

Non-one-dimensional dynamics of vertical Bloch lines in domain walls of ferrite garnets

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Profiles of the solitary waves that accompany moving vertical Bloch lines in domain walls in ferrite garnets have been numerically calculated using the two-dimensional Slonczewski equations. In the in-plane magnetic field (i.e., the field in the plane of the sample), which is of the order of the demagnetization field and perpendicular to the domain wall plane, vertical Bloch lines are stationary. In weak planar fields, the dynamics of vertical Bloch lines is significantly affected by the motion of horizontal Bloch lines. This leads to decay of a vertical Bloch line with a topological charge of $N\pi$ to a packet of N separate lines with charges of π (π -lines). The velocity of separate π -lines of the packet is approximately equal to that of a solitary vertical Bloch line with charge π in one dimension. The velocity of the packet as a whole is lower than that of a vertical Bloch line with charge $N\pi$ in one dimension because of the continual annihilation of leading lines and creation of trailing lines in the packet. Experiments in in-plane magnetic fields have confirmed the calculations. Stable solitary waves with narrow leading fronts, which accompany vertical Bloch lines, are generated at low domain-wall velocities in the absence of in-plane magnetic fields, and at high domain-wall velocities when they are present. This fact allowed the success of earlier measurements of dynamics and soliton-like behavior of vertical Bloch lines in collisions. In the presence of a sufficiently high in-plane magnetic field, one can use the one-dimensional Slonczewski equations to interpret measurements of motion and collisions of vertical Bloch lines in an in-plane magnetic field. © 1996 American Institute of Physics. [S1063-7761(96)02211-1]

1. INTRODUCTION

Investigations of the dynamical properties of vertical and horizontal Bloch lines and Bloch points have aroused the interest of both experimenters and theorists. These elements of domain walls in ferrite garnets are stable topological formations^{1,2} that can be detected, as will be demonstrated below, through changes in the shape of the solitary flexural waves that accompany vertical Bloch lines on mobile domain walls. Solitary flexural waves on domain walls related to vertical Bloch lines have thus far been the only experimental means of detecting vertical lines in real time.² The technique of anisotropic dark-field light diffraction allows one to detect static vertical Bloch lines with the topological charge π (π -lines).³

In previous time-resolved measurements of dynamics, pair collisions, and soliton-like properties of vertical Bloch lines, neither the presence nor the effect of horizontal Bloch lines on these processes was detected. All the data of the experiments cited above were interpreted in terms of the one-dimensional Slonczewski equations without taking account of the twisting of domain walls.⁴⁻⁷

Numerical calculations of the dynamics of vertical Bloch lines based on the three-dimensional Landau–Lifshitz equations for relatively thin ferrite-garnet films were attempted by Rejda and Humphrey.⁸ They obtained spatial distributions of the azimuthal magnetization angle in a moving vertical Bloch line, but did not publish calculations of the profiles of solitary flexural waves that accompany vertical Bloch lines. It is interesting, therefore, to calculate profiles of these waves numerically using the two-dimensional Slonczewski

equations. Such calculations have thus far not been published, and they are reported in this paper. Given these calculations, we can carry out an experiment to study the effect of the motion of horizontal Bloch lines on profiles of solitary flexural waves on a domain wall, which accompany moving vertical Bloch lines, to examine the effect of external in-plane magnetic fields on their stability, and to explain why numerical solutions of the spatial one-dimensional Slonczewski equations previously used to analyze the dynamics and collisions of vertical Bloch lines in domain walls of ferrite garnets apply to our case.

2. NUMERICAL CALCULATION OF THE DYNAMICS OF VERTICAL BLOCH LINES BASED ON THE TWO-DIMENSIONAL SLONCZEWSKI EQUATIONS

We have numerically calculated the motion of vertical Bloch lines in domain walls of garnets. To this end, we have used the two-dimensional Slonczewski equations¹:

$$\begin{aligned} -\dot{q} + \alpha\dot{\varphi} &= \Delta\varphi - \frac{1}{2} \sin 2\varphi + v_0 - H_x \sin \varphi \\ &+ (H_y + H_d) \cos \varphi, \\ \dot{\varphi} + \alpha\dot{q} &= \Delta q - b^2 q, \end{aligned} \quad (1)$$

