot{P}^\mu_{em}}{d\Omega}=\sum_{n=1}^{\infty}\frac{d\dot{P}^\mu(\omega_n)}{d\Omega}, \tag{43}$$

where

$$\frac{d\dot{P}^\mu_{em}(\omega)}{d\Omega}=n^\mu\frac{\omega^2}{2\pi}P^{pq}\tilde{j}^*_p\tilde{j}_q. \tag{44}$$

We now calculate electromagnetic radiation of the angular momentum. The angular momentum rate per unit solid angle is given by [20]

$$\frac{d\dot{\mathbf{L}}^{em}}{d\Omega}=\frac{r^3}{4\pi}\left[[\mathbf{n}\times\mathbf{E}](\mathbf{n}\cdot\mathbf{E})+[\mathbf{n}\times\mathbf{H}](\mathbf{n}\cdot\mathbf{H})\right]. \tag{45}$$

In calculating $[\mathbf{n}\times\mathbf{E}]$ and $[\mathbf{n}\times\mathbf{H}]$, it suffices to keep only terms of the order of $1/r$. But the longitudinal components $\mathbf{n}\cdot\mathbf{E}$ and $\mathbf{n}\cdot\mathbf{H}$ arise from terms of the order of $1/r^2$. As a result, the term r^3 is canceled in (45). This implies that the distance from the system r does not enter the final formula, as should be the case. Using (41) and (40), we obtain

$$\begin{aligned} &[\mathbf{n}\times\mathbf{E}]^i=-\epsilon^{ijk}n_jA_{k,0}=\\ &=-\sum_{l=1}^{\infty}\frac{i\omega_l}{r}\exp\{-i\omega_l(t-r)\}\,\epsilon^{ijk}n_j\tilde{j}_k+\text{c.c.},\\ &[\mathbf{n}\times\mathbf{H}]^i=\sum_{l=1}^{\infty}\frac{i\omega_l}{r}\exp\{-i\omega_l(t-r)\}\,P^{ij}\tilde{j}_k+\text{c.c.},\\ &\mathbf{n}\cdot\mathbf{E}=-\sum_{l=1}^{\infty}\frac{i\omega_l}{r^2}\exp\{-i\omega_l(t-r)\}\,P^{pq}\tilde{j}_{pq}+\text{c.c.},\\ &\mathbf{n}\cdot\mathbf{H}=\sum_{l=1}^{\infty}\frac{i\omega_l}{r^2}\exp\{-i\omega_l(t-r)\}\,\epsilon^{pqr}n_p\tilde{j}_{rq}+\text{c.c.} \end{aligned} \tag{46}$$

Substituting (46) in (45), we obtain

$$\frac{d\dot{\mathbf{L}}^{em}}{d\Omega}=\sum_{n=1}^{\infty}\frac{d\dot{\mathbf{L}}^{em}(\omega_n)}{d\Omega}, \tag{47}$$

where

$$\frac{d\dot{L}^{em}_i(\omega)}{d\Omega}=$$
$$=\frac{\omega^2}{4\pi}\left[(\epsilon^{ijk}P_{pq}-P_{ik}\epsilon_{jpq})\,n_j\,\tilde{j}^*_k\,\tilde{j}_{pq}+\text{c.c.}\right]. \tag{48}$$

As for the gravitational field, we rewrite (44) and (48) in the corotating basis $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)=(\mathbf{n},\mathbf{v},\mathbf{w})$,

$$\frac{d\dot{P}^\mu_{em}(\omega)}{d\Omega}=n^\mu\frac{\omega^2}{2\pi}\tilde{\iota}^*_p\tilde{\iota}_p, \tag{49}$$

$$\frac{d\dot{\mathbf{L}}^{em}(\omega)}{d\Omega}=\frac{d\dot{L}^{em}_2}{d\Omega}\mathbf{v}+\frac{d\dot{L}^{em}_3}{d\Omega}\mathbf{w}, \tag{50}$$

where

$$\begin{aligned} \frac{d\dot{L}^{em}_2}{d\Omega}&=-\frac{\omega^2}{4\pi}[\iota^*_3\iota_{pp}+\iota^*_2(\iota_{23}-\iota_{32})+\text{c.c.}],\\ \frac{d\dot{L}^{em}_3}{d\Omega}&=\frac{\omega^2}{4\pi}[\iota^*_2\iota_{pp}-\iota^*_3(\iota_{23}-\iota_{32})+\text{c.c.}], \end{aligned} \tag{51}$$

and ι^p and ι^{pq} are components of j^μ and $j^{\mu p}$ in this corotating basis.

For superconducting chiral strings, we obtain from expression (7) for the current that

$$\iota_i(\omega_l,\mathbf{n})=\frac{\mathcal{L}q\sqrt{\mu}}{2}[I_i(l)X(l)], \tag{52}$$

where the function $I_i(l)$ is given by (20) and $X(l)$ is

$$X(l)\equiv\frac{1}{2\pi}\int_0^{2\pi}d\eta\,e^{il(\eta-\mathbf{n}\cdot\mathbf{b})}\sqrt{1-|\mathbf{b}'|^2}. \tag{53}$$

Similarly, for the first moment ι_{pq}, we obtain

$$\iota_{pq}(\omega_l, \mathbf{n}) = \frac{\mathcal{L}^2 q\sqrt{\mu}}{8\pi}[I_p(l)Z_q(l) + X(l)M_{pq}(l)], \quad (54)$$

where M_{pq} is given by (22) and Z_q is

$$Z_i(l) \equiv \frac{1}{2\pi}\int_0^{2\pi} d\eta\, e^{il(\eta - \mathbf{n}\cdot\mathbf{b})}\sqrt{1-|\mathbf{b}'|^2}\,(\mathbf{b}\cdot\mathbf{e}_i). \quad (55)$$

We now integrate expressions (53) and (55) by parts to obtain an additional l in the denominator,

$$X = -\frac{1}{2\pi i l}\int_0^{2\pi} d\eta\, \mathcal{X}\exp\{il(\eta - \mathbf{n}\cdot\mathbf{b})\},$$
$$Z_j = -\int_0^{2\pi} d\eta\left(\frac{1}{2\pi i l}\mathcal{Z}_j + \frac{1}{2\pi l^2}\tilde{\mathcal{Z}}_j\right)\times$$
$$\times\exp\{il(\eta - \mathbf{n}\cdot\mathbf{b})\}, \quad (56)$$

where

$$\mathcal{X} = \left[\frac{\sqrt{1-|\mathbf{b}'|^2}}{1-\mathbf{n}\cdot\mathbf{b}'}\right]',$$
$$\mathcal{Z}_j = \left[\frac{\sqrt{1-|\mathbf{b}'|^2}}{1-\mathbf{n}\cdot\mathbf{b}'}\right]'(\mathbf{b}\mathbf{e}_j), \quad (57)$$
$$\tilde{\mathcal{Z}}_j = \left[\frac{\sqrt{1-|\mathbf{b}'|^2}(\mathbf{b}'\cdot\mathbf{e}_j)}{1-\mathbf{n}\cdot\mathbf{b}'}\right]'.$$

