ЖЭТФ, 2001, том 119, вып. 2, стр. 321–330 

# CONTACT EFFECTS ON THE MAGNETORESISTANCE OF FINITE SEMICONDUCTORS

*G. González de la Cruz, Yu. G. Gurevich**

*Departamento de Fisica, Centro de Investigación y Estudios Avanzados del Instituto Politécnico Nacional Apartado Postal 14-740 México, D.F.*

*V. V. Kucherenko, E. Ramirez de Arellano*

*Departamento de Matemáticas ESFM del IPN;*
*Departamento de Matematicas, Centro de Investigación y Estudios Avanzados del Instituto Politecnico Nacional Apartado Postal 14-740 México, D.F.*

Submitted 8 February 2000

We propose a new theoretical method to study galvanomagnetic effects in bounded semiconductors. The general idea of this method is as follows. We consider the electron temperature distribution and the electric potential as consisting of two terms, one of which represents the regular solution of the energy balance equation obtained from the Boltzaman transport equation at steady-state conditions and the Maxwell equation respectively, and the other is the effect of the specimen size that is significant near the contacts (the boundary layer function). With the distribution of the electric potential at the contacts and the electron temperature distribution at the surface of the sample taken into account, we find that the magnetoresistance is different from the one in the standard theory of galvanomagnetic effects in boundless media. We show that besides the usual quadratic dependence on the applied magnetic field $B$, the magnetoresistance can have a linear dependence on $B$ under certain conditions. We obtain new formulas for the linear and quadratic terms of the magnetoresistance in bounded semiconductors. This linear contribution of the magnetic field to the magnetoresistance is essentialy due to the spatial dependence of the potential at the electric contacts. We also discuss the possibility to obtain the distribution of the electric potential at the contacts from standard magnetoresistance experiments. Because the applied magnetic field acts differently on carriers of different mobilities, a redistribution of the electron energy occurs in the sample and thus, the Ettingshausen effect on the magnetoresistance must be considered in bounded semiconductors.

PACS: 02.30.Jr, 72.15.Gd, 75.70.Ak

## 1. INTRODUCTION

Physically, the magnetoresistance phenomenon consists in an increase of the electric resistance of a metal or semiconductor subject to an external magnetic field applied transversally to the electric field direction. We obtain a complete formula for the magnetoresistance in the bounded semiconductor involving several previously unknown terms. Using the expression for the magnetoresistance in bounded semiconductors, it is possible to obtain some information about the electron energy relaxation, the carrier density, and the electron temperature distribution in the semiconductor. Currently, the innovation of some sensitive magnetic field detectors is based on the magnetoresistance effect in semiconductors. This means that the linear contribution of the magnetic field to the magnetoresistance obtained in this paper, which arises due to the spatial dependence of the potential at electric contacts, can improve the sensitivity of the devices. Furthermore, the experimental measurements of magnetoresistance allow one to describe the homogeneity of the electric potential at the contacts and therefore also the homogeneity of the current density in the sample, which is very important for semiconductor devices.

Most of the theoretical works, as far as galvanomagnetic effects in bulk semiconductors are concerned, have been addressed to boundless media where the electric

*E-mail: gurevich@fis.cinvestav.mx

field is constant in all directions and the only contribution to the magnetoresistance is related to the dependence of the electric conductivity on the magnetic field [1, 2]. However, this assumption implicitly involves the effect of the sample surface, because the electrostatic Hall field and thus the magnetoresistance cannot be found otherwise. It is worth mentioning that in reality, it is usual to fix some specific boundary conditions at the surface of the sample; as a consequence, in general, magnetoresitance depends on the electric potential, which is a linear function of the coordinates [3]. Moreover, this linear term can be only calculated if the surface effects on the electric potential are considered through an additional function of coordinates. The coefficients characterizing the potential also depend strongly on these boundaries and as the result, they are different from the coefficients obtained in the standard magnetoresistance theory.

Size-dependent contributions to the magnetoresistance of an isotropic semiconductor in a uniform electric field $E_x$ and a transverse magnetic field $B$ (in the $y$-direction) have been discussed in [4–7]. The discussion is given for systems bounded along only one direction (the $z$-axis) and boundless in the direction of the electric field. The current density is taken to vanish at the surface of the sample, which is viewed as a boundary condition (i.e., $j_z = 0$ at $z = \pm b$) in contrast with the standard magnetoresistance theory, where $j_z = 0$ in the semiconductor sample. In this case, the electron temperature gradient $\partial T_e/\partial z$ arises because the magnetic field acts in a different way on carriers of different mobilities (the Ettingshausen effect) [8], which leads to a linear dependence of the electron temperature distribution on the electric field. The experimental evidence of these theoretical results has shown a strong influence of the semiconductor thickness on the magnetoresistance. When the Ettingshausen effect in bounded semiconductors is taken into account, a size-dependent term appears in the magnetoresitance. However, when the transverse dimensions of the semiconductor are very large compared to the electron–phonon energy relaxation length ($k^{-1}$) [9], the usual result of the conventional magnetoresistance theory is recovered, with the Ettingshausen effect bieng important if $kb \leq 1$. On the other hand, the size-dependent contribution to the magnetoresistance does not disappear in the limit as $kb \to 0$ [10] and is in fact of the same order as the physical magnetoresistance term in the standard theory.

As can be seen, the surfaces of the sample play an important role in the theory of magnetoresistance in thin-film semiconductors. However, in real physical experiments on magnetoresistance, besides the effect of the size, the effects due to the inhomogeneity of the potentials at the contacts must be considered.

Magnetoresistance and the electric potential distribution in a bounded metal (degenerate electron gas) have been investigated in [11, 12]; in [12], in particular, it was studied using a conformal transformation in the complex plane. This approach is only valid when the electric potential is constant at the contacts, i.e., is independent of the coordinates; the approach cannot be applied to semiconductors where the current depends on the potential and the temperature and satisfies the Helmholtz equation.

In the limit of small electric and magnetic fields, size-dependent contributions of the magnetoresistance of an isotropic semiconductor have been discussed in [13, 14] using a perturbative method. The relevant discussion is given for systems bounded in all directions, with the current density vanishing at $z = \pm b$. It is found that magnetoresistance exists even if the relaxation time is independent of the electron energy. However, when the distance between the contacts is very large, the perturbative approach of Refs. [13, 15] looses its applicability.

Recently, magnetoresitance in bulk semiconductors that are bounded in all directions was investigated within a new mathematical approach [3] for a degenerate electron gas, the result being a simple analytical expression. Moreover, it was shown in [7] that the carrier temperature distribution for a nondegenerate semiconductor (the Ettingshausen effect) plays an important role in the study of galvanomagnetic effects.

