Emission of low-frequency electromagnetic waves by a short laser pulse in stratified rarefied plasma

L. M. Gorbunov

P. N. Lebedev Physical Institute, Russian Academy of Sciences, 117924 Moscow, Russia

A. A. Frolov

Scientific Research Center of the Thermophysics of Pulsed Effects, Combined Institute of High Temperatures, Russian Academy of Sciences, 127412 Moscow, Russia

(Submitted 13 February 1996)
Zh. Éksp. Teor. Fiz. **110**, 1757–1769 (November 1996)

This paper discusses the possibility that a short laser pulse propagating in a periodically varying (stratified) plasma can generate relatively low-frequency electromagnetic radiation. The spectral, angular, and energy characteristics of the radiation are studied. It is shown that the radiation at the plasma frequency is most intense when the resonance condition is satisfied, when the plasma-density modulation period equals the wavelength of a wake plasma wave excited by the pulse. [S1063-7761(96)01411-4]

1. INTRODUCTION

When a short laser pulse propagates in a disperse plasma under the action of averaged ponderomotive forces, the electron density of the plasma is perturbed, and a charge-density wave (a plasma wave) is excited.[1] Being potential forces, ponderomotive forces in the linear approximation generate irrotational electric fields and currents in a homogeneous plasma. In an inhomogeneous plasma, however, even in the linear approximation, a laser pulse excites rotational as well as irrotational electromagnetic fields. This effect was apparently explained for the first time in Ref. 2, which discussed the one-dimensional problem of the passage of an amplitude-modulated electromagnetic wave through an inhomogeneous plasma. It was shown that, in the neighborhood of the resonant density of the plasma, where the plasma frequency is close to the wave-modulation frequency, not only electrostatic electric fields but also rotational electromagnetic fields are excited. This mechanism for the generation of a magnetic field in an inhomogeneous plasma by an amplitude-modulated laser beam was discussed in Ref. 3.

It is easy to explain the physical cause of the generation of rotational currents by potential ponderomotive forces in an inhomogeneous plasma. In the linear approximation, the current density equals $\mathbf{j}=eN(\mathbf{r})\mathbf{V}$, where $N(\mathbf{r})$ is the electron density of the plasma in the absence of the laser pulse, and $\mathbf{V}$ is the irrotational velocity of the electrons, resulting from the action of the ponderomotive forces. It is obvious that curl $\mathbf{j}=e\nabla\times(N\mathbf{V})$, and the rotational part of the current is nonzero if the density gradient is not parallel to the velocity vector.

In this paper, we shall consider a plasma whose density varies periodically in one direction (a so-called stratified plasma), through which a laser pulse of axisymmetric shape passes in the direction of the density variation. We have shown that, under these conditions, the electromagnetic fields excited by a pulse in the plasma can form a wave of radiation whose total energy flux in the radial direction is conserved in the wave zone. We have studied the spectral, angular, and energy characteristics of such low-frequency (from the viewpoint of the laser-radiation frequency) waves for various ratios between the plasma-density modulation period and the wavelength of the plasma wave excited by the pulse.

It should be noted that the question of Cherenkov and transition radiations of a short laser pulse in a material medium have been discussed in the literature[4] and have been studied experimentally.[5] Cherenkov radiation from a short laser pulse was also recently detected in a weakly ionized plasma[6] created by the pulse itself. However, in the case of interest to us, of a fully ionized plasma, where the phase velocities of the electromagnetic waves exceed the velocity of light, Cherenkov radiation is impossible. Transition radiation is possible in principle, appearing when a pulse crosses a jump in the plasma density. However, the study of this question requires not only taking into account the reflection of the laser radiation from the jump, but also analyzing the effect of the pulse on the density jump itself. We shall not consider the question of the transition radiation of a laser pulse in the plasma in this article.

2. BASIC RELATIONSHIPS

We consider a plasma whose density varies periodically along the z axis:

$$N(\mathbf{r})=n_0(1+\alpha\sin k_r z). \tag{2.1}$$

Here n_0 is the background density, and k_r and α are, respectively, the wave number and the dimensionless density-modulation amplitude, which is considered small ($\alpha<1$). The question of creating a periodically inhomogeneous plasma was recently discussed in Refs. 7 in connection with the development of a new design for a free-electron laser (a so-called ripple laser). References 7 pointed out two ways to create a periodic structure of the plasma density. One way is to excite a sound wave in a neutral gas and then to ionize this gas by a laser pulse. The second way to create a periodic density lattice is to excite an ion–sound wave in the plasma.

We shall assume that a laser pulse with a given axisymmetric intensity distribution propagates along the z axis with group velocity v_g. The low-frequency action of the pulse on the plasma electrons is associated with the averaged ponderomotive force, which, referred to one electron, has the form

$$\mathbf{f}=-\nabla\Phi(\mathbf{r},t), \quad (2.2)$$

where $\Phi=mV_E^2/4$ is the high-frequency potential, expressed in terms of the high-frequency electric field amplitude $\mathbf{E}_0(\mathbf{r},t)$ ($V_E=eE_0/m\omega_0$, where ω_0 is the frequency of the laser radiation, and e and m are the electron charge and mass).