Substituting (20), (22), and (56) in (52) and (54), we obtain

$$\iota_i = \frac{\mathcal{L}q\sqrt{\mu}}{8\pi^2 l^2}\int_0^{2\pi}\int_0^{2\pi} d\xi d\eta\, \mathcal{J}_i\times$$
$$\times\exp\{-il[\xi - \eta + \mathbf{n}\cdot(\mathbf{a}+\mathbf{b})]\},$$
$$\iota_{ij} = \frac{\mathcal{L}^2 q\sqrt{\mu}}{32\pi^3 l^2}\int_0^{2\pi}\int_0^{2\pi} d\xi d\eta\left(\mathcal{J}_{ij} + \frac{1}{il}\tilde{\mathcal{J}}_{ij}\right)\times$$
$$\times\exp\{-il[\xi - \eta + \mathbf{n}\cdot(\mathbf{a}+\mathbf{b})]\}, \quad (58)$$

where

$$\mathcal{J}_i = \mathcal{I}_i\mathcal{X}, \quad \mathcal{J}_{ij} = \mathcal{I}_i\mathcal{Z}_j + \mathcal{X}\mathcal{M}_{ij},$$
$$\tilde{\mathcal{J}}_{ij} = -\mathcal{I}_i\tilde{\mathcal{Z}}_j + \mathcal{X}\tilde{\mathcal{M}}_{ij}. \quad (59)$$

Finally, substituting (58) in (49) and (51), we find the expressions for electromagnetic radiation rates of the energy, momentum, and angular momentum in a unit solid angle at the frequency ω_l,

$$\frac{d\dot{P}^{\mu}_{em}(\omega_l)}{d\Omega} = n^{\mu}\frac{q^2\mu}{8\pi^3 l^2}\int d^4\xi\,\mathcal{P}^{em}\cos(l\Delta x), \quad (60)$$

$$\frac{d\dot{L}^{em}_v}{d\Omega} = \frac{\mathcal{L}q^2\mu}{32\pi^4}\times$$
$$\times\int d^4\xi\left[\frac{\sin(l\Delta x)}{l^3}\tilde{\Lambda}^{em}_2 - \frac{\cos(l\Delta x)}{l^2}\Lambda^{em}_2\right],$$
$$\frac{d\dot{L}^{em}_w}{d\Omega} = -\frac{\mathcal{L}q^2\mu}{32\pi^4}\times \quad (61)$$
$$\times\int d^4\xi\left[\frac{\sin(l\Delta x)}{l^3}\tilde{\Lambda}^{em}_3 - \frac{\cos(l\Delta x)}{l^2}\Lambda^{em}_3\right],$$

where

$$\mathcal{P}^{em} = \mathcal{J}'_p\mathcal{J}_p,$$
$$\Lambda^{em}_2 = \mathcal{J}'_3\mathcal{J}_{pp} + \mathcal{J}'_2(\mathcal{J}_{23} - \mathcal{J}_{32}),$$
$$\tilde{\Lambda}^{em}_2 = \mathcal{J}'_3\tilde{\mathcal{J}}_{pp} + \mathcal{J}'_2(\tilde{\mathcal{J}}_{23} - \tilde{\mathcal{J}}_{32}), \quad (62)$$
$$\Lambda^{em}_3 = \mathcal{J}'_2\mathcal{J}_{pp} - \mathcal{J}'_3(\mathcal{J}_{23} - \mathcal{J}_{32}),$$
$$\tilde{\Lambda}^{em}_3 = \mathcal{J}'_2\tilde{\mathcal{J}}_{pp} - \mathcal{J}'_3(\tilde{\mathcal{J}}_{23} - \tilde{\mathcal{J}}_{32}).$$

As for the gravitational radiation, we use the values for infinite series (31) to obtain the total electromagnetic radiation rates of the energy, momentum, and angular momentum [19],

$$\frac{d\dot{P}^{\mu}_{em}}{d\Omega} = n^{\mu}\frac{q^2\mu}{32\pi^3}\int d^4\xi\,\mathcal{P}^{em}(\Delta x \bmod 2\pi - \pi)^2, \quad (63)$$

$$\frac{d\dot{L}^{em}_v}{d\Omega} = \frac{\mathcal{L}q^2\mu}{128\pi^4}\int d^4\xi\,\times$$
$$\times\left[\frac{1}{3}\left((\Delta x \bmod 2\pi - \pi)^3 - \pi^2\Delta x \bmod 2\pi\right)\tilde{\Lambda}^{em}_2 - \right.$$
$$\left. - (\Delta x \bmod 2\pi - \pi)^2\Lambda^{em}_2\right],$$
$$\frac{d\dot{L}^{em}_w}{d\Omega} = -\frac{\mathcal{L}q^2\mu}{128\pi^4}\int d^4\xi\,\times \quad (64)$$
$$\times\left[\frac{1}{3}\left((\Delta x \bmod 2\pi - \pi)^3 - \pi^2\Delta x \bmod 2\pi\right)\tilde{\Lambda}^{em}_3 - \right.$$
$$\left. - (\Delta x \bmod 2\pi - \pi)^2\Lambda^{em}_3\right].$$

As a result, we found expressions for the electromagnetically radiated energy, momentum, and angular momentum from chiral string loops in which the summation over modes l is carried out.

5. RADIATION OF NEARLY STATIONARY LOOPS

We can now consider small-amplitude oscillations of the chiral string loop (i. e., the string that is close to its vorton state) in more detail. An arbitrary function $b(\eta)$ in the solution for string motion (2) is then such that $b'(\eta) = k(\eta) \ll 1$. If three-dimensional coordinates are chosen such that the b-loop is near the origin of the coordinate system (e.g., exactly intersects the origin of the coordinate system), then $b(\eta) \ll 1$. We now return to expressions (13). The expressions for the functions $Y_j(l)$ and $N_{ij}(l)$ can be integrated by parts twice to increase the power of l in the denominator, while the expressions for $I_j(l)$ and $M_{ij}(l)$ are left unchanged. We assume that $b(\eta)$ is twice continuously differentiable and $b'''(\eta)$ is piecewise continuous. Integrating in (20) and (22) by parts twice and using the smallness of $b'(\eta)$, we obtain

$$\begin{aligned} Y_i(l) &= -\frac{1}{2\pi l^2}\int_0^{2\pi} d\eta e^{il\eta}\mathbf{b}'''\cdot\mathbf{e}_i, \\ N_{ij}(l) &= -\frac{1}{2\pi l^2}\int_0^{2\pi} d\eta e^{il\eta}\left[(\mathbf{b}'\cdot\mathbf{e}_i)\,(\mathbf{b}\cdot\mathbf{e}_j)\right]''. \end{aligned} \tag{65}$$

As has been noted, we are free to add any numerical coefficient to $(\Delta x \bmod 2\pi - \pi)^2$ in (32) without changing the value of the integral. Using this, we add $-\pi^2/2$, and then Eq. (32) implies

$$\left|\frac{d\dot{P}^\mu}{d\Omega}\right| = \\ = \left|n^\mu \frac{G\mu^2}{16\pi^3}\int d^4\xi \mathcal{P}\left[(\Delta x \bmod 2\pi - \pi)^2 - \pi^2/2\right]\right| \le \\ \le \frac{G\mu^2\pi^3}{2}\left|\mathcal{P}\right|. \tag{66}$$

It only remains to estimate the function $\mathcal{P}$ in (66). Using (20), (27), (30), and (65) we easily find

$$|\mathcal{P}| \le 12 b_3^2, \tag{67}$$

where b_3 is the maximum value of $|b'''(\eta)|$ on the segment $\eta \in (0, 2\pi)$. From (66) and (67), we then estimate the energy losses as

$$\left|\dot{E}^{gr}\right| \le 24 G\mu^2\pi^4 b_3^2, \quad \left|\dot{\mathbf{P}}^{gr}\right| \le 24 G\mu^2\pi^4 b_3^2. \tag{68}$$

