*ЖЭТФ, 1998, том 114, вып. 2(8), стр. 640–653* ©1998

# COULOMB EFFECTS IN A BALLISTIC ONE-CHANNEL $S-S-S$ DEVICE

*D. A. Ivanov*[a,b], *M. V. Feigel'man*[a]

[a] *L. D. Landau Institute for Theoretical Physics*
*117940, Moscow, Russia*
[b] *12-127 M.I.T., Cambridge, MA 02139, USA*

Submitted 20 November 1997

We develop a theory of Coulomb oscillations in superconducting devices in the limit of small charging energy $E_C \ll \Delta$. We consider a small superconducting grain with finite capacitance connected to two superconducting leads by nearly ballistic single-channel quantum point contacts. The temperature is assumed to be very low, so there are no single-particle excitations on the grain. Then the behavior of the system can be described in terms of the quantum mechanics of the superconducting phase on the island. The Josephson energy as a function of this phase has two minima that become degenerate when the phase difference on the leads equals to $\pi$, the tunneling amplitude between them being controlled by the gate voltage on the grain. We find the Josephson current and its low-frequency fluctuations, and predict their periodic dependence with period $2e$ on the induced charge $Q_x = CV_g$.

## 1. INTRODUCTION

Coulomb effects in several different types of three-terminal devices consisting of an island connected to external leads by two weak-link contacts, and capacitively coupled to an additional gate potential, have been extensively studied in the last few years. Systems with a normal metal island and leads were studied theoretically both in the tunnel-junction limit [1] and in the case of a quantum point contact with almost perfect transmission [2]. The theory of charge-parity effects and Coulomb modulation of the Josephson current was investigated in detail in [3]. All of the above systems at present are realized experimentally.

Recently, it was shown to be possible to produce a quantum point contact between two superconductors via a normally conductive region made of two-dimensional electron gas [4]; smeared step-wise behavior of the critical current was observed, in qualitative agreement with predictions [5] for a superconductive quantum contact with a few conduction channels of high transmittivity. Observation of a nonsinusoidal current-phase relation in superconducting mechanically controllable break junctions has been reported in Ref. [6], again in agreement with Ref. [5].

Another interesting experimental achievement was reported in Ref. [7], where $S-N-S$ contact with a size comparable to the de Broglie wavelength in the N region made of BiPb was realized and nonmonotonic behaviour of the critical current with the thickness of normal region was found. This remarkable development of technology suggests the feasibility of making a system of a small superconductive (SC) island connected to the superconductive leads by two quantum point contacts (QPC). In such a system, macroscopic quantum effects due to competition between Josephson coupling energy and Coulomb (charging) energy could be realized, together with quantization (due to the small number of conductive channels) of the Josephson critical current.

In the present paper we develop a theory for a limiting case of such a system, namely, two almost ballistic one-channel QPCs connecting a small SC island with two SC leads. We consider the limit of the characteristic charging energy much smaller than the superconducting gap, $E_C \ll \Delta$; therefore, Coulomb effects are small. We derive the dependence of the average Josephson current across the system and its fluctuations (noise power) as functions of the SC phase difference between the leads $\alpha$, and of the electric gate potential $V_g$. Coulomb effects show up at phase differences $\alpha$ close to $\pi$, where the two lowest states are almost degenerate. We show that such a system realizes a tunable quantum two-level system (pseudospin 1/2) which may be useful for the realization of quantum computers (see, e.g., Refs. [8–11]).

The paper is organized as follows. We start by considering a single QPC connecting a superconducting island to a single lead (Sec. 2). We find the oscillations of the effective capacitance of the island as a function of the gate potential (in some analogy with Matveev's results [2] for a normal QPC). Depending on the backscattering probability in the contact, it can be described either in the adiabatic or in the diabatic approximation. We find the condition for diabatic-adiabatic crossover. Then in Sec. 3 we formulate a simple model for a double-contact system in the adiabatic approximation. We replace the full many-body problem by a quantum-mechanical problem for the dynamics of the SC phase on the middle island. In Sec. 4 we calculate the average Josephson current through the system as a function of $\alpha$ and $V_g$, with particular emphasis on phase differences $\alpha$ close to $\pi$ (where our effective two-level system is almost degenerate). Section 5 is devoted to the analysis of Josephson current noise; we calculate total intensity $S_0$ of the «zero»-frequency noise (an analog of the noise calculated in Refs. [12–14] for a single superconductive QPC), as well as finite-frequency noise $S_\omega$ due to transitions between the two almost-degenerate levels. Finally, we present our conclusions in Sec. 6.

## 2. ADIABATIC-DIABATIC CROSSOVER IN A SINGLE QUANTUM POINT CONTACT

Consider a small superconducting island connected to an external superconducting lead by an one-channel, nearly ballistic quantum point contact [5, 15]. The electric potential of the grain can be adjusted via a gate terminal (Fig. 1*a*). Following Ref. [5], we assume that the contact is much wider than the Fermi wavelength (so that transport through the constriction can be treated adiabatically), but much smaller than the coherence length $\xi_0 \equiv \hbar v_F/\pi\Delta$ (where $v_F$ is the Fermi velocity, and $\Delta$ is the superconducting gap).