The linear response of the plasma to the action of the ponderomotive force given by Eq. (2.2) is described by the system of equations of electron hydrodynamics,

$$\frac{\partial\delta n}{\partial t}+\mathrm{div}(N\mathbf{V})=0, \quad m\frac{\partial\mathbf{V}}{\partial t}=e\mathbf{E}+\mathbf{f} \quad (2.3)$$

and the system of Maxwell's equations,

$$\mathrm{curl}\,\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \quad \mathrm{curl}\,\mathbf{B}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}+\frac{4\pi e}{c}N(\mathbf{r})\mathbf{V}, \quad (2.4)$$

where $\mathbf{V}$ and $\delta n=n-N$ are, respectively, the velocity and the density perturbation of the electrons, and $\mathbf{E}$ and $\mathbf{B}$ are the low-frequency electric and magnetic fields. The system of Eqs. (2.3) and (2.4) is valid when the inequalities $|\mathbf{V}|\ll c$ and $\delta n\ll N$ are satisfied. Moreover, in this system of equations, the thermal motion of the electrons is neglected, which assumes that the inequality $v_g\simeq c\gg v_T$ is satisfied (v_T is the thermal velocity of the electrons), and the displacement of the ions is neglected, which is valid when the pulsewidth is much less than ω_{pi}^{-1} (ω_{pi} is the ion plasma frequency). As applied to an inhomogeneous plasma, it is necessary to make one more stipulation for the validity of the system of Eqs. (2.3) and (2.4). Because the plasma is inhomogeneous, high-frequency electron-density perturbations $\delta\tilde{n}$ can arise in it, along with an associated averaged current $\langle\mathbf{j}\rangle=e\langle\delta\tilde{n}\tilde{\mathbf{V}}\rangle$, where $\tilde{\mathbf{V}}$ is the oscillator velocity of the electrons. Estimates show that the contribution of such a current to the generation of low-frequency radiation is a factor of $(\omega_0 T)^2\gg 1$ less than that from the current that we have taken into account, associated with the action of ponderomotive forces (T is the characteristic period of the low-frequency waves).

The equations for the low-frequency electric and magnetic fields follow from Eqs. (2.3) and (2.4). They are

$$\frac{\partial^2\mathbf{E}}{\partial t^2}+\Omega_p^2(\mathbf{r})\mathbf{E}-c^2\Delta\mathbf{E}=4\pi e\left(\frac{N(\mathbf{r})}{m}\nabla\Phi-c^2\nabla\,\delta n\right), \quad (2.5)$$

$$\frac{\partial^2\mathbf{B}}{\partial t^2}+\Omega_p^2(\mathbf{r})\mathbf{B}-c^2\Delta\mathbf{B}=4\pi ec[\nabla N\mathbf{V}], \quad (2.6)$$

where $\Omega_p(\mathbf{r})=(4\pi e^2N(\mathbf{r})/m)^{1/2}$ is the local plasma frequency. Equations (2.5) and (2.6) contain the quantities δn and $\mathbf{V}$, which are also determined via the high-frequency potential:

$$\frac{\partial^2\mathbf{V}}{\partial t^2}+c^2\mathrm{curl\,curl}\,\mathbf{V}+\Omega_p^2(\mathbf{r})\mathbf{V}=-\frac{1}{m}\nabla\left(\frac{\partial\Phi}{\partial t}\right), \quad (2.7)$$

$$\frac{\partial^2\delta n}{\partial t^2}+\mathrm{div}\left[N(\mathbf{r})\left(\frac{e}{m}\mathbf{E}-\frac{1}{m}\nabla\Phi\right)\right]=0. \quad (2.8)$$

We shall solve the system of Eqs. 2.5–(2.8) by perturbation theory, using the smallness of the plasma-density modulation amplitude ($\alpha<1$). In the zeroth approximation, the plasma is considered homogeneous, and it follows from Eq. (2.6) that $\mathbf{B}^{(0)}=0$. In this case, according to Eqs. (2.5) and (2.8), the electric field is determined from

$$\frac{\partial^2\mathbf{E}^{(0)}}{\partial t^2}+\omega_p^2\mathbf{E}^{(0)}=\frac{\omega_p^2}{e}\nabla\Phi, \quad (2.9)$$

where $\omega_p^2=4\pi e^2n_0/m$. A solution of the equation that satisfies the condition of the absence of electric fields in the plasma as $t\to-\infty$ has the form[1]

$$\mathbf{E}^{(0)}(\mathbf{r},t)=\frac{\omega_p}{e}\int_{-\infty}^{t}dt'\,\sin[\omega_p(t-t')]\nabla\Phi(\mathbf{r},t'). \quad (2.10)$$

To first order in α, the equations for the fields $\mathbf{E}^{(1)}$ and $\mathbf{B}^{(1)}$, according to Eqs. 2.5–(2.7), transform into

$$\left(\frac{\partial^2}{\partial t^2}+\omega_p^2-c^2\Delta\right)\mathbf{E}^{(1)}=\omega_p^2\nu(z)\nabla\varphi-\omega_pc^2\nabla\,\mathrm{div}\left\{\nu(z)\int_{-\infty}^{t}dt'\times\sin[\omega_p(t-t')]\nabla\varphi(\mathbf{r},t')\right\}, \quad (2.11)$$

$$\left(\frac{\partial^2}{\partial t^2}+\omega_p^2-c^2\Delta\right)\frac{\partial\mathbf{B}^{(1)}}{\partial t}=-c\omega_p^2\,\mathrm{curl}(\nu(z)\nabla\varphi), \quad (2.12)$$

where the quantity $\nu(z)$ equals $\alpha\sin(k_rz)$ and characterizes the density modulation,

$$\nabla\varphi(\mathbf{r},t)=\frac{1}{e}\nabla\Phi(\mathbf{r},t)-\mathbf{E}^{(0)}(\mathbf{r},t). \quad (2.13)$$

