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THE THEORY OF SHOT NOISE IN THE SPACE-CHARGE LIMITED DIFFUSIVE CONDUCTION REGIME

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As is well known, fluctuations from a stable stationary nonequilibrium state are described by the linearized inhomogeneous Boltzmann–Langevin equation. The stationary state itself can be described by the nonlinear Boltzmann equation. The ways of its linearization sometimes seem to be not unique. We argue that there is actually a unique way to obtain a linear equation for the fluctuations. As an example, we consider an analytical theory of nonequilibrium shot noise in a diffusive conductor under the space-charge limited regime. Our approach is compared to that in Ref. [11]. We find some difference between the present theory and the approach in [11] and discuss a possible origin of the difference. We believe that it is related to the fundamentals of the theory of fluctuation phenomena in a nonequilibrium electron gas.

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1. INTRODUCTION

The present paper is devoted to the theory of shot noise in space-charge limited diffusive conduction regime. The motivation can be formulated as follows. It is well known that fluctuations from a stable stationary nonequilibrium state are described by the linearized inhomogeneous Boltzmann–Langevin equation (see, e.g., [1–7]). At the same time, the stationary state itself is described by the nonlinear Boltzmann equation. There are instances where the ways of linearization of the nonlinear Boltzmann equation seem to be not unique. We believe, however, that in each such case, there is a unique way to obtain the linearized Boltzmann equation for the fluctuations and we give general considerations to find this linearization and indicate it for the particular case treated in the present paper.

We develop a theory of nonequilibrium shot noise in a nondegenerate diffusive conductor under space-charge limited regime. This regime is extensively discussed in the literature (see, e.g., Refs. [8, 9]). The current noise under such a regime was recently studied by Monte Carlo simulation by González et al. [10]. Quite recently, the noise was analytically studied under the same conditions by Schomerus, Mishchenko, and Beenakker [11]. Their general finding was that because of the Coulomb correlation between electrons, the shot noise is reduced below the classical Poisson value. The authors of both Refs. [10] and [11] came to the conclusion that under certain conditions, the suppression factor in the nondegenerate $3D$ case can be close to $1/3$.

Later on, Nagaev [12] has shown in a special example that unlike the $1/3$ noise reduction in degenerate systems, the noise suppression by the Coulomb interaction is nonuniversal in nondegenerate systems. The noise suppression in such systems may depend on the details of the electron scattering.

We agree with the conclusion in [10, 11] that the reduction of the shot noise power in nondegenerate diffusive conductors can sometimes be close to the value $1/3$ theoretically predicted for the three-dimensional degenerate electron gas. As mentioned above, we also arrive at some conclusions that may prove important for the general theory of fluctuations in nonequilibrium systems. As is well known, the fluctuation phenomena in nonequilibrium stable systems are described by a linearized Boltzmann equation. We use the example analyzed in detail in the present paper to show that the linearization must be performed with care. In particular, there is a difference between the analytical pro-

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cedures used in Ref. [11] and in the present paper for the calculation of the shot noise power. We discuss the origin of this difference and its implications. Because the point leading to the discrepancy is very subtle, it demands a rather detailed analysis, which we perform in the present paper partly repeating the calculations in Ref. [11] with some modifications. Our starting point is the Boltzmann equation formulated for the description of the stationary state; it is then applied to the analysis of fluctuations.

2. BOLTZMANN EQUATIONS

We consider the simplest model, used in Ref. [11], for the diffusion-controlled and space-charge limited transport. As the starting point, we use the Boltzmann equation in the presence of an electric field,

$$\left(\frac{\partial}{\partial t} + \mathcal{J}_{\mathbf{p}}\right) f_{\mathbf{p}} = 0, \tag{2.1}$$

$$\mathcal{J}_{\mathbf{p}} f_{\mathbf{p}} \equiv \left(\mathbf{v}\frac{\partial}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial}{\partial \mathbf{p}} + I_{\mathbf{p}}\right) f_{\mathbf{p}}, \tag{2.2}$$

where we have introduced the collision integral $I_{\mathbf{p}}$ describing the electron scattering,

$$I_{\mathbf{p}} f_{\mathbf{p}} = \sum_{\mathbf{p}'} \left(W_{\mathbf{p}'\mathbf{p}} f_{\mathbf{p}} - W_{\mathbf{p}\mathbf{p}'} f_{\mathbf{p}'}\right) \tag{2.3}$$

(we deal with the nondegenerate statistics, and therefore, $f_{\mathbf{p}} \ll 1$).

Splitting the distribution function into the even and odd parts with respect to $\mathbf{p}$, we obtain

$$f_{\mathbf{p}}^{\pm} = \frac{1}{2}\left(f_{\mathbf{p}} \pm f_{-\mathbf{p}}\right).$$

We assume that the collision operator acting on the even (odd) part of the distribution function gives an even (odd) function. This can be the case either because of the central symmetry of the crystal itself and the scatterers or because of the possibility to use the Born approximation in calculating the scattering probability. The first split equation is

$$\frac{\partial f_{\mathbf{p}}^{-}}{\partial t} + \mathbf{v}\frac{\partial f_{\mathbf{p}}^{+}}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f_{\mathbf{p}}^{+}}{\partial \mathbf{p}} = -I_{\mathbf{p}} f_{\mathbf{p}}^{-}. \tag{2.4}$$

Being interested in relatively small frequencies of fluctuations $\omega\tau_{\mathbf{p}} \ll 1$, where $\tau_{\mathbf{p}}$ is the characteristic value of $I_{\mathbf{p}}^{-1}$, we can neglect the time derivative and express $f_{\mathbf{p}}^{-}$ as

$$f_{\mathbf{p}}^{-} = -I_{\mathbf{p}}^{-1}\left(\mathbf{v}\frac{\partial f_{\mathbf{p}}^{+}}{\partial \mathbf{r}} + e\mathbf{E}\cdot\mathbf{v}\frac{\partial f_{\mathbf{p}}^{+}}{\partial \varepsilon_{\mathbf{p}}}\right). \tag{2.5}$$

