Nonlocal electron heat transport in plasma with ion-acoustic turbulence

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A theory of nonlocal electronic heat transport in plasma with ion-acoustic turbulence has been developed. Two different thermal conductivities characterizing plasma response to inverse bremsstrahlung absorption of high-frequency radiation and to quasistatic low-frequency electric field have been determined. Since the electron free paths in turbulent and laminar plasmas are described by similar functions of the velocity, the effective thermal conductivities are also described by similar functions of the wave vector magnitude. But owing to anisotropy of electron scattering by acoustic fluctuations of ion charge density in turbulent plasma, the effective thermal conductivity is essentially anisotropic. The nonlocal nature of the heat transport in turbulent plasma appears when the scale of the electron temperature perturbation is considerably smaller than in laminar plasma. At the same time, the decrease in the heat flux is notably larger because of an essentially higher effective collision frequency. [S1063-7761(96)00812-8]

1. INTRODUCTION

In recent years some progress has been made in understanding the nature of inhibition of electron thermal conductivity of plasma. This phenomenon[1–3] has been ascribed as a result of computer simulations of collisional heat transport in fully ionized plasma[4–9] to peculiarities of transport in weakly collisional plasma, where the electron free path is longer than the typical length scale of perturbations of the electron density distribution. The progress in analytical techniques applied to the kinetic theory of weakly collisional plasmas[10,11] has allowed us to interpret[12] the results of computer simulations.[5–9] It turned out that energy transport in weakly collisional plasma is due to thermal collisionless electrons, and an increase in the electron energy in the case of either inverse bremsstrahlung absorption of high-frequency waves or due to low-frequency electric field may be caused by subthermal colliding electrons. Plasma electrons can be classified as collisionless and highly collisional because the electron mean free path is a function of its velocity v, namely, the mean free path is proportional to its fourth power. As a result of this peculiar property of weakly collisional plasma, where heat is conducted by one (majority) group of thermal electrons and energy supplied from outside is absorbed by another group, firstly, electron heat transport is essentially nonlocal, and secondly, it has become clear now that nonlocal electron thermal conductivity cannot be described by a single parameter. The latter conclusion was supported by a computer simulation.[13] Hence, the effective nonlocal thermal conductivity in plasma heated by inverse bremsstrahlung absorption of radiation calculated analytically[10] is notably different from the similar parameter in the case of plasma driven by a quasistatic electric field.[14,15]

All these results of the theory of nonlocal electron heat transport explaining the cause of the inhibition of electron heat transport in laser produced plasmas have been derived in recent years from the kinetics of laminar plasmas. On the other hand, weakly collisional plasma is often easily transformed to a turbulent state,[16] where intense oscillations of plasma fields substaintially modify scattering of charged particles, so the equations which describe heat transport are notably different from those in laminar plasma. Note that initially the inhibition of the heat transport in laser produced plasmas was ascribed to the generation of ion-acoustic instability.[17] However, at that time, firstly, the quantitative theory of ion-acoustic turbulence had not been developed, and secondly, there was no theory of anomalous transport in weakly collisional plasma. The progress than has been made in these fields at present has allowed us to formulate in this paper principles of the theory of the nonlocal-field effect on weakly collisional plasma with developed ion-acoustic turbulence. As a result, we can identify, in particular, an anomalous decrease in the electronic thermal conductivity of plasma when, beside electron scattering by charged particles, scattering by low-frequency ion-acoustic fluctuations of the charge density is also important.

In the next section we formulate the basic kinetic equation (2.3) of our theory describing both plasma turbulence and plasma perturbations which, according to Eq. (2.12), are responsible for heating by both high-frequency radiation and low-frequency (quasistatic) electric fields. Section 3 contains essential information about the spectrum of turbulent ion-acoustic fluctuations. Section 4 gives a solution of the kinetic equation for the antisymmetrical component of the electron distribution function taking account of the scattering of slow electrons by turbulent oscillations. An equation for the symmetrical component of the distribution function and its solution are given.

Section 5 contains a description of the perturbed density and temperature of cold (subthermal) electrons due to the nonlocal nature of their interaction with electric field. This section also gives an expression for the effective nonlocal thermal conductivity responsible, according to Ref. 13, for plasma heating by inverse bremsstrahlung absorption. Section 6 describes results derived from the theory of plasma perturbed by an low-frequency electric potential, and the complex dielectric function and nonlocal effective thermal conductivity controlled by interaction between the electrons and field are given. Finally, conclusions and discussion of the results are given in Sec. 7.

2. BASIC KINETIC EQUATION

The subject of our study is fully ionized nonisothermal plasma with ion-acoustic turbulence in a high-frequency electromagnetic field defined by the formula

$$\frac{1}{2}\mathbf{E}(\mathbf{r},t)\exp(-i\omega_0 t)+\text{c.c.}, \tag{2.1}$$

where the amplitude $\mathbf{E}=\mathbf{E}(\mathbf{r},t)$ changes little during the period $2\pi/\omega_0$. We assume that the frequency ω_0 is much higher than the Langmuir electron frequency ω_{Le} and the effective collision frequencies between electrons and either ions or ion-acoustic fluctuations of the charge density. Under these conditions, the effect of the ion-acoustic turbulence on the high-frequency motion of electrons driven by high-frequency electric field is relatively weak[18,19] so that it can be neglected in deriving the perturbed component of the electron distribution function linear in the high-frequency field intensity and oscillating at the frequency ω_0. In contrast, contrary, the motion of subthermal electrons is largely controlled by the turbulence and in what follows is described by a quasilinear collision operator

$$\mathrm{St}_{\mathrm{QL}}(f)=\frac{e^2}{2\pi m^2}\int d\mathbf{k}\left(\mathbf{k}\frac{\partial}{\partial\mathbf{v}}\right)\delta(\omega_s-\mathbf{kv})\frac{\omega_s^3 N(\mathbf{k})}{k^2\omega_L^2}\left(\mathbf{k}\frac{\partial}{\partial\mathbf{v}}\right)f. \tag{2.2}$$

Here e and m are the electron charge and mass, $N(\mathbf{k})$ is the population of the ion-acoustic waves with the wave vector $\mathbf{k}$ and frequency $\omega_s=kv_s/\sqrt{1+k^2r_D^2}$, $v_s=\omega_L r_D$, and r_D is the electronic Debye radius. The formula for the frequency ω_L depends on the plasma composition. In plasma with two species of ions $\omega_L^2=\omega_{L1}^2+\omega_{L2}^2$ where $\omega_{L\alpha}$ is the Langmuir frequency of the species identified by α, and if all ions are identical, ω_L is their Langmuir frequency. Finally, $f=f(\mathbf{v},\mathbf{r},t)$ is the electron distribution function which changes little over the period $2\pi/\omega_0$.