To solve Eqs. (2.11) and (2.12), we use a Fourier transformation in time t and in the variable $\mathbf{r}_\perp$, which defines the coordinate that is transverse with respect to the z axis.

$$\mathbf{E}^{(1)}(\mathbf{r},t)=\frac{1}{(2\pi)^3}\int_{-\infty}^{+\infty}d\omega\,e^{-i\omega t}\int d\mathbf{k}_\perp e^{i\mathbf{k}_\perp\mathbf{r}_\perp}\mathbf{E}^{(1)}\times(\omega,\mathbf{k}_\perp,z),$$

$$\mathbf{E}^{(1)}(\omega,\mathbf{k}_\perp,z)=\int_{-\infty}^{+\infty}dt\,e^{i\omega t}\int d\mathbf{r}_\perp e^{-i\mathbf{k}_\perp\mathbf{r}_\perp}\mathbf{E}^{(1)}(\mathbf{r},t).$$

The expansion for $\mathbf{B}^{(1)}$ is similar.

According to Eqs. (2.11) and (2.12), we find for the Fourier component of the fields

$$\mathbf{E}^{(1)}(\omega,\mathbf{k}_\perp,z)=\frac{\alpha\omega_p^2}{2\omega^2\epsilon_0}\varphi_0(\mathbf{k}_\perp,\omega)\left\{\mathbf{k}_\perp\left[e^{-ik_-z}\left(1\right.\right.\right.$$

$$+\frac{k_r k_-}{k_0^2-k_-^2}\bigg)-e^{ik_+z}\bigg(1+\frac{k_r k_+}{k_0^2-k_+^2}\bigg)\bigg]$$

$$+\mathbf{e}_z\bigg[e^{-ik_-z}\bigg(\frac{\omega}{v_g}+\frac{k_r k_\perp^2}{k_0^2-k_-^2}\bigg)$$

$$-e^{ik_+z}\bigg(\frac{\omega}{v_g}-\frac{k_r k_\perp^2}{k_0^2-k_+^2}\bigg)\bigg]\bigg\}, \tag{2.14}$$

$$\mathbf{B}^{(1)}(\omega,\mathbf{k}_\perp,z)=\frac{\alpha k_r\omega_p^2}{2\omega c}\varphi_0(\mathbf{k}_\perp,\omega)(\mathbf{e}_x k_y-\mathbf{e}_y k_x)$$

$$\times\bigg(\frac{e^{ik_+z}}{k_0^2-k_+^2}+\frac{e^{-ik_-z}}{k_0^2-k_-^2}\bigg), \tag{2.15}$$

where

$$k_0^2=\frac{\omega^2}{c^2}\epsilon_0-k_\perp^2,\quad \epsilon_0=1-\frac{\omega_p^2}{\omega^2},\quad k_\perp^2=k_x^2+k_y^2,$$

$$k_\pm=k_r\pm\frac{\omega}{v_g},$$

and $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are the unit vectors directed along the x, y, and z axes, respectively. In deriving Eqs. (2.14) and (2.15), we assume that the pulse has a constant given shape and propagates along the z axis with group velocity v_g. The high-frequency potential therefore depends on the variables $\mathbf{r}_\perp$ and $\xi=z-v_g t$, while the function in Eq. (2.13), taking into account Eq. (2.10), transforms to

$$\varphi(\mathbf{r}_\perp,\xi)=\frac{1}{e}\Phi(\mathbf{r}_\perp,\xi)-\frac{k_p}{e}\int_{+\infty}^{\xi}d\xi'\sin[k_p(\xi-\xi')]\Phi(\mathbf{r}_\perp,\xi'), \tag{2.16}$$

where $k_p=\omega_p/v_g$. The function $\varphi_0(\mathbf{k}_\perp,\omega)$ in Eqs. (2.14) and (2.15) is related to the Fourier component of the function in Eq. (2.16) by

$$\varphi(\omega,\mathbf{k}_\perp,z)=\exp\bigg(\frac{i\omega z}{v_g}\bigg)\varphi_0(\mathbf{k}_\perp,\omega)$$

and is

$$\varphi_0(\mathbf{k}_\perp,\omega)=\frac{1}{v_g}\int_{-\infty}^{+\infty}d\xi\exp\bigg(-i\frac{\omega}{v_g}\xi\bigg)\int d\mathbf{r}_\perp\times\exp(-i\mathbf{k}_\perp\mathbf{r}_\perp)\varphi(\mathbf{r}_\perp,\xi). \tag{2.17}$$

The explicit form of the function given by Eq. (2.16) and, consequently, the function given by Eq. (2.17) depends on the pulse shape. Let us consider as an example an axisymmetric pulse with a Gaussian shape in both the longitudinal and the transverse directions:

$$\Phi(\mathbf{r}_\perp,\xi)=\Phi_0\exp\bigg(-\frac{\xi^2}{L^2}-\frac{r^2}{R^2}\bigg), \tag{2.18}$$

where $r=\sqrt{x^2+y^2}$; L and R are, respectively, the longitudinal and transverse dimensions of the pulse; and Φ_0 is the maximum value of the high-frequency potential, which is expressed in terms of the total energy W of the pulse:

$$\Phi_0=\frac{2e^2W}{\sqrt{\pi}R^2Lm\omega_0^2}. \tag{2.19}$$

Using Eqs. (2.16) and (2.17), we find by means of Eqs. (2.18) and (2.19) that

$$\varphi_0(\mathbf{k}_\perp,\omega)=\frac{2\pi eW}{m\omega_0^2\epsilon_0 v_g}\exp\bigg(-\frac{\omega^2\tau^2}{4}-\frac{k_\perp^2R^2}{4}\bigg), \tag{2.20}$$

where $\tau=L/v_g$ is a time characterizing the pulsewidth.