Our low-temperature assumption is that the average number of one-electron excitations on

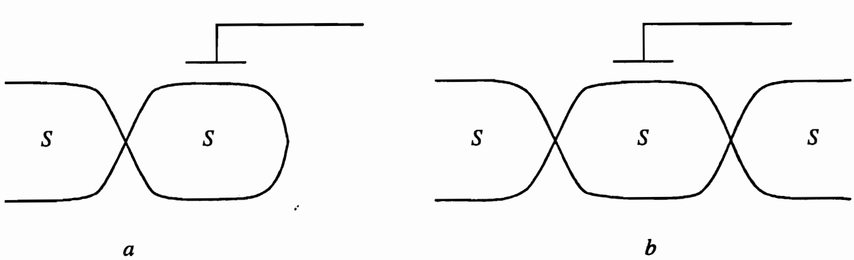


**Fig. 1.** *a*) Single QPC. The system consists of a SC grain connected to a SC lead via a QPC. A gate terminal is used to control the electric potential of the grain. *b*) Double-contact $S-S-S$ system. The second terminal is added to the single-QPC setup

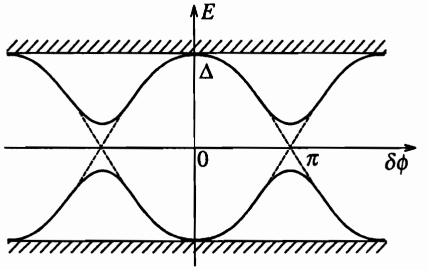


**Fig. 2.** Single-contact energy spectrum. The spectrum consists of the continuum of delocalized states and the two Andreev (subgap) states. Dashed lines denote Andreev states in the absence of backscattering (diabatic terms). Solid lines are the states split by backscattering (adiabatic terms)

the island is much less than one. Then they cannot contribute to the total charge of the grain, and we may restrict our Coulomb blockade problem to the evolution of the superconducting phase only. The low-temperature condition is then $T < \Delta/\ln(V\nu(0)\Delta)$, where $V$ is the volume of the grain and $\nu(0)$ is the density of electron states at the Fermi level.

We neglect phase fluctuations in the bulk of the island and describe the whole island by a single superconducting phase $\chi$. At a fixed value of the phase on the island, the spectrum of the junction consists of the two Andreev states localized on the junction and the continuum spectrum above the gap [15] $\Delta$ (Fig. 2). The energies of the Andreev states lie below the gap:

$$E(\chi) = \pm\Delta\sqrt{1 - t\sin^2(\chi/2)}, \qquad (1)$$

where $\chi$ is the phase difference at the contact and $t$ is the transmission coefficient.

At $t = 1$, the spectrum of Andreev states (1) has a level crossing point at $\chi = \pi$. At this point, the left and right Andreev states have equal energies, but in the absence of backscattering ($t = 1$), transitions between them are impossible. Therefore, we expect that an ideal ballistic contact cannot adiabatically follow the ground state as the phase $\chi$ changes, but remains in the same left or right Andreev state as it passes the level-crossing point $\chi = \pi$. We borrow the terminology from the theory of atomic collisions [16] and call the (crossing) Andreev levels at $t = 1$ diabatic terms (dashed lines in Fig. 2), and the split levels — adiabatic terms (solid lines in Fig. 2). Instead of a transmission coefficient $t$, it will be more convenient to speak of the reflection coefficient $r = 1 - t$. At $r = 0$, the contact is described by diabatic terms. As $r$ increases, transitions occur between the terms, and at sufficiently large $r$ the system will mostly adiabatically follow the split Andreev levels. In this section we study the adiabatic-diabatic crossover, and find the crossover scale for the reflection coefficient $r$.

We assume that the reflection probability $r \ll 1$ (almost perfect transmission) and that the charging energy $E_C \ll \Delta$ (the charging energy is defined by $E_C = (2e)^2/C$). The latter assumption appears natural, because as in tunnel junctions [17], we expect that the capacitance $C$ of the grain has an additional contribution from the capacitance of the point contact. This capacitance is of order $\Delta/e^2$. A more detailed discussion of this phenomenon will be given elsewhere. For now, we just mention that this contribution to the capacitance leads to the inequality $E_C \leq \Delta$.

To probe the degree of adiabaticity, we study the periodic dependence of the ground state energy $E_0$ on the gate voltage. Because of the weakness of charging effects, this dependence will be sinusoidal:

$$E_0(V_g) = \varepsilon\cos(2\pi N) \qquad (2)$$

(where $N = V_g C/2e$ is the dimensionless voltage), and we are interested in the amplitude $\varepsilon$ of these oscillations. The physical origin of this periodicity is oscillations of the induced charge on the grain; this follows immediately from the relation

$$\delta Q = \frac{C}{2e}\frac{\partial E_0}{\partial N}. \tag{3}$$

There is a simple physical explanation of the sinusoidal dependence (2). The ground-state energy modulation is determined by phase-slip processes in the contact. Such processes are phase tunneling events with phase changing by $\pm 2\pi$. While the magnitudes of the clockwise and counterclockwise tunneling amplitudes are the same, their phases are $\pm 2\pi N$. This results in the expression (2). Higher-order tunneling processes would give rise to higher-order harmonics in the periodic $N$-dependence. This argument shows that the amplitude of oscillations $\varepsilon$ coincides with the phase-tunneling amplitude, and therefore provides a good measure of adiabaticity in the phase dynamics.

Assuming $E_C \ll \Delta$, we can describe the contact by the dynamics of the phase on the grain, and thus reduce the problem to single-particle quantum mechanics. Since we restrict our attention to low-lying excitations, it is only necessary to include the two Andreev levels on the junction. The potential term is the Josephson energy of the Andreev levels, and the kinetic term is the charging energy. After a simple computation of the backscattering matrix elements (the off-diagonal entries in the potential term), we arrive at the Hamiltonian:

$$H = H(\chi) + \frac{1}{2}E_C(\pi_\chi - N)^2, \tag{4}$$

where

$$H(\chi) = \Delta \begin{pmatrix} -\cos\frac{\chi}{2} & r^{1/2}\sin\frac{\chi}{2} \\ r^{1/2}\sin\frac{\chi}{2} & \cos\frac{\chi}{2}. \end{pmatrix} \tag{5}$$

Here $\chi$ is the phase difference across the contact, and $r$ is the reflection coefficient. Obviously, the eigenvalues of $H(\chi)$ reproduce the result (1). The number of Cooper pairs at the grain $\pi_\chi$ is the momentum conjugate to $\chi$, $[\chi, \pi_\chi] = i$. Notice that $\chi$ takes values on the circle $\chi = \chi + 2\pi$, and accordingly $\pi_\chi$ is quantized to take integer values. We can also write $\pi_\chi = -i\partial/\partial\chi$.