In this work, we analyze the magnetoresistance in bounded isotropic nondegenertate semiconductors and consider the effect of the inhomogeneous electric potentials at the contacts and the thickness $b$ and the length $a$ of the thin-film semiconductor. This analysis is based on representing the potential and the temperature as the sum of a term that is regular (analytical) in the small parameters $b/a$ and $\omega_H \tau_0$ and a term involving the boundary layer functions corresponding to vortex currents. The boundary layer functions are essential near the contacts. They vanish as the magnetic field $B \to 0$ for a constant potential at the contacts, are regular in the small parameter $\omega_H \tau_0$, and decay exponentially along the sample. The analysis shows that it should be possible to observe an interesting electronic transport phenomenon caused by the electric field and the electron temperature distributions; moreover, the magnetoresistance that we find is different from the one in the standard theory.

## 2. THEORETICAL MODEL

We assume that the semiconductor has the shape of a parallelepiped bounded by the $x = 0, a$; $y = 0, c$ and $z = 0, b$ planes and the electric contacts with the respective distributions $\varphi^0(y,z)$ and $\varphi^a(y,z)$ are in the $x = 0, a$ planes, while the applied uniform magnetic field is directed along the $y$-axis. The normal components of the current density vanish at the $y = 0, c$ and $z = 0, b$ planes of the sample (open circuit at these surfaces). If the potential distributions $\varphi^0(y,z)$ and $\varphi^a(y,z)$ are only functions of $z$, the transport problem is obviously two dimensional (all the physical parameters depend only on $x$ and $z$). We consider the effect that the redistribution of carriers according to their energy across the sample has on the magnetoresistance (the Ettingshausen effect). Assuming that the electric and magnetic fields are weak, and therefore, $T_e - T_0 \approx jB$, where $T_0$ is the ambient temperature, we can use the Maxwell and the thermal balance equations to find the electron temperature distribution and the electrostatic potential in the sample as functions of coordinates and the magnetic field. At the steady-state conditions, the equations for the coupled electron temperature and the electric potential can be written as [7, 14]

$$\begin{gathered} \nabla^2\varphi(x,z) + \frac{q+1}{e}\nabla^2 T_e(x,z) = 0, \\ \nabla^2 T_e(x,z) + \frac{e}{q+2}\nabla^2\varphi(x,z) = k^2\left(T_e(x,z) - T_0\right), \end{gathered} \tag{1}$$

where $k^{-1}$ is the scale length of the electron–phonon energy relaxation, referred to as the cooling length ($k^{-1} \approx 10^{-3}$–$10^{-4}$ cm for nondegenerate semiconductors), and $q$ is the parameter characterizing the dependence of the momentum relaxation time $\tau$ on the energy $\varepsilon$ via $\tau(\varepsilon) = \tau_0(\varepsilon/T_0)^q$. The values of $q$ for various momentum relaxation mechanisms are given in [16] (it is important that $|q| < 3/2$). In this work, we assume that the temperature of the phonon system is equal to the ambient temperature $T_0$.

To arrive at Eqs. (1), we have assumed that the electron gas is nondegenerate (satisfies the Maxwell statistics), the energy–momentum relation is quadratic and isotropic, and the current density is sufficiently small for the nonlinear effects to be negligible, i.e., the kinetic coefficients do not depend on the electric field. We also consider a weak magnetic field such that $\omega_H\tau_0 \ll 1$, where $\omega_H$ is the cyclotron frequency.

The continuity and the energy balance equations for the potential $\varphi(x,z)$ and the electron temperature $T_e(x,z)$ must be supplemented by boundary conditions describing the distribution of the potential at the electric contacts and the normal components of the current density at the lateral surfaces:

$$\begin{gathered} \varphi(x,z)\big|_{x=0} = \varphi^0(z), \quad \varphi(x,z)\big|_{x=a} = \varphi^a(z), \\ j_z\big|_{z=0,b} = 0. \end{gathered} \tag{2}$$

The coupled equations for the potential and the electron temperature must be supplemented by boundary conditions describing the absorption of the carrier energy at the surface of the sample. These conditions can be written as [17]

$$Q_n\big|_s = \eta_s(T_e - T_0)\big|_s, \tag{3}$$

where $Q_n$ is the electron normal component of the heat flux at the surface of the sample and the parameter $\eta_s$ represents the inelastic scattering of electrons at the boundaries (surface heat conductivity), with $\eta_s = 0$ corresponding to the absence of surface mechanisms, that is,

$$Q_z\big|_{z=0,b} = 0 \tag{4}$$

in our geometry, and with the infinite $\eta_s$ corresponding to a good thermal conductivity across the surface. We consider this latter boundary condition for the electron temperature at the contacts, i.e.,

$$T_e\big|_{x=0,a} = T_0. \tag{5}$$

Under the above assumptions, we see from the expressions for $\mathbf{j}$ and $\mathbf{Q}$ given in [18] that the potential and the temperature distributions satisfy the following equations at the surface of the sample, where $j_z|_{z=0,b} = Q_z|_{z=0,b} = 0$:

$$\begin{gathered} \frac{\partial\varphi}{\partial z} + \frac{q+1}{e}\frac{\partial T_e}{\partial z} + \frac{\Gamma(2q+5/2)}{\Gamma(q+5/2)} \times \\ \times (\omega_H\tau_0)\left(\frac{\partial\varphi}{\partial x} + \frac{2q+1}{e}\frac{\partial T_e}{\partial x}\right)\Bigg|_{z=0,b} = 0, \\ \frac{\partial\varphi}{\partial z} + \frac{q+2}{e}\frac{\partial T_e}{\partial z} + \frac{\Gamma(2q+7/2)}{\Gamma(q+7/2)} \times \\ \times (\omega_H\tau_0)\left(\frac{\partial\varphi}{\partial x} + \frac{2q+2}{e}\frac{\partial T_e}{\partial x}\right)\Bigg|_{z=0,b} = 0, \end{gathered} \tag{6}$$

with $\Gamma(x)$ being the Gamma function.

