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REDUCTION OF THE FINITE GRAND UNIFICATION THEORY TO THE MINIMAL SUPERSYMMETRIC STANDARD MODEL

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The recently proposed mechanism for reducing the finite $SU(5)$ grand unification theory (GUT) to the minimal supersymmetric standard model (MSSM) is reanalyzed and simplified. For the scalar $SU(2) \times U(1)$ invariant Higgs doublet potential that results from $SU(5)$ symmetry breaking to have no dangerous directions, a restriction on the parameters of the unified theory should be imposed. At the same time, this restriction guarantees that the scalar Higgs doublet potential has a minimum at zero at the GUT scale, and the low-energy theory appears to be exactly the MSSM.

1. INTRODUCTION

The supersymmetric (SUSY) field theories have remarkable properties in the ultraviolet range. The no-renormalization theorem for $N = 1$ SUSY theories [1] guarantees absence of divergences in quantum corrections to the superpotential. The only possible divergences in these theories (in the background field method) are the logarithmic divergences of the two-point Green functions of gauge and chiral superfields. If the group and the multiplet contents of the $N = 1$ theory are chosen in some particular way, these divergences disappear at the one-loop level too. This occurs as the result of mutual cancellation of the divergent contributions from gauge and Yukawa interactions [2]. The gauge groups and the multiplet contents of the theories for which this cancellation is possible have been classified in [3]. In Refs. [2, 4] it has been shown that one-loop finiteness guarantees two-loop finiteness of the theory without imposing new conditions, but this requirement appeared to be insufficient for the theory to be free from ultraviolet divergences at the three-loop level [5]. However, an algorithm for constructing an $N = 1$ SUSY field theory finite in all orders of perturbation theory has been proposed and a finite $SU(5)$ grand unification theory (GUT) was constructed [6]. The method used there was based on fine-tuning of the Yukawa coupling constants in each new order of perturbation theory. The only requirement imposed on the theory for this algorithm to work is one-loop finiteness (and, automatically, two-loop finiteness) [6].

The idea of complete finiteness of the unified theory is very attractive, and it is not surprising that many efforts have been made to derive low-energy predictions of the finite unified theory and compare them with modern experimental data [7]. For this purpose the standard approach is used: it is assumed that $SU(5)$ symmetry is spontaneously broken at the unification scale, and the unified theory is reduced to a low-energy supersymmetric theory with the corresponding boundary conditions for the coupling constants of the low-energy Lagrangian at the GUT scale.

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Then, the renormalization group equation method is applied to get quantities of the Lagrangian at the electroweak scale, where spontaneous breaking of the electroweak symmetry occurs [8, 9].

Although the one-loop finiteness conditions fix the gauge groups and the multiplet contents of the finite $N = 1$ SUSY theory, they allow considerable arbitrariness in the Yukawa and mass matrices [10]. In this situation the main guiding principles in choosing the finite GUT are simplicity and aesthetic attractiveness of the unified theory. In Ref. [11], the finite GUT satisfying these requirements was completely investigated in consistent way. The model was based on the $SU(5)$ gauge group and is the simplest finite GUT compatible with the low-energy phenomenology. Its distinction from the minimal SUSY $SU(5)$ GUT model is in the extension of the Higgs sector: it contains eight Higgs superfields instead of two in the minimal $SU(5)$ GUT [12]. The Yukawa and mass parts of the Lagrangian are chosen in the most economic way. Soft supersymmetry breaking takes place at the Planck scale M_P due to the appearance of soft terms in the Lagrangian [13, 14]. A universal form for these terms at the Planck scale is assumed [14]. In Ref. [11], the condition of finiteness was extended to them, which resulted in the completely finite theory between M_P and M_{GUT}. Complete finiteness in this case means that no charge or mass coupling of the theory changes in this energy range. To get the small initial mass parameters of the low-energy theory from the large mass parameters of the unified theory, the usual fine-tuning procedure was used in Ref. [11]. This procedure generates the hierarchy of the mass scales in the doublet part of the $SU(5)$ superpotential which decouples from the triplet part of the latter after spontaneous breaking of the $SU(5)$ symmetry.