The possibility that electromagnetic waves will be emitted by a laser pulse in a stratified plasma is associated with the condition that the denominators $k_0^2-k_\pm^2$ in Eqs. (2.14) and (2.15) go to zero. Let us study this condition in more detail.

3. CONDITIONS FOR THE EMISSION

The electromagnetic waves emitted by the pulse must satisfy the dispersion law

$$\frac{\omega^2}{c^2}\epsilon_0=k_\perp^2+k_\parallel^2, \tag{3.1}$$

where $k_\parallel$ is the longitudinal component of the wave vector of the emitted wave, which must coincide with the corresponding value of the Fourier component of the radiation source that has the frequency ω. In our case, the radiation source is the rotational current that creates a pulse moving with velocity $v_g\simeq c$ at the periodic density variations. According to Eq. (2.11) or Eq. (2.12), this current is proportional to the product of $\nu(z)$ and $\exp(i\omega z/v_g)$, and it is obvious that this corresponds to the value of $k_\parallel=\omega/v_g\pm k_r$. Substituting this expression into Eq. (3.1), we find the condition $k_0^2=k_\pm^2$, which corresponds to the denominator in Eqs. (2.14) and (2.15) going to zero.

In analyzing the relationship $k_0^2=k_\pm^2$, we shall assume that $\omega>0$. Then the sign of the quantity $k_\parallel$ determines the propagation direction of wave relative to the z axis.

It is easy to convince oneself that, for $v_g\simeq c$, the condition $k_0^2=k_\pm^2$ can be satisfied only for $k_-=k_r-\omega/v_g$, and, in this case,

$$k_\parallel=k_z=-k_-=k_r\frac{\omega-2\omega_r}{2\omega_r},\quad k_\perp=k_r\sqrt{\frac{\omega-\omega^*}{\omega_r}}, \tag{3.2}$$

where $\omega_r=(1/2)k_r v_g$ and $\omega^*=\omega_r(1+k_p^2/k_r^2)$. According to Eq. (3.2), $k_\perp$ is real only for frequencies that exceed ω^*.

Eliminating the frequency ω from Eqs. (3.2), we easily find the equation for the curve that bounds the emitted wave vectors on the $(k_\perp, k_\parallel)$ plane:

$$k_\parallel=\frac{k_\perp^2+k_p^2-k_r^2}{2k_r}. \tag{3.3}$$

Plots of this function are shown in Fig. 1 for different ratios of k_p to k_r.

If the angle θ that the emitted wave makes with the z axis is fixed, and if a ray is drawn at this angle on the $(k_\perp, k_\parallel)$ plane, the points where it intersects the graphs of the function given by Eq. (3.3) determine the values of $k_\perp$

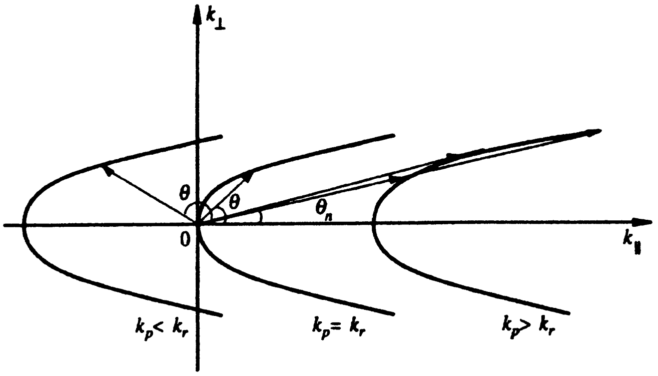

FIG. 1. Graphs of the functions that determine the wave vectors of the electromagnetic waves emitted by a laser pulse.

and $k_\parallel$ (and, consequently, the frequency ω) of the electromagnetic waves emitted by the pulse. Several conclusions follow from Fig. 1.

For plasma-density modulations with a length that exceeds the length of a plasma wave excited by a pulse ($k_p>k_r$ or $\omega_p>2\omega_r$), waves are emitted only in the angle interval $\theta<\pi/2$ (forward), and their frequencies exceed $2\omega_r$. An interesting feature of this case is the possibility of emitting waves with two different frequencies at the same angle. Using Eq. (3.2) for k_z, it is easy to find the relationship between the frequency and the angle θ:

$$\omega(\theta)=\frac{2\omega_r}{\sin^2\theta}\left(1\pm\cos\theta\sqrt{1-\frac{k_p^2}{k_r^2}\sin^2\theta}\right). \qquad (3.4)$$

When the inequality $k_p>k_r$ is satisfied, there is a maximum angle of emission $\theta_n=\arcsin(k_r/k_p)$ (see Fig. 1), which corresponds to the frequency $\omega_n=\omega_p^2/2\omega_r$. The minus sign in Eq. (3.4) corresponds to the frequency interval $\omega^*<\omega<\omega_n$, while the plus sign corresponds to $\omega>\omega_n$. Figure 2a shows how the radiation frequency depends on angle θ.

For shorter-wavelength modulation of the plasma density ($k_p<k_r$), there is a one-to-one relationship between the angle θ and radiation frequency ω (see Fig. 2b), but the waves can be emitted either forward ($\theta<\pi/2$) or backward ($\theta>\pi/2$) with respect to the pulse-propagation direction. The waves are emitted forward with higher frequencies ($\omega>2\omega_r$), and backward with lower frequencies ($\omega^*<\omega<2\omega_r$). The wave emitted directly backwards ($\theta=\pi$) has frequency ω^*.