Inserting this expression into the second split equation for $f_{\mathbf{p}}^{+} \approx f(\varepsilon, \mathbf{r}, t)$ and averaging over the constant-energy surface in the quasimomentum space, we arrive at

$$\begin{aligned}\nu(\varepsilon)\frac{\partial f}{\partial t} - \left(\frac{\partial}{\partial x_\alpha} + eE_\alpha\frac{\partial}{\partial \varepsilon}\right)\nu(\varepsilon)D_{\alpha\beta}(\varepsilon) \times \\ \times \left(\frac{\partial}{\partial x_\beta} + eE_\beta\frac{\partial}{\partial \varepsilon}\right) f = \\ = -\sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) I_{\mathbf{p}}^{(inel)} f,\end{aligned} \tag{2.6}$$

where the term in the right-hand side describes the inelastic collisions, while the density of states $\nu(\varepsilon)$ and the diffusion tensor $D_{\alpha\beta}(\varepsilon)$ are defined as

$$\begin{aligned}\nu(\varepsilon)D_{\alpha\beta}(\varepsilon) = \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}) v_\alpha I_{\mathbf{p}}^{-1} v_\beta, \\ \nu(\varepsilon) = \sum_{\mathbf{p}} \delta(\varepsilon - \varepsilon_{\mathbf{p}}).\end{aligned} \tag{2.7}$$

The electric field obeys the Poisson equation

$$\begin{aligned}\kappa\nabla\mathbf{E} = 4\pi e\left[n(\mathbf{r}, t) - n^{eq}\right], \\ n(\mathbf{r}, t) = \int_0^\infty d\varepsilon \nu(\varepsilon) f(\varepsilon, \mathbf{r}, t),\end{aligned} \tag{2.8}$$

where κ is the dielectric susceptibility and n^{eq} is the equilibrium concentration (equal to the concentration of donors). In what follows, we neglect n^{eq} compared to the nonequilibrium concentration n.

The part of the distribution function contributing to the current consists of two terms that are proportional to the spatial and energy derivatives of $f(\varepsilon, \mathbf{r}, t)$ respectively,

$$\begin{aligned}j_\alpha = e\sum_{\mathbf{p}} \mathbf{v} f_{\mathbf{p}}^{-} = \\ = -e\nu(\varepsilon)D_{\alpha\beta}(\varepsilon)\left(\frac{\partial}{\partial x_\beta} + eE_\beta\frac{\partial}{\partial \varepsilon}\right) f.\end{aligned} \tag{2.9}$$

We consider the case where $D\tau_\varepsilon \gg L^2$, where L is the sample length and τ_ε is the energy relaxation time (of the order $\left[I_{\mathbf{p}}^{(inel)}\right]^{-1}$). In the right-hand side of Eq. (2.6), we can then omit the term that describes the energy relaxation. Under the same conditions, we obtain the Boltzmann equation for the fluctuations of the distribution function (we remind the reader that here we consider low-frequency fluctuations with

$$\omega \ll I_{\mathbf{p}} \approx 1/\tau_{\mathbf{p}},$$

where $\tau_{\mathbf{p}}$ is the characteristic time of elastic collisions),

$$\left(\frac{\partial}{\partial x_\alpha}+eE_\alpha\frac{\partial}{\partial\varepsilon}\right)\delta j_\omega^\alpha+e\delta E_\omega^\alpha\frac{\partial}{\partial\varepsilon}j_\alpha = ey_\omega(\varepsilon,x), \quad (2.10)$$

$$\delta j_\omega^\alpha = e\sum_{\mathbf{p}} v_\alpha\delta f_{\mathbf{p}}^- = g_\omega^\alpha - e\nu(\varepsilon)D_{\alpha\beta}(\varepsilon)\times$$
$$\times\left(\left[\frac{\partial}{\partial x_\beta}+eE_\beta\frac{\partial}{\partial\varepsilon}\right]\delta f_\omega + e\delta E_\omega^\beta\frac{\partial}{\partial\varepsilon}f\right), \quad (2.11)$$

and the source of the current fluctuations g_ω^α is related to the Langevin forces $y_{\mathbf{p}}^\omega$ as

$$g_\omega^\alpha = e\sum_{\mathbf{p}}\delta(\varepsilon-\varepsilon_{\mathbf{p}})v_\alpha I_{\mathbf{p}}^{-1}y_{\mathbf{p}}^\omega, \quad (2.12)$$

$$y_\omega(\varepsilon,x) = \sum_{\mathbf{p}}\delta(\varepsilon-\varepsilon_{\mathbf{p}})y_{\mathbf{p}}^\omega = 0. \quad (2.13)$$

The last equality is a consequence of the elasticity of scattering, which leads to the particle conservation within the constant-energy surface in the quasimomentum space.

The correlation function of the Langevin forces is well known [7],

$$\langle y_{\mathbf{p}}(\mathbf{r})y_{\mathbf{p}'}(\mathbf{r}')\rangle_\omega = (\mathcal{J}_{\mathbf{p}}+\mathcal{J}_{\mathbf{p}'})\delta_{\mathbf{rr}'}\delta_{\mathbf{pp}'}f_{\mathbf{p}}. \quad (2.14)$$

Integrating Eq. (2.10) over ε, we obtain the continuity equation

$$A\frac{d}{dx}\int_0^\infty d\varepsilon\,\delta j_\omega(\varepsilon,x) = \frac{d}{dx}\delta J_\omega(x) = 0, \quad (2.15)$$

which implies that the low-frequency current fluctuations are spatially homogeneous.