In the linear approximation with respect to the radiation intensity, the distribution function is governed by the kinetic equation

$$\begin{aligned}\frac{\partial f}{\partial t}&+\left(\mathbf{v}\frac{\partial}{\partial\mathbf{r}}\right)f+\frac{e}{m}\mathbf{E}_0\frac{\partial f}{\partial\mathbf{v}}=\mathrm{St}(f)+\mathrm{St}(f,f)+\mathrm{St}_{\mathrm{QL}}(f)\\&+\frac{1}{4}\left(\frac{\partial}{\partial\mathbf{r}}v_E^2\right)\frac{\partial f}{\partial\mathbf{v}}+\frac{1}{8}\frac{\partial^2 f}{\partial v_i\partial v_j}\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}\right)V_{ij}\\&+\frac{1}{4}V_{ij}\frac{\partial^2 f}{\partial r_i\partial v_j}+\frac{1}{4}V_{ij}\left(\frac{\partial}{\partial t}+\mathbf{v}\frac{\partial}{\partial\mathbf{r}}\right)\frac{\partial^2 f}{\partial v_i\partial v_j}\\&-\frac{1}{4}V_{ij}\frac{\partial}{\partial v_i}\mathrm{St}\left(\frac{\partial f}{\partial v_j}\right)-\frac{1}{4}V_{ij}\,\mathrm{St}\left(\frac{\partial f}{\partial v_i},\frac{\partial f}{\partial v_j}\right),\end{aligned} \tag{2.3}$$

where $\mathbf{E}_0=\mathbf{E}_0(\mathbf{r},t)$ is the quasistationary ambipolar electric field in the plasma, we have written $v_E^2=|\mathbf{v}_E|^2$, $\mathbf{v}_E=e\mathbf{E}/m\omega_0$, and

$$V_{ij}=\frac{e^2}{m^2\omega_0^2}(E_iE_j^*+E_jE_i^*) \tag{2.4}$$

is the tensor of oscillating velocities. Beside the quasilinear collision operator $\mathrm{St}_{\mathrm{QL}}(f)$ defined by Eq. (2.2), the right-hand side of Eq. (2.3) contains the electron–ion operator $\mathrm{St}(f)$ and the electron–electron collision operator $\mathrm{St}(f,f)$:

$$\mathrm{St}(f)=\frac{1}{2}\nu(v)\frac{\partial}{\partial v_i}(v^2\delta_{ij}-v_iv_j)\frac{\partial}{\partial v_j}f, \tag{2.5}$$

$$\mathrm{St}(f,f)=\frac{1}{2n}\frac{\partial}{\partial v_i}\int d\mathbf{v}'\,\nu_{ee}(|\mathbf{v}-\mathbf{v}'|)[(\mathbf{v}-\mathbf{v}')^2\delta_{ij}-(v_i-v_i')(v_j-v_j')]\left(\frac{\partial}{\partial v_j}-\frac{\partial}{\partial v_j'}\right)f(\mathbf{v},\mathbf{r},t)f(\mathbf{v}',\mathbf{r},t). \tag{2.6}$$

The collision frequencies in Eqs. (2.5) and (2.6) are expressed as

$$\nu_{ee}(v)=4\pi e^4 n\Lambda/m^2v^3, \tag{2.7}$$

$$\nu(v)=Z_{\mathrm{eff}}\nu_{ee}(v), \tag{2.8}$$

where n is the electron density, Z_{eff} is the effective ion charge,

$$Z_{\mathrm{eff}}=\sum_\alpha e_\alpha^2 n_\alpha\Lambda_\alpha/e^2n\Lambda, \tag{2.9}$$

e_α and n_α are the charge and density of ions identified by the index α, and the Coulomb logarithms Λ and Λ_α are considered to be flat functions of velocity.

The kinetic equation (2.3), which is linear with respect to the radiation intensity, is sufficient for description of the low-frequency response due to electrons if the amplitude of electron velocity oscillations driven by high-frequency electric field v_E is much smaller than the velocity of thermal electron motion v_T. Let us consider only solutions of Eq. (2.3) at a relatively low radiation intensity, when $v_E^2<v_T^2/Z_{\mathrm{eff}}$ holds. Then we can neglect the perturbations of the local Maxwellian electron distribution function f_m due to the inverse bremsstrahlung absorption of radiation,

$$f_m=\frac{n}{2\pi\sqrt{2\pi}v_T^3}\exp\left(-\frac{v^2}{2v_T^2}\right).$$

Let us assume that the length scale of the plasma density and temperature is much larger than the thermal electron mean free path but smaller than the length scale of the high-frequency field multiplied by the large factor v_T^2/v_E^2 This condition allows us to neglect the effect of the high-frequency field on the spectrum of ion-acoustic turbulence due to the gradient of the electron pressure and the quasistatic field $\mathbf{E}_0$, which is itself proportional to the gradient of the electron distribution function. We assume that the frequency of electron–electron scattering is so high that the deviation of the isotropic component of the electron distribution function from the Maxwellian distribution can be ignored.

Under these conditions, we express the solution of Eq. (2.3) as a sum of the local Maxwellian electron distribution function and two small nonequilibrium components:

$$f = f_m + \delta f_n + \delta f. \qquad (2.10)$$

The perturbation δf_n is generated by the quasistationary electric field $\mathbf{E}_0$ and the nonuniformity of the local Maxwellian distribution function. If we neglect the small change in the electron energy caused by the Cherenkov interaction with ion-acoustic waves and the weak Coulomb scattering of electrons at times much longer than the reciprocal frequency of electron scattering by turbulence, the function δf_n is determined by the equation

$$\left(\mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}}\right) f_m + \frac{e}{m}\mathbf{E}_0\cdot\frac{\partial f_m}{\partial\mathbf{v}} = \mathrm{St}_{\mathrm{QL}}(\delta f_n). \qquad (2.11)$$

This is the basic equation for our description of the basic turbulent plasma state, which is not affected by high-frequency radiation. The perturbation δf, on the contrary, is fully dependent on the weak high-frequency electric field. Under the conditions defined above, it is governed by the linear equation

$$\frac{\partial}{\partial t}\delta f + \left(\mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}}\right)\delta f - \mathrm{St}(\delta f, f_m) - \mathrm{St}(f_m, \delta f) - \mathrm{St}_{\mathrm{QL}}(\delta f)$$
$$= \frac{e}{m}\nabla\delta\varphi\frac{\partial f_m}{\partial\mathbf{v}} + \frac{1}{4}\left(\frac{\partial}{\partial\mathbf{r}}v_E^2\right)\cdot\frac{\partial f_m}{\partial\mathbf{v}} + \frac{1}{8}\frac{\partial^2 f_m}{\partial v_i\partial v_j}\left(\mathbf{v}\cdot\frac{\partial}{\partial\mathbf{r}}\right)V_{ij}$$
$$-\frac{1}{6}v_E^2 v_T^2 \nu(v_T)\frac{\partial}{\partial v_i}\left(f_m\frac{v_i}{v^3}\right) - \frac{1}{4}\left(V_{ij}\right.$$
$$\left.-\frac{2}{3}\delta_{ij}v_E^2\right)v_T\nu(v_T)\frac{\partial}{\partial v_i}\left(f_m\frac{v_j}{v^3}\right), \qquad (2.12)$$

where $\delta\varphi$ is the perturbation of the quasistationary field potential ($\delta\mathbf{E}_0 = -\nabla\delta\varphi$). Equation (2.12) allows us to study the kinetics of electrons in plasma with ion-acoustic turbulence driven by a high-frequency field. Before proceeding to this investigation, let us recall the basic properties of the spectrum of ion-acoustic turbulence and quasilinear collision operator.