Under resonance conditions ($k_p=k_r$), only forward emission is possible ($0<\theta<\pi/2$), and the radiation frequency increases as the angle θ decreases (Fig. 2b).

In concluding this section, we point out that the minimum emitted frequency ω^* depends on the ratio k_p/k_r but always exceeds the plasma frequency ω_p, except for the resonant case. Figure 3 shows how ω^*/ω_p depends on k_p/k_r and indicates the curve that separates the frequency region of the waves emitted forward ($0<\theta<\pi/2$) and backward ($\pi/2<\theta<\pi$).

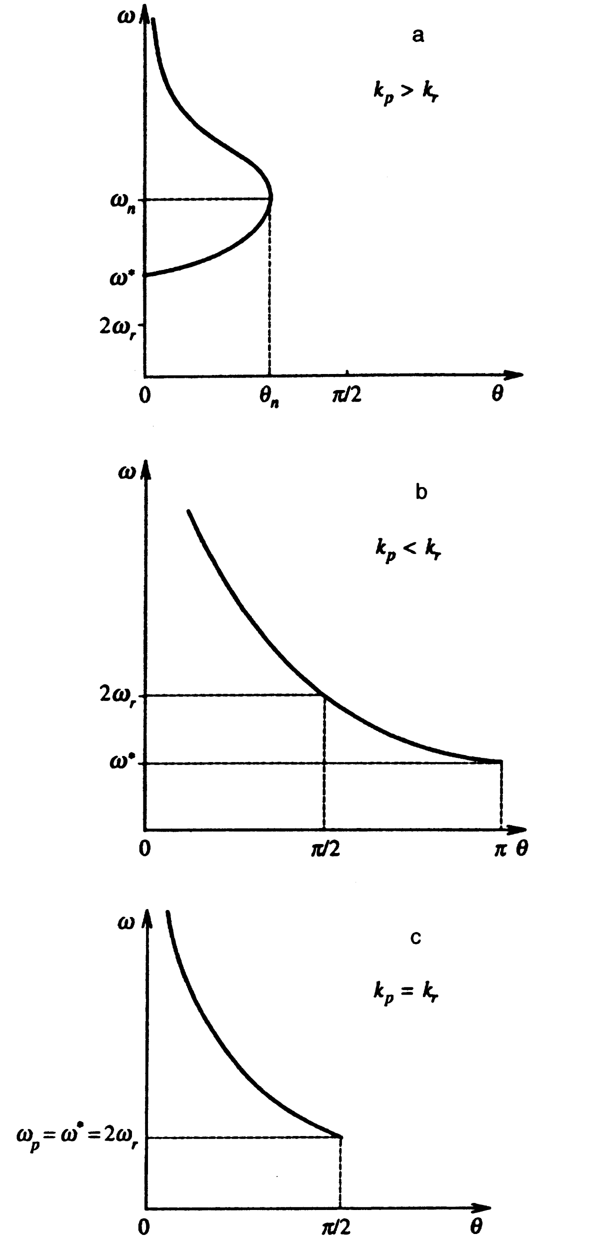

FIG. 2. Frequency ω of an emitted wave vs its propagation angle θ relative to the direction of motion of the pulse: (a) large-scale density modulations ($k_p>k_r$), (b) small-scale density modulations ($k_p<k_r$), (c) resonance ($k_p=k_r$).

4. SPECTRAL CHARACTERISTICS OF THE EMISSION

Let us consider the time-integrated energy flux density of the radiation at a certain point of space:

$$\mathbf{P}(\mathbf{r})=\int_{-\infty}^{+\infty}dt\frac{c}{4\pi}\mathbf{E}(\mathbf{r},t)\times\mathbf{B}(\mathbf{r},t). \qquad (4.1)$$

Expanding the fields in a Fourier expansion in time and using the relationships $\mathbf{E}^*(\omega,\mathbf{r})=\mathbf{E}(-\omega,\mathbf{r})$ and $\mathbf{B}^*(\omega,\mathbf{r})=\mathbf{B}(-\omega,\mathbf{r})$, which follow from the condition that they are real, we write Eq. (4.1) in the form

$$\mathbf{P}(\mathbf{r})=\int_0^\infty d\omega\ \mathbf{p}(\omega,\mathbf{r}), \qquad (4.2)$$

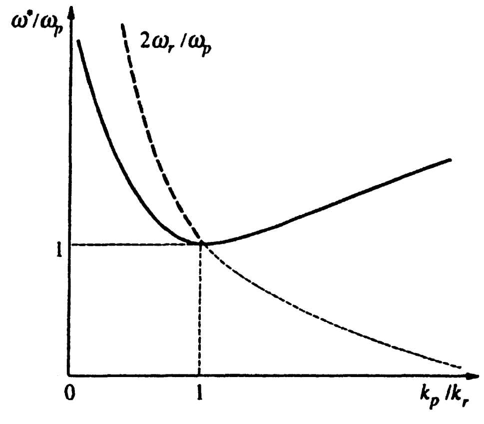

FIG. 3. Minimum frequency of the waves emitted by a pulse vs the ratio between the wavelength of the plasma wave and the density-modulation length. The dashes indicate the frequency emitted at an angle of $\theta=\pi/2$.

