Antiferromagnetic resonance and spin superfluidity in $CsNiCl_3$

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A theory of a nonlinear antiferromagnetic resonance in the antiferromagnetic $CsNiCl_3$ containing six sublattices has been developed. Solutions of the equations of spin dynamics describing inhomogeneous coherently precessing magnetization distributions similar to those detected in the superfluid and normal phases of a liquid helium-3 due to the nondissipative transfer of magnetization (spin superfluidity) have been found. The effect of the antiferromagnet domain structure on the coherently oscillating magnetization distributions has been investigated.
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1. INTRODUCTION

Magnetic domains coherently precessing in nonuniform magnetic field were detected in a superfluid ^{3}He-B more than a decade ago.[1] Their existence is related to nondissipative spin currents, which redistribute the magnetization through the helium volume.[2] Like conventional superfluidity, which is due to the phase degeneracy of the condensate wave function, the spin superfluidity in ^{3}He-B is due to the phase degeneracy of the magnetization precession. Spin analogues of the effects typical of conventional superfluids have been detected in ^{3}He-B, such as the Josephson effect, phase slipping in the magnetization flow through a channel, fourth sound, and quantized spin vortices.[3,4]

Coherent precession of inhomogeneous magnetization distributions has been also detected recently in normal Fermi liquids, such as the solution of ^{3}He in ^{4}He (Refs. 5 and 6) and pure ^{3}He (Ref. 7), in the collisionless regime. Unlike ^{3}He-B, in which the phase coherence of the precession results from the coherence of spin states in the superfluid Fermi-liquid with the triplet Cooper pairing, the precession phase coherence in a normal Fermi-liquid in the collisionless regime in magnetic field (as well as in superfluid phases of helium-3 near T_c[8,9]) is due to the Landau spin molecular field, which generates a nondissipative spin current of quasiparticles. Silin waves in a normal Fermi liquid, in other words, zeroth spin sound in a magnetic field arise for the same reason.[10]

Nonuniform magnetization distributions may precess wherently not only in quantum liquids, but also in magnetically ordered materials where relativistic interactions do not fully lift the degeneracy with respect to the alignment of the spin system as a whole relative to the crystal axes.

In addition to ^{3}He-B and ^{3}He-A, the theory of coherently precessing nonuniform distributions of magnetization has been developed only for one magnetically ordered material, namely the solid antiferromagnetic ^{3}He (Ref. 11). Solid helium-3 is not quite convenient for experiments in this field. Besides the purely technical problems encountered in experiments at temperatures $T<T_N\approx 1$ mK, there is a fundamental difficulty deriving from the presence of three types of antiferromagnetic domain in the body-centered cubic lattice. Each type of domain has its specific antiferromagnetic resonant frequency as a function of the external magnetic field. It seems to us that it is more convenient to experiment with hexagonal antiferromagnets with several sublattices and relatively high temperatures of magnetic ordering, namely $CsNiCl_3$ ($T_N=4.4$ K), $RbNiCl_3$ ($T_N=11$ K), and $CsMnBr_3$ ($T_N=8.3$ K), which have been extensively studied in connection with the quasi-one-dimensional ordering in them at temperatures $T>T_N$. These materials have been studied both theoretically and experimentally for small deviations of the magnetization and order parameter from their equilibrium values.[12] The solutions of spin dynamical equations for arbitrary angles between the magnetization and external magnetic field reported in this paper demonstrate that the antiferromagnetic resonant frequencies in domains of different types are equal. Besides, it is probably true that, although the transitional regions between domains create additional inhomogeneities in distributions of magnetization and order parameter, they do not destroy the coherence of the precessions over the entire sample.

The paper is organized as follows. Section 2 describes the structure of ABX_3 antiferromagnetic materials and Hamilton's equations applied to the nonuniform spin dynamics. In Sec. 3 we will discuss periodic solutions of the nonlinear equations in the spatially nonuniform case. Section 4 gives a coherently precessing domain-wall solution for the region between domains with magnetizations parallel and antiparallel to the external magnetic field. In Sec. 5 we will discuss a solution describing a quantized spin vortex. Section 6 is devoted to the study of coherent precessing solutions for mulitdomain samples. The results are summarized in the Conclusion. The present theory applies to the entire class of ABX_3 antiferromagnets, but numerical data are discussed only for the case of $CsNiCl_3$.

2. MAGNETIC STRUCTURE AND HAMILTONIAN DYNAMICS

Below the Néel temperature, $CsNiCl_3$ is a hexagonal antiferromagnet with six sublattices. In each basal plane, Ni atoms with $S=1$ form a triangular lattice. In the ground state the exchange interaction orders the spins so that the total spin of each triangle should be zero, and the spin configura-

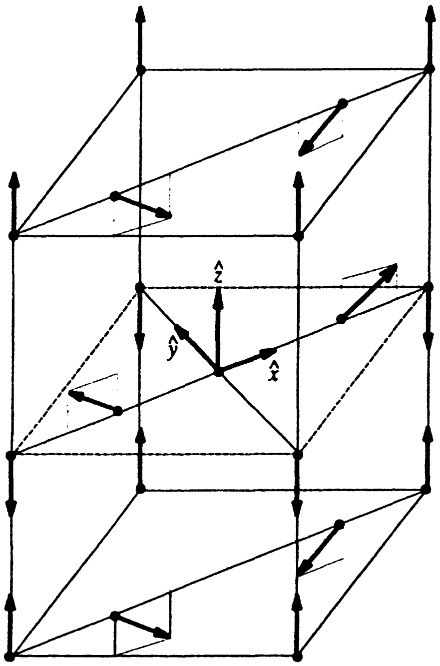

FIG. 1. Spin order in $CsNiCl_3$. The spins lie in the vertical plane: $\mathbf{l}_1=\hat{z}$, $\mathbf{l}_2=\hat{x}$, and $\mathbf{n}=\hat{y}$.

tion is reproduced in translations in the basal plane through spatial vectors which are multiples of the basis vectors. In the adjacent basal plane the spin vectors are reversed (Figs. 1 and 2). Given that the mutual orientation of spins is the same, the antiferromagnetic order can be described in terms of a pair of mutually orthogonal unit vectors $\mathbf{l}_1$ and $\mathbf{l}_2$, which define the basis in the plane of spin vectors. The vector $\mathbf{n}=\mathbf{l}_1\times\mathbf{l}_2$ is orthogonal to this plane.

Owing to the relativistic interaction, the vector $\mathbf{n}$ has a fixed orientation with respect to the hexagonal axis $\hat{z}$:

$$F_a=\frac{a}{2}n_z^2. \tag{1}$$

In $CsNiCl_3$ the anisotropy constant satisfies $a>0$, and the spins are aligned as shown in Fig. 1. In $CsMnBr_3$, on the other hand, we have, $a<0$, and the resulting spin configuration is shown in Fig. 2. The anisotropy constant a is notably smaller than the exchange interaction amplitude.

There is also anisotropy in the basal plane defining the orientation of the projections of $\mathbf{l}_1$ and $\mathbf{l}_2$ on the xy-plane (i.e., the projections of spin directions on the xy-plane). It is quite obvious that there are six different equilibrium orientations of $\mathbf{l}_1$ and $\mathbf{l}_2$ projections on the basal plane, i.e., six types of antiferromagnetic domains. The corresponding energy density is expressed as

$$F_a'=-b\ \mathrm{Re}(l_{1x}+il_{2x})^6. \tag{2}$$

The basal-plane anisotropy is very weak in comparison with that described by Eq. (1), so we will discuss effects deriving from Eq. (2) only in Sec. 6, devoted to the antiferromagnetic domains.