Assuming the potential difference at the contacts to be small, which means restricting to the transport effects that are linear in the electric field, we see from [14, 19] that in the theory of galvanomagnetic phenomena with the electron temperature distribution taken into account, the $x$ component of the current density is given by

$$\begin{aligned} j_x = -\sigma_0 \frac{\partial \varphi}{\partial x} - \frac{(q+1)\sigma_0}{e}\frac{\partial T_e}{\partial x} + \sigma_0(\omega_H\tau_0) \times \\ \times \frac{\Gamma(2q+5/2)}{\Gamma(q+5/2)}\left[\frac{\partial \varphi}{\partial z} + \frac{(2q+1)}{e}\frac{\partial T_e}{\partial z}\right] + \\ + \sigma_0(\omega_H\tau_0)^2\frac{\Gamma(3q+5/2)}{\Gamma(q+5/2)}\left[\frac{\partial \varphi}{\partial z} + \frac{(3q+1)}{e}\frac{\partial T_e}{\partial z}\right], \end{aligned} \quad (7)$$

where

$$\sigma_0 = \frac{4\Gamma(q+5/2)}{3\sqrt{\pi}}\frac{ne^2\tau_0}{m}.$$

The first term in (7) corresponds to the usual current; the second term corresponds to the thermoelectric current; the third term corresponds to the Hall effect and the transverse Nernst–Ettingshausen effect. The last term in Eq. (7) describes the longitudinal Nernst–Ettingshausen effect.

## 3. ASYMPTOTIC APPROXIMATION FOR MAGNETORESISTANCE

For small magnetic fields such that $(\omega_H\tau_0)^2 \ll 1$, we naturally seek solutions of Eqs. (1) in the form

$$\begin{aligned} \varphi(x,z) = \varphi_0(x,z) + \varphi_1(x,z)(\omega_H\tau_0) + \\ + \varphi_2(x,z)(\omega_H\tau_0)^2 + \dots, \\ T_e(x,z) = T_0 + T_1(x,z)(\omega_H\tau_0) + \\ + T_2(x,z)(\omega_H\tau_0)^2 + \dots \end{aligned} \quad (8)$$

To calculate the terms $\varphi_j(x,z)$ and $T_j(x,z)$, we propose a new nonstandard perturbation theory with respect to the small magnetic field. This theory is uniform with respect to the small parameter $b/a$. Inserting Eqs. (8) in Eq. (7), we can write the $x$ component of the current density to the second order of the magnetic field as

$$\begin{aligned} j_x(x,z) = j_0(x,z) + j_1(x,z)(\omega_H\tau_0) + \\ + j_2(x,z)(\omega_H\tau_0)^2 + \dots, \end{aligned} \quad (9)$$

where

$$\begin{aligned} j_0(x,z) = -\sigma_0\frac{\partial \varphi_0}{\partial x}, \\ j_1(x,z) = -\sigma_0\left(\frac{\partial \varphi_1}{\partial x} + \frac{q+1}{e}\frac{\partial T_1}{\partial x}\right)\frac{\Gamma(2q+5/2)}{\Gamma(q+5/2)}, \\ j_2(x,z) = -\sigma_0\left(\frac{\partial \varphi_2}{\partial x} + \frac{q+1}{e}\frac{\partial T_2}{\partial x} - \frac{\Gamma(2q+5/2)}{\Gamma(q+5/2)} \times \right. \\ \left. \times \left\{\frac{\partial \varphi_1}{\partial z} + \frac{2q+1}{e}\frac{\partial T_1}{\partial z}\right\} - \frac{\Gamma(3q+5/2)}{\Gamma(q+5/2)}\frac{\partial \varphi_0}{\partial x}\right). \end{aligned} \quad (10)$$

The average value of the current density over the semiconductor cross-section that is significant for the magnetoresistance is given by

$$\bar{j} = \frac{1}{b}\int_0^b j_x(x,z)dz. \quad (11)$$

Because $\operatorname{div}\mathbf{j} = 0$, $\bar{j}$ is $x$-independent.

It is clear from the above that a detailed analysis of $\bar{j}$ is a very complicated problem. As we see in what follows, however, an analytical expression for the average current density can be obtained in the limit where $b/a \ll 1$. This condition allows us to study galvanomagnetic effects in semiconductors; depending on the results, we can decide whether it is possible to talk about the effects of the finite dimension of the sample on the magnetoresistance.

We now restrict ourselves to thin-film semiconductors with $a \gg b$. Because the cooling length is of the order 1 $\mu$m, we can use the relation

$$a \gg k^{-1}, b. \quad (12)$$

Alternatively, if the geometry of the sample is such that $a \ll b$, the distribution of the current density $j_z$ corresponds to the closed Hall contacts [19].

We introduce the average potential at the contacts $x = 0$ and $x = a$ as

$$\overline{\varphi}^0 = \frac{1}{b}\int_0^b \varphi^0(z)dz, \quad \overline{\varphi}^a = \frac{1}{b}\int_0^b \varphi^a(z)dz. \quad (13)$$

We note that if the distribution of the potential is constant at the contacts of the sample, we have $\varphi^0(z) = \overline{\varphi}^0$ and $\varphi^a(z) = \overline{\varphi}^a$, otherwise it depends on the $z$-coordinate.

For a constant potential at the contacts and in the presence of a weak magnetic field, the magnetoresistance can be defined as

$$\delta = \frac{(\bar{j} - \bar{j}_0)\, a}{(\overline{\varphi}^a - \overline{\varphi}^0)\, \sigma_0}.$$

In the case where $\varphi^0(z) = \overline{\varphi}^0$ and $\varphi^a(z) = \overline{\varphi}^a$, the magnetoresistance is given by

$$\delta = \left[\delta_0 - \frac{b}{a}\left(K + F(kb)\right)\right](\omega_H\tau_0)^2 \quad (14)$$

(the proof of this formula is given in Sec. 6). It follows that

$$\delta_0 = \frac{\Gamma(5/2+q)\Gamma(5/2+3q) - \Gamma^2(5/2+2q)}{\Gamma^2(5/2+q)} - \\ - \frac{\dfrac{q^2}{5/2+q}\Gamma^2(2q+5/2)}{\dfrac{kb}{2}(q+2)^{1/2}\Gamma^2(q+5/2)} \operatorname{th}\left[(q+2)^{1/2}kb/2\right]$$

is the magnetoresistance for samples such that the dimension along the $x$ direction is infinite ($a \to \infty$) and the transverse dimension $b$ is finite. The formulas for the coefficient $K$ and the function $F(kb)$ have not been known previously. We obtain that

$$K = \frac{\Gamma^2(2q+5/2)}{\Gamma^2(q+5/2)} \frac{16}{\pi^3} \sum_{l=0}^{\infty} \frac{1}{(2l+1)^3}, \tag{15}$$

$$F(kb) = \frac{8q^2}{q+5/2} \frac{\Gamma^2(2q+5/2)}{\Gamma^2(q+5/2)} \times \\ \times \sum_{l=0}^{\infty} \left[\pi^2(2l+1)^2 + (kb)^2(q+2)\right]^{-3/2}. \tag{16}$$