This Hamiltonian loses its validity at the top of the upper band at $\chi = 2\pi n$, where the upper Andreev state mixes with the continuous spectrum (Fig. 2). However, the probability of the phase $\chi$ reaching the top of the upper band of $H(\chi)$ is exponentially small at $E_C \ll \Delta$ (smaller than the tunneling probability). The adiabatic-diabatic crossover is determined by the properties of the system near the minimal-gap point $\chi = \pi$. We can therefore neglect transitions to the continuous spectrum at $\chi = 2\pi n$. At the same time, we must disregard tunneling processes via the top of the upper Andrees band (next-nearest-neighbor tunneling), which are present in the Hamiltonian (4)-(5), but not in the original system. Nearest-neighbor tunneling is a feature of our model, and is beyond the precision of our approximation.

There are two opposite limits of the problem: small and «strong» reflection.

At zero reflection, the Hamiltonian splits into lower and upper components. Within each component the potential is periodic with period $4\pi$. As explained above, we must neglect next-nearest-neighbor tunneling via the top of the bands. Therefore, the potential minima of $H(\chi)$ are disconnected and cannot tunnel to each other ($\varepsilon = 0$).

The opposite limit is the case of «large» reflection (the precise meaning of «strong reflection» consistent with $r \ll 1$ will be clarified below). In this limit, a gap opens in the spectrum of Andreev states, and the system adiabatically follows the lower state. We can replace the two-level Hamiltonian $H(\chi)$ by its lowest eigenvalue and arrive to the quantum-mechanical problem of a particle in a periodic potential. The semiclassical limit of this problem is solved in Ref. [18]. In our notation, the result is

$$\varepsilon_{ad} = \text{const}\sqrt{E_C \Delta} \exp(-S_{cl}), \tag{6}$$

where

$$S_{cl} = B_1 \sqrt{\frac{\Delta}{E_C}} - \frac{1}{4} \ln \frac{\Delta}{E_C} + O(1) \tag{7}$$

is the classical action connecting two nearest minima (or more precisely, the two turning points). The numerical constant $B_1$ is of order unity (at $r \to 0$, $B_1 = 4.69 + 1.41 r \ln r + \ldots$).

To study how the adiabaticity is destroyed, it is useful to introduce the dimensionless «coherence factor» $f(r)$ defined by

$$\varepsilon = f(r) \varepsilon_{ad}, \tag{8}$$

where $\varepsilon_{ad}$ is the amplitude of oscillation of the ground-state energy derived in the adiabatic approximation (with only the lowest Andreev state included). We see that $f(0) = 0$ and $f(r \gg r_{ad}) = 1$. The crossover scale $r_{ad}$ can be derived by computing the corrections to $f(r)$ in these two limits.

We first consider the limit of weak backscattering ($r \ll r_{ad}$). In this limit, we take the wavefunction to be the ground state of the Hamiltonian with zero $r$ (at a given wavevector $N$), and then compute the first-order correction in $r^{1/2}$ to the energy. The wavefunction is of «tight-binding» type, and is generated by the «ground-state» wavefunctions $\Psi_i$ localized in the potential minima (diabatic terms). The components of the two-dimensional vectors $\Psi_i$ alternate:

$$\Psi_i = \begin{pmatrix} \Psi_i(\chi) \\ 0 \end{pmatrix}, \qquad \Psi_{i+1} = \begin{pmatrix} 0 \\ \Psi_{i+1}(\chi) \end{pmatrix}. \tag{9}$$

We then find

$$\varepsilon = 2\langle \Psi_i | H_{12}(\chi) | \Psi_{i+1} \rangle = 2 r^{1/2} \Delta \int d\chi \, \Psi_i^*(\chi) \Psi_{i+1}(\chi) \sin \frac{\chi}{2} \tag{10}$$

(we assume the wavefunctions $\Psi_i$ to be normalized). It is important to note that $\Psi_i$ and $\Psi_{i+1}$ are wavefunctions for different potentials ($-\Delta_0 \cos(\chi/2)$ and $\Delta_0 \cos(\chi/2)$); the overlap integral (10) has a saddle point at the minimal-gap point $\chi = \pi$, and it reduces the effective region of integration to $|\chi - \pi| \leq (E_C/\Delta)^{1/4}$. The normalization of the semiclassical tail of the wavefunctions $\Psi_i(\chi)$ yields

$$\Psi(\chi = \pi) = \exp\left(-S_{cl}(\chi = \pi)\right) \tag{11}$$

(up to a numerical factor independent of $E_C/\Delta$). We thus obtain

$$\varepsilon \sim r^{1/2} \Delta \left( \frac{E_C}{\Delta} \right)^{1/4} \exp(-S_{cl}), \tag{12}$$

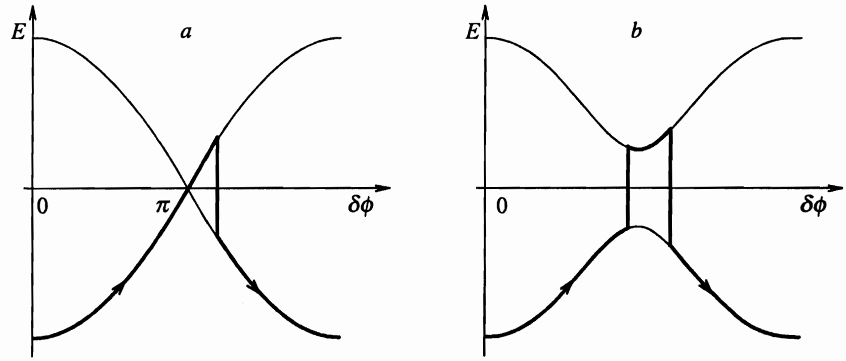


**Fig. 3.** Tunneling paths in the diabatic (*a*) and adiabatic (*b*) limits. These diagrams represent the lowest-order corrections to the phase-tunneling amplitudes in the diabatic and adiabatic limits, respectively

i.e., in terms of the «coherence factor» $f(r)$,

$$f(r) \sim r^{1/2} \left(\frac{\Delta}{E_C}\right)^{1/4}. \tag{13}$$

Physically, meaning of the integral (10) is the sum over all paths shown in Fig. 3*a*.