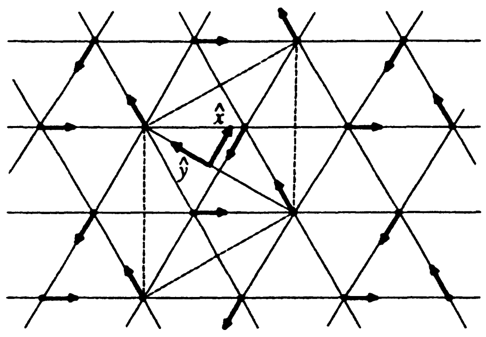

FIG. 2. Spin order in $CsMnBr_3$. The spins lie in the basal plane: $\mathbf{l}_1=\hat{x}$, $\mathbf{l}_2=\hat{y}$, and $\mathbf{n}=\hat{z}$. The dashed lines encloses an area of the basal plane corresponding to that shown in Fig. 1.

Since the spins are aligned with the plane perpendicular to $\mathbf{n}$, the magnetic susceptibility in such antiferromagnets is described by a uniaxial tensor:

$$\chi_{\alpha\beta}=\chi_{\|}n_\alpha n_\beta+\chi_\perp(\delta_{\alpha\beta}-n_\alpha n_\beta), \tag{3}$$

and the Hamiltonian density in an applied magnetic field has the form

$$\mathscr{H}=\frac{\gamma^2}{2}\chi_{\alpha\beta}^{-1}S_\alpha S_\beta-\gamma\mathbf{S}\cdot\mathbf{H}+\frac{a}{2}n_z^2$$
$$=\frac{\gamma^2}{2\chi_\perp}\mathbf{S}^2+\frac{\gamma^2}{2}\left(\frac{1}{\chi_\|}-\frac{1}{\chi_\perp}\right)(\mathbf{S}\cdot\mathbf{n})^2-\gamma\mathbf{S}\cdot\mathbf{H}+\frac{a}{2}n_z^2. \tag{4}$$

Here γ is the gyromagnetic ratio and $\gamma\mathbf{S}=\mathbf{M}$ is the magnetic moment per unit volume.

Since we have $\chi_\|>\chi_\perp$ in $CsNiCl_3$, a magnetic field directed in the basal plane stabilizes the equilibrium orientation of the vector $\mathbf{n}$ aligned with the field direction. If the field satisfies $\mathbf{H}\|\hat{z}$, then the vector $\mathbf{n}$ lies in the basal plane for $H<H_0=\sqrt{a/(\chi_\|-\chi_\perp)}$, and the equilibrium spin per unit volume is $\mathbf{S}=\chi_\perp H\hat{z}/\gamma$. For $H>H_0$ the sublattices are realigned (spin-flop takes place), so that the spins are directed in the basal plane, $\mathbf{n}=\hat{z}$, and the equilibrium spin per unit volume is $\mathbf{S}=\chi_\| H\hat{z}/\gamma$. The spin-flop field in $CsNiCl_3$ is $H_0=1.9$ T. Compare this to the field $H_s=73.5$ T in which the antiferromagnetic order is destroyed (a spin-flip occurs).[13]

If the spin configuration is not uniform in space, the Hamiltonian of a unit volume defined by Eq. (4) should be supplemented with the gradient energy term

$$F_\nabla=\frac{1}{2}\rho^s_{\alpha i\beta j}\omega_{\alpha i}\omega_{\beta j}. \tag{5}$$

Here the spin velocities

$$\omega_{\alpha i}=\nabla_i\varphi_\alpha, \tag{6}$$

and φ is the vector of rotation of the order-parameter reference. The reference orientation $(\mathbf{l}_1,\mathbf{l}_2,\mathbf{n})$ at a point $(\mathbf{r},t)$ is obtained by the rotation $\varphi(\mathbf{r},t)$ of the initial orientation $(\mathbf{l}_{10},\mathbf{l}_{20},\mathbf{n}_0)$ through the angle $|\varphi|$ around the direction of $\hat{\varphi}$. The spin density tensor

$$\rho^s_{\alpha i\beta j}=\frac{1}{\gamma^2}[\chi_1 n_\alpha n_\beta+\chi_2(\delta_{\alpha\beta}-n_\alpha n_\beta)][c_\parallel^2\hat{z}_i\hat{z}_j+c_\perp^2(\delta_{ij}-\hat{z}_i\hat{z}_j)] \quad (7)$$

is uniaxial with respect to the spin indices α and β, and (owing to the hexagonal symmetry of the crystal) the orbital indices i and j. The parameters χ_1 and χ_2 have dimensions of susceptibility, and $c_\parallel$ and $c_\perp$ are the velocities of the spin waves.

The evolution of the system parameters is described by Hamilton's equations of motion for the conjugate variables $\boldsymbol{\varphi}$ and $\mathbf{S}$:

$$\dot{\boldsymbol{\varphi}}=\frac{\partial \mathcal{H}}{\partial S}, \quad (8)$$

$$\dot{\mathbf{S}}=-\frac{\delta(\mathcal{H}+F_\nabla)}{\delta\boldsymbol{\varphi}}. \quad (9)$$

Equation (9) can be rewritten in the form

$$\dot{S}_\alpha+\nabla_i j_{\alpha i}=-\frac{\partial(\mathcal{H}+F_\nabla)}{\partial\varphi_\alpha}, \quad (10)$$

where

$$j_{\alpha i}=-\frac{\partial F_\nabla}{\partial\omega_{\alpha i}}=-\rho^s_{\alpha i\beta j}\omega_{\beta j}; \quad (11)$$

is the spin current.

The motion of the spin degrees of freedom in the system can be described, of course, in terms of the evolution of the magnetization density and rotation of the order-parameter reference only if variations in time and space do not lead to notable distortions of the magnetic structure. This is the case when the following conditions are satisfied: the typical frequencies of motion are smaller than γH_s, and the distances over which the magnetic parameters change significantly are larger than the interatomic distances.

It is convenient to describe the motion of the order-parameter reference $(\mathbf{l}_1,\mathbf{l}_2,\mathbf{n})$ with respect to its initial orientation at $t=0$, $(\mathbf{l}_{10},\mathbf{l}_{20},\mathbf{n}_0)$, in terms of Euler rotations. For example,

$$\mathbf{n}=\hat{R}(\alpha,\beta,\gamma)\mathbf{n}_0=\hat{R}_z(\alpha)\hat{R}_y(\beta)\hat{R}_z(\gamma)\mathbf{n}_0. \quad (12)$$

Here $\hat{R}_z(\alpha)$ is the operator for rotation around the z-axis through the angle $\alpha(t)$, etc.; $\hat{R}(\alpha,\beta,\gamma)$ is the operator of three-dimensional rotations described by Eq. (12). The angles α, β, and γ are canonically conjugate to the projections of $\mathbf{S}$ on the $\hat{z}$, $\beta=\hat{\zeta}\times\hat{z}$, and $\hat{\zeta}=R(\alpha,\beta,\gamma)\hat{z}$, axes respectively. For $\mathbf{H}\|\hat{z}$ we can solve the problem of the periodic motion of the magnetization $\gamma\mathbf{S}$ for any angle β between $\mathbf{S}$ and the magnetic field $\mathbf{H}$. At the same time, if $H<H_0$ holds and the vector $\hat{\mathbf{n}}_0$ lies in the xy-plane, we can obtain a solution of this problem only numerically. We will limit our analysis to the field range $H>H_0$ ($\mathbf{H}\|\hat{z}$). In this case we have $\mathbf{n}_0=\hat{z}$ and the Hamiltonian density has the form

$$\mathcal{H}=\frac{\gamma^2}{2\chi_\perp}\left[S_\beta^2+\left(\frac{S_z-S_\zeta\cos\beta}{\sin\beta}\right)^2\right]+\frac{\gamma^2}{2\chi_\parallel}S_\zeta^2-\gamma S_z H+\frac{a}{2}\cos^2\beta. \quad (13)$$