where $\mathbf{p}(\omega,\mathbf{r})$ is the time-integrated spectral energy-flux density of the radiation:

$$\mathbf{p}(\omega,\mathbf{r})=\frac{c}{8\pi^2}\{\mathbf{E}(\omega,\mathbf{r})\times\mathbf{B}^*(\omega,\mathbf{r})+\mathbf{E}^*(\omega,\mathbf{r})\times\mathbf{B}(\omega,\mathbf{r})\}. \tag{4.3}$$

To compute the quantities entering into Eq. (4.3), we use Eqs. (2.14) and (2.15), into which we substitute Eq. (2.20), which is valid for an axisymmetric pulse of Gaussian shape:

$$\mathbf{E}^{(1)}(\omega,\mathbf{r})=\frac{\pi\alpha e\omega_p^2 W}{mc\omega_0^2\omega^2\epsilon_0^2}\times\exp\left(-\frac{\omega^2\tau^2}{4}\right)\int\frac{d\mathbf{k}_\perp}{(2\pi)^2}\exp\left(i\mathbf{k}_\perp\cdot\mathbf{r}_\perp-\frac{k_\perp^2R^2}{4}\right)\left\{\mathbf{k}_\perp\left[e^{-ik_-z}\left(1+\frac{k_rk_-}{k_0^2-k_-^2}\right)-e^{ik_+z}\left(1+\frac{k_rk_+}{k_0^2-k_+^2}\right)\right]+\mathbf{e}_z\left[e^{-ik_-z}\left(\frac{\omega}{v_g}+\frac{k_\perp^2k_r}{k_0^2-k_-^2}\right)-e^{ik_+z}\left(\frac{\omega}{v_g}-\frac{k_\perp^2k_r}{k_0^2-k_+^2}\right)\right]\right\}, \tag{4.4}$$

$$\mathbf{B}^{(1)}(\omega,\mathbf{r})=\frac{i\pi\alpha ek_r\omega_p^2W}{mc^2\omega_0^2\omega^2\epsilon_0}e^{-\omega^2\tau^2/4}\mathbf{e}_\varphi\frac{\partial}{\partial r}\int\frac{d\mathbf{k}_\perp}{(2\pi)^2}\times\exp\left(i\mathbf{k}_\perp\cdot\mathbf{r}_\perp-\frac{k_\perp^2R^2}{4}\right)\left\{\frac{e^{ik_+z}}{k_0^2-k_+^2}+\frac{e^{-ik_-z}}{k_0^2-k_-^2}\right\}. \tag{4.5}$$

Taking into account in Eqs. (4.4) and (4.5) the contribution from the pole $k_0^2=k_-^2$ and estimating it by the saddle-point method, we find the following expressions for the fields when $r\gg R$ holds:

$$\mathbf{E}^{(1)}(\omega,\mathbf{r})=\frac{\pi\alpha e\omega_p^2W}{2mc\omega_0^2\omega^2\epsilon_0^2}e^{-\omega^2\tau^2/4}\frac{\partial}{\partial r}\left(\mathbf{e}_r\frac{\partial M}{\partial z}-\mathbf{e}_z\frac{\partial M}{\partial r}\right), \tag{4.6}$$

$$\mathbf{B}^{(1)}(\omega,\mathbf{r})=\frac{i\pi\alpha e\omega_p^2W}{2mc^2\omega_0^2\omega\epsilon_0^2}e^{-\omega^2\tau^2/4}\mathbf{e}_\varphi\frac{\partial M}{\partial r}, \tag{4.7}$$

where $\mathbf{e}_r$, $\mathbf{e}_\varphi$, and $\mathbf{e}_z$ are the unit vectors in a cylindrical coordinate system; and

$$M(\omega,r)=\sqrt{\frac{k_r}{2\pi r}}\times\frac{\exp\left(i\frac{\pi}{4}-ik_-z+ik_rr\sqrt{\frac{\omega-\omega^*}{\omega_r}}-\frac{\omega-\omega^*}{\omega_r}\frac{k_r^2R^2}{4}\right)}{\left(\frac{\omega-\omega^*}{\omega_r}\right)^{1/4}}\times\theta(\omega-\omega^*), \tag{4.8}$$

where $\theta(z)$ is the Heaviside step function: $\theta(z)=1$ for $z>0$ and $\theta(z)=0$ for $z<0$.

Substituting Eqs. (4.6) and (4.7) into Eq. (4.3), we get

$$\mathbf{p}(\omega,\mathbf{r})=\frac{e^2k_r^2}{4\pi r}\left(\frac{\alpha\omega_p^2W}{2\omega_0^2mc^2}\right)^2\frac{\omega_r}{\omega^2\epsilon_0^{5/2}}\sqrt{\frac{\omega-\omega^*}{\omega_r}}\times\exp\left(-\frac{\omega^2\tau^2}{2}-\frac{\omega-\omega^*}{\omega_r}\frac{k_r^2R^2}{2}\right)\left(\mathbf{e}_r\frac{k_r}{k}\times\sqrt{\frac{\omega-\omega^*}{\omega_r}}+\mathbf{e}_z\frac{k_r}{k}\frac{\omega-2\omega_r}{2\omega_r}\right)\theta(\omega-\omega^*), \tag{4.9}$$

where $k=(\omega/c)\sqrt{\epsilon_0}$ is the wave number of the emitted wave. Comparing the expression in parentheses on the right-hand side of Eq. (4.9) with Eqs. 3.2, we see that, as expected, the vector $\mathbf{p}$ coincides in direction with the vector $\mathbf{k}$, and we have $\mathbf{p}=p\mathbf{k}/k$, where

$$p(\omega,r)=\frac{\mathbf{k}\cdot\mathbf{p}}{k}=\frac{e^2k_r^2}{4\pi r}\left(\frac{\alpha\omega_p^2W}{2\omega_0^2mc^2}\right)^2\frac{\omega_r}{\omega^2\epsilon_0^{5/2}}\sqrt{\frac{\omega-\omega^*}{\omega_r}}\times\exp\left(-\frac{\omega^2\tau^2}{2}-\frac{\omega-\omega^*}{\omega_r}\frac{k_r^2R^2}{2}\right)\theta(\omega-\omega^*). \tag{4.10}$$