3. SPECTRUM OF TURBULENT FLUCTUATIONS

According to the principles of the quasilinear theory, Eq. (2.11) yields the derivative of the function δf_n without the specific spectrum of ion-acoustic turbulence. Then the growth rate of the ion-acoustic instability due to the Cherenkov radiation of acoustic waves by drifting electrons is calculated. Thereafter, the distribution of ion acoustic waves over wave vectors is derived from the balance condition for waves generated by electrons with due account of both the Cherenkov absorption of these waves by superthermal resonant ions and induced scattering by thermal ions. This theory is described in detail elsewhere.[20] In this section we only recall the basic results of the self-consistent nonlinear theory of ion-acoustic turbulence. If the plasma contains only one species of ions, the distribution of acoustic waves has the form

$$N(\mathbf{k}) = N(k)\Phi(\cos\theta_k), \qquad (3.1)$$

$$N(k) = 3\pi^2\sqrt{2\pi}\,\omega_{Le}\omega_L^{-2}Rr_D^4 F_1(kr_D), \qquad (3.2)$$

$$F_1(y) = y^{-4}(1+y^2)^{-3/2}[\ln(1+y^{-2}) - (1+y^2)^{-1}$$
$$-0.5(1+y^2)^{-2}], \qquad (3.3)$$

where θ_k is the angle between $\mathbf{k}$ and $\mathbf{R}$, $\mathbf{R}$ is the effective density of the force generating the instability, and

$$\mathbf{R} = en\mathbf{E}_0 - \frac{\partial}{\partial\mathbf{r}}nmv_T^2. \qquad (3.4)$$

The shape of the angular distribution of the wavevector depends on the effective force density. When

$$R \ll R_1(1+\delta)^2. \qquad (3.5)$$

and

$$R_1 = nmv_s\omega_L r_D^2/6\pi r_{Di}^2, \qquad (3.6)$$

where δ is the ratio between the Cherenkov sound damping rates due to ions and electrons, and r_{Di} is the ion Debye radius, we have

$$\Phi(x) = \frac{4}{3\pi x(1+\delta)}\frac{d}{dx}[x^4(1-x+\varepsilon)^{a-1}]. \qquad (3.7)$$

In Eq. (3.7) we have put $x = \cos\theta_k$, and small dimensionless parameters a and ε are functions of the force density R:

$$a = -\ln 2/\ln A_1 \ll 1, \qquad (3.8)$$

$$\varepsilon = -A_1\ln A_1 \ll 1, \qquad (3.9)$$

$$A_1 = (4/3\pi\ln 2)R/R_1(1+\delta)^2 \ll 1. \qquad (3.10)$$

In the case of a large force density, when the condition (3.5) is reversed, the function $\Phi(x)$ has the form

$$\Phi_1(x) = \frac{2}{\pi x^2}\sqrt{\frac{R_1}{R}}\frac{d}{dx}\int_0^x\frac{t^5dt}{\sqrt{x^2-t^2}}\left[a_1 + a_2t^2 + a_3t^4\right.$$
$$\left. + t^2\sqrt{1-t^2}(a_4 - a_3t^2)\ln\frac{1+\sqrt{1-t^2}}{t}\right]^{-1}, \qquad (3.11)$$

where $a_1 = 0.26$, $a_2 = -0.19$, $a_3 = 0.31$, and $a_4 = 0.09$.

In a plasma with two species of ions characterized by different charge to mass ratios, the variety of distribution functions for acoustic waves is considerably wider because the dynamic separation of charged ions in the field of interacting ion-acoustic waves changes the probability of induced scattering.[21,22] This effect is not essential if

$$\sum_\alpha\frac{r_{D\alpha}^2\omega_{L\alpha}^6}{n_\alpha m_\alpha} \gg \pi\left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right)^2\frac{r_D^2\omega_L^2}{(r_{D1}^2 + r_{D2}^2)^2}\sum_\alpha\omega_{L\alpha}^2 r_{D\alpha}^4, \qquad (3.12)$$

where $r_{D\alpha}$ is the Debye radius for ions labeled by the index α. In this case the ion-acoustic turbulence distribution with respect to the absolute value of the wave vector is still described by Eqs. (3.2) and (3.3). The shape of the angular distribution in the wavevector space is little changed and can be described by Eqs. 3.5–3.11 with slightly different parameters, namely δ and R_1 in Eq. (3.6) are substituted by $\delta_1+\delta_2$ and

$$R_2^{(1)}=\frac{v_s\omega_L^3}{6\pi\omega_{Le}^2}\left(\sum_\alpha \frac{r_{D\alpha}^2}{r_D^2}\frac{\omega_{L\alpha}^6}{\omega_L^6}\frac{1}{n_\alpha m_\alpha}\right)^{-1}, \tag{3.13}$$

respectively, where δ_α characterizes the Cherenkov absorption of waves due to ions labeled by the index α.

The effect of dynamic charge separation, in contrast, considerably changes turbulent oscillations when the condition (3.12) is reversed. In this case the distribution of turbulent noise with respect to the magnitude of the wave vector is described by an equation like Eq. (3.2) with the function $F_1(kr_D)$ replaced by

$$F_2(y)=\frac{1}{\lambda}y^{-4}(1+y^2)^{-1}\left[\ln\frac{1+\sqrt{1+y^2}}{y}-\frac{1}{\sqrt{1+y^2}}-\frac{1}{3(1+y^2)^{3/2}}\right], \tag{3.14}$$

where $\lambda=0.55$. The angular distribution of waves is also modified. In this case the condition for the smallness of the effective force density is defined by modified relations similar to Eqs. (3.5) and (3.6):

$$R\ll R_2^{(2)}(1+\delta_1+\delta_2)^2, \tag{3.15}$$

$$R_2^{(2)}=\frac{\lambda v_s}{6\pi^2}\frac{\omega_L^7}{\omega_{Le}^2}\left(\frac{e_1}{m_1}-\frac{e_2}{m_2}\right)^{-2}\frac{(r_{D1}^2+r_{D2}^2)^2}{(\omega_{L1}^2r_{D1}^4+\omega_{L2}^2r_{D2}^4)}. \tag{3.16}$$

The angular distribution is described by expressions similar to Eqs. (3.7)–(3.10) with δ replaced by $\delta_1+\delta_2$ and R_1 by $2R_2^{(2)}$. If the condition (3.15) is reversed, the function $\Phi(x)$ has the form

$$\Phi_2(x)=\frac{2}{\pi x^2}\sqrt{\frac{R_2^{(2)}}{R}}\frac{d}{dx}\int_0^x\frac{t^5dt}{\sqrt{x^2-t^2}}\times\left[b_1+b_2t^2+b_3t^4+b_4t^2\sqrt{1-t^2}\ln\frac{1+\sqrt{1-t^2}}{t}\right]^{-1}, \tag{3.17}$$

where $b_1=0.51$, $b_2=0.08$, $b_3=-0.33$, and $b_4=-0.92$. Given the above information about the spectrum of ion-acoustic turbulence, we can explicitly express the quasilinear collision operator in Eq. (2.12). After omitting small perturbations of the order of $\omega_s/kv\ll 1$, we obtain in the spherical coordinate system where the polar axis is aligned with the effective force density $\mathbf{R}$