where the functions q and φ describe the profile of the domain wall flexure normalized to the wall thickness $\Delta_0 = \sqrt{A/K}$ (A is the bulk stiffness constant and K is the constant of uniaxial anisotropy) and the azimuthal rotation angle of the magnetic moment; the x -axis is aligned with the

domain wall, and the y - and z -axes are perpendicular to the domain wall and film, respectively; H_x and H_y are external fields normalized to $8M_s$, where M_s is the saturation magnetization; α is the damping factor; $\Delta = \partial_x^2 + \partial_z^2$; the coordinates x and z are normalized to $\Lambda_0 = \sqrt{A/2\pi M_s^2}$, and the time to $(4\pi\gamma M_s)^{-1}$; $b^2 = \Delta_0 H_z^2 / 4\pi M_s$; H_z^i is the magnetic field gradient stabilizing the domain wall; $v_0 = H_z^i / 4\pi\alpha M_s$ is the normalized magnetic field perpendicular to the film surface, and equal to the domain wall velocity (normalized to $4\pi M_s \gamma \Delta_0$) in the case $\varphi(x, y, z) = \text{const}$; the velocity along the domain wall is normalized to $4\pi M_s \gamma \Delta_0$; H_d is the demagnetization field¹:

$$H_d = \frac{1}{4} \ln \frac{z + 1/2\sqrt{Q}}{h - z + 1/2\sqrt{Q}}, \quad (2)$$

where $Q = (\Lambda_0/\Delta_0)^2$ is the quality-factor and h is the film thickness.

Calculations were performed for parameter values $\alpha = 0.4$, $b^2 = 10^{-4}$, $h = 35\Lambda_0$, and $H_z = 0.05$, which are in good agreement with real experimental conditions. The dimensions in all the graphs of this section are measured in micrometers, and the curves are plotted for the following values of parameters: $\Delta_0 = 0.03 \mu\text{m}$, $Q = 45$, $4\pi M_s = 100 \text{ Oe}$, and $\gamma = 1.8 \times 10^7 (\text{Oe} \cdot \text{s})^{-1}$.

The calculations were performed in two steps. In the first step, Eq. (1) was solved at $v_0 = 0$ with the initial conditions

$$q(x, z, 0) = 0, \quad \varphi(x, z, 0) = \sum_{i=1}^N 2 \arctan \left[\exp \left(\frac{x - x_i - i\delta}{\Lambda_0} \right) \right], \quad (3)$$

where $\delta \approx \Lambda_0$. In a time of the order of some hundred $1/4\pi M_s \gamma$, the functions $q(x, z, t)$ and $\varphi(x, z, t)$ converged to functions which were used as initial conditions in subsequent calculations.

We found that the behavior of a vertical Bloch line at $H_y = 0$ is strongly affected by the field H_z driving the wall. At $v_0 > 0.138$, horizontal Bloch lines were generated in the wall. The value $v_0 = 0.138$ is in good agreement with the formula for the critical velocity¹

$$v_c = (9.5\Lambda_0/h)v_w,$$

where $v_w = 0.5$ is the Walker velocity. Horizontal Bloch lines damp the wall propagation, so the wall velocity v is smaller than v_0 . The wall velocity v is plotted in Fig. 1 as a function of v_0 . The critical velocity v_c also increases with the external field H_y perpendicular to the domain wall. At a high field $H_y = 1$, horizontal Bloch lines are not generated, and the motion of the vertical line is stable until its velocity is close to the critical value.

At $v_0 < v_c$ the motion of a vertical Bloch line driven by the gyroforce is similar to the one-dimensional case. Figure 2 shows the solution of Eq. (1) for $v_0 = 0.0926$ and $N = 4$ at time $t = 480 \text{ ns}$. The vertical Bloch line moves from left to right. The wall flexure has an extremely narrow leading front at which the entire topological charge of the vertical Bloch

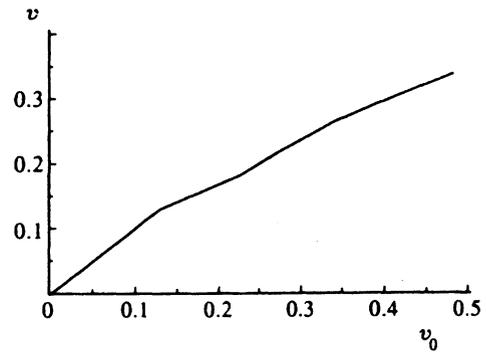


FIG. 1. Domain wall velocity v in a ferrite garnet film versus external driving field $H_z/4\pi\alpha M_s = v_0$ at $\alpha = 0.4$ and film thickness $h = 35\Lambda_0$.

line is concentrated. Similar solutions are obtained at large domain wall velocities in a strong external magnetic field H_y .