The above calculation shows that the crossover scale to adiabatic behavior is

$$r_{ad} \sim \left(\frac{E_C}{\Delta}\right)^{1/2}. \tag{14}$$

In fact, we neglected the effect of change in the classical action $S_{cl}$ due to the gap opening; this effect is estimated to be of order

$$\delta S_{cl} \sim \sqrt{\frac{\Delta}{E_C}}\, r \ln r, \tag{15}$$

i. e., it is a higher-order effect than the change in $f(r)$ proportional to $r^{1/2}$. Notice that the characteristic scale of this change in the classical action is again $r_{ad} \sim \sqrt{E_C/\Delta}$ (corresponding to $\delta S_{cl} \sim 1$).

We can alternatively find the crossover scale $r_{ad}$ by computing the lowest order correction to the «coherence factor» $f(r)$ in the adiabatic limit. In this limit the Hamiltonian (4), (5) can be rewritten in adiabatic terms (for simplicity the voltage $N$ is introduced into the boundary condition $\Psi(\chi + 2\pi) = e^{2i\pi N}\Psi(\chi)$ by a gauge transformation) as

$$H = -\frac{E_C}{2}\left(\frac{\partial}{\partial\chi}\right)^2 + D(\chi) - \frac{E_C}{2}\left[G(\chi)\frac{\partial}{\partial\chi} + \frac{\partial}{\partial\chi}G(\chi)\right] - \frac{E_C}{2}\,G^2(\chi), \tag{16}$$

where

$$D(\chi) = \begin{pmatrix} E_1(\chi) & 0 \\ 0 & E_2(\chi) \end{pmatrix} \tag{17}$$

is the diagonalized form of the matrix (5),

$$G(\chi) = \begin{pmatrix} 0 & g(\chi) \\ -g(\chi) & 0 \end{pmatrix}, \quad g(\chi) = \langle 0| \frac{\partial}{\partial \chi} |1\rangle, \tag{18}$$

and $|0\rangle$ and $|1\rangle$ are the eigenvectors of the matrix (5). The last term in the Hamiltonian (16) can be shown to yield smaller corrections than the first-order term in $G(\chi)$. A careful second-order perturbation calculation in $g(\chi)$ yields

$$1 - f(r) \sim \int\limits_{\chi_1 < \chi_2} \exp\{S_1(\chi_1, \chi_2) - S_2(\chi_1, \chi_2)\}\, g(\chi_1) g(\chi_2)\, d\chi_1\, d\chi_2, \tag{19}$$

where $S_{1,2}(\chi_1, \chi_2)$ are the classical actions along the lower and upper adiabatic branches between the points $\chi_1$ and $\chi_2$. This integral corresponds to summation over all tunneling paths shown in Fig. 3*b*. The function $g(\chi)$ for the given matrix $H(\chi)$ is a Lorentzian peak at $\chi = \pi$ of height $r^{-1/2}$ and width $r^{1/2}$. Putting everything together, the integral (19) is calculated to be

$$1 - f(r) \sim \frac{1}{r} \sqrt{\frac{E_C}{\Delta}}. \tag{20}$$

This asymptotic behaviour agrees with the crossover scale (14) found previously.

To summarize the results of this section, the characteristic scale for adiabatic-diabatic crossover in a nearly-ballistic single contact is found to be $r_{ad} \sim \sqrt{E_C/\Delta}$. The phase tunneling amplitude is proportional to the gate-voltage modulation of the effective capacitance of the island, and thus can be directly measured. At low reflection coefficients, these oscillations are proportional to $\sqrt{r}$, as in the normal one-channel QPC [2].

## 3. ADIABATIC APPROXIMATION OF A DOUBLE-JUNCTION SYSTEM

We now turn to the case of a double-junction system (Fig. 1*b*). As before, we assume that the reflection probabilities in both contacts are small, $r_i \ll 1$, that the charging energy $E_C \ll \Delta$, and that the temperature is sufficiently low to preclude single-electron excitations on the grain. To adjust the electrostatic potential of the grain we again use a gate terminal; $N = V_g C/2e$ denotes the dimensionless gate voltage, as before.

For the moment, to simplify the discussion we assume that the reflection coefficients in the contacts are greater than the crossover scale $r_{ad}$ found in the previous section; we can therefore consider only the lower adiabatic branch of the Andreev states. In fact, the results can be extended further to the case $r_i < r_{ad}$ by using appropriate «coherence factors» $f(r)$, similar to those in the previous section.

We set the superconducting phase on one of the leads to zero; the phase on the other lead $\alpha$ is assumed to be fixed externally. Then the total Josephson energy of the two contacts is (Fig. 4):

$$U(\chi) = U_1(\chi) + U_2(\alpha - \chi), \tag{21}$$

where

$$U_i(\delta\phi) = -\Delta\sqrt{1 - t_i \sin^2(\delta\phi/2)} \tag{22}$$

are the lower adiabatic Andreev terms in the two junctions.

At $t_1 = t_2 = 1$, the potential $U(\chi)$ obviously has two minima — at $\chi = \alpha/2$ and at $\chi = \alpha/2 + \pi$ — and sharp peaks at $\chi = \pi$ and $\chi = \pi + \alpha$ (Fig. 4). At small nonzero $r_i$, gaps open at the crossing points of Andreev levels, which smoothes the peaks of $U(\chi)$. Still, the bottom of the potential remains essentially unchanged.