$$\mathrm{St}_{\mathrm{QL}}(\delta f)=\nu_t\frac{v_T^3}{v^3}\left\{\frac{\partial}{\partial\xi}\left[(1-\xi^2)\mathrm{X}(\sqrt{1-\xi^2})\frac{\partial}{\partial\xi}\delta f\right]+\frac{1}{1-\xi^2}\Xi(\sqrt{1-\xi^2})\frac{\partial^2}{\partial\varphi^2}\delta f\right\}, \tag{3.18}$$

where $\xi=\cos\theta$, θ and φ are the angles defining the velocity vector,

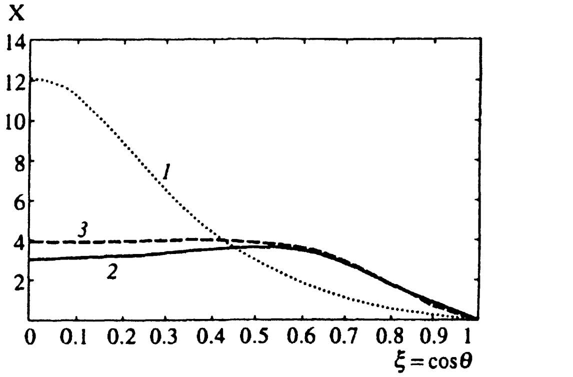

FIG. 1. Electron collision frequency which governs electron flows along the anisotropy axis of the ion-acoustic turbulence as a function of angle. Curve *1* corresponds to the limit of low force density R [Eq. (3.5)], when the distribution in the ion-acoustic turbulence is described by Eqs. (3.7) and (3.22), $A_1=0.02$ [Eq. (3.10)]. Curve *2* corresponds to the limit of large R, when the distribution in the turbulence is described by Eqs. (3.11) and (3.24). Curve *3*, corresponding to Eqs. (3.17) and (3.26), is realized in the limit of large R in plasma with two ion species.

$$\mathrm{X}=\mathrm{X}(1-\xi^2)=\int_0^{\sqrt{1-\xi^2}}dx\,\psi(x)\frac{x^2}{1-\xi^2}\frac{1}{\sqrt{1-\xi^2-x^2}}, \tag{3.19}$$

$$\Xi=\Xi(\sqrt{1-\xi^2})=\frac{1}{1-\xi^2}\int_0^{\sqrt{1-\xi^2}}dx\,\psi(x)\sqrt{1-\xi^2-x^2}. \tag{3.20}$$

The specific forms of the turbulent collision frequency ν_t and the function $\psi(x)$ describing the anisotropy of electron scattering depend on the plasma composition and the value of the effective force density.

For a plasma containing one species of ions and the value of R limited by the condition (3.5), we have

$$\nu_t=\sqrt{9\pi/8}\,R/nmv_s(1+\delta), \tag{3.21}$$

$$\psi(x)=(1+\delta)\Phi(x), \tag{3.22}$$

where $\Phi(x)$ is defined by Eq. (3.7). In the opposite case, when the condition (3.5) is reversed, we have

$$\nu_t=\sqrt{9\pi/8}\sqrt{RR_1}/nmv_s, \tag{3.23}$$

$$\psi(x)=\psi_1(x)=\sqrt{R/R_1}\Phi_1(x). \tag{3.24}$$

The dimensionless collision frequencies X and Ξ corresponding to Eqs. (3.22) and (3.24), respectively, are plotted in Figs. 1 and 2.

In a plasma with two species of ions with approximately equal charge-to-mass ratios [see Eq. (3.12)] ν_t and $\psi(x)$ are determined by Eqs. (3.21–3.24) with δ replaced by $\delta_1+\delta_2$ and R_1 by $R_2^{(1)}$.

If the dynamic separation of ions is strong, i.e., the condition opposite Eq. (3.12) is satisfied, then Eqs. (3.21), (3.22), and (3.7) with δ replaced by $\delta_1+\delta_2$ and R_1 by $R_2^{(2)}$ are valid at small R limited by the condition (3.15). In the case of a large effective force density, when the condition (3.15) is reversed, we obtain the following equations for ν_t and $\psi(x)$ [compare to Eqs. (3.23) and (3.24)]:

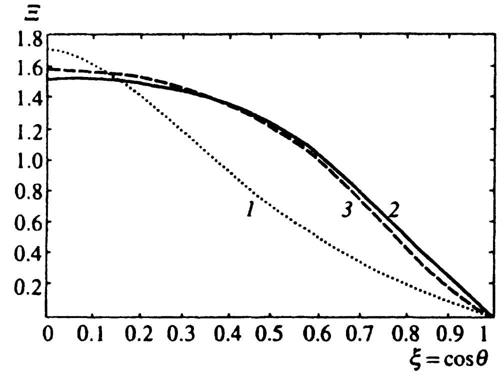

FIG. 2. Effective frequency of electron collisions which determines the electron flow perpendicular to the axis of anisotropy of ion-acoustic turbulence as a function of angle. Curves *1*, *2*, and *3* correspond to the same equations as the respective curves in Fig. 1.

$$\nu_t=\sqrt{9\pi/8}\sqrt{RR_2^{(2)}}/nmv_s, \tag{3.25}$$

$$\psi(x)=\psi_2(x)=\sqrt{R/R_2^{(2)}}\Phi_2(x). \tag{3.26}$$

Figures 1 and 2 show curves of the functions X and Ξ for a plasma with two species of ions with very different charge-to-mass ratios.

4. KINETICS OF SUBTHERMAL ELECTRONS

Let us consider Eq. (2.12). Assume that the tensor of oscillating velocities V_{ij} and perturbations $\delta\varphi$ of the electric potential and δf of the distribution function are proportional to the following function of time and coordinates:

$$\exp(i\mathbf{q}\cdot\mathbf{r}-i\omega t). \tag{4.1}$$

The perturbed distribution function is expressed as a sum of components independent of the frequency ω and collision frequencies, and an additional term δf_c:

$$\delta f=\delta f_c-\frac{e\delta\varphi}{mv_T^2}f_m-\frac{v_E^2}{4v_T^2}f_m+\frac{1}{8}V_{ij}\frac{\partial^2 f_m}{\partial v_i\partial v_j}. \tag{4.2}$$

Since the frequency of electron–electron collisions is small as compared to the frequency of scattering by turbulence, we will ignore the effect of electron–electron collisions on the anisotropic addition to the distribution function. Then using Eqs. (4.1) and (4.2), we derive from Eq. (2.12)

$$\begin{aligned}-i(\omega-\mathbf{q}\cdot\mathbf{v})\delta f_c&-\mathrm{St}(\delta f_c,f_m)-\mathrm{St}(f_m,\delta f_c)-\mathrm{St}_{\mathrm{QL}}(\delta f_c)\\&=-\frac{1}{6}v_E^2v_T^2\nu(v_T)\frac{\partial}{\partial v_i}\left(f_m\frac{v_i}{v^3}\right)+\mathrm{St}_{\mathrm{QL}}\left(\frac{1}{8}V_{ij}\frac{\partial^2 f_m}{\partial v_i\partial v_j}\right)\\&-\frac{1}{4}\left(V_{ij}-\frac{2}{3}\delta_{ij}v_E^2\right)v_T\nu(v_T)\frac{\partial}{\partial v_i}\left(f_m\frac{v_j}{v^3}\right)\\&-\frac{i\omega e\delta\varphi}{mv_T^2}f_m-i\omega\frac{v_E^2}{4v_T^2}f_m+\frac{i\omega}{8}V_{ij}\frac{\partial^2 f_m}{\partial v_i\partial v_j}.\end{aligned} \tag{4.3}$$