In the treatment of the low-energy part of the finite GUT model in Ref. [11], in addition to the matter superfields of the minimal supersymmetric standard model (MSSM) three Higgs doublets were included in the low-energy Lagrangian. To get the Higgs potential at the electroweak scale, the renormalization group equations for the parameters of the potential were used. According to the radiative symmetry-breaking scenario [8], the parameters of the scalar Higgs potential yield nontrivial vacuum expectation values of the scalar Higgs fields at the electroweak scale. Due to the degeneracy of the Yukawa couplings with respect to the generations of the matter superfields [11, 15], the quark and lepton mass spectrum at the GUT scale in this model is completely determined by the spectrum of vacuum expectation values of the Higgs fields at the electroweak scale.

In this paper, the low-energy part of this finite theory is analyzed in a simpler and more efficient way than that of Ref. [11]. Namely, the finite GUT is shown to reduce to the MSSM after spontaneous breaking of the $SU(5)$ symmetry at the unification scale. The parameters of the electroweak Lagrangian need not be evolved down to low energies in this approach.

2. FINITE GUT

In this section, a brief review of the main points of the softly broken supersymmetric finite $SU(5)$ model constructed in [11] is proposed. The multiplet contents of the model and its Lagrangian were described there. The sector of the chiral matter and Higgs superfields has the following contents (in terms of the irreducible representations of $SU(5)$):

$$\text{Matter fields:} \quad \Psi_i - \bar{5}, \quad \Lambda_i - 10, \quad i = 1,2,3,$$
$$\text{Higgs fields:} \quad \Phi_a - 5, \quad \overline{\Phi}_a - \bar{5}, \quad \Sigma - 24, \quad a = 1,2,3,4,$$

where i and a are the generation indices of the matter and Higgs superfields, respectively.

The most general form of the superpotential for the theory having this field content is [6]

$$W = A^a_{ij}\overline{\Phi}_{a\alpha}\Psi_{i\beta}\Lambda^{\alpha\beta}_j + \frac{1}{8}B^a_{ij}\Phi^\alpha_a\Lambda^{\beta\gamma}_i\Lambda^{\delta\sigma}_j\epsilon^{\alpha\beta\gamma\delta\sigma} + C_{ab}\overline{\Phi}_{a\alpha}\Phi^\alpha_b\Sigma^\alpha_\beta +$$
$$+ \frac{1}{3}D\Sigma^\alpha_\beta\Sigma^\beta_\gamma\Sigma^\gamma_\alpha + \frac{1}{2}E^i_{ab}\overline{\Phi}_{a\alpha}\overline{\Phi}_{b\beta}\Lambda^{\alpha\beta}_i + F_{i\alpha}\Psi_{i\alpha}\Phi^\beta_a\Sigma^\alpha_\beta + \frac{1}{2}G^k_{ij}\Psi_{i\alpha}\Psi_{j\beta}\Lambda^{\alpha\beta}_k,$$

where $\epsilon^{\alpha\beta\gamma\delta\sigma}$ is a completely antisymmetric tensor. The last three terms, which would violate the baryon and lepton numbers at the tree levels and lead to $(B-L)$-nonconservation, are usually ignored.

The one-loop finiteness conditions of the two-point Green's functions in the theory with the above potential are [6]

$$4\sum_{i,j} A^a_{ij}(A^b_{ij})^* + \frac{24}{5}\sum_e C_{ae}(C_{be})^* + 4\sum_{i,e} E^i_{ae}(E^i_{be})^* = \frac{12}{5}g^2\delta_{ab},$$
$$3\sum_{i,j} B^a_{ij}(B^b_{ij})^* + \frac{24}{5}\sum_e C_{ea}(C_{eb})^* + \frac{24}{5}\sum_i F_{ia}(F_{ib})^* = \frac{12}{5}g^2\delta_{ab},$$
$$4\sum_{a,k} A^a_{ik}(A^a_{jk})^* + \frac{24}{5}\sum_a F_{ia}(F_{ja})^* + 4\sum_{l,k} G^k_{il}(G^k_{jl})^* = \frac{12}{5}g^2\delta_{ij}, \quad (1)$$
$$2\sum_{a,k} A^a_{ki}(A^a_{kj})^* + 3\sum_{a,k} B^a_{ik}(B^a_{jk})^* + 2\sum_{a,b} E^i_{ab}(E^j_{ab})^* + 2\sum_{k,l} G^i_{kl}(G^j_{kl})^* = \frac{18}{5}g^2\delta_{ij},$$
$$\sum_{a,b} C_{ab}(C_{ab})^* + \frac{21}{5}DD^* + \sum_{i,a} F_{ia}(F_{ia})^* = 5g^2.$$