If the field lies in the basal plane of the crystal, the problem of the periodic motion of the magnetization can be also solved. In this case it is more convenient to align the $\hat{z}$-axis with the magnetic field $\mathbf{H}$ and the $\hat{x}$-axis with the hexagonal crystal axis. Then we have $\mathbf{n}_0=\hat{z}$, the anisotropy energy equals $(1/2)an_x^2$ and the Hamiltonian density is

$$\mathcal{H}=\frac{\gamma^2}{2\chi_\perp}\left[S_\beta^2+\left(\frac{S_z-S_\zeta\cos\beta}{\sin\beta}\right)^2\right]+\frac{\gamma^2}{2\chi_\parallel}S_\zeta^2-\gamma S_z H+\frac{a}{2}\sin^2\beta\cos^2\gamma. \quad (14)$$

In magnetic fields $H\gg H_0$, the last term in this equation can be treated as a perturbation. At resonance, for $\alpha+\gamma\approx$const, its contribution has a low amplitude $[\sim(H_0/H)^2]$ and a frequency twice that of the fundamental harmonic, $\alpha=-\omega_p t$. Therefore this contribution can be averaged over the precession period $2\pi/\omega_p$, which is equivalent to the substitution $\cos^2\gamma\to 1/2$. This means that for $H>H_0$ the Hamiltonians containing the magnetic field aligned with the hexagonal crystal axis and with the basal plane differ only in the sign of the anisotropy constant a. Given this relation, we will only consider the Hamiltonian defined by Eq. (13).

If we use the relation

$$n_\alpha\omega_{\alpha i}=\hat{R}_{\alpha\mu}n_{0\mu}\omega_{\alpha i}=n_{0\mu}\tilde{\omega}_{\mu i}, \quad (15)$$

where $\hat{R}_{\alpha\mu}$ is the operator introduced by Eq. (11), it is convenient to transform the gradient energy density in Eq. (5) to

$$F_\nabla=\frac{1}{2}\rho^{so}_{\alpha i,\beta j}\tilde{\omega}_{\alpha i}\tilde{\omega}_{\beta j}, \quad (16)$$

where

$$\rho^{so}_{\alpha i,\beta j}=\frac{1}{\gamma^2}[\chi_1 n_{0\alpha}n_{0\beta}+\chi_2(\delta_{\alpha\beta}-n_{0\alpha}n_{0\beta})][c_\parallel^2\hat{z}_i\hat{z}_j+c_\perp^2(\delta_{ij}-\hat{z}_i\hat{z}_j)]. \quad (17)$$

After substituting into Eq. (16) spin velocities expressed in the explicit form[4]

$$\tilde{\omega}_{xi}=-\sin\beta\cos\gamma\nabla_i\alpha+\sin\gamma\nabla_i\beta,$$

$$\tilde{\omega}_{yi}=\sin\beta\sin\gamma\nabla_i\alpha+\cos\gamma\nabla_i\beta,$$

$$\tilde{\omega}_{zi}=\cos\beta\nabla_i\alpha+\nabla_i\gamma, \quad (18)$$

we finally obtain

$$F_\nabla=\frac{c_\perp^2}{2\gamma^2}\{\chi_1[(\alpha_x'\cos\beta+\gamma_x')^2+(\alpha_y'\cos\beta+\gamma_y')^2]+\chi_2\times[[(\alpha_x')^2+(\alpha_y')^2]\sin^2\beta+(\beta_x')^2+(\beta_y')^2]\}+\frac{c_\parallel^2}{2\gamma^2}\times\{\chi_1(\alpha_z'\cos\beta+\gamma_z')^2+\chi_2[(\alpha_z'\sin\beta)^2+(\beta_z')^2]\}. \quad (19)$$

The gradient energy density is derived from Eq. (19) in the case when the z-axis lies in the basal plane and the x-axis is aligned with the hexagonal crystal axis after the exchange $x \leftrightarrow z$.

3. PERIODIC SOLUTIONS IN THE SPATIALLY UNIFORM CASE

The Hamiltonian in Eq. (13) (when the field **H** is parallel to the hexagonal crystal axis and we have $H > H_0$) is independent of the angles α and γ, which means conservation of the total spin projections on the axes $\hat{z}$ and $\hat{\zeta}$:

$$\dot{S}_z = -\frac{\partial \mathcal{H}}{\partial \alpha} = 0, \tag{20}$$

$$\dot{S}_\zeta = -\frac{\partial \mathcal{H}}{\partial \gamma} = 0, \tag{21}$$

and allows us to seek periodic (cyclic) solutions

$$\alpha = -\omega_p t + \alpha_0, \tag{22}$$

$$\gamma = \omega_\gamma t + \gamma_0 \tag{23}$$

of the Hamilton equations

$$\dot{\alpha} = \frac{\partial \mathcal{H}}{\partial S_z} = \frac{\gamma^2}{\chi_\perp} \frac{S_z - S_\zeta \cos \beta}{\sin^2 \beta} - \gamma H, \tag{24}$$

$$\dot{\gamma} = \frac{\partial \mathcal{H}}{\partial S_\zeta} = -\frac{\gamma^2}{\chi_\perp} \frac{S_z - S_\zeta \cos \beta}{\sin^2 \beta} \cos \beta + \frac{\gamma^2}{\chi_\parallel} S_\zeta, \tag{25}$$

such that the remaining variables S_β and β are independent of time:

$$\dot{\beta} = \frac{\partial \mathcal{H}}{\partial S_\beta} = \frac{\gamma^2}{\chi_\parallel} S_\beta = 0, \tag{26}$$

$$\dot{S}_\beta = -\frac{\partial \mathcal{H}}{\partial \beta} = -\frac{\gamma^2}{\chi_\perp} \frac{S_z - S_\zeta \cos \beta}{\sin \beta} \frac{S_\zeta - S_z \cos \beta}{\sin^2 \beta} + a \cos \beta \sin \beta = 0. \tag{27}$$

It follows from Eq. (26) that $S_\beta = 0$ holds, i.e., the vector **S** is in the $\hat{z}\hat{\zeta}$-plane. Equation (27) relates the conserved values S_z, S_ζ, and β to each other. After deriving the expression for $S_z = S_z(S_\zeta, \beta)$ and substituting into Eqs. (24) and (25), we obtain, with due account of Eqs. (22) and (23), expressions for the precession frequency ω_p of the magnetization and the rotation frequency ω_γ of the order-parameter reference around the magnetization vector. It is easier to perform this procedure analytically in the limit $H \gg H_0 \sim \sqrt{a}$. If this inequality holds, Eq. (27) has two solutions:

$$1) \; S_z = S_\zeta \cos \beta + \frac{\chi_\perp a}{\gamma^2 S_\zeta} \sin^2 \beta \cos \beta, \tag{28}$$

$$2) \; S_\zeta = S_z \cos \beta + \frac{\chi_\perp a}{\gamma^2 S_z} \sin^2 \beta \cos \beta. \tag{29}$$