It follows from Eqs. (14)–(16) that when the distribution of the potential is uniform at the contacts, the correction term to the magnetoresistance depends on the ratio $b/a \ll 1$ linearly rather than exponentially via $\exp(-a/b)$, as is assumed in the standard theory of galvanomagnetic effects in semiconductors. On the other hand, if the electric potential is inhomogeneous at the contacts, the magnetoresistance is given by

$$\delta = -\frac{4}{\pi b\left(\overline{\varphi}^a - \overline{\varphi}^0\right)} \left\{ \int_0^b \left[\varphi^0(z) + \varphi^a(z) - \overline{\varphi}^0 - \overline{\varphi}^a\right] \times \\ \times \sum_{l=0}^{\infty} \frac{\cos\left[(2l+1)\pi z/b\right]}{2l+1} dz \right\} \frac{\Gamma(2q+5/2)}{\Gamma(q+5/2)} (\omega_H \tau_0) \tag{17}$$

(the proof of this formula is given in Sec. 5). In this case, the magnetoresistance depends on the magnetic field linearly rather than quadratically as in the usual theory of galvanomagnetic effects in semiconductors. In addition, it changes sign when the magnetic field is reversed. Thus, the resistance in the sample decreases with the magnetic field before reversing its sign. We note that the sign in Eq. (17) strongly depends on the potential distribution at the contacts and is independent of the length $a$ of the sample in the first approximation with respect to the magnetic field. Size effects on the magnetoresistance occur in the second-order approximation with respect to $B$. For example, if $\varphi^0(z) + \varphi^a(z) - \overline{\varphi}^0 - \overline{\varphi}^a = C(z - b/2)$, it follows from Eq. (17) that

$$\delta = \frac{8C\omega_H\tau_0}{\pi^3\left(\overline{\varphi}^a - \overline{\varphi}^0\right)} \frac{\Gamma(2q+5/2)}{\Gamma(q+5/2)} \sum_{l=0}^{\infty} (2l+1)^{-3}.$$

We note that Eq. (14) gives the magnetoresistance with the precision $[(\omega_H\tau_0)^3 + e^{(-\pi a/2b)}(\omega_H\tau_0)^2]$, and Eq. (17) with the precision $(\omega_H\tau_0)^2$. Therefore, Eq. (14) gives the correct results in case where $\omega_H\tau_0 \ll 1$ and $b/a \ll e^{(-\pi a/2b)}$; this does not necessarily imply the constraint $b/a \ll 1$. Equation (17) is applicable in the cases where $\omega_H\tau_0 \ll 1$ and $\omega_H\tau_0 \ll |\delta|$. We see that for the potential that is homogeneous at the contacts, we have $\delta_0 = 0$ for the degenerate electron gas, that is, for $q = 0$. This implies that the standard mechanisms of creating magnetoresistance do not work and the magnetoresistance is the result only of the mechanism proposed in this paper. However, if the linear part of magnetoresistance in the magnetic field does not vanish, it does not vanish for all values of $q$. This means that inhomogeneity of the potential at the contact plane is a new mechanism of creating magnetoresistance. The linear dependence coefficient in (17) is a product of two factors. The first factor depends only on the potential distributions at the contact planes. The second factor results in the Ettingshausen effect and is independent of the potential distribution. It follows from Eq. (17) that if we know the potential distributions at the contacts, we can calculate the parameter $q$ of the relaxation mechanism using the magnetoresistance.

It is worth mentioning that if the magnetoresistance is calculated in all orders in the magnetic field, the potential distribution at the contacts can be evaluated explicitly. The solution in the form of a Taylor expansion has been exactly obtained only for the degenerate electron gas (metals) [15, 20]. Thus, experimental measurements of magnetoresistance allow one to shed some light on the distribution of the potential at the contacts.

## 4. MAGNETORESISTANCE CALCULATION FOR RECTANGULAR SAMPLES

We now proceed to describe a method of solving the two-dimensional potential and electron temperature distribution for magnetoresistance in the presence of a weak magnetic field. The geometry considered is again that of a rectangular semiconductor. We introduce a new function $\Phi$ depending on the potential and the electron temperature distribution such that the current $J_x$ is expressed through this function up to the order $(\omega_H\tau_0)^2$ (see Eq. (7)) as

$$\Phi = \varphi + \frac{q+1}{e}T_e, \quad T = T_e - T_0 \tag{18}$$

and the dimensionless variables $x' = x/b$ and $z' = z/b$ are such that $0 < x' < \beta^{-1}$ and $0 < z' < 1$, where $\beta = b/a$. With these new functions, Eq. (1) can be written as (we omit the prime on the variables)

$$\nabla^2\Phi = 0, \quad \nabla^2 T - (q+2)(kb)^2 T = 0 \tag{19}$$

and the boundary conditions in Eqs. (4)–(6) become

$$\Phi\big|_{x=\beta^{-1},0} = \varphi^a(z) + \frac{q+1}{e}T_0, \quad T\big|_{x=0,\beta^{-1}} = 0,$$

$$\begin{aligned} &\frac{\partial\Phi}{\partial z} + \alpha(\omega_H\tau_0)\frac{\partial\Phi}{\partial x} + \frac{\alpha q}{e}(\omega_H\tau_0)\left.\frac{\partial T}{\partial x}\right|_{z=0,1} = 0,\\ &\frac{\partial T}{\partial z} + \chi(\omega_H\tau_0)\frac{\partial T}{\partial x} + \gamma(\omega_H\tau_0)\left.\frac{\partial\Phi}{\partial x}\right|_{z=0,1} = 0, \end{aligned} \tag{20}$$

$$\begin{aligned} &\alpha = \frac{\Gamma(2q+5/2)}{\Gamma(q+5/2)},\\ &\chi = (q+1)\frac{\Gamma(2q+5/2)}{\Gamma(q+7/2)} - q\frac{\Gamma(2q+5/2)}{\Gamma(q+5/2)},\\ &\gamma = e\left[\frac{\Gamma(2q+7/2)}{\Gamma(q+7/2)} - \frac{\Gamma(2q+5/2)}{\Gamma(q+5/2)}\right]. \end{aligned} \tag{21}$$