We next estimate the upper bounds on the radiated angular momentum. Similarly to the case of energy and momentum radiation, we use Eqs. (33), (30), (27), and (65) to find the upper bounds on losses of the angular momentum to gravitational waves,

$$\left|\dot{L}^{gr}\right| \le 12\sqrt{2}\pi^4\left(1 + \frac{4}{3\sqrt{3}}\right) G\mathcal{L}\mu^2 b_3^2. \tag{69}$$

We now consider the electromagnetic radiation in the case of a large current. To find the first-order expansion with respect to k in (53), we must take not only zero, but also the first term in the expansion of $\exp(-il\mathbf{n}\cdot\mathbf{b})$ into account. Subsequent integration of the resulting expression by parts gives

$$X(l) \simeq -\frac{1}{2\pi l^2}\int_0^{2\pi} d\eta e^{il\eta}\mathbf{n}\cdot\mathbf{b}'''. \tag{70}$$

For the function Z_i, we have

$$Z_i(l) \approx -\frac{1}{2\pi l^2}\int_0^{2\pi} d\eta\, e^{il\eta}(\mathbf{b}''\cdot\mathbf{e}_i). \tag{71}$$

Similarly to the gravitational case, we can find the bounds on the electromagnetic radiation for a large current. Using (70), (71), (64), (63), (62), and (59), we obtain

$$\begin{aligned} &\left|\dot{E}^{em}\right| \le \pi^4 q^2\mu b_3^2, \quad \left|\dot{P}^{em}\right| \le \pi^4 q^2 \mu b_3^2, \\ &\left|\dot{L}^{em}\right| \le \sqrt{2}\pi^3\left(1 + \frac{4\pi}{9\sqrt{3}}\right)\mathcal{L} q^2\mu b_3^2. \end{aligned} \tag{72}$$

The presence of the third derivative $\mathbf{b}'''(\eta)$ in (68), (69), and (65) is not surprising and resembles the quadruple gravitational radiation formula (see, e. g., [20])

$$\dot{E} = \frac{G}{45}\dddot{D}_{ij}^{2} \tag{73}$$

involving the third time derivative of the quadruple moment D_{ij}. Electromagnetic radiation involves $\ddot{\mathbf{d}}$ in the dipole approach ($\mathbf{d}$ is the dipole moment). Arguing similarly, we can conclude that in this case, the second derivative of $\mathbf{b}(\eta)$, not the third, must be restricted. But in the first order of the expansion in k, the dipole radiation is equal to zero, the first nonzero term is quadruple, and we therefore again obtain the dependence on $\mathbf{b}'''$.

We note that it is not necessary to restrict the third derivative $\mathbf{b}'''$ in general. For example, if the string has kinks (see below), the first derivative $\mathbf{b}'$ is discontinuous (and consequently, $Y_p(l) \propto 1/l$, $M(l) \propto 1/l$). Convergence of series (13), (43), and (47) is then ensured by the behavior of fundamental integrals $I_p(l) \propto 1/l$ at $l \gg 1$.

It is possible to derive rather simple expressions for the total energy, momentum, and angular momentum radiated by chiral loops in the limit as loops are very close to their stationary states, i. e., $k \ll 1$ in (4). Additionally, it is supposed that k is independent of η and the current j^μ is therefore constant along the string.

Using expansions of (32), (33), (63), and (64) in powers of k, we can write the corresponding gravitational and electromagnetic rates as

$$\dot{E}^{gr} = K_E^{gr} G\mu^2 k^2, \quad |\dot{\mathbf{P}}^{gr}| = K_P^{gr} G\mu^2 k^2,$$
$$|\dot{\mathbf{L}}^{gr}| = K_L^{gr} \mathcal{L} G\mu^2 k^2, \quad \dot{E}^{em} = K_E^{em} q^2 \mu k^2, \tag{74}$$
$$|\dot{\mathbf{P}}^{em}| = K_P^{em} q^2 \mu k^2, \quad |\dot{\mathbf{L}}^{em}| = K_L^{em} \mathcal{L} q^2 \mu k^2,$$

where K^{em} and K^{gr} are numerical coefficients depending only on the loop geometry. We see that radiation rates of nearly stationary chiral loops are proportional to k^2. The geometrical numerical factors K in Eq. (74) are in turn related to the corresponding coefficients Γ in Eq. (1) as

$$\Gamma = K k^2. \tag{75}$$

We now evaluate the damping time of small-amplitude oscillations of nearly stationary chiral strings corresponding to the limit $k \ll 1$. For simplicity, we again assume that k is independent of η in the considered limit (this assumption is valid in the solvable examples considered above). The total loop charge conservation in (7) then gives

$$\frac{q\sqrt{\mu}}{2} L\sqrt{1-k^2} = \text{const}. \tag{76}$$

From this equation, we find the relation between the energy E and the parameter k of the chiral string with small-amplitude oscillations,

$$E \simeq E_v \left(1 + \frac{k^2}{2}\right), \tag{77}$$

where $E_v = L\mu$ is the energy of the stationary (vorton) chiral loop configuration at $k = 0$. Comparing (77) with (74), we estimate the damping time of string oscillations [16] as

$$\tau \sim \frac{E_v}{2(K^{gr}G\mu^2 + K^{em}q^2\mu)}. \tag{78}$$

We next express (78) through the vorton length. We have $E_v = L\mu$, where L is the invariant length, and the physical length of a stationary string is equal to half the invariant length $L_{ph} = L/2$ [22]. We find

$$\tau \sim \frac{L_{ph}}{K^{gr}G\mu + K^{em}q^2}. \tag{79}$$

Also assuming for simplicity that k depends only on time and using Eqs. (74) and (77), we find the oscillation damping law

$$k^2 \sim k_0^2 \exp\left\{-t\left(\frac{1}{\tau_c^{gr}} + \frac{1}{\tau_c^{em}}\right)\right\}, \tag{80}$$

where $k_0 = k(t=0)$; therefore, the damping time due to gravitational radiation is

$$\tau_c^{gr} \sim \frac{E_v}{2K^{gr}G\mu^2} = \frac{L_{ph}}{K^{gr}G\mu} \tag{81}$$

and that due to electromagnetic radiation is

$$\tau_c^{em} \sim \frac{E_v}{2K^{em}q^2\mu} = \frac{L_{ph}}{K^{em}q^2}. \tag{82}$$

Substituting (80) in (77), we obtain

$$E \sim E_v\left[1 + \frac{k_0^2}{2}\exp\left\{-t\left(\frac{1}{\tau_c^{gr}} + \frac{1}{\tau_c^{em}}\right)\right\}\right]. \tag{83}$$

The effective number of oscillations during the damping time (oscillator quality) is

$$Q = \frac{\tau}{T} \sim \frac{2}{L}\frac{\tau^{gr}\tau^{em}}{\tau^{gr} + \tau^{em}}. \tag{84}$$

To restore the standard CGS units, we replace $G\mu^2 \to G\mu^2 c$, $q^2\mu \to q^2\mu c^2/\hbar$ and choose the standard normalization for the string mass per unit length $G\mu/c^2 = 10^{-6}\mu_6$ and $q_e = q/e$ for the dimensionless charge carrier on the string, where the elementary electric charge is $e = 4.8\cdot 10^{-10}$. As a result, the damping times are expressed as

$$\tau^{gr} \sim \frac{L_{ph}c}{K^{gr}G\mu}, \quad \tau^{em} \sim \frac{L_{ph}\hbar}{K^{em}q^2}. \tag{85}$$

Oscillator quality (84) for the gravitational and electromagnetic radiation is given by the respective formulas

$$Q^{gr} \sim \frac{1}{K^{gr}}\frac{c^2}{G\mu}, \quad Q^{em} \sim \frac{1}{K^{em}}\frac{1}{\alpha_{em}q_e^2}, \tag{86}$$

with $\alpha_{em} = e^2/c\hbar$. The ratio of the damping times is

$$\frac{\tau^{gr}}{\tau^{em}} \sim \frac{q^2}{G\mu\hbar}\left(\frac{K^{em}}{K^{gr}}\right) \approx 1.4\cdot 10^{-4}\frac{q_e^2}{\mu_6}\frac{K^{em}}{K^{gr}}. \tag{87}$$

If $q_e^2/\mu_6 \gtrsim 1.4\cdot 10^{-3}$, the electromagnetic radiation prevails in the chiral loop evolution (this is valid for the standard values $\mu_6 \sim 1$ and $q_e \sim 1$). If on the contrary $q_e^2/\mu_6 \lesssim 1.4\cdot 10^{-3}$ (for example, if the current is neutral and there is no electromagnetic radiation at all), then the pure gravitational radiation determines the evolution.