The adiabatic Hamiltonian for the double junction becomes

$$H(\alpha, N) = U(\chi) + U(\alpha - \chi) + \frac{1}{2}E_C\left(-i\frac{\partial}{\partial\chi} - N\right)^2. \tag{23}$$

The potential term of the Hamiltonian is the sum of Josephson energies of the contacts, and the kinetic term is the Coulomb energy of the charge at the grain.

## 4. JOSEPHSON CURRENT

The condition $E_C \ll \Delta$ enables us to treat the Coulomb term in the Hamiltonian perturbatively. First, neglecting the Coulomb term, we obtain a classical system defined on the circle $\chi \in (0, 2\pi)$ in the potential (21) with two minima. The energies of the minima are $V_1(\alpha) = -2\Delta|\cos(\alpha/4)|$ and $V_2(\alpha) = -2\Delta|\sin(\alpha/4)|$ (see Fig. 4). To very high accuracy, we can neglect backscattering in determining the minima, except near $\alpha = 0$. Since all Coulomb effects occur near the resonance point $\alpha = \pi$, this approximation is justified. At zero temperature, our classical system prefers the lowest of the minima. Thus the energy of the $S-S-S$ system in the absence of the Coulomb term is given by

$$E(\alpha) = -2\Delta\cos(\alpha/4) \qquad \text{for} \quad -\pi < \alpha < \pi \tag{24}$$

(see Fig. 5). Differentiating this energy with respect to the phase $\alpha$ gives the Josephson current

$$I(\alpha) = 2e\frac{\partial E(\alpha)}{\partial\alpha} = \Delta\sin\frac{\alpha}{4} \qquad \text{for} \quad -\pi < \alpha < \pi \tag{25}$$

(Fig. 6). Notice that the current has large jumps at the points of level crossing $\alpha = \pi + 2\pi n$. Qualitatively this picture is very similar to the case of a single $S-S$ ballistic junction, but the shape of the current-phase dependence $I(\alpha)$ is different.

If we assume a nonzero temperature $T \ll \Delta$, the occupation of the upper minimum is exponentially small except in the vicinity of the level-crossing point $|\alpha - \pi| \sim T/\Delta$. Thus, the effect of the temperature is to smear the singularity in $I(\alpha)$ at $\alpha = \pi$.

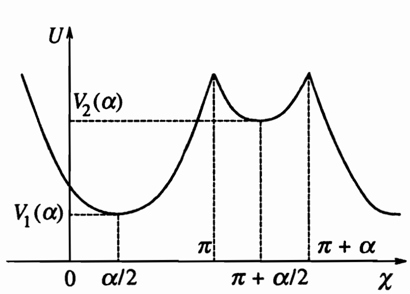


**Fig. 4.** Potential $U(\chi)$. At $\alpha \neq 0$ it has two minima. Finite backscattering in the contacts smoothes the summits of the potential, but leaves the bottom of the wells unchanged

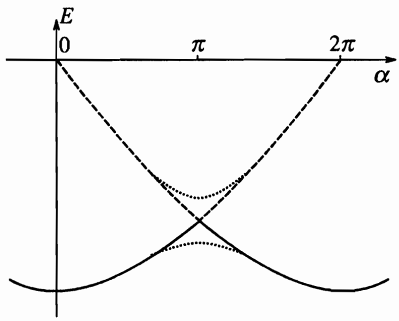


Рис. 5

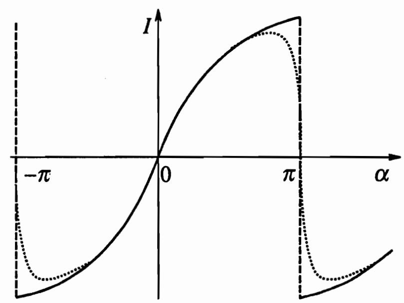


Рис. 6

**Fig. 5.** Classical minimum of the potential $U(\chi)$ as a function of the external phase difference $\alpha$. Dotted line shows the quantum gap opened by the Coulomb term

**Fig. 6.** Josephson current as a function of the external phase difference $\alpha$. Dotted line shows smearing of the singularity due to the Coulomb term

Another source of level mixing near the singular point $\alpha = \pi$ is quantum fluctuations, i.e., fluctuations arising from the kinetic term in the Hamiltonian (23). They result in nonzero tunnelling amplitudes through the two potential barriers between the potential minima. Due to the shift in the «angular momentum» by $N$, the wave functions in the two potential wells aquire an additional factor $\exp(iN\chi)$. This results in the relative phase of the two tunneling amplitudes differing by $2\pi N$. The net tunneling amplitude (defining the level splitting) can be written

$$H_{12}(N) \equiv \Delta\gamma(N) = \Delta(\gamma_1 e^{i\pi N} + \gamma_2^{-i\pi N}), \tag{26}$$

where $\gamma_1$ and $\gamma_2$ are the two amplitudes of phase tunneling in the two different directions (i.e., of phase slip processes in the two different contacts). Below we assume that these amplitudes are computed at the level-crossing point $\alpha = \pi$, where they are responsible for level splitting.