In most of the theoretical works related to galvanomagnetic effects in bulk semiconductors, solutions of Eqs. (19) are represented as infinite series in $\omega_H\tau_0$ for weak magnetic fields; to obtain approximations for the coefficients $\Phi_k$ and $T_k$ of the orders $k = 0, 1, \ldots$, the authors neglect the terms $(\omega_H\tau_0)\partial\Phi_k/\partial x$ and $(\omega_H\tau_0)\partial T_k/\partial x$ in boundary conditions (20). However, the exact solutions for the degenerate electron gas [15] demonstrate that this series diverges for large samples, i.e., for $a \gg b$. For this reason, we now seek solutions of Eqs. (19) in the form

$$\begin{aligned} \Phi = {} & \Phi_0(x,z,\omega_H\tau_0) + \Phi_1(x,z,\omega_H\tau_0)(\omega_H\tau_0)^1 + \\ & + \Phi_2(x,z,\omega_H\tau_0)(\omega_H\tau_0)^2 + O\left((\omega_H\tau_0)^3\right),\\ T = {} & T_1(x,z,\omega_H\tau_0)(\omega_H\tau_0)^1 + \\ & + T_2(x,z,\omega_H\tau_0)(\omega_H\tau_0)^2 + O\left((\omega_H\tau_0)^3\right). \end{aligned} \tag{22}$$

The functions $\Phi_j$ and $T_j$ with $j = 0, 1, \ldots$ satisfy Eqs. (19). The boundary conditions for $\Phi_0$ and $T_0$ in the planes $x = 0, \beta^{-1}$ are the same as for the functions $\Phi$ and $T$, and we have $\Phi_j|_{x=0,\beta^{-1}} = 0$, $T_j|_{x=0,\beta^{-1}} = 0$ for $j \ge 1$. The boundary conditions for $\Phi_j(x,z,\omega_H\tau_0)$ and $T_j(x,z,\omega_H\tau_0)$ on the planes $z = 1, 0$ were obtained from boundary conditions (20) using perturbation theory with one exception. For $\Phi_j$, we keep the term $(\omega_H\tau_0)\partial T_j/\partial x$ in boundary condition (20) and omit the term $(\omega_H\tau_0)\partial\Phi_j/\partial x$. For $T_j$, on the contrary, we keep the term $(\omega_H\tau_0)\partial T_j/\partial x$ in boundary condition (20) and omit the term $(\omega_H\tau_0)\partial\Phi_j/\partial x$. The terms $\partial T_{j-1}/\partial x$ and $\partial\Phi_{j-1}/\partial x$ enter the boundary conditions for the respective functions $T_j$ and $\Phi_j$ and make them heterogeneous. We then see that the zero-order term $T_0$ satisfies Eq. (19) and zero boundary conditions in the planes $x = 0, \beta^{-1}$; $z = 0, 1$. Therefore, $T_0 = 0$, which is why we started with the term $T_1$ in Eq. (22). The functions $\Phi_j$ and $T_j$ are analytical in $\omega_H\tau_0$ and can also be expressed in terms of the natural low-field expansion for $\omega_H\tau_0 \ll 1$. Within this approximation, we can obtain the solution of Eq. (19) and, thus, the magnetoresistance. The equations and boundary conditions for the coefficients in Eq. (22) are formulated in what follows. Since the average current in Eqs. (7), (9), and (10) depends on $\Phi_0$, $\Phi_1$, $\Phi_2$, and $T_1$ and is independent of $T_2$ with the accuracy up to the $(\omega_H\tau_0)^3$ terms, it is not necessary to calculate it. We then consider the boundary problems for $\Phi_0$, $\Phi_1$, $\Phi_2$, and $T_1$. Similarly to the above, we obtain the following boundary problem for $\Phi_0$ and $T_1$:

$$\begin{aligned} &\nabla\Phi_0 = 0, \quad \Phi_0\big|_{x=0} = \varphi^0(z) + \frac{q+1}{e}T_0,\\ &\Phi_0\big|_{x=\beta^{-1}} = \varphi^a(z) + \frac{q+1}{e}T_0,\\ &\frac{\partial\Phi_0}{\partial z} + \alpha\omega_H\tau_0\left.\frac{\partial\Phi_0}{\partial x}\right|_{z=0,1} = 0, \end{aligned} \tag{23}$$

$$\begin{aligned} &\nabla^2 T_1 - (q+2)(kb)^2 T_1 = 0, \quad T_1\big|_{x=0,\beta^{-1}} = 0,\\ &\frac{\partial T_1}{\partial z} + \chi\omega_H\tau_0\frac{\partial T_1}{\partial x} + \gamma\left.\frac{\partial\Phi_0}{\partial x}\right|_{z=0,1} = 0. \end{aligned} \tag{24}$$

With $T_0 = 0$, the function $\Phi_1$ satisfies Eq. (19) with zero boundary conditions, and hence, $\Phi_1 = 0$. The function $\Phi_2$ satisfies the boundary problem

$$\begin{aligned} &\nabla^2\Phi_2 = 0, \quad \Phi_2\big|_{x=0,\beta^{-1}} = 0,\\ &\frac{\partial\Phi_2}{\partial z} + \alpha\omega_H\tau_0\frac{\partial\Phi_2}{\partial x} + \frac{\alpha q}{e}\left.\frac{\partial T_1}{\partial x}\right|_{z=0,1} = 0. \end{aligned} \tag{25}$$

## 5. THE WEAK-FIELD $\Phi_0$ SOLUTION

To derive the first term of the expansion of (23) for a weak magnetic field, we represent the solution $\Phi_0$ with the precision $O(e^{-\pi/\beta})$ as the sum of a regular and a boundary layer functions

$$\Phi_0 = \Phi_{reg} + \Pi_0 + \Pi_1 + O(e^{-\pi\beta^{-1}}), \tag{26}$$

where

$$\Phi_{reg} = C_0 + (x - \alpha\omega_H\tau_0 z)C_1 \tag{27}$$

satisfies the boundary condition

$$\left.\frac{\partial\Phi_{reg}}{\partial z} + \alpha\omega_H\tau_0\,\frac{\partial\Phi_{reg}}{\partial x}\right|_{z=0,1} = 0;$$

and $\Pi_i$ (with $i = 0, 1$) are two boundary layer functions that are exact solutions of the problem

$$\nabla^2\Pi_i = 0 \quad \text{and} \quad \left.\frac{\partial\Pi_i}{\partial z} + \alpha\omega_H\tau_0\,\frac{\partial\Pi_i}{\partial x}\right|_{z=0,1} = 0$$

such that $\Pi_0$ and $\Pi_i$ exponentially decrease as $x \to \infty$ and $x \to -\infty$ respectively. Separating the variables, we can write solutions for the last equations as

$$\begin{aligned} \Pi_0 &= \sqrt{2}\sum_{n=1}^{\infty} A_n(\cos\pi nz + \alpha\omega_H\tau_0\sin\pi nz) \times \\ &\quad \times e^{-\pi nx}, \\ \Pi_1 &= \sqrt{2}\sum_{n=1}^{\infty} B_n(\cos\pi nz - \alpha\omega_H\tau_0\sin\pi nz) \times \\ &\quad \times e^{-\pi n(\beta^{-1}-x)}. \end{aligned} \tag{28}$$