3. THE DISTRIBUTION FUNCTION

We consider a semiconductor with a uniform cross-section A connecting two identical metallic electrodes. The length L of the sample is assumed to be much larger than the elastic scattering length l and much smaller than the inelastic one. We use the $1D$ versions of the Boltzmann equations describing the distribution function evolution along the dc current direction.

To obtain the stationary solution of Eq. (2.6) in the accepted approximation, we rewrite it as

$$\left(\frac{\partial}{\partial x}+eE\frac{\partial}{\partial\varepsilon}\right)j(\varepsilon,x) = \delta(x)j(\varepsilon). \quad (3.1)$$

We here assume that the current density at $x=0$, $j(\varepsilon)$, is nonvanishing only for $\varepsilon>0$. In the absence of tunneling, $j(\varepsilon)$ must have the property that

$$j(\varepsilon)\to 0 \quad \text{as} \quad T\to 0 \quad (3.2)$$

at the contact $x=0$, with T being the temperature. This condition must be valid, irrespective of whether a Schottky barrier or an Ohmic contact occurs. Evidently, the total current J given by Eq. (3.3) below must have the same property.

The solution of Eq. (3.1) is a function of the total energy $\mathcal{E}$,

$$\mathcal{E} = \varepsilon + U(x),$$

where

$$U(x) = e\varphi(x) - e\varphi(0).$$

It can be found using, e.g., the inverse differential operator

$$\frac{1}{\partial_x}\Phi(x) = \int_0^x d\xi\,\Phi(\xi).$$

We have

$$j(\varepsilon,x) = \frac{1}{\partial_x+eE(x)\partial_\varepsilon}\delta(x)j(\varepsilon) =$$
$$= \exp[e\varphi(x)\partial_\varepsilon]\frac{1}{\partial_x}\exp[-e\varphi(x)\partial_\varepsilon]\delta(x)j(\varepsilon) = j(\mathcal{E})$$

and $j(\varepsilon,x)$ takes nonzero values at a given x only if $\varepsilon > -U(x)$ ($\mathcal{E}\ge 0$). The total current through the sample is

$$J = A\int_0^\infty d\varepsilon\,j(\varepsilon,x) =$$
$$= A\int_{-U(x)}^\infty d\varepsilon\,j[\varepsilon+U(x)] = A\int_0^\infty d\mathcal{E}\,j(\mathcal{E}). \quad (3.3)$$

From Eq. (2.9), we now obtain

$$f(\varepsilon,x) = -\frac{1}{\partial_x+eE(x)\partial_\varepsilon}\frac{j(\varepsilon,x)}{e\lambda(\varepsilon)} + f[\varepsilon+U(x)], \quad (3.4)$$

or

$$f(\varepsilon,x) = -j[\mathcal{E}]\int_0^x d\xi\frac{1}{e\lambda[\mathcal{E}-U(\xi)]} + f[\mathcal{E}], \quad (3.5)$$

where

$$\lambda(\varepsilon) \equiv \nu(\varepsilon)D(\varepsilon).$$

We have taken the boundary condition at the source into account. Equation (3.5) can be rewritten as

$$f[\mathcal{E}-U(x),x] = \left[f[\mathcal{E}-U(L)] \int\limits_0^x d\xi \frac{1}{\lambda[\mathcal{E}-U(\xi)]} + \right. \left. + f(\mathcal{E}) \int\limits_x^L d\xi \frac{1}{\lambda[\mathcal{E}-U(\xi)]} \right] \times \left[\int\limits_0^L d\xi \frac{1}{\lambda[\mathcal{E}-U(\xi)]} \right]^{-1}, \quad (3.6)$$

where $j(\varepsilon)$ is expressed through the difference of the distribution functions at $x=0$ and $x=L$,

$$j(\mathcal{E}) \int\limits_0^L dx \frac{1}{e\lambda[\mathcal{E}-U(x)]} = f(\mathcal{E}) - f[\mathcal{E}-U(L)]. \quad (3.7)$$

An advantage of the form chosen for Eq. (3.6) is its physical transparency. The first term in the right-hand side gives the contribution of the right boundary and the second term gives the contribution of the left boundary. The solution clearly demonstrates that the thermally excited carriers injected from the contact at $x=L$ make a negligible contribution to the distribution function $f[\mathcal{E}-U(x),x]$, because

$$f(\mathcal{E}) \gg f[\mathcal{E}-U(L)] \quad (\mathcal{E} \geq 0)$$

for the parameter $|U(L)|/k_BT$ is assumed to be large. Neglecting this term in our solution of Eq. (3.6), we arrive at the solution already obtained in [11] by assuming absorbing boundary conditions at the current drain.

4. THE FIELD DISTRIBUTION

We use the Poisson equation to determine the self-consistent electric field that can be expressed through the obtained distribution function. We consider the values of x such that $x > x_{\overline{\varepsilon}}$, where

$$-U(x_{\overline{\varepsilon}}) \gg \overline{\mathcal{E}} \sim k_BT,$$

$$-\frac{\kappa}{4\pi e^2}\frac{d^2U}{dx^2} = \int\limits_0^\infty d\mathcal{E}\,\nu[\mathcal{E}-U(x)] \times f[\mathcal{E}-U(x),x] = \int\limits_0^\infty d\mathcal{E}\,\nu[\mathcal{E}-U(x)] j(\mathcal{E}) \times \int\limits_x^L \frac{d\xi}{e\lambda[\mathcal{E}-U(\xi)]} \approx \nu[-U(x)]\frac{J}{eA}\int\limits_x^L \frac{d\xi}{\lambda\,[-U(\xi)]}. \quad (4.1)$$