The amplitudes $\gamma_1$ and $\gamma_2$ have the asymptotic behaviour derived in the previous section (except for numerical factors). When the backscattering in the contacts is such that $r \gg r_{ad}$, they can be found in the semiclassical approximation:

$$\gamma_{1,2} \sim \left(\frac{E_C}{\Delta}\right)^{1/4} \exp\left(-B_2\sqrt{\frac{\Delta}{E_C}}\right) \ll 1, \tag{27}$$

where $B_2 \sim 1$ is determined by the classical action connecting the two potential minima (at $r \ll 1$, $B_2 \simeq 1.45 + 2.20 r \ln r + \ldots$). At $r \ll r_{ad}$, the tunneling amplitudes are

$$\gamma_{1,2} \sim r^{1/2} \exp\left(-B_2\sqrt{\frac{\Delta}{E_C}}\right). \tag{28}$$

For the best observation of Coulomb oscillations, $\gamma_1$ and $\gamma_2$ must be of the same order, but not too small. In the ideal case $\gamma_1 = \gamma_2 = \gamma$, the total amplitude is

$$\gamma(N) = 2\gamma\cos(\pi N). \qquad (29)$$

Although the periodic dependence (29) has a period of $4e$ as function of the «external charge» $Q_x = CV_g \equiv 2eN$, the Josephson current and its fluctuations only depend on $|\gamma(N)|^2$ (cf. Eqs. (32), (34) below), and their period is $2e$, as expected [3].

The characteristic scale for the $r$-dependence of $B_2$ is $\delta r \sim \sqrt{E_C/\Delta}$, so for $\gamma_1$ and $\gamma_2$ to be of the same order, the transparencies of the two contacts must differ by no more than $|r_1 - r_2| \le \sqrt{E_C/\Delta}$.

Here we should comment on the difference between our result (26)–(28) and the normal two-channel system discussed in Ref. [2]. In the normal system, the two tunneling amplitudes multiply, and the net ground-state energy oscillations are proportional to $r\ln r$ at small $r$. In the superconducting system, the external leads have different superconducting phases, and the tunneling in the two contacts occurs at different values of the phase on the grain. Therefore, the tunneling amplitudes add with certain phase factors, and yield the asymptotic behavior $\sqrt{r}$ at $r \to 0$. In fact, the oscillations in the superconducting system will be proportional to $r$ (as in the normal system [2]) in a different limit — at a phase difference $\alpha = 0$, where the potential $U(\chi)$ has a single minimum and a single barrier.

The hybridized energy levels in the vicinity of $\alpha = \pi$ are given by the eigenvalues of the $2\times 2$ Hamiltonian

$$H(\alpha, N) = \begin{pmatrix} V_1(\alpha) & H_{12}(N) \\ H_{12}(N) & V_2(\alpha) \end{pmatrix}. \qquad (30)$$

Diagonalization yields the two energy levels:

$$E_{1,2}(\alpha, N) = -\Delta\left[|\sin\frac{\alpha}{4}| + |\cos\frac{\alpha}{4}| \pm \sqrt{\left(|\sin\frac{\alpha}{4}| - |\cos\frac{\alpha}{4}|\right)^2 + \gamma^2(N)}\right]. \qquad (31)$$

The off-diagonal matrix elements of the Hamiltonian open a gap at the level-crossing point $\alpha = \pi$ (Fig. 5). This gap depends periodically on the gate voltage $V_g$, and these oscillations comprise the Coulomb effects in the $S - S - S$ junction.

We can obtain the Josephson current by differentiating the energy levels with respect to the phase $\alpha$. The gap results in smearing the singularity in $I(\alpha)$, even at zero temperature (Fig. 6):

$$I(\alpha) = \frac{\Delta}{\sqrt{2}}\sin\left(\frac{\alpha-\pi}{4}\right)\left[1 - \frac{\cos\left(\frac{\alpha-\pi}{4}\right)}{\sqrt{\sin^2\left(\frac{\alpha-\pi}{4}\right) + \frac{1}{2}\gamma^2(N)}}\right] \quad \text{for} \quad \alpha \sim \pi. \qquad (32)$$

The width of the crossover at $\alpha = \pi$ depends periodically on $V_g$: $|\alpha - \pi| \sim |\gamma(N)|$.

In the above discussion we neglected excited oscillator states. The interlevel spacing for the excitations in the potential wells is of order $\sqrt{\Delta E_C} \gg \Delta\gamma$. Therefore, Coulomb effects have a much smaller energy scale and the excited states do not participate in mixing the ground states of the two potential wells.

At nonzero temperature, these Coulomb effects compete with temperature-induced smearing, so that the width of the singularity at $\alpha = \pi$ is given at nonzero temperature $T \ll \Delta$ by $|\alpha - \pi| \sim \max(\gamma(N), T/\Delta)$. In order for Coulomb effects to dominate thermal fluctuations, we must therefore have $T \le \gamma\Delta$.

It is instructive to compare this picture with the case of a multi-channel $S-S-S$ tunnel junction (in contrast to the results of Ref. [3], note that we consider the opposite limit, with $\Delta \gg E_C$). If we develop a similar theory for tunnel Josephson junctions, we find that the potentials (21) and (22) are both sinusoidal, and therefore the total potential (21) has only one minimum (versus two in the nearly ballistic system). In the tunnel $S-S-S$ system, the current-phase relation $I(\alpha)$ is smeared at $\alpha = \pi$ due to the difference between the critical currents of the two Josephson junctions. Coulomb effects compete with this smearing, and in order to prevail, the charging energy $E_C$ must be greater than the difference of the critical currents. In the tunnel system, the corresponding splitting $\gamma$ is linear in $E_C$, while in the nearly ballistic system it is exponentially small. Otherwise, Coulomb oscillations in $I(\alpha)$ will appear similar in these two cases.

To summarize this section, we observed that the Coulomb effects in the one-channel $S-S-S$ junction smears the singularity in the Josephson current $I(\alpha)$ at the critical value $\alpha = (2n+1)\pi$. This smearing depends periodically on the potential of the grain with period $2e/C$, and is exponentially small in the adiabatic parameter $E_C/\Delta \ll 1$. The smearing is the result of mixing the two states in the potential minima of the Josephson energy.