The first solution describes uniform precession of the magnetization with a frequency

$$\omega_p = \omega_L - \frac{a}{S_\zeta} \cos \beta \tag{30}$$

around the magnetic field vector, and $\omega_L = \gamma H$. In this case the integral of motion S_ζ is expressed in terms of ω_γ as follows:

$$\omega_\gamma = \frac{\gamma^2}{\chi_\parallel} S_\zeta - \frac{a}{S_\zeta} \cos^2 \beta. \tag{31}$$

At resonance, when $\omega_p = \omega_\gamma$ holds, the parameter S_ζ is close to its equilibrium value

$$S_\zeta = \frac{\chi_\parallel \omega_L}{\gamma^2} + \frac{a \cos \beta}{\omega_L} (\cos \beta - 1), \tag{32}$$

and to the same accuracy the precession frequency is equal to

$$\omega_p = \omega_L - \frac{\gamma^2 a}{\chi_\parallel \omega_L} \cos \beta. \tag{33}$$

In the case of the second solution we have

$$\omega_p = \omega_L - \frac{\gamma^2 S_\zeta}{\chi_\perp} + \frac{a \cos^2 \beta}{S_z}, \tag{34}$$

$$\omega_\gamma = \gamma^2 \left(\frac{1}{\chi_\parallel} - \frac{1}{\chi_\perp} \right) S_z \cos \beta + \frac{a \cos^3 \beta}{S_z}. \tag{35}$$

Under the resonant condition, $\omega_\mathrm{p} = \omega_\gamma$, we have the approximate expression

$$S_z = \frac{\omega_L / \gamma^2}{1/\chi_\perp + (1/\chi_\parallel - 1/\chi_\perp) \cos \beta}. \tag{36}$$

It demonstrates that the magnetization component along the magnetic field equals the equilibrium value, and the vector $\mathbf{n} = \hat{\zeta}$ rotates around the $\hat{z}$-axis with the frequency ω_p. The vectors $\mathbf{l}_1$ and $\mathbf{l}_2$ rotate around **n** with the same frequency. Although the magnetization in the second mode does not precess around the magnetic field vector, a precessing magnetization component $S_\beta \propto H_1$ is generated if there is a magnetic field component $\mathbf{H}_1(t)$ rotating in the xy-plane with the frequency ω_p. This means that, as in the first mode, high-frequency electromagnetic power should be absorbed at the resonance.

4. COHERENTLY PRECESSING DOMAIN WALL

Let us consider the situation when the field satisfies $\mathbf{H} \| \hat{z} = \mathbf{n}$ and is aligned with the hexagonal crystal axis. Suppose that $H > H_0$ holds and there is a field gradient along the $\hat{z}$-axis. The equation system takes the form

$$\begin{aligned} \dot{S}_z &= -\frac{\delta(\mathcal{H} + F_\nabla)}{\delta \alpha}, & \dot{\alpha} &= \frac{\partial \mathcal{H}}{\partial S_z}, \\ \dot{S}_\zeta &= -\frac{\delta(\mathcal{H} + F_\nabla)}{\delta \gamma}, & \dot{\gamma} &= \frac{\partial \mathcal{H}}{\partial S_\zeta}, \\ \dot{S}_\beta &= -\frac{\delta(\mathcal{H} + F_\nabla)}{\delta \beta}, & \dot{\beta} &= \frac{\partial \mathcal{H}}{\partial S_\beta}, \end{aligned} \tag{37}$$

where $\mathcal{H}$ is defined by Eq. (13), and F_∇ defined by Eq. (19) is considered as a function of only the coordinate z. Like anisotropy, weak nonuniformity acts as a small perturbation of the fundamental precession mode with the Larmor frequency. Therefore, as in the case of the uniform configuration, we seek a solution of Eq. (37) periodic in α and γ and stationary with respect to the rest of the variables:

$$\alpha=-\omega_p t+\alpha(z), \tag{38}$$

$$\gamma=\omega_\gamma t+\gamma(z), \tag{39}$$

$$\dot S_z=\dot S_\zeta=\dot S_\beta=\dot\beta=0. \tag{40}$$

According to Hamilton's equations,

$$\dot S_z=+\frac{c_\parallel^2}{\gamma^2}\chi_1[(\alpha_z' \cos\beta+\gamma_z' \cos\beta)]_z' + \frac{c_\parallel^2}{\gamma^2}\chi_2(\alpha_z' \sin^2\beta)_z', \tag{41}$$

$$\dot S_\zeta=\frac{c_\parallel^2}{\gamma^2}\chi_1(\alpha_z' \cos\beta+\gamma_z')_z'. \tag{42}$$

The stationarity condition for S_ζ and Eq. (42) yield

$$\alpha_z' \cos\beta+\gamma_z'=\text{const}, \tag{43}$$

i.e., the corresponding component of the spin current along the z-axis is constant. The spin current should be zero on the sample boundaries, i.e., at $z=\pm d/2$, which results in the equation

$$\alpha_z' \cos\beta+\gamma_z'=0 \tag{44}$$

that holds at all z. By substituting Eq. (44) into Eq. (41) and using the stationarity condition $\dot S_z=0$, we obtain the relationship

$$\alpha_z' \sin^2\beta=\text{const}. \tag{45}$$

This constant must also be zero because the spin current is zero on the boundary. Since we are looking for a distribution that is nonuniform with respect to the angle β, the left-hand sides of Eqs. (44) and (45) can equal zero only if

$$\alpha_z'=\gamma_z'=0. \tag{46}$$

With due account of Eqs. (38), (39), and (46), we derive from the rest of Hamilton's equations the following relations:

$$\omega_p=\omega_L(z)-\frac{\gamma^2}{\chi_\perp}\frac{S_z-S_\zeta\cos\beta}{\sin^2\beta}, \tag{47}$$

$$\omega_\gamma=\frac{\gamma^2 S_\zeta}{\chi_\parallel}-\frac{\gamma^2}{\chi_\perp}\frac{S_z-S_\zeta\cos\beta}{\sin^2\beta}\cos\beta, \tag{48}$$

$$\frac{\gamma^2}{\chi_\perp}S_\beta=0, \tag{49}$$

$$-\frac{\gamma^2}{\chi_\perp}\frac{S_z-S_\zeta\cos_\beta}{\sin\beta}\frac{S_\zeta-S_z\cos\beta}{\sin^2\beta}+a\cos\beta\sin\beta + \frac{\chi_2 c_\parallel^2}{\gamma^2}\beta''=0, \tag{50}$$

where $\omega_L(z)=\gamma H(z)$.

The solution that will be given below indicates that the last two terms on the right-hand side of Eq. (50) (those due to the anisotropy and inhomogeneity) are of the same order of magnitude. Therefore, as in Eq. (28), we derive from Eq. (50) at $H\gg H_0$ the expression

$$S_z=S_\zeta\cos\beta+\frac{\chi_\perp a}{\gamma^2 S_\zeta}\cos\beta\sin^2\beta+\frac{\chi_\perp\chi_2 c_\parallel^2}{\gamma^4 S_\zeta}\beta''\sin\beta. \tag{51}$$

The substitution of Eq. (51) into Eqs. (47) and (48) yields

$$\frac{\chi_2}{\gamma^2}c_\parallel^2\beta''+a\cos\beta\sin\beta-S_\zeta[\omega_L(z)-\omega_p]\sin\beta=0, \tag{52}$$

$$\omega_\gamma=\frac{\gamma^2 S_\zeta}{\chi_\parallel}-[\omega_L(z)-\omega_p]\cos\beta, \tag{53}$$

respectively. Equation (53) determines the spatial distribution of the parameter $S_\zeta(z)$, which is a constant in the lowest-order approximation:

$$S_\zeta=\frac{\chi_\parallel\omega_\gamma}{\gamma^2}. \tag{54}$$

We recall that in the resonant conditions, when $\omega_\gamma=\omega_p$ holds, S_ζ is close to its equilibrium value.