Let us consider corollaries of Eq. (4.3) in the case of relatively small perturbations, when

$$\omega\ll\nu_t(v)=\nu_t v_T^3/v^3. \tag{4.4}$$

We will concentrate our attention on the kinetics of slow electrons whose mean free path in turbulent plasma is smaller than the typical scale of the low-frequency perturbations:

$$qv\ll\nu_t(v). \tag{4.5}$$

The function δf_c for slow electrons can be expressed as a sum of the dominant, isotropic component

$$\delta f_0=\int\frac{d\Omega}{4\pi}\delta f_c, \tag{4.6}$$

where $d\Omega$ is a solid angle of the velocity vector, and a smaller, anisotropic term $\delta f_a=\delta f_c-\delta f_0$. In what follows we need only δf_-, the part of δf_a which is odd with respect to the velocity vector. Taking into account the smallness of the electron–electron collision frequency $\nu_{ee}(v)$ as compared to the frequency $\nu_t(v)$ of scattering by the turbulence in the case of low-frequency [Eq. (4.4)], long-wave [Eq. (4.5)] perturbations and using the explicit expression for the quasilinear collision operator given by Eq. (3.18), we derive from Eq. (4.3)

$$\begin{aligned}&[q_z\xi+q_\perp\sqrt{1-\xi^2}\cos(\varphi-\varphi_q)]iv\delta f_0/\nu_t(v)\\&=\frac{\partial}{\partial\xi}\left[(1-\xi^2)\mathrm{X}(\sqrt{1-\xi^2})\frac{\partial}{\partial\xi}\delta f_-\right]\\&+\frac{1}{1-\xi^2}\Xi(\sqrt{1-\xi^2})\frac{\partial^2}{\partial\varphi^2}\delta f_-,\end{aligned} \tag{4.7}$$

where φ_q is the azimuthal angle of the vector $\mathbf{q}$, $q^2=q_\perp^2+q_z^2$ $q_z=\mathbf{q}\cdot\mathbf{n}$, $\mathbf{n}=\mathbf{R}/R$ is the unit vector aligned with the vector of the effective force density.

The equation for the function δf_0, in its turn, is derived by averaging Eq. (4.3) over the angles in velocity space. Using Eqs. (4.4) and (4.5) and the smallness of the function δf_a in comparison with δf_0, we obtain the following equation for δf_0:

$$\begin{aligned}&-i\omega\delta f_0+i\int\frac{d\Omega}{4\pi}\mathbf{q}\cdot\mathbf{v}\delta f_--\frac{4\pi}{3n}v_T^3\nu_{ee}(v_T)\\&\times\left(\frac{1}{v^2}\frac{\partial}{\partial v}\right)\left(v^3\int_v^\infty dv'v'+\int_0^v dv'v'^4\right)\left(\frac{1}{v}\frac{\partial}{\partial v}\right.\\&\left.-\frac{1}{v'}\frac{\partial}{\partial v'}\right)[f_m(v')\delta f_0(v)+f_m(v)\delta f_0(v')]\\&=-\frac{i\omega}{mv_T^2}e\delta\varphi f_m(v)-\frac{v_E^2}{6vv_T}\nu(v_T)f_m(v)\\&\times\left[\delta\left(\frac{v^2}{2v_T^2}\right)-1\right].\end{aligned} \tag{4.8}$$

In order to analyze this equation, we have to calculate the function δf_- using Eq. (4.7). A solution of the partial differential equation (4.7) is sought in the form of the sum of two independent functions:

$$\delta f_-=-i\frac{q_zv}{\nu_t(v)}\delta f_0g_1(\xi)-i\frac{q_\perp v}{\nu_t(v)}\delta f_0g_2(\xi)\cos(\varphi-\varphi_q). \tag{4.9}$$

We obtain an ordinary second-order differential equation system for the functions g_1 and g_2:

$$\frac{d}{d\xi}\left[(1-\xi^2)\mathrm{X}(\sqrt{1-\xi^2})\frac{d}{d\xi}g_1(\xi)\right]=-\xi, \tag{4.10}$$

$$\frac{d}{d\xi}\left[(1-\xi^2)\mathrm{X}(\sqrt{1-\xi^2})\frac{d}{d\xi}g_2(\xi)\right]-\frac{g_2(\xi)}{1-\xi^2}\Xi(\sqrt{1-\xi^2})$$
$$=-\sqrt{1-\xi^2}. \tag{4.11}$$

For analysis of Eq. (4.8) only the integrals

$$\beta_\parallel=2\int_{-1}^{1}d\xi\,\xi g_1(\xi), \tag{4.12}$$

$$\beta_\perp=\int_{-1}^{1}d\xi\sqrt{1-\xi^2}g_2(\xi), \tag{4.13}$$

of the functions g_1 and g_2 are needed. Let us calculate the parameters $\beta_\parallel$ and $\beta_\perp$ in the most interesting cases. First we calculate $\beta_\parallel$. Using the condition of regularity of the derivative of $g_1(\xi)$ at $\xi=\pm1$, we have from Eq. (4.10) the following equation:

$$\frac{d}{d\xi}g_1(\xi)=1/2\mathrm{X}(\sqrt{1-\xi^2}). \tag{4.14}$$

After integrating the expression on the right of Eq. (4.12) by parts with due account of Eqs. (4.14) and (3.19), we obtain

$$\beta_\parallel=\int_0^1 d\xi(1-\xi^2)\left[\int_0^1 dy\frac{y^2}{\sqrt{1-y^2}}\psi(y\sqrt{1-\xi^2})\right]^{-1}. \tag{4.15}$$

The numerical value of $\beta_\parallel$ depends on the angular distribution of the ion-acoustic turbulence in the space of wave vectors $\psi(x)$. If the distribution is given by Eqs. (3.7) and (3.22), for $a\ll1$ and $\varepsilon\ll1$ we have $\beta_\parallel=0.18$. The values of $\beta_\parallel$ for the distributions defined by Eqs. (3.11), (3.24) and Eqs. (3.17), (3.26) are fairly close: $\beta_\parallel^{(1)}=0.25$ and $\beta_\parallel^{(2)}=0.23$. In calculating $\beta_\perp$ we need a solution to Eq. (4.11) with the boundary conditions

$$g_2(\xi=-1)=g_2(\xi=1)=0. \tag{4.16}$$

The numerical integration of Eq. (4.11) under the condition (4.16) with a view to calculating $\beta_\perp$ using Eq. (4.13) yields $\beta_\perp=0.02$ if the turbulent noise distribution is defined by Eqs. (3.7) and (3.22) for $a\ll1$ and $\varepsilon\ll1$; $\beta_\perp^{(1)}=0.80$ and $\beta_\perp^{(2)}=0.85$ hold for the distributions defined by Eqs. (3.11), (3.22) and Eqs. (3.17), (3.26), respectively.