At $v_0 > v_c$ the domain wall always contains one or more horizontal Bloch lines, which leads to a decay of the vertical Bloch line into individual π -components. Figures 3–5 illustrate this decay at $N = 8$ and $v_0 = 0.185$. The motion of the horizontal Bloch line leads to annihilation of the leading vertical Bloch line and simultaneous creation of a π -line behind the packet. The total charge of the vertical Bloch line is conserved. After this decay, we have a packet of N individual vertical Bloch lines with charge π , as shown in Fig. 5. The separation between neighboring π -lines is close to $10 \mu\text{m}$, which is much greater than the value obtained for a vertical Bloch line with topological charge of 2π in Ref. 8 (albeit the time interval and film thickness, were considerably smaller than in our model). The total width of the central part of the vertical Bloch line packet is about $100 \mu\text{m}$. The saw-tooth shape of the packet is due to $H_d(z)$ being an odd function. The total decay time increases with N and decreases with the velocity v_0 . For $N = 8$ and $v_0 = 0.185$ the total decay time is $5 \mu\text{s}$, for $v_0 = 0.463$ the decay time is $1 \mu\text{s}$. For $N = 2$ and $v_0 = 0.185$ the vertical Bloch line decays in $1 \mu\text{s}$, and for $v_0 = 0.463$ it decays in $0.5 \mu\text{s}$.

The velocity of individual π -components of the packet in steady-state motion is very high, and approximately equal to that of a π -line in the one-dimensional case. The velocity of a packet as a whole after its disintegration is lower than that of a vertical Bloch line with topological charge $N\pi$ in the one-dimensional case, owing to the ongoing annihilation of leading lines and creation of new trailing lines. Figure 6 shows the velocity of the packet as a whole versus v_0 for several values of N .

3. EXPERIMENTAL TECHNIQUES

The dynamics of vertical Bloch lines was studied in real time. Two or three images of solitary flexural waves accompanying moving vertical Bloch lines at different positions on a domain wall were taken by a TV camera, displayed on a computer monitor, and stored in the computer memory. The images enabled us to study profiles of solitary flexural waves on domain walls, and to determine the velocities of both domain wall and vertical Bloch line in real time.⁹

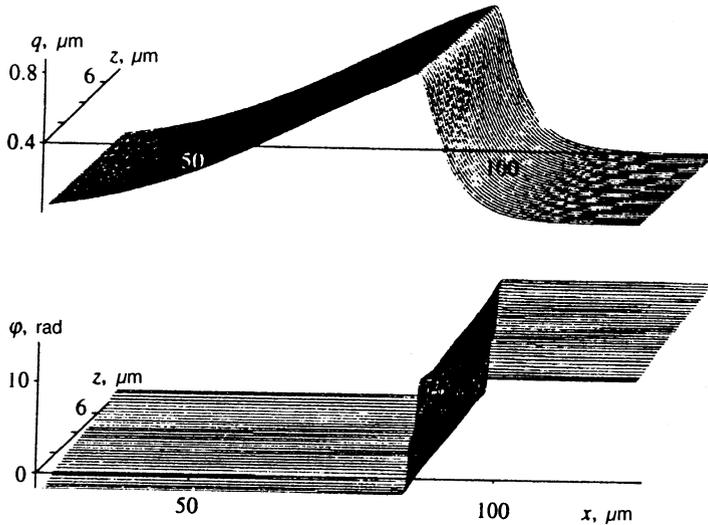


FIG. 2. Solution of the Slonczewski equation for a solitary flexural wave q in a domain wall accompanying a vertical Bloch line, and azimuthal angle φ for $N=4$, $\alpha=0.4$, $v_0=0.0926$, and $H_y=0$ at time $t=480$ ns.