Equation (52) can be conveniently transformed to

$$\beta''=\frac{z-z_0}{\lambda^3}\sin\beta-\frac{1}{\lambda_a^2}\sin\beta\cos\beta. \tag{55}$$

Here we have $\omega_L(z)-\omega_p=\nabla\omega(z-z_0)$,

$$\lambda^3=\frac{\chi_2 c_\parallel^2}{\chi_\parallel\omega_\gamma\nabla\omega}, \tag{56}$$

$$\lambda_a^2=\frac{\chi_2 c_\parallel^2}{\gamma^2 a}. \tag{57}$$

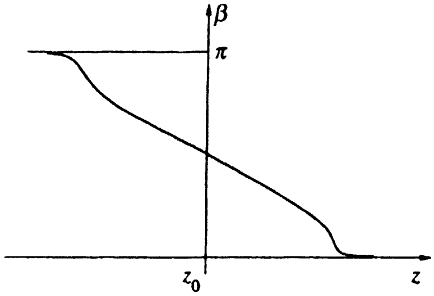

FIG. 3. Approximate form of the solution of Eq. (55).

The boundary condition for Eq. (55) is zero spin current on the upper and lower boundaries of the sample, i.e., $\beta'=0$ at $z=\pm d/2$. If the sample dimension along the z-axis is much larger than the inhomogeneity dimension in the distribution of the angle β, this boundary condition is equivalent to $\beta\to 0$ for $z\to\pm\infty$. A solution of Eq. (55) satisfying these conditions is plotted in Fig. 3. This distribution of the angle β describes a domain wall between a domain with magnetization antiparallel to the applied magnetic field (the region of lower fields, $z\to-\infty$, $\beta=\pi$) and a domain with magnetization parallel to the applied field (region of higher fields, $z\to+\infty$, $\beta=0$). The function $\beta(z)$ has three inflection points. They can be found by solving the equation

$$z-z_0=\frac{\lambda^3}{\lambda_a^2}\cos\beta. \qquad (58)$$

Inside the domain wall the magnetization precesses coherently with the frequency $\omega_p=\omega_L(z_0)$, $\beta(z_0)=\pi/2$. The wall thickness is determined to order of magnitude by the expression

$$\Lambda=\frac{\lambda^3}{\lambda_a^2}=\frac{\gamma^2 a}{\chi_{\parallel}\omega_\gamma\nabla\omega}. \qquad (59)$$

This parameter can be easily estimated using the approximate relation

$$\Lambda\approx\frac{H_0^2}{H\nabla H}. \qquad (60)$$

Let us recall that $H_0\approx 1.9$ T. If we take $H\approx 10H_0$ and $\nabla H\approx 10^{-1}H_0/\mathrm{cm}$, then $\Lambda\approx 1$ cm. Note that in samples with thickness $d\sim\Lambda$ the width of the inhomogeneous distribution of the coherently precessing magnetization is determined by the sample thickness d.

In a uniform magnetic field (or in a sample with $d<\Lambda$), the domain wall thickness is infinite, i.e., the coherent precession of the two-domain structure transforms to a uniform precession of magnetization. The domain wall position z_0 is determined by the initial conditions, i.e., the total longitudinal magnetization, $\int S_z dz$. A $CsNiCl_3$ domain wall coherently precessing in a nonuniform magnetic field can be observed in experiments with continuous antiferromagnetic resonance. In this case the precession frequency ω_p should equal that of the transverse high-frequency field, and the wall position can be tuned through duration and amplitude of the driving pumping.

Now recall that in $CsMnBr_3$ the anisotropy constant satisfies $a<0$. A similar situation takes place in $CsNiCl_3$ when both the magnetic field and its gradient lie in the basal plane [the Hamiltonian (14)]. Equation (55) is expressed in this case as

$$\beta''=\frac{\omega_L(z)-\omega_p}{\lambda^3\nabla\omega}\sin\beta+\frac{1}{\lambda_a^2}\sin\beta\cos\beta, \qquad (61)$$

where

$$\lambda_a^2=\frac{\chi_2 c_{\parallel}^2}{\gamma^2|a|}. \qquad (62)$$

The solution of Eq. (61) with the same boundary condition also describes a domain wall between domains with magnetizations parallel and antiparallel to the applied magnetic field (Fig. 4). The distribution $\beta(z)$ has in this case only one inflection point determined by the equation

$$z-z_0=-\frac{\lambda^3}{\lambda_a^2}\cos\beta, \qquad (63)$$

i.e., $\beta(z_0)=\pi/2$. The wall has a thickness $\lambda_a\approx 10^{-6}$ cm. The precession frequency in the wall equals the Larmor frequency at $z=z_0$, i.e., $\omega_p=\omega_L(z_0)$. Such walls also occur in absolutely uniform magnetic fields.

To sum up, coherently precessing domain walls in a nonuniform magnetic field ($\mathbf{H}$ and ∇H) in the same material

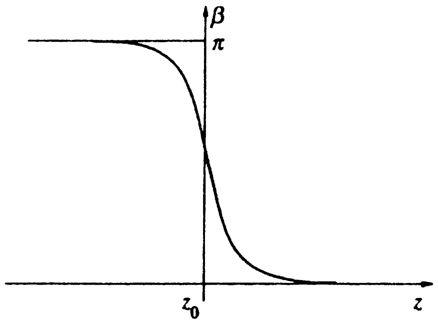

FIG. 4. Approximate form of the solution of Eq. (61).

$CsNiCl_3$ may have thicknesses differing by several orders of magnitude, depending on the magnetic field alignment with respect to the hexagonal axis, so emitted induction signals may also differ enormously. In a field aligned with the hexagonal axis and with the gradient also directed along this axis, the wall thickness is controlled by the field gradient [Eqs. (59) and (60)], and in a field directed in the basal plane a wall can be generated even when the field is uniform, and the wall thickness is controlled by the anisotropy parameter [the last term in Eq. (61)].

5. SPIN VORTEX

Beside the solutions describing coherently precessing domain walls, the equation system has quite different coherently precessing solutions with circular nondissipative currents, called spin vortices. Such solutions were originally derived in the case of superfluid ^{3}He,[14] and later such vortices were detected in experiments.[15]

Let a uniform magnetic field $H>H_0$ be aligned with the hexagonal axis. We look for axially symmetrical coherently precessing solutions of Eq. (37) in the form

$$\alpha=-\omega_p t+\alpha(\varphi), \qquad (64)$$

$$\gamma=\omega_\gamma t+\gamma(\varphi), \qquad (65)$$

$$\beta=\beta(\rho), \qquad (66)$$

$$\dot S_z=\dot S_\zeta=\dot S_\beta=\dot\beta=0. \qquad (67)$$

Here ρ and φ are polar coordinates in the basal plane. The parameters α and γ as functions of coordinates are determined by the equations

$$\dot S_\alpha=-\frac{\delta(\mathscr{H}+F_\nabla)}{\delta\alpha}=\frac{c_\perp^2}{\gamma^2}[(\chi_1\cos^2\beta+\chi_2\sin^2\beta)\Delta\alpha+\chi_1\cos\beta\Delta\gamma]=0, \qquad (68)$$

$$\dot S_\zeta=-\frac{\delta(\mathscr{H}+F_\nabla)}{\delta\gamma}=\frac{c_\perp^2}{\gamma^2}[\cos\beta\Delta\alpha+\Delta\gamma]=0. \qquad (69)$$