Let us use the solution in the form given by Eq. (4.9) and Eqs. (4.12), (4.13) to transform Eq. (4.8). After introducing the new function $F(x)=F(v^2/2v_T^2)$ and using the definition

$$\delta f_0(v)=(Z_{\mathrm{eff}}/2N_t)f_m(v)F(v^2/2v_T^2), \tag{4.17}$$

where the parameter N_t is expressed as

$$N_t=\frac{3}{2}\sqrt{\pi}\frac{v_T^2}{\nu_t\nu_{ee}(v_T)}(\beta_\parallel q_z^2+\beta_\perp q_\perp^2), \tag{4.18}$$

we derive from Eq. (4.8) the equation

$$\frac{1}{N_t}\left\{F''(x)\frac{3}{2}\int_0^x dy\sqrt{y}e^{-y}+\sqrt{x}\left(\frac{3}{2}-x\right)\int_x^\infty dye^{-y}[F'(x)\right.$$
$$\left.-F'(y)]-\int_0^x dy\,y^{3/2}e^{-y}[F'(x)-F'(y)]\right\}-x^3F(x)$$
$$=I\sqrt{\pi}[\delta(x)-1]+i\omega e\delta\varphi\sqrt{x}/\nu_{ei}mv_T^2. \tag{4.19}$$

In this equation we have used the notation $I=v_E^2/4v_T^2$ and $\nu_{ei}=\nu(v_T)\sqrt{2}/3\sqrt{\pi}$ and omitted the term $i\omega\delta f_0$, which was present in Eq. (4.8). Since for our further analysis only electrons with velocities $v\sim v_TN_t^{-1/7}\ll v_T$ are essential, the term $i\omega\delta f_0$ can be omitted if the following condition on the perturbation frequency is satisfied:

$$\omega\ll\nu_{ei}N_t^{2/7}/Z_{\mathrm{eff}}.$$

Equation (4.19) differs from the similar equation in the kinetic theory of laminar plasma[10,11] in the parameter N, which is expressed in the laminar model as

$$N=\frac{4}{9\sqrt{\pi}}Z_{\mathrm{eff}}q^2l_{ei}^2, \tag{4.20}$$

where $l_{ei}=v_T/\nu_{ei}$, and this parameter is replaced in our model by a smaller quantity

$$N_t=\frac{9\sqrt{\pi}}{4\sqrt{2}}(\beta_\parallel\cos^2\theta_q+\beta_\perp\sin^2\theta_q)\left(\frac{l_t}{l_{ei}}\right)N\ll N. \tag{4.21}$$

Here $l_t=v_T/\nu_t$ is the mean free path of a thermal electron due to scattering by turbulence, θ_q is the angle between the vectors $\mathbf{q}$ and $\mathbf{R}$.

Following the technique proposed in Refs. 10 and 11, a solution to Eq. (4.19) asymptotically exact at $N_t\gg1$ can be expressed as

$$F(x)=IF_0(x)+i\frac{\omega\,e\delta\varphi}{\nu_{ei}mv_T^2}F_{1/2}(x), \tag{4.22}$$

$$F_0(x)=-\frac{4}{\sqrt{\pi}}\left(\frac{2}{7}\right)^{1/7}\Gamma\left(\frac{6}{7}\right)\sin\frac{\pi}{7}N_t^{15/14}x^{-1/4}K_{1/7}$$
$$\times\left(\frac{4}{7}\sqrt{N_t}x^{7/4}\right)+N_t^{6/7}\Psi(xN_t^{2/7},[\sqrt{\pi}]), \tag{4.23}$$

$$F_{1/2}(x)=-N_t^{5/7}\Psi(xN_t^{2/7},[\sqrt{\zeta}]), \tag{4.24}$$

where $\Gamma(z)$ is the gamma-function, $K_{1/7}(z)$ and $I_{1/7}(z)$ are modified Bessel functions, and

$$\Psi(z,[\rho(\zeta)])=\frac{4}{7}z^{-1/4}\left\{I_{1/7}\left(\frac{4}{7}z^{7/4}\right)\right.$$
$$\times\int_z^\infty d\zeta\,\zeta^{-1/4}\rho(\zeta)K_{1/7}\left(\frac{4}{7}\zeta^{7/4}\right)$$
$$\left.+K_{1/7}\left(\frac{4}{7}z^{7/4}\right)\int_0^z d\zeta\zeta^{-1/4}\rho(\zeta)I_{1/7}\left(\frac{4}{7}\zeta^{7/4}\right)\right\}. \tag{4.25}$$

In subsequent sections, the correction to the distribution function due to collisions given by Eqs. (4.17), (4.22)–(4.25) is used in calculating perturbations of the electron density and temperature caused by either inverse bremsstrahlung absorption or perturbation of the electric potential. Perturbations of the density and temperature are determined by relatively slow subthermal electrons with velocities

$$v \sim v_T N_t^{-1/7} \ll v_T. \tag{4.26}$$

The condition of small anisotropy of the perturbed distribution function [see Eqs. (4.5) and (4.9)] takes the form $ql_t \gg (l_t/l_{ee})^4$, where $l_{ee} \sim Z_{\rm eff} l_{ei}$ is the electron free path due to electron–electron collisions. At the same time, the distribution defined by Eqs. (4.22–4.25) occurs for $N_t \gg 1$, when

$$ql_t \gg \sqrt{l_t/l_{ee}}. \tag{4.27}$$

Since $l_{ee} \gg l_t$ holds in turbulent plasma, the condition of the weak anisotropy of subthermal electron distribution defined by Eq. (4.26) is satisfied automatically if Eq. (4.27) holds. Thus our model applies if the mean free path of thermal electrons is sufficiently long:

$$l_{ee} \gg \sqrt{l_{ee} l_t} \gg l, \tag{4.28}$$

where $L \sim 1/q$ is the typical length scale of perturbations. According to Eq. (4.28), we have $l_{ee} \gg L$, whereas the mean free path due to electron scattering by turbulent oscillations of charge density can be either larger, $l_t > L$, or smaller, $l_t < L$, than the typical density variation because L_t is smaller than l_{ee} if the condition (4.28) is satisfied. But even in the case $l_{ee} \gg l_t \gg L$, i.e., when thermal electrons propagate without collisions, the subthermal electrons, which are essential in our model, have an effective mean free path smaller than the scale of the density variation owing to the relation $l_t(v) \simeq (v/v_T)^4 l_t$ between the mean free path and velocity typical of the Coulomb scattering:

$$l_t(v_T N_t^{-1/7}) \sim \left(\frac{l_t}{l_{ee}}\right)^{1/2} \left(\frac{l}{\sqrt{l_{ee} l_t}}\right)^{1/7} L \ll L. \tag{4.29}$$

In connection with the above discussion, we should recall that, although the perturbations of the charge density and temperature are controlled by subthermal colliding electrons with a short mean free path given by Eq. (4.29), heat is conducted by thermal electrons with $v \sim v_T$ and longer mean free path [Eq. (4.28)] (Ref. 12).