6. NUMERICAL EXAMPLES OF RADIATING LOOPS

In this section, we apply analytic formulas (32), (33), (63), and (64) derived above for gravitational and electromagnetic radiation to some particular examples of chiral loops. At the final steps, numerical calculations of four-dimensional integrals are used to find the energy, momentum, and angular momentum radiation rates as functions of the current on the string.

We first consider the class of piecewise linear kinky loops. Let $\mathbf{a}(\xi)$ and $\mathbf{b}(\eta)$ be piecewise linear functions; that is, vector functions $\mathbf{a}(\xi)$ and $\mathbf{b}(\eta)$ are closed loops consisting of connected straight parts. The join points of segments of a- and b-loops, where $\mathbf{a}'(\xi)$ and $\mathbf{b}'(\eta)$ are discontinuous, are called «kinks». We take the a-loop consisting of N_a and b-loop consisting of N_b segments (parts). Kinks are labeled by $i = 0, 1, \ldots, N_a - 1$, and the value of ξ on the kink labeled by i is denotes as ξ^i. In what follows, we use superscripts for the segment labels and subscripts for tensor components. Because we use only spatial tensor components, there should be no confusion. Without the loss of generality, we can set $\xi^0 = 0$. We note that $\xi^{i+N_a} = \xi^i + 2\pi$ because of periodicity. Using the notation $\Delta\xi^i = \xi^{i+1} - \xi^i$, $\mathbf{A}^i = a(\xi^i)$, and $\mathbf{a}^i = (\mathbf{A}^{i+1} - \mathbf{A}^i)/\Delta\xi^i$, and similarly for the b-loop, we find

$$\begin{aligned} \mathbf{a}(\xi) &= \mathbf{A}^i + (\xi - \xi^i)\mathbf{a}^i, \quad \xi \in [\xi^i, \xi^{i+1}], \\ \mathbf{b}(\eta) &= \mathbf{B}^j + (\eta - \eta^j)\mathbf{b}^j, \quad \eta \in [\eta^j, \eta^{j+1}]. \end{aligned} \tag{88}$$

For piecewise linear loops, the functions $\mathcal{I}_p$, $\mathcal{Y}_p$, $\mathcal{M}_{pq}$, $\mathcal{N}_{pq}$, $\mathcal{X}$, and $\mathcal{Z}_p$ in (25) and (57) become the sums of delta functions because of the discontinuity of $\mathbf{a}'$ and $\mathbf{b}'$ at the kinks. For example, the function $\mathcal{I}_p$ in (25) is given by

$$\mathcal{I}_p = \sum_i \left(\frac{\mathbf{a}^i \cdot \mathbf{e}_p}{1 + \mathbf{a}^i \cdot \mathbf{n}} - \frac{\mathbf{a}^{i-1} \cdot \mathbf{e}_p}{1 + \mathbf{a}^{i-1} \cdot \mathbf{n}} \right) \delta(\xi - \xi^i). \tag{89}$$

Similar expressions can be obtained for the other functions. Due to the presence of delta functions in $\mathcal{I}_p$, $\mathcal{Y}_p$, $\mathcal{M}_{pq}$, $\mathcal{N}_{pq}$, $\mathcal{X}$, and $\mathcal{Z}_p$, the integrations in (32), (33), (63), and (64) can be carried out easily. To obtain the expressions for the gravitational and electromagnetic radiation from the general formulas, we must replace integrations in (32), (33), (63), and (64) by summations over the kinks and make the substitutions

$$\begin{aligned} &\Delta x \to x^{ijkl} = \xi^i - \xi^k - (\eta^j - \eta^l) + \\ &\quad + \mathbf{n} \cdot (\mathbf{a}^i - \mathbf{a}^k + \mathbf{b}^j - \mathbf{b}^l), \\ &\mathcal{I}_p \to \mathcal{I}_p^i = \frac{\mathbf{a}^i \cdot \mathbf{e}_p}{1 + \mathbf{a}^i \cdot \mathbf{n}} - \frac{\mathbf{a}^{i-1} \cdot \mathbf{e}_p}{1 + \mathbf{a}^{i-1} \cdot \mathbf{n}}, \\ &\mathcal{Y}_p \to \mathcal{Y}_p^j = \frac{\mathbf{b}^j \cdot \mathbf{e}_p}{1 - \mathbf{b}^j \cdot \mathbf{n}} - \frac{\mathbf{b}^{j-1} \cdot \mathbf{e}_p}{1 - \mathbf{b}^{j-1} \cdot \mathbf{n}}. \end{aligned} \tag{90}$$

Similar substitutions must be performed for the functions M_{pq}, N_{pq}, X, and Z_p.

6.1. 2-4 piecewise loop

As the first example, we consider the chiral string loop shown in Fig. 1. In this example, the a-loop consists of 2 segments and lies along the z axis. One kink of the a-loop is positioned at the origin ($\xi = 0$) and the other kink ($\xi = \pi$) has the coordinates $\pi(\cos\alpha, 0, \sin\alpha)$. The positions of the b-loop kinks are as follows: the first kink at $\eta = 0$ has the coordinates $(\pi k/2\sqrt{2})(1, 0, 0)$, the second kink at $\eta = \pi/2$ has the coordinates $(\pi k/2\sqrt{2})(0, 1, 0)$, the third kink at $\eta = \pi$ has the coordinates $(\pi k/2\sqrt{2})(-1, 0, 0)$, and the position of the fourth kink at $\eta = 3\pi/2$ is given by $(\pi k/2\sqrt{2})(0, -1, 0)$. We call this loop the 2-4 piecewise loop. The dependence of the radiated gravitational and electromagnetic energy on the mode number l is shown in Fig. 2 for the 2-4 piecewise loop with $\alpha = \pi/2$. The decrease of the radiated energy with the mode number l is more pronounced for the larger current, as it should be physically, because the maximal speed of the string decreases as the current increases. In Fig. 3, the dependence of the total radiated energy on the parameter k is shown for $\alpha = \pi/2$. We can see a monotonic increase of the gravitational energy radiation with k (i. e., with the decrease of the string current). At the same time, the electromagnetic energy radiated by the string has a maximum near $k \sim 0.9$. The value of $k = k^{gr}_{max}$ at which the maximum for the gravitational radiation rate is reached is exactly 1, and for the electromagnetic radiation rate, $k^{em}_{max} \sim 0.9$.

The corresponding rates for the angular momentum as a function of the mode number are shown in Fig. 4. For the electromagnetic radiation, we can also see weak oscillations of the angular momentum rate in addition to the overall decrease of the radiated angular momentum with the mode number.