Here Δ is the two-dimensional Laplace operator. The determinant of this linear equation system is nonzero, so its only solution is

$$\Delta\alpha=0, \qquad (70)$$

$$\Delta\gamma=0. \qquad (71)$$

Hence we have

$$\alpha(\varphi)=N_1\varphi+\alpha_0, \quad (72)$$

$$\gamma(\varphi)=N_2\varphi+\gamma_0. \quad (73)$$

N_1 and N_2 are integers because the orientation of the order-parameter reference at a given point φ specified by rotations through the angles $\alpha(\varphi)$ and $\gamma(\varphi)$ is uniquely determined. The distributions of the angular variables α and γ correspond to vortex distributions of spin velocities with quantized circulation:

$$\nabla\alpha=\frac{N_1}{\rho}\hat{\varphi}, \quad (74)$$

$$\nabla\gamma=\frac{N_2}{\rho}\hat{\varphi}. \quad (75)$$

The rest of Hamilton's equations yield

$$\omega_p=\omega_L-\frac{\gamma^2}{\chi_\perp}\frac{S_z-S_\zeta\cos\beta}{\sin^2\beta}, \quad (76)$$

$$\omega_\gamma=\frac{\gamma^2 S_\zeta}{\chi_\parallel}-\frac{\gamma^2}{\chi_\perp}\frac{S_z-S_\zeta\cos\beta}{\sin^2\beta}\cos\beta, \quad (77)$$

$$\frac{\gamma^2}{\chi_\perp}S_\beta=0, \quad (78)$$

$$-\frac{\gamma^2}{\chi_\perp}\frac{S_z-S_\zeta\cos\beta}{\sin\beta}\frac{S_\zeta-S_z\cos\beta}{\sin^2\beta}+a\cos\beta\sin\beta$$
$$+\frac{c_\perp^2N_1\sin\beta}{\gamma^2\rho^2}[\chi_1(N_1\cos\beta+N_2)-\chi_2N_1\cos\beta]$$
$$+\frac{c_\perp^2}{\gamma^2}\chi_2\Delta\beta=0. \quad (79)$$

As in the spatially uniform case, we derive from Eq. (79)

$$S_z=S_\zeta\cos\beta+\frac{\chi_\perp}{\gamma^2}\frac{\sin\beta}{S_\zeta}\Bigg\{a\cos\beta\sin\beta$$
$$+\frac{c_\perp^2N_1\sin\beta}{\gamma^2\rho^2}[\chi_1(N_1\cos\beta+N_2)-\chi_2N_1\cos\beta]$$
$$+\frac{c_\perp^2}{\gamma^2}\chi_2\Delta\beta\Bigg\}. \quad (80)$$

The substitution of this expression into Eq. (77) yields a relation between the spatial distribution $S_\zeta(\rho)$ and the distribution of the angle $\beta(\rho)$. In the case of resonance, $\omega_\gamma=\omega_p$, and in the lowest-order approximation S_ζ may be considered constant:

$$S_\zeta=\frac{\chi_\parallel\omega_L}{\gamma^2}. \quad (81)$$

After substituting Eq. (80) into Eq. (76) with due account of Eq. (81), we obtain

$$\Delta\beta+\frac{\chi_1}{\chi_2}\frac{\sin\beta}{\rho^2}N_1(N_1\cos\beta+N_2)-\frac{N_1^2\sin\beta\cos\beta}{\rho^2}$$
$$+\frac{\gamma^2a}{\chi_2c_\perp^2}\sin\beta\cos\beta-\frac{\chi_\parallel\omega_L}{\chi_2c_\perp^2}(\omega_L-\omega_p)\sin\beta=0. \quad (82)$$

Equation (82) describes the spatial distribution of the angle β in a spin vortex similar to the order-parameter distribution in a quantized vortex in a traditional superfluid. The precession with a definite frequency

$$\omega_p=\omega_L-\frac{\gamma a}{\chi_\parallel\omega_L}\cos\beta_0 \quad (83)$$

determines a constant value of the angle β at a large distance from the vortex axis:

$$\beta(\rho\to\infty)=\beta_0.$$

Inside the vortex core with radius

$$\lambda_a\approx\sqrt{\frac{c_\perp^2\chi_2}{a\gamma^2}} \quad (84)$$

the angle β tends to zero as $\rho\to 0$:

$$\beta\propto\rho^A, \quad (85)$$

where

$$A=\sqrt{N_1^2-\frac{\chi_1}{\chi_2}(N_1^2+N_1N_2)}. \quad (86)$$

The latter formula, however, is exact only if N_1+N_2 is an odd number. In this case β goes to zero on the vortex axis, and we have singular vortices, i.e., vortices in which the antiferromagnetic order is destroyed on the axis. If N_1+N_2 is an even number, at distances $\rho<\lambda$ the distribution of the order parameter may "leak into the third dimension," i.e., α, β, and γ can depnd on on ρ and φ and the gradients of the angles α and γ will not have singularities on the vortex axis, like the vortices in superfluid helium-3.[16] This property derives from the fact that for $\rho>\lambda_a$ the region within which the order parameter varies is $S^1\times S^1$ (the domain of the variation of the angles α and γ), and at $\rho<\lambda_a$ the orientations of the order parameter are determined by arbitrary rotations in a three-dimensional space generating an SO_3 group.

6. EFFECT OF ANTIFERROMAGNETIC DOMAINS

In previous sections we have developed the theory of nonlinear antiferromagnetic resonance in an ideal single-domain crystal of an antiferromagnetic material. Although the difficulties in fabricating single-domain samples can be overcome, it is interesting to investigate a more realistic case of a multidomain antiferromagnetic crystal. As was noted above, the degeneracy with respect to the spin directions in the basal plane is lifted owing to the interaction described by Eq. (2). If we have $\mathbf{H}\|\hat{z}$, $H>H_0$, then by selecting $\hat{l}_{10}=\hat{x}$ and $\hat{l}_{20}=\hat{y}$ we obtain

$$F_a'=-b\,\mathrm{Re}[R_{xx}(\alpha,\beta,\gamma)+iR_{xy}(\alpha,\beta,\gamma)]^6. \quad (87)$$

Here R_{xx} and R_{xy} are matrix elements of the rotation operator defined by Eq. (12). In explicit form,

$$F'_a=-\frac{b}{2^6}\mathrm{Re}[(\cos\beta+1)e^{-i(\alpha+\gamma)}+(\cos\beta-1)e^{i(\alpha-\gamma)}]^6. \quad (88)$$

In the case of resonance, when $\alpha+\gamma=\Phi$ changes little in one precession period $2\pi/\omega_p$ and the variable $\alpha-\gamma=-2\omega_p t$ gives rise to low-amplitude motion $[\sim(H_0/H)^2 b/a]$ at double frequency, averaging Eq. (88) over precession period yields

$$F'_a=-\frac{b}{2^6}(\cos\beta+1)^6\cos 6\Phi. \quad (89)$$

The minima of the function on the right-hand side at $\Phi_i=0,\ \pi/3,\ 2\pi/3,\ \pi,\ 4\pi/3,\ 5\pi/3$ correspond to six different equilibrium configurations of the vectors $\mathbf{l}_1$ and $\mathbf{l}_2$ in the basal plane, i.e., to different antiferromagnetic domains.