One example of such double-exposure images that shows a domain wall with a vertical Bloch line is given in Fig. 7, where two positions of the domain wall, moving from top to bottom, and two positions of the solitary flexural wave, moving from left to right, are clearly seen. The first and second images were recorded by exposing a sample of ferrite-garnet film with only one domain wall to two 8-ns optical pulses. A nonuniform magnetic field with a gradient of 2000 Oe/cm was generated by two ferrite magnets with magnetization vectors perpendicular to their surfaces, so that the film contained only one domain wall. These magnets, from which the tested film was separated by a 0.5-mm substrate, generated a field of about 100 Oe in the sample plane perpendicular to the domain wall plane. This field was essential to the stability of the vertical Bloch line up to critical values of the magnetic fields propelling the domain wall. This assumption was confirmed by computer simulations of the vertical Bloch line dynamics based on the two-dimensional Slonczewski equations. Note that double exposures of solitary flexural waves recorded in the experiment and given in Fig. 7 have no better resolution than images on

photographic film produced in previous experiments by double-exposure high-speed photography. But given the possibility of displaying images on a monitor and storing them in computer memory, this technique is more convenient and up-to-date than double-exposure high-speed photography. In our analysis of the evolution of solitary flexural wave profiles, we used triple-exposure images on the monitor. In these images, the second position was detected in the domain wall contrast, as in previous experiments.^{7,10}

4. EXPERIMENTAL RESULTS

4.1. Dynamics of a vertical Bloch line with topological charge 2π in a magnetic field perpendicular to the domain wall

Double-exposure images of solitary flexural waves at a domain wall, recorded using the technique described in the previous section at a certain delay between two optical pulses, enabled us to derive from one frame both the domain wall velocity and the velocities of the minimum-amplitude

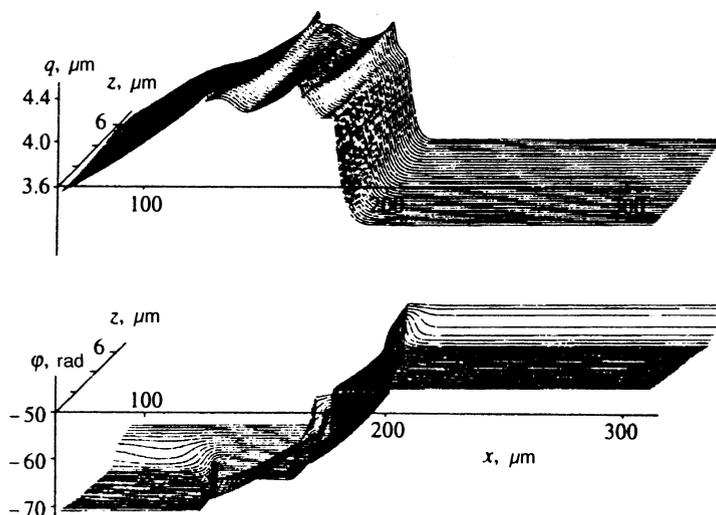


FIG. 3. Solution of the Slonczewski equation for a solitary flexural wave q in a domain wall accompanying a vertical Bloch line, and azimuthal angle φ for $N=8$, $\alpha=0.4$, $v_0=0.185$, and $H_y=0$ at time $t=2560$ ns.

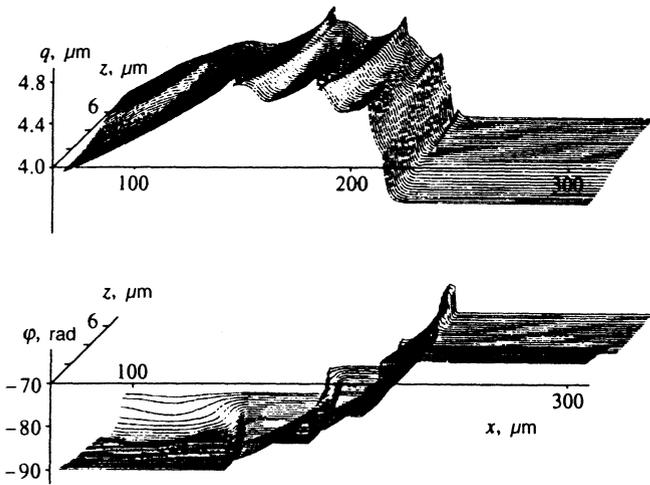


FIG. 4. Same as Fig. 3, at time $t=3200$ ns.

solitary flexural waves observable on the computer monitor accompanying Bloch lines with a certain topological charge. The domain wall velocity was measured as a function of the driving magnetic field. The domain wall mobility and dimensionless damping parameter $\alpha=0.2$ in the film under study were derived from the linear portion of this curve.