The total angular momentum radiation to electromagnetic and gravitational waves is shown in Fig. 5. The graphs for the angular momentum rates look very similar to the graphs for the energy radiation. The corresponding gravitational radiation rates increase monotonically with k and the electromagnetic radiation of momentum has maxima near $k = 0.9$. Using the general expressions for gravitational and electromagnetic radiation in Eqs. (32), (33), (63), and (64), we can easily calculate the coefficients K in the case of large currents. For $\alpha = \pi/2$, we find that $K^{gr}_E = 28.36$, $K^{gr}_L = 1.41$, $K^{gr}_E = 4$, and $K^{gr}_L = 0.25$. The radiated gravitational $\dot{E}^{gr}$ and electromagnetic $\dot{E}^{em}$ powers are approximately equal to Kk^2 in accordance with Eq. (75).

Durrer [14] found that for some particular class of

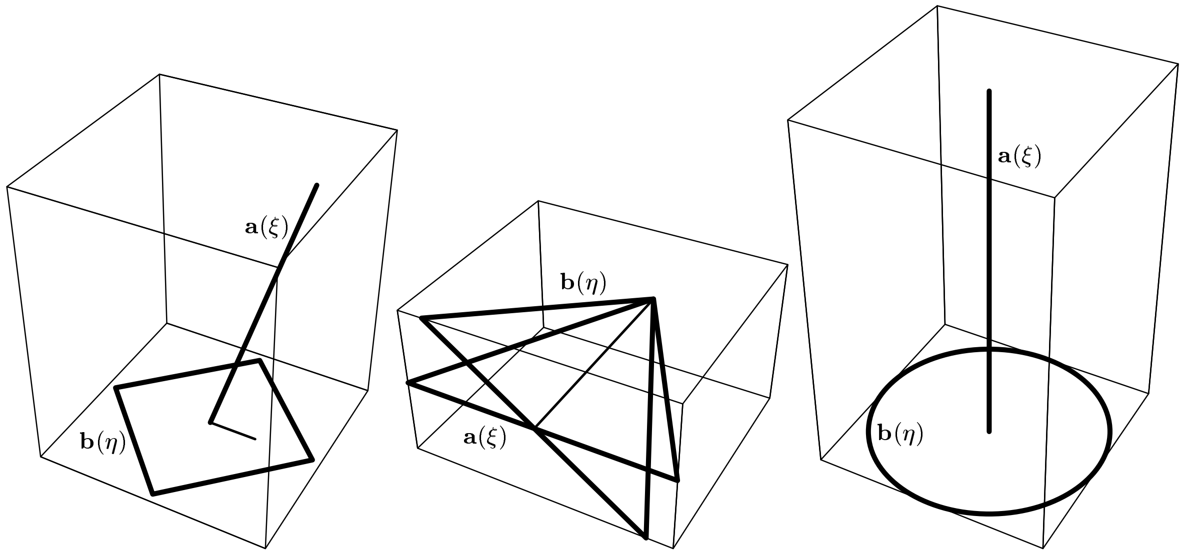

Fig. 1. Schematic view of the vector functions $\mathbf{a}(\xi)$ and $\mathbf{b}(\eta)$ for radiating loop examples considered in Secs. 6.1, 6.2, and 6.3

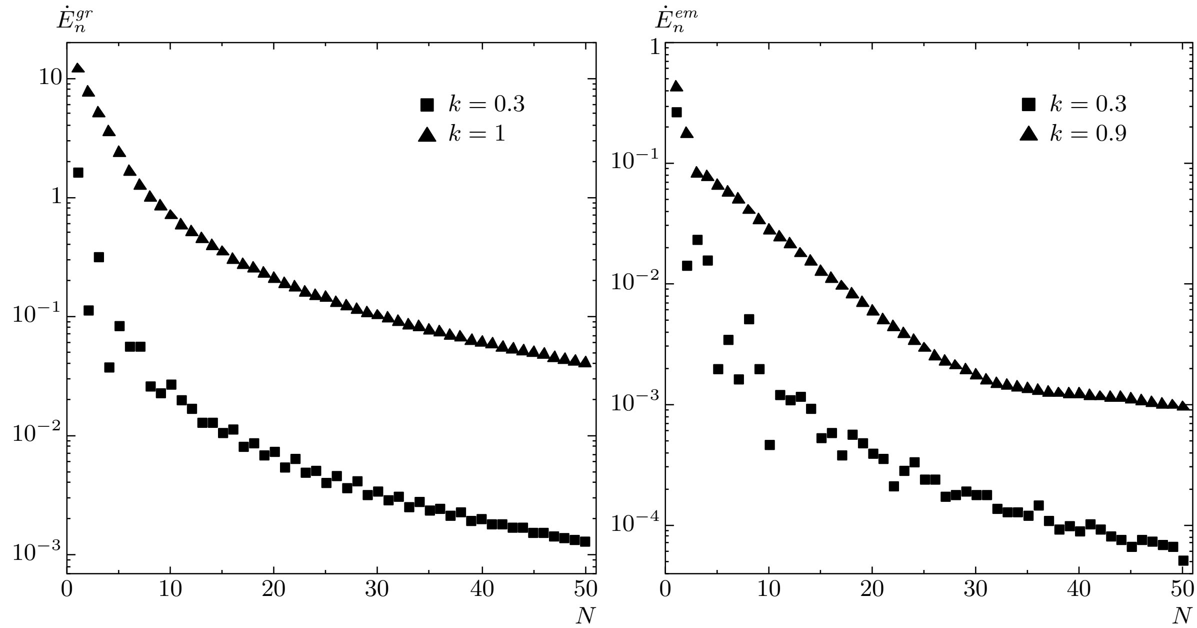

Fig. 2. Radiated gravitational energy rate (left graph), $\dot{E}_n^{gr}$ in the units $G\mu^2$ and the electromagnetic energy rate (right graph) $\dot{E}_n^{em}$ in the units $q^2\mu$. For the 2-4 kinky loop, the energy radiation is drawn as a logarithmic function of the mode number N for different values of the parameter k

ordinary cosmic string loops, the radiated angular momentum $\dot{\mathbf{L}}^{gr}$ is antiparallel to the stationary angular momentum $\mathbf{L}_{st}$ of the loop. This implies that the angular momentum of the loops always decreases with time due to gravitational radiation. Our results for the angular momentum radiation to electromagnetic and gravitational waves for string loops with the chiral current agree with the results of Durrer in general. The chiral loops considered in this paper also lose angular momentum with time. But in contrast