Note that in the nonresonant case we have $\omega_p\neq\omega_\gamma$. The averaging over the variables α and γ can be performed independently, the average energy density $\langle F'_a\rangle$ vanishes, and the effect of the domains is completely absent.

Oscillations of Φ around its equilibrium values Φ_i lead to longitudinal oscillations of the magnetization, i.e., oscillations of S_ζ about its equilibrium value $\chi_\parallel H/\gamma$. Their frequencies can be calculated using the equations

$$\dot S_\zeta=-\frac{\partial\tilde{\mathscr{H}}}{\partial\Phi}=-\frac{6b}{2^6}(\cos\beta+1)^6\sin 6\Phi, \quad (90)$$

$$\dot\Phi=\frac{\partial\tilde{\mathscr{H}}}{\partial S_\zeta}=\frac{\gamma^2}{\chi_\parallel}S_\zeta-\gamma H. \quad (91)$$

Here we have written $\tilde{\mathscr{H}}=\mathscr{H}+F'_a$ and the Hamiltonian density determined by Eq. (13) can be represented as a function of pairs of the new canonically conjugate variables $P=S_z-S_\zeta$ and α, S_β and β, S_ζ and Φ (cf. Ref. 4). From Eqs. (90) and (91) we derive the frequency of small oscillations:

$$\omega_\parallel^2=\frac{\gamma\sqrt{6b}|\cos\beta+1|^3}{2^3\sqrt{\chi_\parallel}}. \quad (92)$$

Thus we have found that, as in the superfluid ^{3}He, the lifting of degeneracy with respect to Φ stabilizes the magnetization against deviations from the equilibrium value. In calculating the equilibrium value, we neglected the small values $\sim(H_0/H)^2$ and $(H_0/H)^2 b/a$.

Now let us analyze the precession motion. It is obvious that if F'_a is added to the Hamiltonian density, the shift of the precession frequency will be equal in all the domains. In particular, we will have instead of Eq. (33) the following formula:

$$\omega_p=\omega_L-\frac{\gamma^2}{\chi_\parallel\omega_L}\left[a\cos\beta-\frac{3b}{2^5}(\cos\beta+1)^5\right]. \quad (93)$$

Hence it does not lead to any difference in precession frequencies among different domains.

The problem is complicated, however, by transitional regions between neighboring antiferromagnetic domains. Let us consider for definiteness a domain wall in the yz-plane between two semi-infinite domains. The angle Φ is a function of the coordinate x and far from the domain boundary it equals

$$\Phi(-\infty)=0,\quad \Phi(+\infty)=\pi/3. \quad (94)$$

Suppose that the magnetic field is directed along the hexagonal axis $\hat z$ and we have $H\gg H_0$. Hamilton's equations take the form

$$\dot P=-\frac{\delta\tilde{\mathscr{H}}}{\delta\alpha}=\frac{c_\perp^2}{\gamma^2}\left\{\chi_1\left[(\alpha'_x(\cos\beta-1)+\Phi'_x)(\cos\beta-1)\right]'_x+\chi_2(\alpha'_x\sin^2\beta)'_x\right\}, \quad (95)$$

$$\dot S_\zeta=-\frac{\delta\tilde{\mathscr{H}}}{\delta\Phi}=\frac{c_\perp^2}{\gamma^2}\left\{\chi_1\left[(\alpha'_x(\cos\beta-1)+\Phi'_x\right]'_x-\frac{3b}{2^5}(\cos\beta+1)^6\sin 6\Phi\right\}, \quad (96)$$

$$\dot S_\beta=-\frac{\partial\tilde{\mathscr{H}}}{\partial\beta}$$
$$=-\frac{\gamma^2}{\chi_\perp}\frac{P+S_\zeta(1-\cos\beta)}{\sin\beta}\frac{S_\zeta(1-\cos\beta)-P\cos\beta}{\sin^2\beta}$$
$$+a\cos\beta\sin\beta-\frac{3b}{2^5}(\cos\beta+1)^5\sin\beta\cos 6\Phi-\frac{c_\perp^2}{\gamma^2}$$
$$\times\{-\chi_1[\alpha'_x(\cos\beta-1)+\Phi'_x]$$
$$\times\alpha'_x\sin\beta+\chi_2[(\alpha'_x)^2\sin\beta\cos\beta-\beta''_{x2}]\}, \quad (97)$$

$$\dot\alpha=\frac{\partial\tilde{\mathscr{H}}}{\partial P}=\frac{\gamma^2}{\chi_\perp}\frac{P+S_\zeta(1-\cos\beta)}{\sin^2\beta}-\gamma H, \quad (98)$$

$$\dot\Phi=\frac{\partial\tilde{\mathscr{H}}}{\partial S_\zeta}=\frac{\gamma^2}{\chi_\perp}\frac{P+S_\zeta(1-\cos\beta)}{\sin^2\beta}(1-\cos\beta)+\frac{\gamma^2}{\chi_\parallel}S_\zeta-\gamma H, \quad (99)$$

$$\dot\beta=\frac{\partial\tilde{\mathscr{H}}}{\partial S_\beta}=\frac{\gamma^2}{\chi_\perp}S_\beta. \quad (100)$$

Let us seek a solution periodic in the variable

$$\alpha=-\omega_p t+\alpha(x) \quad (101)$$

and stationary with respect to the rest of the variables:

$$\dot\beta=\dot S_\beta=\dot S_\zeta=\dot P=\dot\Phi=0. \quad (102)$$

As in the case of a spin vortex discussed in the previous section, the precession at a long distance from the domain boundary with a fixed frequency [see Eq. (93)]

$$\omega_p=\omega_L-\frac{\gamma a}{\chi_\parallel\omega_L}\cos\beta_0-\frac{3\gamma b}{2^5\chi_\parallel\omega_L}(\cos\beta_0+1)^5 \quad (103)$$

uniquely determines the angle β:

$$\beta_0=\beta(\pm\infty). \quad (104)$$

As we will see below, deviations of $\beta(x)$ from a constant value,

$$\delta\beta=\beta(x)-\beta_0, \quad (105)$$

are very small ($\sim b/a$). Therefore in solving Eqs. (95) and (96) we may consider $\beta(x)$ to be constant. Given the stationarity condition, this equation is transformed to

$$\left[(1-\cos\beta_0)+\frac{\chi_2}{\chi_1}(1+\cos\beta_0)\right]\alpha''-\Phi''=0, \quad (106)$$

$$-(1-\cos\beta_0)\alpha''+\Phi''=\frac{3b\gamma^2}{2^5c_\perp^2\chi_1}(1+\cos\beta_0)^6\sin 6\Phi, \quad (107)$$

so the function $\Phi(x)$ is determined by the sine-Gordon equation

$$\Phi''=A\sin 6\Phi, \quad (108)$$

$$A=\frac{3b\gamma^2}{2^5c_\perp^2\chi_1}\times\frac{(1+\cos\beta_0)^6[(1-\cos\beta_0)+(\chi_2/\chi_1)(1+\cos\beta_0)]}{(1-\cos\beta_0)[1+(\chi_2/\chi_1)(1+\cos\beta_0)]}. \quad (109)$$

Its solution with the boundary conditions (94) is

$$\Phi(x)=\frac{2}{3}\arctan[\exp(\sqrt{6A}x)]. \quad (110)$$

The function α is calculated by integrating Eq. (106):

$$\alpha(x)=\frac{\Phi(x)}{(1-\cos\beta_0)+(\chi_2/\chi_1)(1+\cos\beta_0)}. \quad (111)$$