The vertical Bloch line velocity u as a function of the domain wall velocity v derived from observed low-amplitude solitary flexural waves in an in-plane magnetic field of 100 Oe perpendicular to the domain wall plane is plotted in Fig. 8. At the lowest velocities u and v , their ratio is about 10. This ratio is several times smaller than the value given by theory when it fails to take account of domain wall flexure about the vertical Bloch line¹:

$$\frac{u}{v} = \frac{\pi\sqrt{Q}}{2\alpha} \approx 50. \quad (4)$$

In our samples $\alpha=0.2$, $Q=45$, and $4\pi M_s=100$ G. Thus, a comparison of Eq. (4) with our experimental data clearly indicates that the profile of a solitary flexural wave has a considerable effect on the ratio u/v . The shape of this

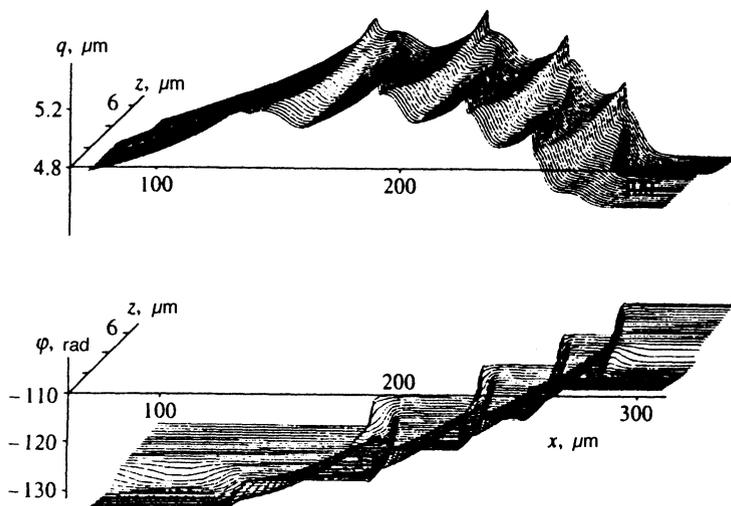


FIG. 5. Same as Fig. 3, at time $t=4480$ ns.

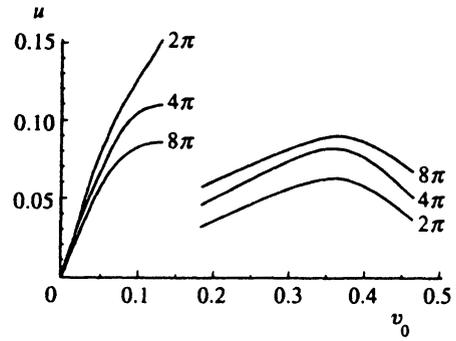


FIG. 6. Velocity u of steady-state motion of a vertical Bloch $N\pi$ -line versus v_0 at $N=2$, $N=4$, and $N=8$. At $v_0<0.138$ the velocity drops with N and grows at $v_0>0.138$.

profile has yet to be derived from three-dimensional solutions of the Landau–Lifshitz equations, so we attribute more importance to analysis of the flexure profile based on the two-dimensional Slonczewski equations. Figure 8 demonstrates that the ratio u/v gradually decreases with domain wall velocity, and at $v=29$ m/s this ratio equals 5.

Solid lines in Fig. 8 show $u(v)$ derived from numerical solutions of the two-dimensional Slonczewski equations for vertical Bloch lines with topological charges of 2π and 4π in an in-plane magnetic field of 100 Oe perpendicular to the domain wall. The agreement between experimental data and calculations for vertical Bloch lines with topological charge 2π is reasonable, and this confirms that the topological charge of the Bloch line in question is really 2π . Figure 7 indicates that there is little hope of observing decay of a 2π -line in our experiment using a TV camera.