## 5. FLUCTUATIONS OF THE JOSEPHSON CURRENT

In this section we compute the low-frequency spectrum of the fluctuations of the Josephson current in our model. We shall be interested in frequencies much less than the oscillator energy scale $\sqrt{\Delta E_C}$, so we consider only transitions between the eigenstates of the reduced ground-state Hamiltonian (30). We also assume that the temperature is lower than $\sqrt{\Delta E_C}$; we can then disregard excited oscillator states and internal noise in the contacts (discussed in Refs. [12–14, 19]). Obviously, under these assumptions we can observe current fluctuations only in the immediate neighbourhood of the resonance point $\alpha = \pm\pi$, where the energies (31) of the two low-lying states are close to each other.

We expect to observe two peaks in the noise spectrum — one at zero frequency (due to thermal excitations above the ground state), and the other at the transition frequency $|E_1 - E_2|$ (due to off-diagonal matrix elements of the current operator). In this section we compute the total weights of these peaks and postpone discussion of their width (determined by dissipative processes).

Consider first the zero-frequency peak. In our approximation it is just the thermal noise of a two-level system. In the vicinity of the resonance point $\alpha = \pi$, we can linearize the spectrum $V_{1,2}(\alpha)$ and make the approximation that one of the two states carries the current $I(\alpha, N)$, and the other $-I(\alpha, N)$. The spectral weight of the noise is then given by a simple formula:

$$S_0(\alpha, N, T) \equiv \langle I^2 \rangle - \langle I \rangle^2 = \frac{I^2(\alpha, N)}{\mathrm{ch}^2 \dfrac{E_1 - E_2}{2T}}. \tag{33}$$

Substituting $I(\alpha, N)$ and $E_{1,2}(\alpha, N)$ from the previous section, we obtain the noise intensity near resonance:

$$S_0(\alpha, N, T) = \frac{\Delta^2}{2} \frac{\left(\dfrac{\alpha-\pi}{2\sqrt{2}}\right)^2}{\left(\dfrac{\alpha-\pi}{2\sqrt{2}}\right)^2 + \gamma^2(N)} \mathrm{ch}^{-2}\left(\frac{\Delta}{T}\sqrt{\left(\frac{\alpha-\pi}{2\sqrt{2}}\right)^2 + \gamma^2(N)}\right). \tag{34}$$

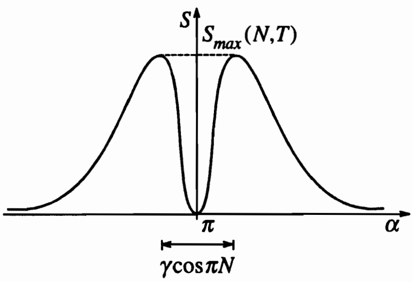


**Рис. 7**

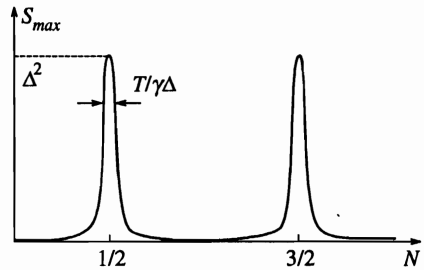


**Рис. 8**

**Fig. 7.** Zero-frequency noise as a function of phase $\alpha$. It decays exponentially for $\alpha$ far from the resonance point $\alpha = \pi$. Right at the very resonance point, the noise is suppressed, because both states carry nearly zero Josephson current

**Fig. 8.** Maximum value of the noise versus the potential of the grain. The period of the peaks corresponds to the period $2e$ of the induced charge $Q = CV_g$. The width of the peaks depends on the capacitance of the grain

For the effect of the Coulomb interaction to be observable, the temperature must be smaller than the Coulomb gap: $T \leq \gamma\Delta$. At constant $T$ and $N$, the noise decreases exponentially as $\alpha$ moves away from its critical value $\alpha = \pi$, and at $\alpha = \pi$ the noise is suppressed in the interval $|\alpha - \pi| < \gamma(N)$ (Fig. 7). The interplay between these two factors results in a strong dependence of the peak noise on the potential of the grain. The peak value of the noise $\max_\alpha S(\alpha, N, T)$ is plotted against $N$ in Fig. 8. Most favorable is the case of identical contacts, where $\gamma_1 = \gamma_2 = \gamma$, and therefore $\gamma(N) = 2\gamma\cos(\pi N)$. In this case, when $\cos(\pi N) \ll T/\gamma\Delta$ (small gap limit) the noise takes its maximal value $S \approx \Delta^2/2$. In the opposite limit of large gap ($\cos(\pi N) \gg T/\gamma\Delta$), the noise decreases exponentially:

$$S \approx \Delta^2 \left[ \frac{T}{\Delta\gamma|\cos\pi N|} \exp\left( -4\frac{\Delta\gamma|\cos\pi N|}{T} \right) \right].$$

The noise has a sharp peak at the resonance point $\cos\pi N = 0$, where two levels on the grain with different electron numbers have equal energies.

Now we turn to the noise peak at the interlevel frequency $\omega = |E_1 - E_2|$. Since $\omega$ can now be large compared to $T$, one needs to discriminate between different kinds of frequency-dependent correlation functions, which can be measured as a noise intensity in different experimental situations [21]; here, by noise we mean the Fourier spectrum of the time-symmetric current-current correlation function. In our approximation of a two-level system, such noise is temperature independent, and its weight is determined purely by the off-diagonal matrix element:

$$S_\omega = \frac{1}{2} \left| \langle 1 | I | 2 \rangle \right|^2. \tag{35}$$

A straightforward computation for the Hamiltonian (30) and $I = 2e(\partial H/\partial\alpha)$ yields (in the vicinity of $\alpha = \pi$)

$$\langle 1|I|2\rangle = \frac{\Delta^2\gamma(N)}{\omega}\left(\cos\frac{\alpha}{4} + \sin\frac{\alpha}{4}\right) \tag{36}$$

and

$$S_\omega(\alpha, N) = \Delta^2 \left(\frac{\Delta\gamma(N)}{\omega}\right)^2 \cos^2\frac{\alpha-\pi}{4}. \tag{37}$$

This result contrasts with the corresponding noise intensity in the single quantum point contact (found in Refs. [12, 13, 19]). In the single quantum point contact, the correponding noise intensity $S_\omega$ is temperature-dependent, because that system has four possible states (or, alternatively, two fermion levels). In the case of the double junction, the system has only two states differing by the phase on the grain, and the quantum fluctuations $S_\omega$ become temperature-independent.