In [11] the following simple ansatz for the Yukawa matrices of the superpotential was proposed:

$$W = y_1\Psi_i K_{ij}\overline{\Phi}_i\Lambda_j + y_1'\Psi_i\overline{\Phi}_4\Lambda_i + \frac{y_2}{8}\Phi_i\Lambda_i\Lambda_i + \frac{y_2'}{8}\Phi_4\Lambda_i\Lambda_i +$$
$$+ y_3\overline{\Phi}_i S_{ij}\Sigma\Phi_j + y_3'\overline{\Phi}_4\Sigma\Phi_4 + \frac{y_4}{3}\Sigma^3 + \overline{\Phi}_i M_{ij}\Phi_j + \overline{\Phi}_4 M\Phi_4 + \frac{M_0}{2}\Sigma^2, \quad (2)$$

where the $SU(5)$ indices are omitted, but can easily be recovered in a covariant manner. The potential (2) is taken in this form so that each generation of the matter interacts with its pair of Higgs fields, while the fourth pair of the Higgs fields is coupled with all the generations of matter as well as the Higgs pair of the minimal SUSY $SU(5)$ GUT. In [6] it is demonstrated that the Yukawa matrices corresponding to this specific ansatz will not be changed by the quantum corrections if the Yukawa couplings in (2) satisfy the conditions of one-loop finiteness (1) and if their necessary fine tuning is performed in each order of perturbation theory step by step.

The presence of unitary K and S matrices does not contradict the finiteness conditions (1). The matrix K is necessary to create the initial mixing of the quark fields at the unification scale (that is, initial values of the Cabibbo–Kobayashi–Maskawa matrix). As for the matrix S, it contains all initial information about the hierarchy of the quark mass spectrum at this scale. This role of S will become clear below.

From the no-renormalization theorem for the superpotential [1] it follows that the mass parameters M_0, M, and M_{ij} are not fixed by the requirement of one-loop finiteness. If M_0

is negative, the unified $SU(5)$ symmetry is broken by the vacuum expectation value of Σ [11]:

$$\langle\Sigma\rangle = \begin{pmatrix} V & & & & \\ & V & & & \\ & & V & & \\ & & & -\frac{3}{2}V & \\ & & & & -\frac{3}{2}V \end{pmatrix},$$

where $V \sim M_0/y_4 \sim 10^{16}$ GeV.

After breaking of the $SU(5)$ symmetry, the Higgs quintets Φ_a and $\overline{\Phi}_a$ split into doublets and triplets. As can be seen from (2), their mass terms look like

$$y_3\overline{\Phi}_i S_{ij}\langle\Sigma\rangle\Phi_j + \overline{\Phi}_i M_{ij}\Phi_j = \overline{\Phi}_i \begin{pmatrix} y_3 S_{ij}V + M_{ij} & \\ & -\frac{3}{2}y_3 S_{ij}V + M_{ij} \end{pmatrix} \Phi_j, \qquad (3)$$

and

$$y_3'\overline{\Phi}_4\langle\Sigma\rangle\Phi_4 + \overline{\Phi}_4 M\Phi_4 = \overline{\Phi}_4 \begin{pmatrix} y_3'V + M & \\ & -\frac{3}{2}y_3'V + M \end{pmatrix} \Phi_4. \qquad (4)$$

All the mass parameters in these relations are on the order of the M_{GUT} scale. To generate light initial boundary values for the masses in the doublet part of the Higgs sector of the unified theory, which should be radiatively corrected to give the mass parameters at the electroweak scale, it is necessary to carry out a fine-tuning procedure. For this purpose, the following trick was used in Ref. [11].