We finally obtain

$$-\frac{\kappa}{4\pi e^2}\frac{1}{\nu[-U(x)]}\frac{d^2U}{dx^2} = \frac{J}{eA}\int\limits_x^L \frac{d\xi}{\lambda\,[-U(\xi)]}. \quad (4.2)$$

We now check that for large x, this equation is consistent with the requirement of a uniform total current. Assuming

$$\nu(\varepsilon) = \nu_0 \varepsilon^{d/2-1}$$

and

$$D(\varepsilon) = D_0 \varepsilon^{s+1},$$

we integrate Eq. (2.9) over the transverse coordinates and energy, with the result

$$\frac{J}{A} = -e\frac{d}{dx}\int\limits_0^\infty d\varepsilon\,\nu(\varepsilon)D(\varepsilon)f(\varepsilon,x) + \frac{eD_0\kappa(d+2s)}{16\pi}[-U(x)]^s\frac{d}{dx}E^2(x). \quad (4.3)$$

We integrate the second term by parts and take into account that at $x > x_{\overline{\mathcal{E}}}$, we can neglect $\mathcal{E}$ compared to $|U(x)|$ and use Poisson equation (2.8). The first term in Eq. (4.3) can be simplified in the same way[1)]

$$\int\limits_{-U(x)}^\infty d\varepsilon\,\nu(\varepsilon)D(\varepsilon)f(\varepsilon,x) = \int\limits_0^\infty d\mathcal{E}\,\nu[\mathcal{E}-U(x)]D[\mathcal{E}-U(x)]f(\mathcal{E}-U(x),x) = D[-U(x)]\int\limits_0^\infty d\mathcal{E}\,\nu[\mathcal{E}-U(x)]f(\mathcal{E}-U(x),x) = D[-U(x)]\frac{\kappa}{4\pi e}\frac{d}{dx}E. \quad (4.4)$$

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1) We note that in view of Eq. (3.5), the distribution function $f(\varepsilon,x)$ takes nonzero values only for $\varepsilon > -U(x)$.

In the second equality, we used that $\mathcal{E} \ll |U(x)|$. Inserting Eq. (4.4) in Eq. (4.3), we obtain the simplified equation

$$\frac{4\pi|J|}{D_0\kappa A} = \frac{d}{dx}\left([-U]^{s+1}\frac{dE}{dx}\right) + \\ + |e|\frac{2s+d}{4}[-U]^s\frac{dE^2}{dx}. \quad (4.5)$$

It can be used to verify the self-consistency of our approach. Indeed, multiplying Eq. (4.2) by $U^{s+d/2}$ and taking the derivative, we arrive at Eq. (4.5) that has been obtained from the equation for the current. A dimensionless version of Eq. (4.5) is

$$\chi^s\left(\frac{d-2}{2}\chi'\chi'' - \chi\chi'''\right) = 1, \quad (4.6)$$

where the dimensionless potential χ is related to φ by

$$\varphi = \left(\frac{4\pi|J|L^3}{D_0\kappa A|e|^{s+1}}\right)^{1/(s+2)} \chi(x/L). \quad (4.7)$$

5. THE CURRENT AND FIELD FLUCTUATIONS

In what follows, we consider the particular cases where

$$s = 0, \quad D(\varepsilon) = D_0\varepsilon;$$

$$s = -1/2, \quad D(\varepsilon) = D_0\varepsilon^{1/2};$$

and

$$s = 1/2, \quad D(\varepsilon) = D_0\varepsilon^{3/2}.$$

We begin with investigating the case of the energy-independent scattering time, $s = 0$. This case can be related to the scattering of electrons by neutral impurities, such as hydrogen-like shallow donor and acceptor states. The scattering is analogous to the scattering of electron by a hydrogen atom [13] (with the effective Bohr radius a_B). The scattering cross-section turns out to be about $2\pi\hbar/pa_B$ times larger than the geometrical cross-section πa_B^2 (that would result in an energy-independent scattering time).

In the case of defects with deep energy levels, we encounter a short-range scattering potential with the scattering length about the atomic length. The scattering cross-section does not depend on the energy. As a result, the scattering rate is proportional to the electron density of states $\varepsilon^{1/2}$ and the diffusion coefficient $v^2\tau$ is proportional to $\varepsilon^{1/2}$, i.e., $s = -1/2$. (This is one of the main scattering mechanisms in metals because the scattering length is then determined by the screening radius, which is of the order of the interatomic distance.) The cases where $s = -1/2$ (which in particular describes elastic scattering by acoustic phonons) and $s = 1/2$ are discussed in the end of this section.

5.1. Energy-independent scattering time

Integrating Eq. (2.11) over ε, we obtain

$$\frac{1}{A}(\delta J_\omega - G_\omega) = -e\frac{d}{dx}\int_{-U(x)}^{\infty} d\varepsilon\, \nu(\varepsilon)D(\varepsilon)\delta f_\omega(\varepsilon,x) + \\ + \frac{eD_0d\kappa}{8\pi}\frac{d}{dx}E(x)\delta E_\omega(x). \quad (5.1)$$

We note that because of Eq. (2.10), the Fourier transform of the current fluctuations δJ_ω is spatially homogeneous. Here, G_ω is the current fluctuation source integrated over the energy and transverse coordinates,

$$G_\omega(x) = \int_0^\infty d\varepsilon\, d\mathbf{r}_\perp g_\omega(\varepsilon,\mathbf{r}), \quad (5.2)$$

$$\langle G(x)G(x')\rangle_\omega = \\ = e^2\int_0^\infty d\varepsilon\int_0^\infty d\varepsilon' \sum_{\mathbf{p},\mathbf{p}'}\delta(\varepsilon-\varepsilon_\mathbf{p})\delta(\varepsilon'-\varepsilon_{\mathbf{p}'})v_x{v_x}'\times \\ \times\frac{1}{I_\mathbf{p}}\frac{1}{I_{\mathbf{p}'}}\int d\mathbf{r}_\perp d\mathbf{r}'_\perp\langle y_\mathbf{p}y_{\mathbf{p}'}\rangle_\omega. \quad (5.3)$$