As in the previous sections, from Eqs. (97) and (102) in the limit $H\gg H_0$ we find

$$P=-S_\zeta(1-\cos\beta)+\frac{\chi_\perp\sin\beta}{\gamma^2S_\zeta}\Big\{a\cos\beta\sin\beta-\frac{3b}{2^5}(\cos\beta+1)^5\sin\beta\cos 6\Phi+\frac{\chi_1c_\perp^2}{\gamma^2}[\alpha_x'(\cos\beta-1)+\Phi_x']\alpha_x'\sin\beta+\frac{\chi_2c_\perp^2}{\gamma^2}\times[-(\alpha_x')^2\sin\beta\cos\beta+\beta_{x^2}'']\Big\}. \quad (112)$$

After substituting this expression into Eq. (98), we obtain with due account of Eq. (101)

$$\omega_L-\omega_p=\frac{1}{S_\zeta\sin\beta}\Big\{a\cos\beta\sin\beta-\frac{3b}{2^5}(\cos\beta+1)^5\sin\beta\cos 6\Phi+\frac{\chi_1c_\perp^2}{\gamma^2}[\alpha_x'(\cos\beta-1)+\Phi_x']\alpha_x'\sin\beta+\frac{\chi_2c_\perp^2}{\gamma^2}[-(\alpha_x')^2\sin\beta\cos\beta+\beta_{x^2}'']\Big\}. \quad (113)$$

Given that, as usual, $S_\zeta\approx\chi_\|H/\gamma$ holds and the angle β_0 is determined by Eq. (103), we obtain by expanding the right-hand side of Eq. (113) in powers of $\delta\beta$

$$\delta\beta(x)=\frac{c_\perp^2}{\gamma^2a}\{\chi_1[\alpha_x'(\cos\beta_0-1)+\Phi_x']\alpha_x'-\chi_2(\alpha_x')^2\cos\beta_0\}. \quad (114)$$

Since, according to Eqs. (108)–(111), the distance over which the angles $\Phi(x)$ and $\alpha(x)$ change significantly is

$$\lambda_b=\frac{c_\perp}{\gamma}\sqrt{\frac{\chi_1}{b}}, \quad (115)$$

the parameter $\delta\beta(x)$ is of order b/a. It is obvious that deviations of β from β_0 vanish far from the domain boundary:

$$\delta\beta(\pm\infty)=0. \quad (116)$$

Thus we have demonstrated the in the presence of a domain wall, which leads to inhomogeneous distributions of the angles $\alpha(x)$, $\Phi(x)$, and $\beta(x)$, and of the values S_ζ and P, there is a coherently precessing solution of the spin-dynamic equation with a precession frequency shifted with respect to the Larmor frequency, in accordance with Eq. (103). The coherent precession, apparently, also occurs in the case of domain boundaries of arbitrary shapes if domain dimensions are larger than the domain wall thickness λ_b [Eq. (115)]. Solutions describing spin vortices and domain walls between domains with magnetization parallel and antiparallel to the external magnetic field should also apply to multidomain antiferromagnetic crystals.

7. CONCLUSIONS

Several solutions of the nonlinear spin dynamical equations describing inhomogeneous coherently precessing distributions of magnetization in $CsNiCl_3$ have been found. A complete theory of such structures should include investigations of their stability and relaxation to equilibrium distributions.

The stability of the discussed solutions against small perturbations is estimated similarly to the stability of the magnetization and order parameter in a uniformly precessing domain in ^{3}He-*B* (Ref. 4), by minimizing the functional

$$\int dz(\mathcal{H}+F_\nabla+\omega_pS_z).$$

The range of parameters in which inhomogeneous coherently precessing structures in antiferromagnets are feasible is also limited by a specific instability mechanism,[17] namely the decay of the magnetization precession with a frequency ω_1 and $\mathbf{k}=0$ to two longitudinal or transverse spin waves:

$$\omega_1(\mathbf{k}=0)=\omega_2(-\mathbf{k})+\omega_2(+\mathbf{k}).$$

This so-called Suhl instability does not affect observation of coherent precession in ^{3}He-*B*, but may be essential in the antiferromagnets discussed in this paper. The problem of the Suhl instability deserves further theoretical and experimental investigation.

As was noted above, observation of coherently precessing magnetization distributions is possible in experiments with continuous antiferromagnetic resonance. By pulsed methods, such structures with coherent precession might be detected only if they could be created in a time shorter than that of longitudinal relaxation. In liquid helium-3 this condition is easily satisfied. The possibility of tipping the magnetization away from the vector of applied magnetic field to a large angle by impulsively exciting precession and of creating inhomogeneous precessing distributions in $CsNiCl_3$ has thus far not been studied experimentally.

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[1] A. S. Borovik-Romanov, Yu. M. Bun'kov, V. V. Dmitriev *et al.*, Zh. Éksp. Teor. Fiz. **88**, 2025 (1985) [Sov. Phys. JETP **61**, 1199 (1985)].
[2] I. A. Fomin, Zh. Éksp. Teor. Fiz. **88**, 2039 (1985) [Sov. Phys. JETP **61**, 1207 (1985)].
[3] A. S. Borovik-Romanov, Yu. M. Bun'kov, V. V. Dmitriev *et al.*, Zh. Éksp. Teor. Fiz. **96**, 956 (1989) [Sov. Phys. JETP **69**, 542 (1989)].
[4] I. A. Fomin, in *Helium 3*, ed. by W. P. Halperin and L. P. Pitaevskii, Elsevier Sci. Publ., Amsterdam (1990), p. 609.
[5] G. Nunes Jr., C. Jin, D. L. Hawthorne *et al.*, Phys. Rev. B **46**, 9082 (1992).
[6] V. V. Dmitriev and I. A. Fomin, JETP Lett. **59**, 378 (1994).
[7] V. V. Dmitriev, S. R. Zakazov, and V. V. Moroz, JETP Lett. **61**, 324 (1995).
[8] Yu. G. Makhlin and V. P. Mineev, Zh. Éksp. Teor. Fiz. **109**, 441 1996 [JETP **82**, 234 (1996)].
[9] V. P. Mineev and Yu. G. Makhlin, JETP Lett. **62**, 595 (1995).
[10] V. P. Silin, Zh. Éksp. Teor. Fiz. **33**, 1227 (1956) [Sov. Phys. JETP **6**, 945 (1958)].
[11] I. A. Fomin and D. V. Shopova, JETP Lett. **42**, 199 (1985).
[12] I. A. Zaliznyak, L. A. Prozorova, and S. V. Petrov, Zh. Éksp. Teor. Fiz. **97**, 359 (1990) [Sov. Phys. JETP **70**, 203 (1990)]; O. A. Petrenko, S. V. Petrov, and L. A. Prozorova, Zh. Éksp. Teor. Fiz. **98**, 727 (1990) [Sov. Phys. JETP **71**, 406 (1990)].
[13] H. A. Katori, J. Ajiro, T. Asano, and T. Goto, J. Phys. Soc. Jap. **64**, 3038 1995.
[14] I. A. Fomin, Zh. Éksp. Teor. Fiz. **94**, 112 (1988) [Sov. Phys. JETP **67**, 1148 (1988)].
[15] A. S. Borovik-Romanov, Jn. M. Bun'kov, V. V. Dmitriev *et al.*, Physica B **165** & **166**, 649 (1990).
[16] M. M. Salomaa and G. E. Volovik, Rev. Mod. Phys. **59**, 533 (1987).
[17] Y. Sasaki, T. Matsuchita, T. Mizusaki, and A. Hirai, Phys. Rev. B **44**, 7362 (1991).

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