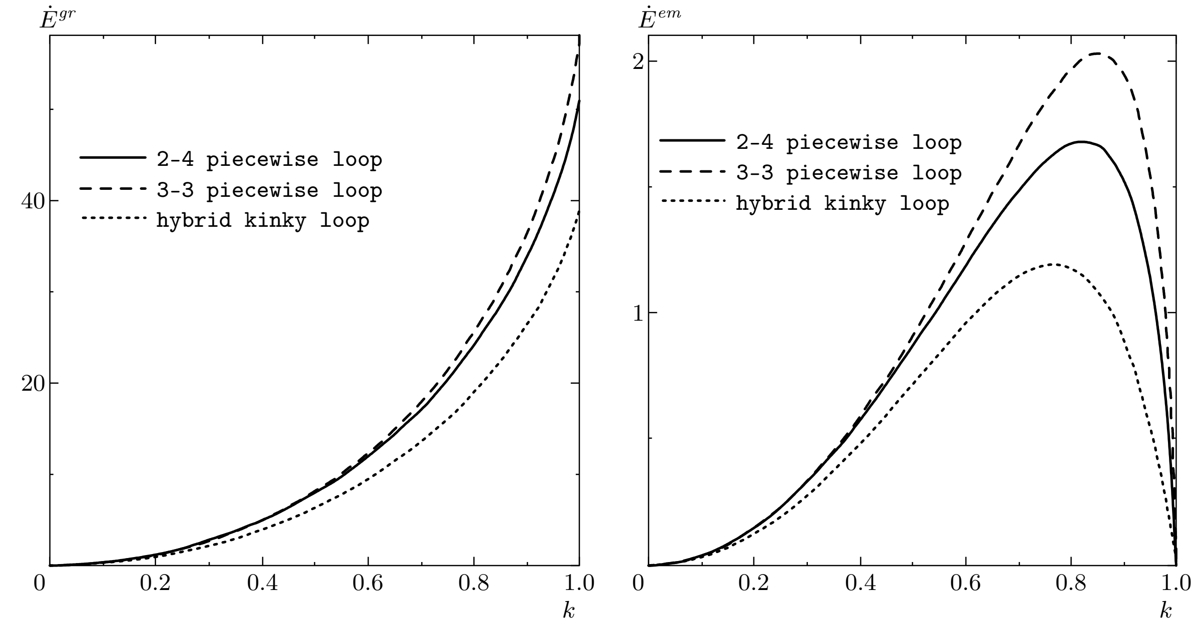

Fig. 3. The total radiated gravitational and electromagnetic energy rates $\dot{E}^{gr}$ in the units $G\mu^2$ (left graph) and $\dot{E}^{em}$ in the units $q^2\mu$ (right graph) correspondingly for the 2-4, 3-3 piecewise and hybrid kinky loops as a function of the parameter k. The following parameters are chosen: $\alpha = \pi/2$, $\beta = \pi/2$, $\gamma = 0$

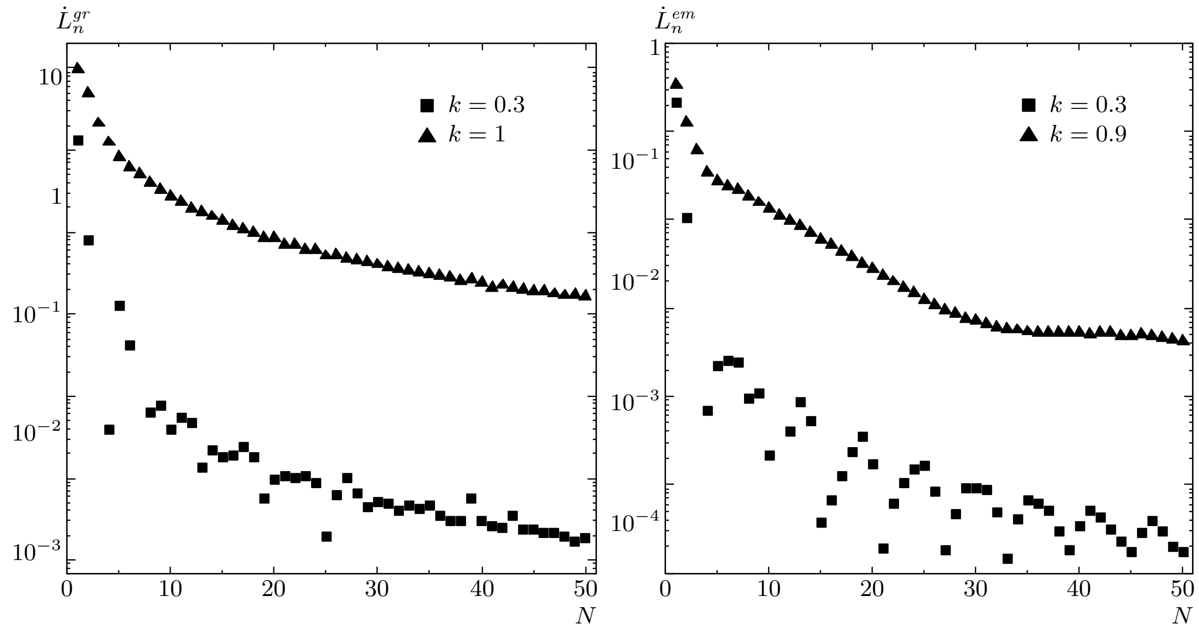

Fig. 4. The angular momentum $\dot{E}_n^{gr}$ radiated to gravitational waves in the units $G\mu^2$ (left graph) and the angular momentum $\dot{E}_n^{em}$ radiated to electromagnetic waves in the units $q^2\mu$ (right graph). For the 2-4 kinky loop, the energy radiation is drawn as a logarithmic function of the mode number N for different values of parameter k

to the examples considered by Durrer, we found that for some configurations of chiral loops, $\dot{\mathbf{L}}^{gr}$ and $\dot{\mathbf{L}}^{em}$ are not exactly antiparallel to the total angular momentum of the loop $\mathbf{L}_{st}$, but deviate by a small angle. In Table 1, the values $\varepsilon^{gr} = (\dot{\mathbf{L}}^{gr} \cdot \mathbf{L}_{st})/|\dot{\mathbf{L}}^{gr}||\mathbf{L}_{st}|$ and $\varepsilon^{em} = (\dot{\mathbf{L}}^{em} \cdot \mathbf{L}_{st})/|\dot{\mathbf{L}}^{em}||\mathbf{L}_{st}|$ determining the angle be-

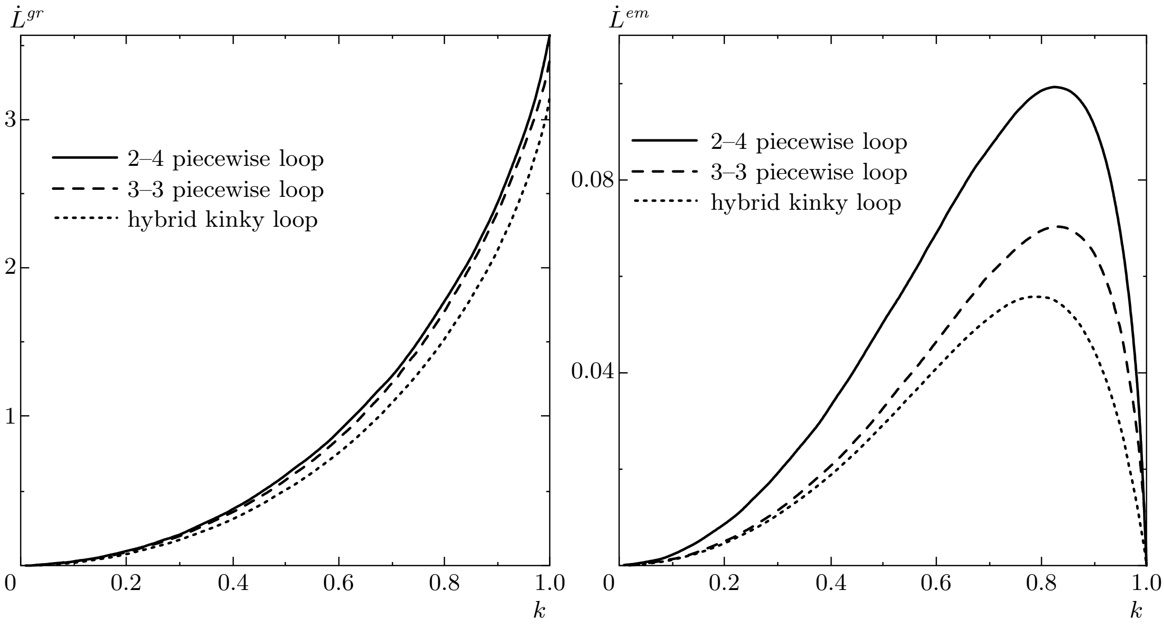

Fig. 5. The total angular momentum radiated to gravitational and electromagnetic waves, $\dot{L}^{gr}$ in the units $G\mu^2$ and $\dot{L}^{em}$ in the units $q^2\mu$, respectively, for the 2-4, 3-3 piecewise and hybrid kinky loops as a function of the parameter k. For the 2-4 loop, $\alpha = \pi/2$, for 3-3 loop, $\beta = \pi/2$, and for the hybrid loop, $\gamma = 0$