The part of the distribution function that is odd with respect to $\mathbf{p}\to-\mathbf{p}$ vanishes after inserting it into correlation function (2.14) of the Langevin forces and subsequently integrating over $\mathbf{p}$ and $\mathbf{p}'$. As a result, we are left with the integral of the even function

$$\langle G(x)G(x')\rangle_\omega = \delta_{xx'}\langle G^2(x)\rangle_\omega, \quad (5.4)$$

$$\langle G^2(x)\rangle_\omega = 2e^2A\int_0^\infty d\varepsilon f(\varepsilon,x)\sum_\mathbf{p}\delta(\varepsilon-\varepsilon_\mathbf{p})v_x\frac{1}{I_\mathbf{p}}v_x = \\ = 2e^2A\int_0^\infty d\varepsilon\,\nu(\varepsilon)D(\varepsilon)f(\varepsilon,x). \quad (5.5)$$

The second term in the right-hand side of Eq. (5.1) can be simplified in the same way as Eq. (4.4),

$$\int_0^\infty d\varepsilon\,\nu(\varepsilon)D(\varepsilon)\delta f_\omega(\varepsilon,x) = \\ = D(-U(x))\frac{\kappa}{4\pi e}\frac{d}{dx}\delta E_\omega, \quad (5.6)$$

and we finally obtain the equation for δE_ω

$$\frac{d}{dx}\left(U(x)\frac{d}{dx}\delta E_\omega(x)\right)+e\frac{d}{2}\frac{d}{dx}E(x)\delta E_\omega(x)= =\frac{4\pi}{AD_0\kappa}(\delta J_\omega-G_\omega). \quad (5.7)$$

In order to justify the simplification in Eq. (5.6), we now show that $\delta f_\omega(\varepsilon,x)$ is also a function taking nonzero values only at $\varepsilon>-U(x)$. Indeed, from Eq. (2.10) and Eq. (2.11), we can obtain the solutions

$$\delta j_\omega[\varepsilon-U(x),x]=\delta U_\omega(x)\frac{\partial}{\partial\varepsilon}j(\varepsilon)+\Delta j(\varepsilon)_\omega, \quad (5.8)$$

$$\delta f_\omega[\varepsilon-U(x),x]=\int_x^L d\xi\, e\delta E_\omega(\xi)\frac{\partial}{\partial\varepsilon}f[\varepsilon-U(\xi),\xi]- -\int_x^L d\xi\,\frac{g_\omega[\varepsilon-U(\xi),\xi]-\delta j_\omega[\varepsilon-U(\xi),\xi]}{e\lambda[\varepsilon-U(\xi)]}, \quad (5.9)$$

which show that δf has the aforementioned property. Here, $\Delta j(\varepsilon)$ are the fluctuations of the current at the left boundary $x=0$. The fluctuations of the distribution function $\Delta f(\varepsilon)$ at the right boundary are assumed to be zero. If we assume $\lambda(\varepsilon)$ to be a constant (independent of the energy), taking Eqs. (5.8) and (5.9) and the equation $\delta f_\omega(\varepsilon,0)=0$ into account, we immediately arrive at the result

$$\Delta J=\frac{1}{L}\int_0^L dx\int d\varepsilon g[\varepsilon-U(x),x] \quad (5.10)$$

obtained by Nagaev [12].

5.2. Comparison with the approach in Ref. [4]

We now embark on setting forth the crucial point of the paper. Equation (5.7) does not coincide with the equation for the field fluctuations obtained in [11] by directly linearizing Eq. (4.5) for $s=0$,

$$\frac{d}{dx}\left[\delta U_\omega(x)\frac{d}{dx}E(x)\right]+\frac{d}{dx}\left[U(x)\frac{d}{dx}\delta E_\omega(x)\right]+ +e\frac{d}{2}\frac{d}{dx}E(x)\delta E_\omega(x)=\frac{4\pi}{AD_0\kappa}(\delta J_\omega-G_\omega). \quad (5.11)$$

The origin of this discrepancy must be understood.

First, we temporarily adopt the scheme of Ref. [11] and reconsider Eq. (4.3) for the current

$$\frac{J}{A}=-e\frac{d}{dx}\int_{-U(x)}^{\infty}d\varepsilon\,\nu(\varepsilon)D(\varepsilon)f(\varepsilon,x)+ +\frac{3}{2}D_0e^2E(x)\int_{-U(x)}^{\infty}d\varepsilon\nu(\varepsilon)f(\varepsilon,x). \quad (5.12)$$

For the total current (the d.c. current plus fluctuations), the equation reads

$$\frac{J+\delta J-G}{A}=-e\frac{d}{dx}\times \times\int_{-U(x)-\delta U(x)}^{\infty}d\varepsilon\,\nu(\varepsilon)D(\varepsilon)[f(\varepsilon,x)+\delta f(\varepsilon,x)]+ +\frac{3}{2}D_0e^2[E(x)+\delta E(x)]\times \times\int_{-U(x)-\delta U(x)}^{\infty}d\varepsilon\nu(\varepsilon)[f(\varepsilon,x)+\delta f(\varepsilon,x)]. \quad (5.13)$$