First, the unitary matrix S was represented as

$$S = \bar{X}\begin{pmatrix} e^{i\theta_1} & 0 & 0 \\ 0 & e^{i\theta_2} & 0 \\ 0 & 0 & e^{i\theta_3} \end{pmatrix} X^T = \bar{X}DX^T, \quad \bar{X}^T\bar{X} = I, \quad X^TX = I,$$

where X and $\bar{X}$ are real orthogonal matrices, D is a unitary diagonal matrix, and I is a unit matrix. The solution of the one-loop finiteness conditions for the specific ansatz of the Yukawa matrices used in the superpotential (2) has still some arbitrariness [11], which can be used to set $y_3' = 0$. This allows one to absorb one common phase into the redefinition of the fields. Therefore, in what follows it is supposed that $\theta_3 = 0$. It is necessary to note that $y_3' = 0$ implies that the fourth pair of the Higgs doublets (4) remains heavy in any case.

Second, the requirement of one-loop finiteness does not restrict the mass matrix M_{ij}. This matrix can be written as

$$M = \bar{X}(RI + T'D)X^T,$$

where R and T' are some heavy mass parameters,

$$R \sim T' \sim V.$$

Now, if the fourth pair of the Higgs doublets and its Yukawa interactions are omitted, the $SU(2)\times U(1)$ invariant superpotential at the M_{GUT} scale can be represented as

$$W = \left(\sqrt{\frac{2}{5}} g Q_j^b K_{ij} \bar{H}_i^a D_i + \sqrt{\frac{2}{5}} g L_i^b \bar{H}_i^a E_i + \sqrt{\frac{8}{15}} g Q_i^b H_i^a U_i \right) \epsilon_{ab} + \\ + \left\{ \bar{H}_i^a \left[\bar{X}(RI + TD) X^T \right]_{ij} H_j^b \right\} \epsilon_{ab}, \tag{5}$$

where $a, b = 1, 2$ are the $SU(2)$ indices and $\epsilon_{12} = 1$, and the following notation is used

$$T = T' - \frac{3}{2} y_3 V.$$

The three pairs of Higgs doublets have the quantum numbers

$$\bar{H}_i \left(1, 2, -\frac{1}{2} \right) = \begin{pmatrix} \bar{H}_i^0 \\ \bar{H}_i^- \end{pmatrix}, \quad H_i \left(1, 2, \frac{1}{2} \right) = \begin{pmatrix} H_i^+ \\ H_i^0 \end{pmatrix}, \tag{6}$$

while the other superfields in (5) are the usual matter superfields of the MSSM [16].

In addition, the following soft supersymmetry breaking terms must be added to the superpotential (5):

$$W_{SSB} = m_0^2 \sum_i |\varphi_i|^2 + \frac{1}{2} \left(m_{1/2} \sum_k \lambda_k \lambda_k + \text{H.c.} \right) + \\ + B \left\{ \bar{H}_i^a \left[\bar{X}(RI + TD) X^T \right]_{ij} H_j^b + \text{c.c.} \right\} \epsilon_{ab} + \\ + \left(A_D y_D \tilde{q}_j^b K_{ij} \bar{H}_i^a \tilde{d}_i + A_L y_L \tilde{l}_i^b \bar{H}_i^a \tilde{e}_i + A_U y_U \tilde{q}_i^b H_i^a \tilde{u}_i + \text{c.c.} \right) \epsilon_{ab}, \tag{7}$$

where φ_i denotes all scalar fields with common mass m_0^2 at the unification scale, and λ_k are the gauginos with common mass $m_{1/2}$ at the same scale. Aside from the gauginos, all other fields in W_{SSB} are the low scalar components of the corresponding superfields. The notation for the scalar Higgs doublets in (7) coincides with the corresponding superfield notation (6). These soft supersymmetry breaking terms can be reduced from the corresponding $SU(5)$ invariant terms of the unified theory after the $SU(5)$ symmetry breaking [11].