The quantity $p(\omega,r)$ determines the spectral density of the time-integrated energy flux of the radiation at a point located a large distance r ($r\gg R$) from the z axis, along which the pulse propagates.

In order to analyze the function given by Eq. (4.10), we introduce the dimensionless frequency $x=\omega/\omega^*$ and the following dimensionless parameters:

$$a=\frac{\omega^*\tau}{\sqrt{2}},\quad b=R\sqrt{\frac{k_p^2+k_r^2}{2}},\quad d=\frac{2k_pk_r}{k_p^2+k_r^2},$$

which characterize, respectively, the duration of the pulse (the longitudinal dimension), the width of the pulse, and the

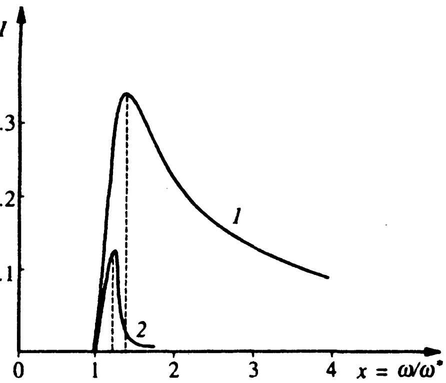

FIG. 4. Normalized spectral energy-flux density of the radiation vs dimensionless frequency $[\omega^*=\omega_r(1+k_p^2/k_r^2)]$ for a short laser pulse $(a=\omega^*\tau/\sqrt{2}\ll 1)$ with different widths: 1—narrow pulse $(b=R\sqrt{(k_p^2+k_r^2)/2}\ll 1)$, 2—wide pulse $(b\gg 1)$.

ratio between the modulation length of the plasma density and the wavelength of the plasma wave. Equation (4.10) then transforms to

$$p(\omega,r)=\frac{e^2k_r^2}{4\pi r\omega_r}\left(\frac{\alpha\omega_p^2W}{2\omega_0^2mc^2}\right)^2\frac{I(x)}{(1+k_p^2/k_r^2)^{3/2}}, \tag{4.11}$$

where

$$I(x)=\frac{x^3\sqrt{x-1}}{(x^2-d^2)^{5/2}}\exp\{-a^2x^2-b^2(x-1)\}. \tag{4.12}$$

The greatest interest is in the frequency x_m at which function $I(x)$ has a maximum. The condition $dI/dx=0$ gives the following equation for determining x_m:

$$(2a^2x+b^2)2x(x-1)(x^2-d^2)=4x^2-3x^3-d^2(7x-6). \tag{4.13}$$

In the two limiting cases $k_r\gg k_p$ and $k_r\ll k_p$, we have $d^2\ll 1$, and Eq. (4.13) simplifies to

$$2x(x-1)(2a^2x+b^2)=4-3x. \tag{4.14}$$

For short laser pulses $[a=\omega^*\tau/\sqrt{2}=(\omega_r\tau/\sqrt{2})\times(1+k_p^2/k_r^2)\ll 1]$ with small transverse dimensions $(b\ll 1)$, the solution of Eq. (4.14) approximately equals

$$x_m=\frac{\omega_m}{\omega^*}=\frac{4}{3}. \tag{4.15}$$

In this case, according to Eq. (4.12), we have $I(x_m)\simeq 0.325\ldots$. Thus, for fairly short laser pulses like these with relatively small transverse dimensions, the wave whose frequency satisfies the condition given by Eq. (4.15) has the maximum amplitude. Figure 4a shows the function $I(x)$ for such short and narrow pulses.

As the transverse dimension (parameter b) of a short laser pulse $(a\ll 1)$ increases, the maximum of function $I(x)$ is displaced toward the value $x_m\simeq 1$, and in the limit $b\gg 1$ it reaches

$$x_m=1+\frac{1}{(k_p^2+k_r^2)R^2}. \tag{4.16}$$

The emission intensity in this case decreases, and the frequency region narrows (Fig. 4b).

Increasing the pulse duration to a value τ that satisfies the inequality $\omega^*\tau\gg\sqrt{2}$ causes the radiation intensity to decrease sharply.

From the dimensionless estimates obtained above, which are valid for both large-scale $(k_r\ll k_p)$ and small-scale $(k_r\gg k_p)$ density modulation, quite different conclusions follow in dimensional form for these limiting cases. Thus, in the limit $k_p\ll k_r$ (small-scale density modulation), the maximum spectral density given by Eq. (4.10) reaches a frequency $\omega_m=(2/3)k_rv_g$, which, according to Eq. (3.4), corresponds to the angle $\theta=2\pi/3$ (backward emission). In the case of large-scale density modulation $(k_p\gg k_r)$, we have $\omega_m\simeq\omega_p^2/3\omega_r$, and the wave propagates almost directly forward, at the small angle $\theta\simeq\omega_r/\sqrt{3}\omega_p$ with respect to the direction of motion of the pulse.