Taking Eq. (5.12) into account, we obtain the linearized equation

$$\frac{\delta J-G}{A}=-e\frac{d}{dx}\int_{-U(x)}^{\infty}d\varepsilon\,\nu(\varepsilon)D(\varepsilon)\delta f(\varepsilon,x)+ +\frac{3}{2}D_0e^2E(x)\left\{\int_{-U(x)}^{\infty}d\varepsilon\nu(\varepsilon)\delta f(\varepsilon,x)+ +\delta U\frac{\delta}{\delta U(x)}\int_{-U(x)}^{\infty}d\varepsilon\nu(\varepsilon)f(\varepsilon,x)\right\}+ +\frac{3}{2}D_0e^2\delta E(x)\int_{-U(x)}^{\infty}d\varepsilon\nu(\varepsilon)f(\varepsilon,x)- -e\frac{d}{dx}\delta U\frac{\delta}{\delta U(x)}\int_{-U(x)}^{\infty}d\varepsilon\,\nu(\varepsilon)D(\varepsilon)f(\varepsilon,x). \quad (5.14)$$

If one linearized the Poisson equation in the spirit of Ref. [11] one would see that the term in the curly brackets in Eq. (5.14) would coincide with $(\kappa/4\pi e)(d\delta E/dx)$, and therefore,

$$\frac{\kappa}{4\pi e}\frac{d\delta E}{dx} = \int\limits_{-U(x)}^{\infty} d\varepsilon\nu(\varepsilon)\delta f(\varepsilon,x) + \\ + \delta U\frac{\delta}{\delta U(x)}\int\limits_{-U(x)}^{\infty} d\varepsilon\nu(\varepsilon)f(\varepsilon,x). \quad (5.15)$$

Simplifying the first, third, and fourth terms in the right-hand side of Eq. (5.14) with the help of Eq. (5.6) and inserting $(\kappa/4\pi e)(d\delta E/dx)$ instead of the term in the curly brackets, we arrive at

$$\frac{\delta J - G}{A} = e\frac{d}{dx}\left(D_0U(x)\frac{\kappa}{4\pi}\frac{d\delta E}{dx}\right) + \\ + \frac{3}{2}D_0e\frac{\kappa}{4\pi}\frac{d}{dx}E\delta E + \\ + eD_0\frac{\kappa}{4\pi}\frac{d}{dx}\delta U\frac{\delta}{\delta U}\left[U\frac{dE}{dx}\right]. \quad (5.16)$$

We can see that the last term in the right-hand side of this equation coincides with the first term in the left-hand side of Eq. (5.11). To avoid confusion, we note that we believe Eq. (5.15) to be also wrong. We have written it here only for the detailed comparison with the approach in Ref. [11]. We believe that the correct Poisson equation for the fluctuation field is

$$\frac{\kappa}{4\pi e}\frac{d\delta E}{dx} = \int\limits_{-U(x)}^{\infty} d\varepsilon\nu(\varepsilon)\delta f(\varepsilon,x). \quad (5.17)$$

In Eq. (4.3) for the d.c. current, we now add the terms that actually vanish because they are proportional to the integrals of the distribution function over ε with the upper limit $-U(x)$, whereas the distribution function $f(\varepsilon,x)=0$ for $\varepsilon<-U(x)$. The point is that when we calculate the fluctuations by the replacement

$$U(x)\to U(x)+\delta U(x),$$

they give a nonvanishing result. We have

$$\frac{J}{A} = -e\frac{d}{dx}\int\limits_{0}^{-U(x)} d\varepsilon\,\nu(\varepsilon)D(\varepsilon)f(\varepsilon,x) - \\ - e\frac{d}{dx}\int\limits_{-U(x)}^{\infty} d\varepsilon\,\nu(\varepsilon)D(\varepsilon)f(\varepsilon,x) + \\ + \frac{3}{2}D_0e^2E(x)\int\limits_{0}^{-U(x)} d\varepsilon\nu(\varepsilon)f(\varepsilon,x) + \\ + \frac{3}{2}D_0e^2E(x)\int\limits_{-U(x)}^{\infty} d\varepsilon\nu(\varepsilon)f(\varepsilon,x). \quad (5.18)$$

Rewriting this equation for the total current, we obtain

$$\frac{J+\delta J-G}{A} = \\ = -e\frac{d}{dx}\int\limits_{0}^{-U(x)-\delta U} d\varepsilon\,\nu(\varepsilon)D(\varepsilon)[f(\varepsilon,x)+\delta f(\varepsilon,x)] - \\ - e\frac{d}{dx}\int\limits_{-U(x)-\delta U(x)}^{\infty} d\varepsilon\,\nu(\varepsilon)D(\varepsilon)[f(\varepsilon,x)+\delta f(\varepsilon,x)] + \\ + \frac{3}{2}D_0e^2[E(x)+\delta E(x)]\times \\ \times\int\limits_{0}^{-U(x)-\delta U(x)} d\varepsilon\nu(\varepsilon)[f(\varepsilon,x)+\delta f(\varepsilon,x)] + \\ + \frac{3}{2}D_0e^2[E(x)+\delta E(x)]\times \\ \times\int\limits_{-U(x)-\delta U(x)}^{\infty} d\varepsilon\nu(\varepsilon)[f(\varepsilon,x)+\delta f(\varepsilon,x)]. \quad (5.19)$$

Linearizing this equation and using relations similar to

$$\delta U(x)\frac{\delta}{\delta U(x)}\int\limits_{-U(x)}^{\infty} d\varepsilon\,\nu(\varepsilon)D(\varepsilon)f(\varepsilon,x) = \\ = -\int\limits_{-U(x)}^{-U(x)-\delta U(x)} d\varepsilon\,\nu(\varepsilon)D(\varepsilon)f(\varepsilon,x), \quad (5.20)$$

we arrive at Eq. (5.1) that has been derived above. We see that the contributions to Eq. (5.19) that are linear in δU cancel because of the terms that vanish in the equation for the d.c. current but must be taken into account in considering fluctuations. This is why the linearization of Eq. (4.5) leads to Eq. (5.11) that we believe to be wrong because it does not take all the sources of fluctuations into account, or in other words, all the terms in Eq. (5.18) containing $U(x)$.