## 6. CONCLUSIONS

We have developed a theory of Coulomb oscillations of the Josephson current and its noise power via the $S-S-S$ system with nearly ballistic quantum point contacts. The period of Coulomb oscillations as a function of the gate potential is $V_g^0 = 2e/C$. These oscillations arise from the semiclassical tunneling of the superconducting phase on the grain, and are therefore exponentially small in $\sqrt{E_C/\Delta}$ at $E_C \ll \Delta$. In addition, we predict a crossover from adiabatic to diabatic tunneling at the backscattering probability $r_{ad} \sim \sqrt{E_C/\Delta}$. At backscattering below $r_{ad}$, the amplitude $\varepsilon$ of the Coulomb oscillations is proportional to the square root of the lesser (of the two contacts) reflection probability $\sqrt{r_{min}}$. This is in contrast to the case of a normal double-contact system [20], in which $\varepsilon$ is proportional to the product $\sqrt{r_1 r_2}$.

The average Josephson current-phase relation $I(\alpha)$ is shown to be strongly nonsinusoidal and roughly similar to the one known for a single nearly ballistic QPC, in the sense that it exhibits abrupt «switching» between positive and negative values of the current as the phase varies via $\alpha = \pi$. The new feature of our system is that it is possible to vary the width of the swithching region $\delta\alpha$ by the electric gate potential $V_g$; in the case of equal reflection probabilities $r_1 = r_2$, this electric modulation is especially pronounced, $\delta\alpha \propto |\cos(\pi C V_g/2e)|$. The noise spectrum of the supercurrent is found to consist mainly of two peaks: the «zero-frequency» peak due to rare thermal excitations of the upper level of the system, and another one centered around the energy difference $\omega_\alpha$ between the two levels. The widths of these peaks are determined by the inverse lifetime $\tau$ of the two states of our TLS, which is due to electron-phonon and electromagnetic couplings. Both sources of level decay are expected to be very weak in the system considered, but the corresponding quantitative analysis is postponed to future studies; we present here only results for the *frequency-integrated* (over those narrow intervals $\sim 1/\tau$) noise power.

The $S-S-S$ device with almost ballistic contacts is a new type of system that can be used to implement an artificial «spin 1/2» — an elemental unit for quantum computation. In comparison with conventional Josephson systems with tunnel junctions, which were proposed for use in adiabatic quantum computation [11], the advantage of our system is that it can operate at considerably higher critical Josephson currents. Moreover, the current-phase characteristics of such a system is almost universal, in the sense that they are determined mainly by the microscopic parameters of the SC materials, and only weakly by the specifics of contact fabrication.

We are grateful to K. A. Matveev, Yu. V. Nazarov, and especially G. B. Lesovik for many useful discussions. The research of M. V. F. was supported by INTAS-RFBR grant № 95-0302, collaboration grant № 7SUP J048531 from the Swiss National Science Foundation, DGA grant № 94-1189, and the Programm «Statistical Physics» of the Russian Ministry of Science.

## References

1. L. I. Glazman and K. A. Matveev, Zh. Exp. Teor. Fiz. **98**, 1834 (1990) (Sov. Phys. JETP **71**, 1031 (1990)).
2. K. A. Matveev, Phys. Rev. B **51**, 1743 (1995).
3. L. I. Glazman, K. A. Matveev et al., Physica B **203**, 316 (1994).
4. H. Takayanagi, T. Akazaki, and J. Nitta, Phys. Rev. Lett. **75**, 3533 (1995).
5. C. W. Beenakker and H. van Houten, Phys. Rev. Lett. **66**, 3056 (1991).
6. M. C. Koops et al., Phys. Rev. Lett. **77**, 2542 (1996).
7. A. Yu. Kasumov et al., Phys. Rev. Lett. **77**, 3029 (1996).
8. A. Yu. Kitaev, E-print quant-ph/9707021 (1997).
9. D. P. Divincenzo, E-print cond-mat/9612126 (1996).
10. A. Shnirman, G. Schoen, and Z. Hermon, E-print cond-mat/9706016 (1997).
11. D. V. Averin, E-print quant-ph/9706026 (1997).
12. A. Martin-Rodero, A. Levy Yeyati, and F. J. Garcia-Vidal, Phys. Rev. B **53**, R8891 (1996).
13. D. Averin and H. T. Imam, Phys. Rev. Lett. **76**, 3814 (1996).
14. G. B. Lesovik and A. Golubov, in *Proc. of the 31 Recontres de Moriond*, ed. by T. Martin et al., Editions Frontieres, Gif-sur-Yvette (1996).
15. C. W. Beenakker, Phys. Rev. Lett. **67**, 3836 (1991).
16. W. Lichten, *Atomic Physics*, Vol. № 4, ed. by G. zu Putlitz et al., Plenum, New York (1975).
17. A. I. Larkin and Yu. N. Ovchinnikov, Phys. Rev. B **28**, 6281 (1983).
18. L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-relativistic Theory*, Pergamon Press, Oxford (1977).
19. A. Martin-Rodero et al., E-print cond-mat/9607079 (1996).
20. A. Furusaki and K. A. Matveev, Phys. Rev. B **52**, 16676 (1995).
21. G. B. Lesovik and R. Loosen, Pis'ma ZhETF **65**, 280 (1997) (JETP Lett. **65**, 295 (1997)); G. B. Lesovik, in *Proceedings of the Conference «Meso-97»*, Chernogolovka; submitted to Usp. Fiz. Nauk (1998).