As noted above, the boundary layer functions $\Pi_0$ and $\Pi_1$ correspond to the vortex current, and therefore, do not contribute to the magnetoresistance. We now demonstrate this. We know that average current (11) is $x$-independent. Therefore, we can calculate it at the point $x = \beta^{-1}/2$. But the exponentials in the boundary layer functions (28) are less than or equal to $e^{-\pi/2\beta}$ at that point. We also have

$$\int_0^b \cos(\pi nz)dz = 0, \quad n = 1, 2, \ldots$$

Hence, the boundary layer contributions to average current (7) and to the magnetoresistance have the order $\omega_H\tau_0 e^{-\pi/2\beta}$. We can sharp this estimate and demonstrate that this contribution is smaller and has the order $(\omega_H\tau_0)^2 e^{-\pi/2\beta}$. Indeed, it follows from (7) that the contribution of $\Pi_i$ (with $i = 0, 1$) to the average current with the precision $(\omega_H\tau_0)^2 e^{-\pi/2\beta}$ is equal to the integral

$$\sigma_0 \int_0^b \left\{ -\frac{\partial}{\partial x}\Pi_i(x,z) + \alpha\omega_H\tau_0\frac{\partial}{\partial z}\Pi_i(x,z)\right\} \partial z\bigg|_{x=1/2\beta}.$$

This is easy to verify for the functions

$$e^{\pm\pi x/b}\left(\cos\frac{\pi nz}{b} \mp \alpha\omega_H\tau_0\sin\frac{\pi nz}{b}\right);$$

in view of decompositions (28) for $\Pi_i$, the above integral is zero for all $x$. The boundary layer contributions to the average current and the magnetoresistance is therefore of the order $(\omega_H\tau_0)^2 e^{-\pi/2\beta}$. Inserting Eqs. (26)–(28) in boundary conditions (23) and neglecting terms of the order $\exp(-\pi\beta^{-1})$, we obtain

$$\begin{aligned} &\sqrt{2}\sum_{n=1}^{\infty} A_n(\cos\pi nz + \alpha\omega_H\tau_0\sin\pi nz) = \\ &\quad = \varphi^0(z) + \alpha\omega_H\tau_0 zC_1 - C_0, \\ &\sqrt{2}\sum_{n=1}^{\infty} B_n(\cos\pi nz - \alpha\omega_H\tau_0\sin\pi nz) = \\ &\quad = \varphi^a(z) - (\beta^{-1} - \alpha\omega_H\tau_0 z)C_1 - C_0. \end{aligned} \tag{29}$$

Equation (29) can be solved using the expansion in $\omega_H\tau_0 \ll 1$. A solution in the zero- and first-order approximation for $A_n$ and $B_n$ exists only if both $C_0$ and $C_1$, which depend on $\omega_H\tau_0$, satisfy special conditions with respect to the potential distribution at the contacts. We, thus, assume that

$$A_n = A_n^0 + A_n^1\omega_H\tau_0 + \ldots; \quad B_n = B_n^0 + B_n^1\omega_H\tau_0 + \ldots;$$

$$C_0 = C_0^0 + C_0^1\omega_H\tau_0 + \ldots; \quad C_1 = C_1^0 + C_1^1\omega_H\tau_0 + \ldots$$

Inserting these series in Eq. (29) and keeping the terms of the zero order in $\omega_H\tau_0$, we obtain

$$\sqrt{2}\sum_{n=1}^{\infty} A_n^0\cos\pi nz = \varphi^0(z) - C_0^0;$$

$$\sqrt{2}\sum_{n=1}^{\infty} B_n^0\cos\pi nz = \varphi^1(z) - (C_0^0 + \beta^{-1}C_1^0).$$

It is well known that the system of functions 1, $\sqrt{2}\cos\pi nz$, $n = 1, 2, \ldots$, is complete and orthogonal on the segment $[0, 1]$. Therefore, every function that is orthogonal to the constant on $[0, 1]$ can be uniquely expanded in the Fourier series with respect to the functions $\sqrt{2}\cos\pi nz$, $n = 1, 2, \ldots$ Hence, to solve the above system for $A_n^0$ and $B_n^0$, it is necessary and sufficient that

$$C_0^0 = \int_0^1 \varphi^0(z)dz = \overline{\varphi}^0,$$

$$\beta^{-1}C_1^0 + C_0^0 = \int_0^1 \varphi^a(z)dz = \overline{\varphi}^a.$$

That is, $C_0^0 = \overline{\varphi}^0$ and $C_1^0 = \beta(\overline{\varphi}^a - \overline{\varphi}^0)$, and therefore,

$$A_n^0 = \sqrt{2}\int_0^1 \left(\varphi^0(z) - \overline{\varphi}^0\right)\cos(\pi n z)\,dz;$$
$$B_n^0 = \int_0^1 \left(\varphi^a(z) - \overline{\varphi}^a\right)\cos(\pi n z)\,dz. \tag{30}$$

Keeping the first-order terms in the magnetic field $\omega_H\tau_0$ in Eq. (28), we then obtain the equations for the coefficients $A_n^1$ and $B_n^1$,

$$\sqrt{2}\sum_{n=1}^{\infty} A_n^1\cos\pi n z = -\sqrt{2}\,\alpha\sum_{n=1}^{\infty} A_n^0\sin\pi n z - C_0^1 + \alpha z C_1^0,$$
$$\sqrt{2}\sum_{n=1}^{\infty} B_n^1\cos\pi n z = -\sqrt{2}\,\alpha\sum_{n=1}^{\infty} B_n^1\sin\pi n z + \alpha z C_1^0 - (C_0^1 + \beta^{-1}C_1^1). \tag{31}$$