5. NONLOCAL THERMAL CONDUCTIVITY RELATED TO INVERSE BREMSSTRAHLUNG ABSORPTION

The perturbation of the distribution function due to collisions given by Eqs. (4.17) and (4.22) contains two independent terms caused by the inverse bremsstrahlung absorption and perturbation of the electric potential. In this section we consider the case when the perturbation of the distribution function is entirely due to inverse bremsstrahlung absorption. Let us calculate the electron density perturbation using the additional term of the distribution function determined by Eqs. (4.2) and (4.17). After retaining only the terms proportional to I in these equations, we obtain

$$\delta n(\mathbf{q}) = \int d\mathbf{v}\, \delta f = -In + \delta n_c(\mathbf{q}), \tag{5.1}$$

$$\delta n_c(\mathbf{q}) = -\beta_0 n I Z_{\rm eff} N_t^{-2/7}, \tag{5.2}$$

where β_0 is a numerical factor of order unity:

$$\beta_0 = \left(\frac{1024}{343}\right)^{1/7} \Gamma\left(\frac{2}{7}\right) \Gamma\left(\frac{3}{7}\right) \Big/ \Gamma\left(\frac{1}{7}\right) = 1.16. \tag{5.3}$$

Taking the term in δf_c proportional to I [Eq. (4.22)], one can calculate the perturbation of the electronic pressure due to absorption:

$$\delta p_c = \int d\mathbf{v}\, m v^2 \delta f_c / 3. \tag{5.4}$$

Here $\delta p_c \ll m v_T^2 \delta n_c$ holds as long as $N_t^{-2/7} \ll 1$. Since we have $p = nk_B T$, where k_B is Boltzmann's constant, and $\delta p_c / p$ is smaller than $\delta n_c / n$, we can write the expression

$$\frac{\delta T_c(\mathbf{q})}{T} = -\frac{\delta n_c(\mathbf{q})}{n} = \beta_0 I Z_{\rm eff} N_t^{-2/7}, \tag{5.5}$$

which determines the electron temperature perturbation due to electron–ion collisions. The Fourier components of the thermal flux and temperature perturbations are related by the equation

$$\mathbf{Q}(\mathbf{q}) = -i\mathbf{q}\, \kappa(\mathbf{q})\, \delta T_c(\mathbf{q}), \tag{5.6}$$

where $\kappa(\mathbf{q})$ is the Fourier transform of the effective thermal conductivity. On the other hand, in steady state we derive from Eq. (4.8) the following relation:

$$i\mathbf{q}\cdot\mathbf{Q}(\mathbf{q}) = 2I \nu_{ei} n \kappa_B T, \tag{5.7}$$

which indicates that the electron heating due to absorption in collisions is balanced by cooling due to electron heat transport. Using Eqs. (5.5–5.7), we calculate the effective thermal conductivity in the strongly nonlocal limit, when $N_t \gg 1$ holds:

$$\kappa(\mathbf{q}) = \kappa_{sh} \frac{\sqrt{\pi}}{48\beta_0} \left(\frac{N_t}{N}\right)^{2/7} N^{-5/7} \ll \kappa_{sh}, \tag{5.8}$$

where $\kappa_{sh} = (128/3\pi) n k_B v_T l_{ei}$ is the electronic thermal conductivity of fully ionized plasma with highly ionized atoms, $Z_{\rm eff} \gg 1$. Unlike the effective nonlocal thermal conductivity of laminar plasma, Eq. (5.8) contains the additional small factor $(N_t/N)^{2/7} \sim (l_t/l_{ei})^{2/7} \ll 1$. This means that ion-acoustic turbulence leads to a further decrease in the nonlocal electron thermal conductivity.

Compare the nonlocal thermal conductivity defined by Eq. (5.8) to the electron thermal conductivity of turbulent plasma in the local limit. Given a certain spectrum of ion-acoustic turbulence, when the collision operator has the form defined by Eq. (3.18), the Fourier component $Q_\parallel(\mathbf{q})$ of the thermal flux along the vector $\mathbf{q}$ is related to the Fourier component of a large-scale temperature perturbation by the local relation

$$Q_\parallel(\mathbf{q}) = (\mathbf{q}\cdot\mathbf{Q}(\mathbf{q}))/q = -iq\kappa_t(\theta_q)\,\delta T(\mathbf{q}), \tag{5.9}$$

where the thermal conductivity $\kappa_t(\theta_q)$ is a function of the angle between the vector $\mathbf{q}$ and the anisotropy axis of the ion-acoustic turbulence $\mathbf{n} = \mathbf{R}/R$:

$$\kappa_t(\theta_q)=\frac{96}{\sqrt{2\pi}}n\kappa_B v_T l_t(\beta_{\|}\cos^2\theta_q+\beta_\perp\sin^2\theta_q). \quad (5.10)$$

Note that directions of the thermal flux and gradient of the temperature perturbation may be different in plasma with ion-acoustic turbulence because the thermal conductivities in directions parallel and perpendicular to the turbulence anisotropy axis, $\kappa_{\|}=\kappa_t(\theta_q=0)$ and $\kappa_\perp=\kappa_t(\theta_q=\pi/2)$, are different owing to the electron scattering anisotropy.

The expressions for $\kappa(\mathbf{q})$ [Eq. (5.8)], $\kappa_t(\theta_q)$ [Eq. (5.10)], and κ_{sh} yield a unified interpolation formula for the effective thermal conductivity in the form

$$\kappa_{\text{eff}}^{(1)}(\mathbf{q})=\kappa_{sh}\{1+2\sqrt{\pi}A_t(\theta_q)l_{ei}/l_t+96\beta_0A_t^{2/7}(\theta_q)\times(l_{ei}/l_t)^{2/7}(q^2l_{ee}l_{ei})^{5/7}\}^{-1}, \quad (5.11)$$

where $l_{ee}=(2Z_{\text{eff}}/9\pi)l_{ei}$ is the effective electron free path due to electron–electron collisions, and the factor $A_t(\theta_q)$ is a function of the angle θ_q defined by the formula

$$A_t(\theta_q)=\frac{\sqrt{8}}{9\pi}(\beta_{\|}\cos^2\theta_q+\beta_\perp\sin^2\theta_q)^{-1}. \quad (5.12)$$

Equation (5.11) describes a transition to the case of laminar plasma, when the effective thermal conductivity is given by the formula[10,11]

$$\kappa_L(q)=\kappa_{sh}[1+(21q\sqrt{l_{ee}l_{ei}})^{10/7}]^{-1}. \quad (5.13)$$

At the same time, in the turbulent state we have $l_t\ll l_{ei}$ and electron–ion collisions may be ignored. In this case Eq. (5.11) takes the form

$$\kappa_{\text{eff}}^{(1)}(\mathbf{q})=\kappa_t(\theta_q)\left[1+\frac{48}{\sqrt{\pi}}\beta_0N_t^{5/7}\right]^{-1}\equiv\kappa_t(\theta_q)\left[1+\frac{48}{\sqrt{\pi}}\beta_0A_t^{-5/7}(\theta_q)(q^2l_tl_{ee})^{5/7}\right]^{-1}. \quad (5.14)$$

According to Eq. (5.14), the effective thermal conductivity is reduced by the nonlocal effects in turbulent plasma for $N_t\gg1$. Since we have $N_t\ll N$, the nonlocal effects in plasma with developed ion-acoustic turbulence become essential for thermal conductivity at temperature perturbations lower than in laminar plasma.