Having rotated the superfields H_i and $\bar{H}_i$ as

$$H_i = (X \ H')_i = X_{i1} H_1' + X_{i2} H_2' + X_{i3} H_3', \tag{8}$$

$$\bar{H}_i = (\bar{X} \ \bar{H}')_i = \bar{X}_{i1} \bar{H}_1' + \bar{X}_{i2} \bar{H}_2' + \bar{X}_{i3} \bar{H}_3', \tag{9}$$

where H_i' and $\bar{H}_i'$ are the new Higgs superfields, one can conveniently rewrite (5) as

$$W = \left(y_D Q_j^b K_{ij} \bar{X}_{ik} \bar{H}_k'^a D_i + y_L L_i^b \bar{X}_{ik} \bar{H}_k'^a E_i + y_U Q_i^b X_{ik} H_k'^a U_i \right) \epsilon_{ab} + \\ + \left[\bar{H}_i'^a (RI + TD)_{ij} H_j'^b \right] \epsilon_{ab}, \tag{10}$$

and (7) as

$$W_{SSB} = m_0^2 \sum_i |\varphi_i|^2 + \frac{1}{2} \left(m_{1/2} \sum_k \lambda_k \lambda_k + \text{H.c.} \right) + \\ + B \left[\bar{H}_i'^a (RI + TD)_{ij} H_j'^b + \text{c.c.} \right] \epsilon_{ab} + \\ + \left(A_D y_D \tilde{q}_j^b K_{ij} \bar{X}_{ik} \bar{H}_k'^a \tilde{d}_i + A_L y_L \tilde{l}_i^b \bar{X}_{ik} \bar{H}_k'^a \tilde{e}_i + A_U y_U \tilde{q}_i^b X_{ik} H_k'^a \tilde{u}_i + \text{c.c.} \right) \epsilon_{ab}. \tag{11}$$

To get the light Higgs doublet pair, a fine tuning procedure should be performed:

$$R + T = \mu \sim 10^3 \text{ GeV}. \tag{12}$$

The fine-tuning procedure is more meaningful than in the other GUTs, since in the finite model none of the parameters is running above the GUT scale.

As can be seen from (10) and (11), the first two components in the decompositions (8) and (9) remain heavy, while the third components H_3' and $\bar{H}_3'$ become light. By the decoupling theorem [17], only this pair need be taken into account in the effective low-energy theory, whose superpotential takes the following form at the unification scale:

$$\begin{aligned} W = {} & \left(y_D \overline{n}_i Q_j^b K_{ij} \bar{H}_3'^a D_i + y_L \overline{n}_i L_i^b \bar{H}_3'^a E_i + y_U n_i Q_i^b H_3'^a U_i\right) \epsilon_{ab} + \\ & + B\left(\mu \bar{H}_3'^a H_3'^b\right) \epsilon_{ab}, \end{aligned} \tag{13}$$

where

$$\overline{n}_i = \bar{X}_{i3}, \quad n_i = X_{i3}, \quad \sum_i \overline{n}_i^2 = 1, \quad \sum_i n_i^2 = 1.$$

The corresponding soft supersymmetry breaking terms are

$$\begin{aligned} W_{SSB} = {} & m_0^2 \sum_i |\varphi_i|^2 + \frac{1}{2}\left(m_{1/2} \sum_k \lambda_k \lambda_k + \text{H.c.}\right) + B\left(\mu \bar{H}_3'^a H_3'^b + \text{c.c.}\right) \epsilon_{ab} + \\ & + \left(A_D y_D \overline{n}_i \widetilde{q}_j^b K_{ij} \bar{H}_3'^a \widetilde{d}_i + A_L y_L \overline{n}_i \widetilde{l}_i^b \bar{H}_3'^a \widetilde{e}_i + A_U y_U n_i \widetilde{q}_i^b H_3'^a \widetilde{u}_i + \text{c.c.}\right) \epsilon_{ab}. \end{aligned} \tag{14}$$

Here φ_i denotes all light scalar fields of the effective low-energy theory. In analogy with (7), the notation for the scalar Higgs doublets in (14) coincides with the notation of the corresponding superfields.

Equations (13) and (14) are the usual superpotential and soft supersymmetry breaking potential of the MSSM [16], respectively. As is well known, in the MSSM there is no problem with unification of the gauge coupling constants of MSSM at a single point at a very high scale [18]. Moreover, only in the supersymmetric model with two Higgs doublets this unification is possible [19].

As can be seen from (13), all information about the quark mass hierarchy at the GUT scale is contained in the Higgs sector of the finite unified theory, namely, in the unitary Higgs mixing matrix S :

$$\begin{aligned} & y_i^U = n_i y^U, \quad y_i^D = \overline{n}_i y^D, \quad y_i^L = \overline{n}_i y^L, \\ & y^U = \frac{4}{\sqrt{15}} g_{GUT}, \quad y^D = y^L = \frac{2}{\sqrt{5}} g_{GUT}, \end{aligned} \tag{15}$$

where g_{GUT} is a gauge coupling constant of the unified theory.