The solution of Eq. (5.7) with the boundary conditions

$$E(x)\delta E_\omega(x)\,|_{x\to 0} \to 0, \quad U(x)\frac{d}{dx}\delta E_\omega(x)\Big|_{x\to 0} \to 0 \qquad (5.21)$$

is given by

$$-\frac{AD_0\kappa}{4\pi}\delta E_\omega(x) = U^{d/2}(x)\times \times\left[C+\int\limits_0^x \frac{d\xi}{U^{d/2+1}(\xi)}\int\limits_0^\xi d\eta\,(\delta J_\omega - G(\eta)_\omega)\right], \qquad (5.22)$$

where C is the integration constant. Requiring a nonfluctuating applied voltage

$$\int\limits_0^L dx\,\delta E_\omega = 0,$$

we obtain from Eq. (5.22) that the constant is

$$C = \int\limits_0^L dx\left(\frac{\psi(x)}{\psi(L)}-1\right)\frac{1}{U^{d/2+1}(x)}\times \times\int\limits_0^x d\xi(\delta J_\omega - G_\omega(\xi)), \qquad (5.23)$$

where

$$\psi(x) = \int\limits_0^x d\xi\,U^{d/2}(\xi). \qquad (5.24)$$

We now require

$$\frac{d}{dx}\delta E_\omega(x)\Big|_{x=L} = 0 \qquad (5.25)$$

at the right boundary and obtain

$$\delta J_\omega = \frac{1}{Z}\int\limits_0^L dx\,\Pi(x)G_\omega(x), \qquad (5.26)$$

where

$$Z = L + \frac{dU'(L)U^{d/2}(L)}{2\psi(L)}\int\limits_0^L dx\,\frac{x\psi(x)}{U^{d/2+1}(x)}, \qquad (5.27)$$

$$\Pi(x) = 1 + \frac{dU'(L)U^{d/2}(L)}{2\psi(L)}\int\limits_x^L d\xi\,\frac{\psi(\xi)}{U^{d/2+1}(\xi)}. \qquad (5.28)$$

The noise power P is then given by

$$P = \frac{2}{Z^2}\int\limits_0^L dx\,\Pi^2(x)\langle G^2(x)\rangle_\omega. \qquad (5.29)$$

In accordance with Eq. (5.5), we have

$$\langle G^2(x)\rangle_\omega = 2e^2A\int\limits_0^\infty d\varepsilon\,\nu(\varepsilon)D(\varepsilon)f(\varepsilon,x) = = 2e^2AD_0U(x)\frac{\kappa}{4\pi e}\frac{d^2U}{dx^2}. \qquad (5.30)$$

We finally arrive at

$$P = \frac{4AD_0\kappa}{4\pi Z^2}\int\limits_0^L dx\,\Pi^2(x)U(x)\frac{d^2U}{dx^2}. \qquad (5.31)$$

The potential distribution can be found following the method in Ref. [11], i.e., by solving Eq. (4.5) with boundary condition (4.2) at $x = L$. Using Eqs. (5.24), (5.27), (5.28), and (5.31), we calculate the suppression factor $P/P_{Poisson}$. For physically relevant different values of the dimensionality d, we obtain

$$\frac{P}{P_{Poisson}} = \begin{cases} 0.3188 & \text{for} \quad d=3, \\ 0.4512 & \text{for} \quad d=2, \\ 0.682 & \text{for} \quad d=1. \end{cases} \qquad (5.32)$$

In this particular case, our results therefore differ from those calculated in Refs. [11] both analytically (which is of principal importance in our opinion) and numerically (although in this particular case, the difference is not great). Naturally, there is essentially no difference from the results calculated within an ensemble Monte Carlo scheme in Ref. [10].

5.3. Energy-dependent scattering time

We here calculate the noise power for $s = \pm 1/2$ and $d = 3$. The equation for the fluctuations is

$$-\frac{4\pi}{\kappa D_0 A}(\delta J_\omega - G_\omega) = \frac{d}{dx}\left[(-U)^{s+1}\frac{d\delta E_\omega}{dx}\right] - - e\frac{2s+d}{2}(-U)^s\frac{d}{dx}\left(E\delta E_\omega\right). \qquad (5.33)$$

Introducing the dimensionless potential χ by Eq. (4.7) and the fluctuation of the field ΔE by

$$\delta E(x) = \frac{1}{L}\left(\frac{4\pi|J|L^3}{\kappa D_0 A|e|^{s+1}}\right)^{1/(s+2)}\Delta E\left(\frac{x}{L}\right), \qquad (5.34)$$

we can rewrite Eq. (5.33) as

$$\Delta E'' + \left(1 - \frac{d}{2}\right)\frac{\chi'}{\chi}\Delta E' - \left(s + \frac{d}{2}\right)\frac{\chi''}{\chi}\Delta E = = \frac{1}{\chi^{s+1}}\frac{(G - \delta J)}{|J|}. \quad (5.35)$$

Setting $s = -1/2$ and $d = 3$, we obtain

$$\Delta E'' - \frac{1}{2}\frac{\chi'}{\chi}\Delta E' - \frac{\chi''}{\chi}\Delta E = \chi^{-1/2}\frac{(G - \delta J)}{|J|}. \quad (5.36)$$