It follows that system (31) has a solution if and only if the average of its right-hand side on $[0,1]$ is equal to zero. These conditions give $C_0^1$, $C_1^1$ and $A_n^1$, $B_n^1$. As can be seen from Eqs. (9)–(11), the magnetoresistance depends only on the parameter $C_1$ given by

$$C_1 = C_1^0 + C_1^1\alpha\omega_H\tau_0 + C_1^2(\alpha\omega_H\tau_0)^2 + O\left((\alpha\omega_H\tau_0)^3\right),$$

where

$$C_1^0 = \beta\left(\overline{\varphi}^a - \overline{\varphi}^0\right),$$

$$C_1^1 = \beta\int_0^1 \left\{\varphi^0(z) + \varphi^a(z) - \overline{\varphi}^0 - \overline{\varphi}^a\right\} I_1(z)dz,$$

$$I_1(z) = \frac{4}{\pi}\sum_{m=0}^{\infty}\frac{\cos\left[(2m+1)\pi z\right]}{(2m+1)\pi},$$
$$I_2(z) = 2\sum_{n=1}^{\infty}\cos\pi n z\int_0^1 \sin(\pi n\xi) I_1(\xi)d\xi, \tag{32}$$

$$C_1^2 = -\beta\int_0^1 \left\{\varphi^0(z) - \varphi^a(z) - \overline{\varphi}^0 + \overline{\varphi}^a\right\} I_2(z)dz - \frac{16\beta^2}{\pi^3}\sum_{m=0}^{\infty}\frac{1}{(2m+1)^3}.$$

Using Eqs. (26)–(28) and recalling Eqs. (32), we now write the solution for as a power series expansion in $\omega_H\tau_0 \ll 1$, i.e.,

$$\Phi_0 = \Phi_0^0 + \Phi_0^1\alpha\omega_H\tau_0 + \Phi_0^2(\alpha\omega_H\tau_0)^2 + O\left((\alpha\omega_H\tau_0)^3\right), \tag{33}$$

with

$$\Phi_0^0 = \frac{q+1}{e}T_0 + \overline{\varphi}^0 + \beta\left(\overline{\varphi}^a - \overline{\varphi}^0\right)x + \Pi_0^0 + \Pi_1^0, \tag{34}$$

where $\Pi_0^0$ and $\Pi_1^0$ are the zero-order approximations in $\omega_H\tau_0$ of the respective functions $\Pi_0$ and $\Pi_1$ given by Eqs. (28) and

$$\Phi_0^1 = \left[C_1^1 x + \beta\left(\overline{\varphi}^0 - \overline{\varphi}^a\right)(z - 1/2)\right] + \Pi_0^1 + \Pi_1^1 + cte, \tag{35}$$

where $\Pi_0^1$ and $\Pi_1^1$ are the first-order approximations of $\Pi_0$ and $\Pi_1$. The corresponding solutions are not given here because the magnetoresistance equations do not depend on them. Finally, the coefficient in the second-order approximation to $\Phi_0$ in the magnetic field is written as

$$\Phi_0^2 = C_1^2(x - z + 1/2) + \Pi_0^2 + \Pi_1^2 + cte,$$

where $\Pi_0^2$ and $\Pi_1^2$ represent the second-order approximations of the functions in Eqs. (28) in the magnetetic field; in this case, the magnetoresistance is also independent of them. Using all these approximations in Eqs. (10), we obtain the magnetoresistance given by Eq. (17), which depends linearly on the magnetic field as a consequence of the $z$-dependence of the potential at the contacts. It is important to note that when the potential distribution is constant at the contacts, the linear term vanishes. In this case, the second-order contribution in the magnetic field must be considered in $\delta$ (see Eq. (14)).

## 6. MAGNETORESISTANCE AND THE HOMOGENEOUS POTENTIAL DISTRIBUTION AT THE CONTACTS

Proceeding to the calculation of the coefficient $T_1(x, z, \omega_H\tau_0)$, we begin with the explicit equations that determine this quantity in the approximation of a constant potential at the contacts, i.e., for $\varphi^0(z) = \overline{\varphi}^0$ and $\varphi^a(z) = \overline{\varphi}^a$. As can be seen, Eqs. (24) depend on the magnetic field, and hence, $T_1(x, z, \omega_H\tau_0)$ also is a function of this parameter. It follows from Eqs. (10) that the magnetoresistance depends only on $T_1(x, z, 0)$, which implies that it is only necessary to consider $T_1(x, z, \omega_H\tau_0)$ in the zero-order approximation in the magnetic field in Eqs. (25) and (10). With these approximations, we write the zero-order term of the potential $\Phi_0^0$ instead of $\Phi_0$ in Eqs. (24). We can then write $T_1(x, z, \omega_H\tau_0)$ as a regular term and two boundary layer terms similar to $\Phi_0$ in Eq. (26). In this specific case, it is possible to obtain the exact expression for $T_1(x, z, \omega_H\tau_0)$ if the term $\chi\omega_H\tau_0\partial T_1/\partial x$ is taken into account in the boundary conditions. We can then express $T_1(x, z, \omega_H\tau_0)$ as a series in $\omega_H\tau_0 \ll 1$; however, the only significant term is $T_1^0$ (the zero-order approximation) that is given by

$$T_1^0 = \frac{\beta\gamma\left(\overline{\varphi}^a - \overline{\varphi}^0\right)}{kb\sqrt{q+2}} \frac{\operatorname{sh}\left[kb\sqrt{q+2}\,(z-1/2)\right]}{\operatorname{ch}\left[\frac{1}{2}kb\sqrt{q+2}\right]} + \\ + \sum_{n=1}^{\infty} A_n^0 \cos\pi n z\left[\exp\left\{-\sqrt{\pi^2n^2 + k^2b^2(q+2)}x\right\} + \\ + \exp\left\{-\sqrt{\pi^2n^2 + k^2b^2(q+2)}(\beta^{-1} - x)\right\}\right], \quad (36)$$

where

$$A_n^0 = \begin{cases} \dfrac{-2\beta\gamma\left(\overline{\varphi}^a - \overline{\varphi}^0\right)}{\pi^2n^2 + k^2b^2(q+2)} & \text{if} \quad n = 2m+1, \\ 0 & \text{if} \quad n = 2m, \end{cases}$$

$$m = 1, 2, 3, \dots$$

We now derive the second-order approximation in the magnetic field for $\Phi_2$, see Eqs. (25). We set $\Phi_2 = \psi_1 + \psi_2$, where the function $\Psi_2$ satisfies the heterogeneous boundary conditions

$$\frac{\partial\psi_2}{\partial z} + \alpha\omega_H\tau_0\frac{\partial\psi_2}{\partial x} = -\left.\frac{\alpha q}{e}\frac{\partial T_1^0}{\partial x}\right|_{z=0,1}$$

and $\Delta\psi_2 = 0$. It is therefore equal to

$$\psi_2 = \frac{\alpha q}{e}\sum_{n=1}^{\infty}\left[\exp\left\{-\sqrt{\pi^2n^2 + k^2b^2(q+2)}\,x\right\} - \\ - \exp\left\{-\sqrt{\pi^2n^2 + k^2b^2(q+2)}\,(\beta^{-1} - x)\right\}\right] \times \\ \times\left(D_n^1 \sin\left\{\left[\pi^2n^2 + k^2b^2(q+2)\right]^{1/2} z\right\} + \\ + D_n^2 \cos\left\{\left[\pi^2n^2 + k^2b^2(q+2)\right]^{1/2} z\right\}\right), \quad (37)$$