The cosine of the angle between $\dot{\mathbf{L}}^{gr}$ and $\mathbf{L}_{st}$ and between $\dot{\mathbf{L}}^{em}$ and $\mathbf{L}_{st}$ for the 2-4 piecewise loop

	k	0.2	0.4	0.6	0.8	1.0
2-4 loop,	ε^{gr}	−0.94	−0.95	−0.95	−0.96	−0.97
$\alpha = \pi/4$	ε^{em}	−0.97	−0.99	−0.99	−0.99	–

tween $\dot{\mathbf{L}}$ and $\mathbf{L}_{st}$ are presented for the 2-4 piecewise loop with $\alpha = \pi/4$. We note that for symmetric configurations with $\alpha = \pi/2$, the angular momentum radiation $\dot{\mathbf{L}}^{gr}$, $\dot{\mathbf{L}}^{em}$ is exactly antiparallel to $\mathbf{L}_{st}$ at any k.

6.2. 3-3 piecewise loop

As the second example, we consider the two-parameter piecewise linear loop with a and b-loops consisting of three segments (Fig. 1). Positions of the a-loop kinks are given by the following coordinates: the first kink at $\eta = 0$ is at the origin, the second kink at $\eta = 2\pi/3$ has the coordinates $-(\pi/3)(\cos\beta_1, \sqrt{3}, \sin\beta_1)$, and the third kink at $\eta = 4\pi/3$ has the coordinates $(\pi/3)(\cos\beta_1, -\sqrt{3}, \sin\beta_1)$. The b-loop is given by almost the same conditions, except for the angle β_1 replaced by β_2. We call this loop the 3-3 piecewise loop. The total radiated energy rates to the gravitational and electromagnetic waves for $\beta_1 = 0$ and $\beta_2 = \pi/2$ are shown in Fig. 3. This loop also radiates momentum and angular momentum. The total angular momentum radiation rates are shown in Fig. 5. In Fig. 6, the total momentum radiation rates to electromagnetic and gravitational waves are shown for different values of the parameters β_1 and β_2. For the momentum radiation, we can see a different situation from that in the case of the energy and angular momentum radiation: for each value of k, the momentum rate has a local maximum on the interval $k \in (0, 2\pi)$.

6.3. Hybrid kinky loop

As the third example, we consider the loop with the configuration

$$\mathbf{a} = \mathbf{A}\begin{cases} \xi, & 0 \le \xi \le \pi, \\ \pi - \xi, & \pi \le \xi \le 2\pi, \end{cases} \tag{91}$$
$$\mathbf{b} = k(\sin\eta,\ -\cos\eta,\ 0).$$

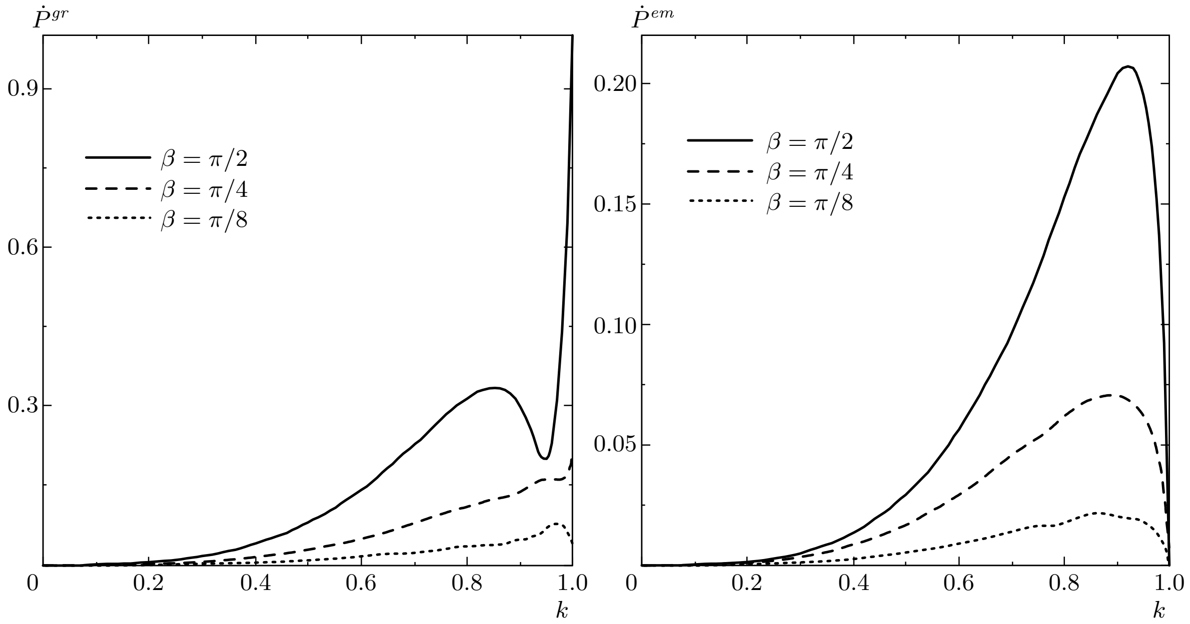

Fig. 6. The total momentum radiated to gravitational and electromagnetic waves, respectively, $\dot{P}^{gr}$ in the units $G\mu^2$ and $\dot{P}^{em}$ in the units $q^2\mu$, for the 3-3 piecewise loop with different parameters β_2 and with $\beta_1 = 0$ as a function of the parameter k. The three cases are considered with $\beta_2 = \pi/2$, $\beta_2 = \pi/4$, and $\beta_2 = \pi/8$

The b-loop in this example is a circle in the (x, y) plane and $\mathbf{A} = (\cos\gamma; 0; \sin\gamma)$ (Fig. 1). For $\gamma = \pi/2$, the gravitational and electromagnetic radiated energy rates and angular momentum rates are shown in Figs. 3 and 5. The total gravitational energy radiation for $k = 1$ coincides with the result of Allen et al. [12] ($\dot{E}^{gr} \approx 39.0 G\mu^2$).

6.4. Weakly oscillating ring loop

As the final example, we consider the radially oscillating loop,

$$\begin{aligned} \mathbf{a} &= (\cos\xi,\ -\sin\xi,\ 0),\\ \mathbf{b} &= k(\cos\eta,\ -\sin\eta,\ 0). \end{aligned} \tag{92}$$

Unfortunately, because the calculation of integrals (32) and (63) would take an enormous amount of computer time, we cannot present the results for radiation rates of oscillating rings for the entire range of currents (we note that the radiated power diverges as the current tends to zero). But in the large-current limit, the radiated power rate is easy to calculate. For loop (92), the first nonzero term in the expansion of the radiated power in k is proportional to k^2 in agreement with (74). It suffices to take only the first term in (13) and (43), the other terms are of higher orders in k. Substituting (92) in (20) and (53) and keeping the leading nonzero term at $k \ll 1$, we obtain

$$\begin{aligned} I_2(1) &= \frac{1}{2}\, e^{-i\phi}[J_2(-\sin\theta) + J_0(-\sin\theta)]\cos\theta,\\ I_3(1) &= \frac{i}{2}\, e^{-i\phi}[J_2(-\sin\theta) - J_0(-\sin\theta)],\\ Y_2(1) &= \frac{k}{2}\, e^{i\phi}\cos\theta, \quad Y_3(1) = i\frac{k}{2}\, e^{i\phi},\\ X(1) &= \frac{k}{2}\, e^{i\phi}\sin\theta. \end{aligned} \tag{93}$$