This equation differs from that derived in Ref. [11], while the equation for the potential χ coincides with

$$\frac{1}{2\chi^{1/2}}\chi'\chi'' - \chi^{1/2}\chi''' = 1. \quad (5.37)$$

To calculate the Green's function of Eq. (5.36), we need the function $\psi_1(x)$ obeying the homogeneous equation

$$\psi_1'' - \frac{1}{2}\frac{\chi'}{\chi}\psi_1' - \frac{\chi''}{\chi}\psi_1 = 0 \quad (5.38)$$

and satisfying the boundary condition $\psi_1'|_{x=0} = 0$. The second function ψ_2 satisfying the boundary condition $\psi_2'|_{x=L} = 0$ can be expressed through the functions χ and ψ_1 as

$$\psi_2(x) = -\psi_1\left[\frac{\chi^{1/2}(1)}{\psi_1(1)\psi_1'(1)} + \int\limits_x^1 d\xi\,\frac{\chi^{1/2}(\xi)}{\psi_1^2(\xi)}\right]. \quad (5.39)$$

The solution of Eq. (5.36) can be written using the Green's function

$$G(x,x') = \frac{1}{\chi^{1/2}(x')}\times \times\left[\theta(x-x')\psi_1(x')\psi_2(x) + \theta(x'-x)\psi_1(x)\psi_2(x')\right] \quad (5.40)$$

as

$$\Delta E = \int\limits_0^1 dx'\,G(x,x')\frac{(G(x') - \delta J)}{\chi^{1/2}(x')|J|}. \quad (5.41)$$

Requiring a nonfluctuating applied voltage, we obtain

$$\delta J = \frac{1}{Z}\int\limits_0^1 dx\,\frac{G(x)}{\chi(x)}\Pi(x), \quad (5.42)$$

where

$$\Pi(x) = \psi_1(x)\int\limits_x^1 d\xi\,\psi_2(\xi) + \psi_2(x)\int\limits_0^x d\xi\,\psi_1(\xi), \quad (5.43)$$

$$Z = \int\limits_0^1 dx\,\frac{\Pi(x)}{\chi(x)}. \quad (5.44)$$

Expressing the correlation function $\langle G^2(x)\rangle$ through χ, we obtain the power supression factor

$$\frac{P}{P_{Poisson}} = \frac{2}{Z^2}\int\limits_0^1 dx\,\frac{\chi''(x)}{\chi^{3/2}(x)}\Pi^2(x) \quad (5.45)$$

for the shot noise. We determine the potential χ following Ref. [11] and numerically find ψ_1 from Eq. (5.38). The functions ψ_2, Π and the constant Z can be found from Eqs. (5.39), (5.43), and (5.44). The suppression factor can be evaluated as

$$\frac{P}{P_{Poisson}} = 0.4257, \quad (5.46)$$

which is about 10 % larger than the result obtained in Ref. [11]. The numerical simulation result in [10] for $s = -1/2$ is

$$\frac{P}{P_{Poisson}} = 0.42\text{–}0.44. \quad (5.47)$$

This interval is noticeably nearer to the value given by Eq. (5.46) than the result in Ref. [11].

In the case where $s = 1/2$, the suppression factor can be evaluated as

$$\frac{P}{P_{Poisson}} = 0.1974, \quad (5.48)$$

which is slightly smaller than the result in Ref. [10].

6. CONCLUSIONS

In summary, we have developed an analytical theory of shot noise in a diffusive conductor under the space charge limited regime. We find that the present theory is different from the approach developed earlier and indicate a possible origin of the difference.

We now make several concluding remarks. The calculated nonequilibrium shot noise power in a nondegenerate diffusive semiconductor for two types of physically relevant elastic scattering mechanisms turned out to be very close to the ones obtained in numerical simulations by the authors of Ref. [10]. The computed noise suppression factor $P/P_{Poisson}$ for the energy-independent scattering time is also sufficiently close to the analytical results obtained earlier by Schomerus et al. [11]. However, for the energy-dependent scattering, the numerical difference between our results and those in Ref. [11] is considerable.

We clarify once more why the authors of Ref. [11] arrived at the equations that differ from ours. As an example, we take the Poisson equation. According to Ref. [11], one could write

$$n = \int_{-U(x)}^{\infty} d\varepsilon \nu(\varepsilon) f(\varepsilon, x), \qquad (6.1)$$

where n and U are the exact total concentration and potential energy and f is the total distribution function (the mean value plus the fluctuating part). The linearization of this equation leads to the equations in Ref. [11]. The authors of Ref. [11] could have argued that because the voltages in the reservoirs do not fluctuate and U is set to zero at the left boundary and because the total energy $\mathcal{E} = \varepsilon + U$ remains positive, the total distribution function is zero for $\varepsilon < -U$.

Our point is that Eq. (6.1) cannot be justifed for the total values of these variables including the stationary and fluctuating parts. This is readily seen from the fact that the fluctuating part of the distribution function itself implicitly depends on the mean value of the distribution function through the correlation function. One should bear in mind that an equation involving both the mean and the fluctuating quantities must be regarded symbolically. Indeed, such an equation is in fact equivalent to two equations, one for the mean values and the other for the fluctuating part. Regarded literally, it can lead to confusion. For example, analyzing the equation

$$\overline{n} + \delta n = \int_{-\overline{U}-\delta U} d\varepsilon \nu(\varepsilon)(\overline{f} + \delta f)$$

one can come to the wrong conclusion that the mean value $\overline{n}$ depends on such an average as $\overline{\delta U \delta f}$.

We add several words about the boundary conditions for the potential. The boundary conditions used here are not applicable within the length

$$R_V = \sqrt{\kappa V / 4\pi e n(0)}$$

near the electrodes. Because the nonequilibrium noise power is a bulk property (we note, e.g., the integration over the coordinate in Eq. (5.45)), this approximation is justified since we assume that the sample length L is much greater than R_V.

Being interested in the analysis of the fluctuation phenomena in the simplest situation of the space-charge limited diffusive conduction regime, we have not taken the electron–electron collisions into account. These collisions can lead to an additional electron–electron correlation [7] that must be considered in analyzing a more general case.

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