These conclusions are natural and correct, and there are no subtle points if the full scalar Higgs doublet potential has no dangerous directions (along these directions it can be unbounded below) and has absolute minimum at zero at the unification scale. Since the full scalar Higgs potential has a rather complicated structure because of the large number of Higgs fields, this is not obvious. In the next section, a condition will be written for the parameters of the unified superpotential which is necessary to guarantee this.

3. DOUBLET POTENTIAL AT THE GUT SCALE

The scalar Higgs doublet potential arises from the superpotential (13) and minimal SUSY gauge interaction, when all nondynamical components of the Higgs and gauge superfields are eliminated, and from the corresponding soft supersymmetry breaking part (14) [16]. Hence, the full potential has the following form at the unification scale:

$$V(\bar{H}'_i, H'_i) = (m_0^2 + R^2 + T^2)\left(\sum_i |\bar{H}'_i|^2 + \sum_i |H'_i|^2\right) +$$
$$+RT(D^* + D)_{ij}\left(\bar{H}'^{\dagger}_i \bar{H}'_j + H'^{\dagger}_i H'_j\right) + B\left[\bar{H}'^{a}_i (RI + TD)_{ij} H'^{b}_j + \text{c.c.}\right]\epsilon_{ab} +$$
$$+\frac{g^2 + g'^2}{8}\left[\sum_i |\bar{H}'_i|^2 - \sum_i |H'_i|^2\right]^2 + \frac{g^2}{4}\left[\left(\bar{H}'^{\dagger}_i \bar{H}'_j\right)^*\left(\bar{H}'^{\dagger}_i \bar{H}'_j\right) - \left(\sum_i |\bar{H}'_i|^2\right)^2 + \right.$$
$$\left. + \left(H'^{\dagger}_i H'_j\right)^*\left(H'^{\dagger}_i H'_j\right) - \left(\sum_i |H'_i|^2\right)^2 + 2\left(\bar{H}'^{\dagger}_i H'_j\right)^*\left(\bar{H}'^{\dagger}_i H'_j\right)\right], \tag{16}$$

where summation over the repeating Higgs generation indices is implied. Also, it is assumed for brevity that

$$|\bar{H}_i|^2 = |\bar{H}^0_i|^2 + |\bar{H}^-_i|^2, \quad |H_i|^2 = |H^+_i|^2 + |H^0_i|^2.$$

It is more convenient to introduce the new notation

$$M_1^2 = m_0^2 + R^2 + T^2 + 2RT\cos\theta_1, \ \mathscr{M}_1 e^{i\gamma_1} = R + Te^{i\theta_1}, \ \mathscr{M}_1 = |R + Te^{i\theta_1}|,$$
$$M_2^2 = m_0^2 + R^2 + T^2 + 2RT\cos\theta_2, \ \mathscr{M}_2 e^{i\gamma_2} = R + Te^{i\theta_2}, \ \mathscr{M}_2 = |R + Te^{i\theta_2}|,$$
$$m^2 = m_0^2 + (R+T)^2, \qquad \mu = R + T,$$

and rewrite the potential (16) as

$$V = M_1^2\left(|\bar{H}'_1|^2 + |H'_1|^2\right) + M_2^2\left(|\bar{H}'_2|^2 + |H'_2|^2\right) + m^2\left(|\bar{H}'_3|^2 + |H'_3|^2\right) +$$
$$+ B\left(\mathscr{M}_1 e^{i\gamma_1}\bar{H}'^{a}_1 H'^{b}_1 + \mathscr{M}_2 e^{i\gamma_2}\bar{H}'^{a}_2 H'^{b}_2 + \mu \bar{H}'^{a}_3 H'^{b}_3\right)\epsilon_{ab} +$$
$$+\frac{g^2 + g'^2}{8}\left[\sum_i |\bar{H}'_i|^2 - \sum_i |H'_i|^2\right]^2 + \frac{g^2}{4}\left[\left(\bar{H}'^{\dagger}_i \bar{H}'_j\right)^*\left(\bar{H}'^{\dagger}_i \bar{H}'_j\right) - \left(\sum_i |\bar{H}'_i|^2\right)^2 + \right.$$
$$\left. + \left(H'^{\dagger}_i H'_j\right)^*\left(H'^{\dagger}_i H'_j\right) - \left(\sum_i |H'_i|^2\right)^2 + 2\left(\bar{H}'^{\dagger}_i H'_j\right)^*\left(\bar{H}'^{\dagger}_i H'_j\right)\right]. \tag{17}$$