Using (93), (19), (52), (14), and (44) and integrating over the unit sphere, we next obtain the coefficients K_E^{gr} and K_E^{em},

$$\begin{aligned} K^{gr} &= \pi^2 \int\limits_0^\pi d\theta \sin\theta \Big\{ [J_2(\sin\theta) - J_0(\sin\theta)]^2 + \\ &+ 2\left[3J_2^2(\sin\theta) - J_0^2(\sin\theta)\right]\cos^2\theta + \\ &+ [J_2(\sin\theta) + J_0(\sin\theta)]^2 \cos^4\theta \Big\},\\ K^{em} &= \int\limits_0^\pi d\theta \sin^3\theta \Big\{ [J_2(\sin\theta) - J_0(\sin\theta)]^2 + \\ &+ [J_2(\sin\theta) + J_0(\sin\theta)]^2 \cos^2\theta \Big\}, \end{aligned} \tag{94}$$

which are numerically given by $K^{gr} = 4.73$ and $K^{em} = 2.28$.

Because of the symmetry of the oscillating ring, the estimations of damping time (79), coefficient k (80), and total string energy (83) become exact in the large-current limit.

7. DISCUSSION

Electromagnetic and gravitational radiation plays an important role in the evolution of the cosmic string network. This network could be produced in the early universe phase transitions and would generate large-scale structures later. Previously, the properties of cosmic string radiation were mainly studied for strings with a small current or without any current. Here, we described the radiation properties of chiral cosmic loops for the entire possible range of the currents. We succeeded in analytically summing the infinite mode series of radiation rates for periodically oscillating string loops. The expressions derived for the energy, momentum, and angular momentum rates contain four-dimensional integrals depending on loop geometry. Such an integral representation is especially convenient for numerical calculations of radiation from relativistically moving loops as compared with the method of summation of a weakly convergent mode series. To find the total rates of the radiated energy, momentum and angular momentum, the expressions obtained were integrated over the unit sphere. Applying the derived formulas to some particular examples of chiral string loop configurations, we numerically calculated the coefficients Γ in Eq. (1) as functions of k. The corresponding calculations of the radiated energy, momentum, and angular momentum rates were done for the following examples (see Fig. 1): (i) a piecewise linear kinky loop with the a-loop consisting of two straight parts and the b-loop consisting of four straight parts (2-4 piecewise loop); (ii) a piecewise linear loop such that the a- and b-loops consist of three segments each (3-3 piecewise loop); (iii) the hybrid loop in which the a-loop consists of two straight parts and the b-loop is a circle (hybrid kinky loop). For the first and second examples, the four-dimensional integrals in our expressions for radiated energy, momentum, and angular momentum become multiple sums over the kinks. These sums can be calculated analytically using symbolic computer manipulations (e. g., the «Mathematica» program packet). To find the radiation in the third example (hybrid loop), we calculated two-dimensional integrals (originating from a smooth a-loop) and summed over the kinks of the b-loop. Unfortunately, we could not perform the calculations for strings with the a and b loops being arbitrary smooth curves because calculations of the four-dimensional integrals would take an enormous amount of time of the computer used.

The total gravitational radiation energy, momentum, and angular momentum rates behave similarly: they slowly increase with k when k is sufficiently small (and respectively the current is large) and rapidly increase at large k (i. e., at small current). Overall, the gravitational radiation rates are increasing functions of k. For the electromagnetic radiation, the situation is quite different: the energy, momentum, and angular momentum losses to electromagnetic waves for all examples considered have a maximum near $k \sim 0.9$, i. e., when the current is rather small. For the examples considered, the maximum values of the coefficients Γ in (1) are approximately equal to

$$\begin{aligned} &\Gamma_E^{gr} \approx 50, \quad \Gamma_P^{gr} \approx 1, \quad \Gamma_L^{gr} \approx 3, \\ &\Gamma_E^{em} \approx 2, \quad \Gamma_P^{em} \approx 0.1, \quad \Gamma_L^{em} \approx 0.1. \end{aligned} \tag{95}$$

We also found that for some nonsymmetric examples of chiral loops, the angular momentum $\dot{\mathbf{L}}$ radiated to electromagnetic and gravitational waves is not exactly opposite to the angular momentum of the loop $\mathbf{L}_{st}$, but slightly differs from it (even when there is no current on the string), unlike in the loop examples considered by Durrer [14].

The asymptotic fading of chiral cosmic string loops into vortons was derived. It was found that the upper bounds on the gravitational and electromagnetic radiation rates of nearly stationary loops is proportional to the squared third derivative of the oscillation amplitude, see Eqs. (68), (69), and (72). We showed that if the oscillation amplitude is small ($k \ll 1$) and the current j^μ is constant along the string, the energy, momentum, and angular momentum radiation rates to gravitational and electromagnetic waves are proportional to k^2 and the proportionality coefficient depends only on the form of the loop, see Eq. (74). In some examples of chiral loops, we calculated the total radiated power in the limit of the small-amplitude oscillations. For the chiral ring with small-amplitude radial oscillations, the radiated power per solid angle $d\Omega$ for the electromagnetic and gravitational radiation is found analytically, Eq. (94). We also estimated the damping time of chiral loops (78) with small-amplitude oscillations. In the case of the gravitational radiation prevalence over the electromagnetic one, this time is $\tau^{gr} \sim L_{ph}/K^{gr}G\mu$, where K^{gr} is a numerical coefficient depending on the string geometry. The damping time due to the gravitational radiation of the chiral loops considered is by the order of magnitude longer than the lifetime of ordinary cosmic strings. On the contrary, if the electromagnetic radiation prevails, the decay time is $\tau^{em} \sim L_{ph}/K^{em}q^2$. For the radially oscillating chiral ring with a large current, expressions (80) and (83) for the temporal evolution of the total energy and the amplitude parameter k become asymptotically exact.

We can find a characteristic size of the string L_v with oscillation damping time (85) equal to the universe lifetime $t_0 \simeq 10^{18}$ s. In the case of the gravitational radiation predominance, we find

$$L_v^{gr} \sim \frac{G\mu K^{gr} t_0}{c} \approx 10^2 \mu_6 \text{ kpc} \qquad (96)$$

for $K^{gr} \sim 1$. Chiral strings with the length $L < L_v^{gr}$ (i. e., with the size of a typical galactic halo or less) therefore have enough time to fade into vortons. On the other hand, if the electromagnetic radiation prevails, we have

$$L_v^{em} \sim \frac{q^2 K^{em} t_0}{\hbar} \approx 70 q_e^2 \text{ Mpc} \qquad (97)$$

for $K^{em} \sim 1$, and the electromagnetically radiated chiral loops with the length shorter than the size of galactic clusters have therefore transformed to vortons. We can see that only sufficiently long superconducting cosmic strings oscillate up to the present time. On the contrary, small-scale chiral loops are transformed into stationary vortons due to the oscillation damping.

It is interesting to estimate the current parameter k at which the electromagnetic and gravitational radiation rates become equal. From [15], we know that at small currents, the electromagnetic radiation is given by $\dot{E}^{em} \sim q^2 \mu \sqrt{1-k}$. For the gravitational radiation of small currents, we have $\dot{E}^{gr} \sim 10^2 G\mu^2$. Comparing these two expressions, we can easily find the string current value at which these radiation rates are equal, $k \sim 1 - (10^2 G\mu/q^2)^2 \sim 1$–$10^{-4}$.

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