4.2. Dynamics of vertical Bloch lines in weak in-plane fields

In our experimental study of the evolution of solitary wave profiles, which were numerically calculated using the two-dimensional Slonczewski equations, we performed measurements in weak magnetic fields of about 10 Oe perpendicular to the domain wall. Triple-exposure images of soli-

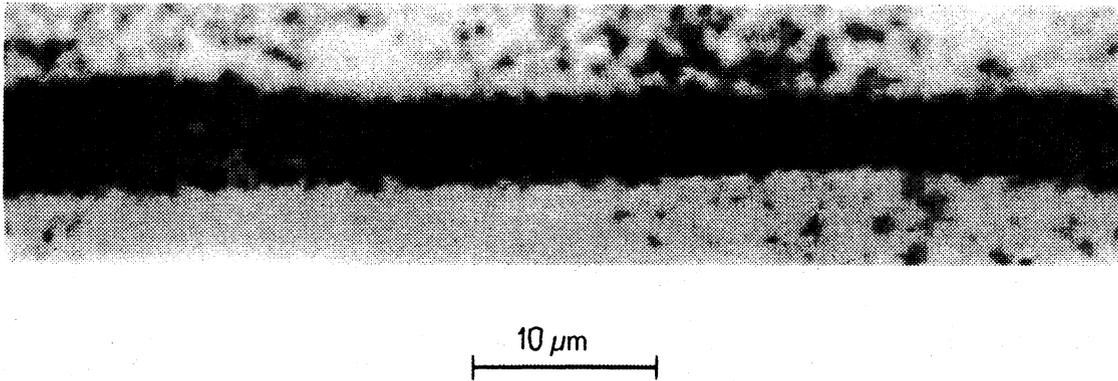


FIG. 7. Double exposure high-speed images of the solitary flexural wave accompanying a vertical Bloch line with a topological charge of 2π moving from left to right along the domain wall, which moves from top to bottom.

tary wave profiles accompanying to vertical Bloch lines with topological charges of $4\pi-8\pi$ are given in Fig. 9. The domain wall moves from top to bottom, and the vertical Bloch line from right to left. In the first position, the solitary wave profile has the narrow leading extended trailing edges observed in previous experiments in a strong in-plane field perpendicular to the domain wall. But in the second position (after $1.0 \mu\text{s}$), and especially in the third position (after another $1.0 \mu\text{s}$), the solitary wave profile has a different shape. It becomes wider, and its central part has a width of some tens of micrometers. The packet velocity drops from 20 to 12 m/s, in qualitative agreement with numerical calculations. Individual π -lines could not be resolved in these profiles because they are distributed over the ferrite-garnet film thickness. They might be resolved, however, via high-speed photography, whose resolution is higher than that of a TV camera.

The linear dependence of the domain wall velocity on magnetic field in the case of a weak in-plane field (~ 10 Oe) terminates at $v = 5$ m/s. This velocity is an order of magnitude lower than in an in-plane field of 100 Oe. The 100-Oe in-plane field perpendicular to the domain wall stabilized

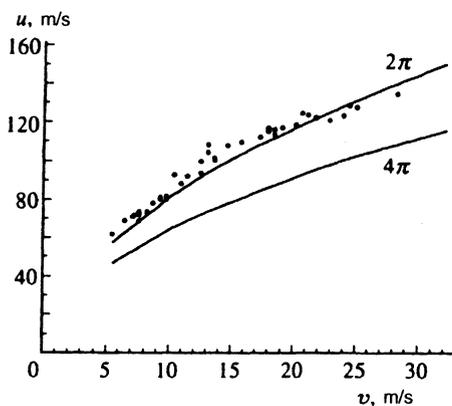


FIG. 8. Measurements of vertical Bloch line velocity u versus domain wall velocity v (dots), and calculations using the Slonczewski equations for vertical Bloch lines with topological charges 2π and 4π in an in-plane magnetic field of 100 Oe perpendicular to the domain wall.

dynamic profiles of solitary waves related to vertical Bloch lines in domain walls of ferrite-garnets in our previous experiments.^{2,7,9} In stronger higher in-plane fields, we could experiment with collisions of vertical Bloch lines and detect their soliton-like properties. Given the calculations and experimental data of this work, we conclude that it would be difficult to detect such properties in low in-plane fields.

5. CONCLUSION

We have performed experiments and numerical calculations based on the two-dimensional Slonczewski equations to study the dynamics of solitary flexural wave profiles that accompany vertical Bloch lines in domain walls of ferrite-garnet films. Our data indicate that stable solitary waves with very narrow leading and extended trailing edges occur at low domain wall velocities when there is no in-plane field, and at high domain wall velocities only in strong in-plane fields. The latter condition enabled us to experimentally study the dynamics of vertical Bloch lines, their pair collisions, and soliton-like behavior in our previous work, and it also enabled us to apply the one-dimensional Slonczewski equations to our experiments. In weak in-plane magnetic fields, the dynamics of solitary wave profiles accompanying vertical Bloch lines is strongly affected by the dynamics of horizontal Bloch lines. Interaction between them leads to the decay of a vertical Bloch line into individual π -components, with leading π -lines being annihilated and new π -lines being created on the trailing edge of the packet. These effects have been confirmed experimentally.