Having parametrized the Higgs doublets as

$$H_i = U_i\begin{pmatrix} 0 \\ v_i \end{pmatrix}, \quad \bar{H}_i = \overline{U}_i\begin{pmatrix} \overline{v}_i \\ 0 \end{pmatrix},$$

where U_i and $\overline{U}_i$ are some $SU(2)$ matrices, and v_i and $\overline{v}_i$ are positive, one can derive that when the quartic terms in (17) vanish the condition for positivity of the quadratic part of (17) is

$$\begin{aligned} & M_1^2(\alpha_1^2+\beta_1^2)+M_2^2(\alpha_2^2+\beta_2^2)+m^2(\alpha_3^2+\beta_3^2)- \\ & -\ 2|B\mathscr{M}_1|\alpha_1\beta_1-2|B\mathscr{M}_2|\alpha_2\beta_2-2|B\mu|\alpha_3\beta_3\geq 0, \end{aligned} \tag{18}$$

where

$$\alpha_1^2+\alpha_2^2+\alpha_3^2=1,\ \ \beta_1^2+\beta_2^2+\beta_3^2=1,$$
$$\overline{v}_i=\alpha_i\sqrt{\textstyle\sum_i \overline{v}_i^2},\quad v_i=\beta_i\sqrt{\textstyle\sum_i v_i^2}.$$

This requirement is necessary to provide stability of (17) in these directions. The quadratic form (18) is obviously positive if the following conditions are satisfied:

$$M_1^2\geq|B\mathscr{M}_1|, \tag{19}$$
$$M_2^2\geq|B\mathscr{M}_2|, \tag{20}$$
$$m^2\geq|B\mu|. \tag{21}$$

The conditions (19) and (20) hold in any case due to our fine-tuning procedure (12) (it is assumed that $B\sim\mu$). Note that if (21) is violated, the quadratic form would be negative when

$$\alpha_1=\alpha_2=\beta_1=\beta_2=0.$$

Thus, the condition

$$m_0^2+\mu^2\geq|B\mu| \tag{22}$$

is necessary for the stability of the potential (17) for large fields. At the same time, as can be seen from (18) and (19)–(21), the restriction (22) guarantees the positivity of the scalar Higgs potential (16) on any field configurations. This means that after spontaneous breaking of the $SU(5)$ symmetry the scalar Higgs doublet potential has its only and absolute minimum at zero at the GUT scale.

4. CONCLUSIONS

In this paper, the SUSY $SU(5)$ finite theory with an R-symmetrical and $(B-L)$-conserving superpotential has been considered. The Yukawa matrices of this theory were chosen in the simplest possible way, and their values were fixed by the condition of finiteness up to some arbitrariness. This arbitrariness, and the arbitrariness in the choice of the mass matrices that are not restricted by the condition of finiteness, can be used to reduce the finite GUT to the MSSM after the $SU(5)$ symmetry breaking at the unification scale. Reduction to the MSSM is necessary because only in a SUSY theory with two Higgs doublets the unification of the gauge couplings of the MSSM at a single point is possible [18]. In the supersymmetric theories with a more extended Higgs sector such a unification is problematical [19].

In this work, the analysis of the reduction of the finite GUT to the MSSM is simpler than that in [11]. For the low-energy theory to be self-consistent, it is necessary that the scalar

Higgs doublet potential have no dangerous directions and have an absolute global minimum at zero at the unification scale after the $SU(5)$ symmetry breaking. These requirements impose the restriction (22) on the parameters of the finite GUT. If this restriction holds, both the requirements are met. As for the rest, after the $SU(5)$ symmetry breaking one gets the MSSM as the low-energy theory with the boundary conditions at the GUT scale for the Yukawa couplings (15).

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