6. SOUND DAMPING AND RELATED THERMAL CONDUCTIVITY

In this section we consider the results of the theory related to low-frequency perturbations of the distribution function driven by the electric potential $\delta\varphi$. Using the collisionless corrections to the Maxwellian distribution function [Eq. (4.2)] and those due to collisions [Eqs. (4.17), (4.22), and (4.24)], either being proportional to $\delta\varphi$, we find the corresponding perturbation of the electron density. In accordance with the definition in Eq. (5.1) and Eqs. (4.2), (4.17), (4.22), (4.24), and (4.25), we obtain

$$\delta n(\mathbf{q})=-n\frac{e\delta\varphi}{mv_T^2}\left[1+i\frac{\omega}{\nu_{ei}}\beta_{1/2}Z_{\text{eff}}N_t^{-5/7}\right], \quad (6.1)$$

where $\beta_{1/2}$ is the numerical factor,

$$\beta_{1/2}=-N_t^{-2/7}\int_0^\infty dx\sqrt{x}e^{-x}F_{1/2}(x)=0.82. \quad (6.2)$$

Using the electron density perturbation, we express the electronic contribution to the longitudinal low-frequency dielectric function of turbulent plasma as

$$\delta\varepsilon(\omega,\mathbf{q})=\frac{1}{q^2r_D^2}\left[1+\frac{i\omega}{qv_T}\left(\sqrt{\frac{\pi}{2}}+\frac{9\pi}{2}\beta_{1/2}A_t^{5/7}(\theta_q)\frac{(l_{ee}/l_t)^{2/7}}{(ql_t)^{3/7}}\right)\right]. \quad (6.3)$$

On the right-hand side of Eq. (6.3) we have added the term $\sqrt{\pi/2}$ responsible for Landau collisionless damping. Equation (6.3) holds for $\omega\ll qv_T$ and small perturbation wavelengths:

$$ql_t\gg1, \quad (6.4)$$

when the conditions $N_t\gg1$ or $q^2l_{ee}l_t\gg A_t(\theta_q)$ are satisfied automatically. Using Eq. (6.3), we derive from the dispersion equation for longitudinal waves the sound damping rate

$$\gamma=\omega_s\frac{\omega_L}{\omega_{Le}}\left(\frac{\omega_s}{qv_s}\right)^3\left\{\sqrt{\frac{\pi}{8}}+\frac{9\pi}{4}\beta_{1/2}A_t^{5/7}(\theta_q)\frac{(l_{ee}/l_t)^{2/7}}{(ql_t)^{3/7}}\right\}. \quad (6.5)$$

The contribution to the damping rate due to collisions is larger than that of the Landau damping if the perturbation scale is sufficiently large:

$$ql_t\ll\left[\frac{81\pi}{2}\beta_{1/2}^2\right]^{7/6}A_t^{5/3}(\theta_q)(l_{ee}/l_t)^{2/3}. \quad (6.6)$$

Since we have $l_{ee}\gg l_t$, the combination of conditions (6.4) and (6.6) defines a fairly wide range of wavelengths in which the sound damping is controlled by collisions. As concerns Eq. (6.5), which is applied to attenuation of ion-acoustic waves with relatively large wavelengths limited by Eq. (6.6) in plasma with ion-acoustic turbulence, let us recall that the turbulent collision frequency ν_t, which controls the effect, is due to electron scattering by ion-acoustic oscillations of charge density with wavelengths comparable to the electron Debye radius. Note also that in the short-wave range defined by Eq. (6.4) the effect of ion–ion collisions on sound damping, including plasmas with two species of ions,[23] is weakened if wavelengths of charge density perturbations are smaller than the effective ion free path.

If multimoment hydrodynamics is applied,[24–26] sound damping is determined by the Fourier component of electronic thermal conductivity, $\kappa(\mathbf{q})$, which characterizes the ratio between Fourier components of thermal flux and temperature (Eq. (5.6)). In this case the electronic contribution to the longitudinal dielectric function has the form[14]

$$\delta\varepsilon(\omega,\mathbf{q})=\frac{1}{q^2r_D^2}\left[1+i\frac{\omega}{qv_T}\frac{n\kappa_Bv_T}{q\kappa(\mathbf{q})}\right]. \quad (6.7)$$

By comparing Eqs. (6.3) and (6.7) we derive the effective thermal conductivity. With a view to describing the transition to the thermal conductivity of laminar plasma κ_{sh} and of turbulent plasma in the local limit, $\kappa_t(\theta_q)$ [Eq. (5.10)], we can write the following interpolation formula for the wavelength range defined by Eq. (6.6):

$$\kappa_{\text{eff}}^{(2)}(\mathbf{q})=\kappa_{sh}\{1+2\sqrt{\pi}A_t(\theta_q)l_{ei}/l_t+192\beta_{1/2}A_t^{5/7}(\theta_q)\times(l_{ei}/l_t)^{5/7}(q^2l_{ee}l_{ei})^{2/7}\}^{-1}. \quad (6.8)$$

In plasma with developed ion-acoustic turbulence, when $L_{ei}\gg L_t$ holds we derive from Eq. (6.8)

$$\kappa_{\text{eff}}^{(2)}(\mathbf{q})=\kappa_t(\theta_q)\left[1+\frac{96}{\sqrt{\pi}}\beta_{1/2}A_t^{-2/7}(\theta_q)(q^2l_{ee}l_t)^{2/7}\right]^{-1}. \quad (6.9)$$

According to Eqs. (6.8) and (6.9), the thermal conductivity of turbulent plasma in the strongly nonlocal limit $q^2l_{ee}l_t\gg 1$ is considerably lower than that of both laminar and turbulent plasmas in the local limit. Note also that, since we have $l_{ei}\gg l_t$, the inhibition of nonlocal thermal conductivity in turbulent plasma is a factor of $(l_{ei}/l_t)^{5/7}\gg 1$ stronger than in laminar plasma, but this inhibition is important for the perturbation scales a factor of $\sqrt{l_{ei}/l_t}\gg 1$ lower than for laminar plasma.

7. CONCLUSION

The proposed model is based on several assumptions about its physical nature. We have assumed that electron scattering by low-frequency turbulent oscillations is inhibited when electrons are driven by an electromagnetic field at a frequency higher than the Langmuir frequency. This means that the effect of ion-acoustic oscillations on the inverse bremsstrahlung absorption due to Coulomb interaction between electrons and ions can be neglected. On the other hand, if the electron distribution changes slowly, the intensity of electron scattering by turbulent oscillations is notably higher than by ions. Therefore, the electron mean free path l_t with respect to scattering by turbulent oscillations is much less than the free path l_{ei} with respect to collisions with ions. It follows from our analysis that the electron mean free paths $l_t(v)$, $l_{ee}(v)$, and $l_{ei}(v)$ as functions of velocity are similar because electron scattering both by particles and by ion-acoustic turbulence is due to the Coulomb interaction. One manifestation of this similarity is that the scaling of Fourier components of the effective thermal conductivity[27] given in this paper is similar to that in the theory of laminar weakly collisional plasma.[10,12,14,15] In contrast, the smallness of the mean free path l_t leads to a considerable decrease in the effective thermal conductivity of turbulent plasma as compared to the model of laminar plasma. The anomalous inhibition of the effective electron thermal conductivity demonstrated in this paper is essentially anisotropic, unlike the prediction by the model of weakly collisional laminar plasma, which is fully controlled by the anisotropy of the distribution of turbulent ion-acoustic oscillations. Using analytical results of the ion-acoustic turbulence theory, we could not only detect the new phenomenon of inhibition of nonlocal electron thermal conductivity, but also calculate quantitative characteristics of this effect.

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