Solitary waves, that accompany Bloch lines and are derived from the two-dimensional Slonczewski equations, are self-organized dynamic structures. The calculated evolution of their shapes and velocities is in approximate agreement with the experimental data. It would be interesting to perform further calculations of the dynamics of the solitary flexural waves that accompany Bloch lines using the three-dimensional Landau-Lifshitz equations, and to conduct experiments with higher spatial resolution.

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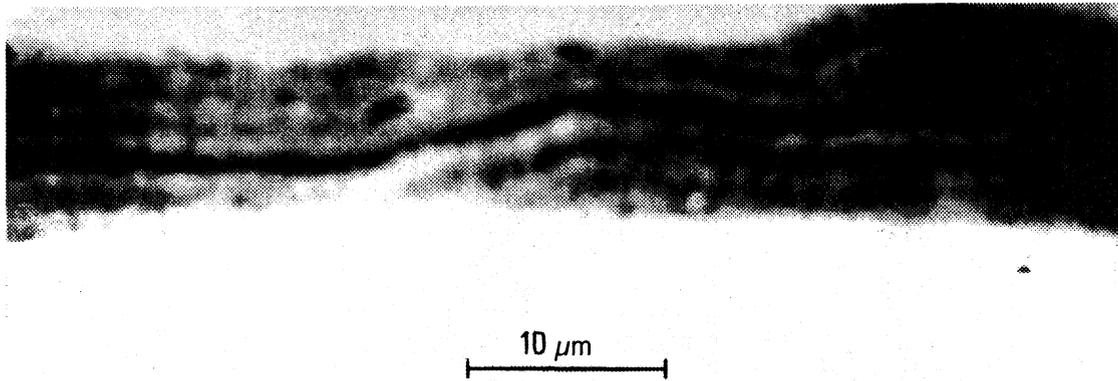


FIG. 9. Triple-exposure high-speed image of a solitary flexural wave accompanying a vertical Bloch line with topological charge 8π on a domain wall in ferrite garnet, in an in-plane magnetic field of 10 Oe perpendicular to the domain wall.

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¹A. Malozemoff and J. Slonczewski, *Magnetic Domain Walls in Bubble Materials*, Academic Press, New York (1979).

²V. G. Bar'yakhtar, M. V. Chetkin, B. A. Ivanov, and S. N. Gadetskiĭ, *Dynamics of Topological Magnetic Solitons*, 182, in *Springer Tracts in Modern Physics*, Vol. 129 (1994).

³A. Thiaville, Ben Youssef, Y. Nakatani, and J. Miltat, *J. Appl. Phys.* **69**, 6090 (1991).

⁴A. K. Zvezdin, A. F. Popkov, and I. P. Yarema, *Zh. Éksp. Teor. Fiz.* **98**, 1070 (1990) [*Sov. Phys. JETP* **71**, 597 (1990)].

⁵E. E. Kotova and V. N. Chetverikov, *Zh. Éksp. Teor. Fiz.* **98**, 2011 (1990) [*Sov. Phys. JETP* **71**, 1131 (1990)].

⁶A. K. Zvezdin, A. F. Popkov, and M. V. Chetkin, *Usp. Fiz. Nauk* **162**, 151 (1992).

⁷M. V. Chetkin, I. V. Parygina, and L. L. Savchenko, *Zh. Éksp. Teor. Fiz.* **105**, 176 (1994) [*Sov. Phys. JETP* **78**, 93 (1994)].

⁸M. Rejda and F. B. Humphrey, *J. Appl. Phys.* **79**, 6464 (1996).

⁹M. V. Chetkin, I. V. Parygina, and L. L. Savchenko, *IEEE Trans. Magn.* **MAG-28**, 2350 (1992).

¹⁰M. V. Chetkin, I. V. Parygina, and L. L. Savchenko, *IEEE Trans. Magn.* **MAG-31**, 4157 (1995).

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