where

$$D_n^1 = \begin{cases} \dfrac{-2\beta\gamma\left(\overline{\varphi}^a - \overline{\varphi}^0\right)}{\pi^2n^2 + k^2b^2(q+2)} & \text{if} \quad n = 2m+1, \\ 0 & \text{if} \quad n = 2m, \end{cases}$$

$$m = 1, 2, 3, \dots,$$

$$D_n^2 = \\ = \begin{cases} D_n^1\dfrac{1 + \cos\left[\pi^2n^2 + k^2b^2(q+2)\right]^{1/2}}{\sin\left[\pi^2n^2 + k^2b^2(q+2)\right]^{1/2}} & \text{if } n = 2m+1, \\ 0 & \text{if } n = 2m, \end{cases}$$

$$m = 1, 2, 3, \dots$$

For $\psi_1$, we obtain

$$\nabla^2\psi_1 = 0, \quad \psi_1\big|_{x=0,\beta^{-1}} = -\psi_2\big|_{x=0,\beta^{-1}}, \\ \left.\frac{\partial\psi_1}{\partial z} + \alpha\omega_H\tau_0\frac{\partial\psi_1}{\partial x}\right|_{z=0,1} = 0. \quad (38)$$

The latter system of equations can be solved in the zero-order approximation in the magnetic field similarly to what was done in Sec. 5. The solution for $\Phi_2$ in the zero-order approximation in the magnetic field is then

$$\Phi_2^0 = -8\frac{\alpha q\gamma\beta^2 x}{e}\left(\overline{\varphi}^a - \overline{\varphi}^0\right) \times \\ \times\left\{\sum_{l=0}^{\infty}\left[\pi^2(2l+1)^2 + k^2b^2(q+2)\right]^{-3/2}\right\} + \\ + \Pi_0^2 + \Pi_1^0 + cte,$$

where the functions $\Pi_0^0$ and $\Pi_1^0$ are the decreasing exponential functions of the distance $\sim 1$ from the contacts at $x = 0$ and $x = \beta^{-1} \gg 1$. It is important to note that the sum $\Pi_0^0 + \Pi_1^0 + cte$ gives a negligible contribution to the magnetoresistance of the order $e^{-\pi/2\beta}(\omega_H\tau_0)^2$. However, these functions must be considered, otherwise the regular function in $\Phi_2^0$ cannot be calculated. Inserting $\Phi_2^0$ in Eq. (10) and taking Eqs. (20) for $\alpha$ and $\gamma$ into account, we obtain expression (16).

## 7. CONCLUSIONS

We have shown that when the electric potential is inhomogeneous at the contacts, the magnetoresistance has a linear dependence on the magnetic field and it is possible to mathematically derive the electric potential distribution on the contacts from the experimental measurements of the magnetoresistance. The magnetoresistance changes its sign when the magnetic field is reversed, i.e., the resistance in the sample decreases with the magnetic field before it changes its direction. It is important to note that the sign in Eq. (17) strongly depends on the potential distribution at the contacts and is independent of the length of the sample in the first-order approximation in the magnetic field.

We emphasize that the correct evaluation of the current contacts for the constant potentials at the contacts leads to the effects of the order $b/a$ but not to the exponential terms $e^{-\pi a/b}$ as was expected from the traditional theory of magnetoresistance.

Finally, it is worth mentioning that the solution of the problems in Eqs. (19) and (20) studied in this paper gives a finite total energy for the system under consideration. This problems can also have a nonphysical solution with an infinite total energy.

This work is partially supported by CONACYT, IPN, and CINVESTAV.

## REFERENCES


**1**. A. I. Anselm, *Introduction to semiconductor theory*, Prentice-Hall, Englewood Cliffs, N.J. (1981).

**2**. O. Madelung, *Physics of III–V compaunds*, Wiley, N.Y. (1964).

**3**. Yu. G. Gurevich and V. V. Kucherenko, Doklady Phys. **43**, 534 (1998).

**4**. P. P. Vilms, V. S. Sardaryan, P. P. Dobrovolskii, and S. V. Kapylova, JETP Lett. **10**, 240 (1969).

**5**. V. S. Bochkov and Yu. G. Gurevich, Sov. Phys. Solid State **11**, 570 (1969).

**6**. A. I. Klimovskaya, O. V. Snitko, and S. I. Kirilova, JETP Lett. **11**, 73 (1970).

**7**. E. I. Rashbaz, Z. S. Gribnikov, and V. Y. Kravchenko, Sov. Phys. Usp. **19**, 361 (1976).

**8**. F. G. Bass, V. S. Bochkov, and Yu. G. Gurevich, Sov. Phys. Semicond. **7**, 1 (1973).

**9**. Z. S. Gribnikov, V. I. Melnikov, and T. S. Sorokina, Sov. Phys. Solid State **8**, 2699 (1967).

**10**. Z. S. Gribnikov and N. A. Prima, Sov. Phys. Semicond. **5**, 1126 (1972).

**11**. R. F. Wick, J. Appl. Phys. **25**, 741 (1954).

**12**. H. Y. Lippman and F. Z. Kahrt, Naturforsch. **13**, 462 (1958).

**13**. G. Gonzalez de la Crus, Yu. G. Gurevich, and V. V. Prosentov, Sol. St. Commun. **97**, 1069 (1996).

**14**. G. Gonzalez de la Cruz, Yu. G. Gurevich, and V. V. Prosentov, Phys. Rev. B **56**, 3863 (1997).

**15**. Yu. G. Gurevich, V. V. Kucherenko, and E. Ramirez de Arellano, Mathematical Notice **65**, 520 (1999).

**16**. V. F. Gantmakher and Y. B. Levinson, *Carrier Scattering in metals and semiconductors*, North-Holland, N.Y. (1987).

**17**. Yu. G. Gurevich and M. I. Kaganov, Sov. Phys. JETP **48**, 1176 (1978).

**18**. Yu. G. Gurevich and O. L. Mashkevich, Phys. Rep. **181**, 328 (1989).

**19**. I. T. Akhiezer, Yu. G. Gurevich, and N. Zakirov, Sov. Phys. Semicond. **27**, 349 (1993).

**20**. V. V. Kucherenko and N. L. Vasilevsky, Matematical Notice **67**, 539 (2000).