Let us now consider the resonant case, in which the density-modulation length coincides with the wavelength of the plasma wave $(k_p=k_r,\ d=1)$ and $\omega^*=\omega_p=k_rv_g$. Equation (4.13) then has no solutions for real x, and the function given in Eq. (4.12) in the region $x>1$ monotonically decreases with increasing x, while it has a singularity for $x\to 1$ $[I(x)\propto(x-1)^{-2}]$. However, in the limit $x\to 1$, or $\omega\to\omega_p$, the Fourier component $\mathbf{E}^{(1)}(\omega,\mathbf{r})$ of the emitted field, given by Eq. (4.4), which is proportional to ϵ_0^{-2}, goes to infinity more rapidly than the corresponding Fourier component $\mathbf{E}^{(0)}(\omega,\mathbf{r})$ of the field of plasma wave, given by Eq. (2.10), which is proportional to ϵ_0^{-1}. Therefore, the condition for the perturbation theory that we have used to be applicable, so that the inequality $\mathbf{E}^{(1)}(\omega,\mathbf{r})\ll\mathbf{E}^{(0)}(\omega,\mathbf{r})$ is valid, is satisfied only for $\epsilon_0\gg\alpha$ or $x-1\gg\alpha/2$. If it assumed that, in the region $x-1\lesssim\alpha/2$, the emission intensity is suppressed because of its nonlinear effect on the plasma wave, we can use $x_m\sim 1+\alpha/2$ as an extremely rough estimate and we get

$$I_{\max}\propto\exp(-\omega_p^2\tau^2/2)/\sqrt{2}\alpha^2.$$

It is obvious that the spectral density given by Eq. (4.11) has a sharp maximum at ω_p in this case, where it is independent of the modulation depth of the plasma density. Such emission is directed almost transverse to the propagation direction of the pulse $(\theta\sim\pi/2)$.

5. ENERGY OF THE WAVES EMITTED BY THE PULSE

Let us consider a cylindrical surface with radius r and length dz, along the axis of which a pulse moves. According to Eq. (4.2), energy dW_r passes through this surface during the entire time the pulse is moving:

$$dW_r=dz\int_{\omega^*}^{+\infty}d\omega\, e^2k_r^2\left(\frac{\alpha\omega_p^2W}{2\omega_0^2mc^2}\right)^2\frac{\omega_r^2}{\omega^3\epsilon_0^3}\left(\frac{\omega-\omega^*}{\omega_r}\right)\times\exp\left(-\frac{\omega^2\tau^2}{2}-\frac{\omega-\omega^*}{\omega_r}\frac{k_r^2R^2}{2}\right). \tag{5.1}$$

It can be stated that dW_r/dz defines the energy lost by the pulse per unit path length in a stratified plasma because of the emission of waves. It is obvious that the energy lost through the wave emission in a frequency interval close to a frequency ω, according to Eq. (5.1), equals

$$\frac{dW_r}{dz\,d\omega}=e^2k_r^2\left(\frac{\alpha\omega_p^2W}{2\omega_0^2mc^2}\right)^2\frac{\omega_r^2}{\omega^3\epsilon_0^3}\left(\frac{\omega-\omega^*}{\omega_r}\right)\times\exp\left(-\frac{\omega^2\tau^2}{2}-\frac{\omega-\omega^*}{\omega_r}\frac{k_r^2R^2}{2}\right). \qquad (5.2)$$

We shall be interested in the total losses of Eq. (5.1), and, introducing the dimensionless frequency $x=\omega/\omega^*$, we write

$$\frac{dW_r}{dz}=\frac{e^2k_r^4}{k_p^2+k_r^2}\left(\frac{\alpha\omega_p^2W}{2\omega_0^2mc^2}\right)^2F(a,b,d), \qquad (5.3)$$

where

$$F(a,b,d)=\int_l^\infty\frac{dx\,x^3(x-1)}{(x^2-d^2)^3}\exp[-a^2x^2-b^2(x-1)], \qquad (5.4)$$

and the parameters a, b, d that characterize the length and width of the pulse and the ratio between k_p and k_r are given above.

In order to estimate the losses, we consider as an example a plasma whose density is modulated by 10% ($\alpha=0.1$) with period $\lambda_r=2\pi/k_r=94$ μm relative to the mean value $n_0=1.5\times10^{16}$ cm^{-3}. We shall assume that a laser pulse propagates in such a plasma, with a carrier frequency $\omega_0=1.88\times10^{15}$ sec^{-1} ($\lambda_0=1$ μm), a duration $\tau=100$ fs (10^{-13} sec), a transverse dimension (the diameter of the focal spot) of $2R=30$ μm, and an energy of $W=1$ J (a Rayleigh length of 710 μm). The wavelength of the excited plasma wave is $\lambda_p=283$ μm, and $k_r/k_p=3$. The parameters a, b, and d in Eq. (5.4) are less than unity, and we have $F\simeq0.5$. According to Eq. (5.3), we find $dW_r/dz\simeq2.4$ erg/cm$=2.4\times10^{-7}$ J/cm. As the mean plasma density increases, λ_p decreases, the ratio k_r/k_p approaches unity, and the effect strengthens.

6. CONCLUSION

A short laser pulse propagating in a periodically inhomogeneous (stratified) low-density plasma can emit relatively low-frequency electromagnetic waves. The frequency spectrum of the emitted waves has a lower limit that exceeds the plasma frequency. There is a definite connection between the frequency of the wave and the angle at which it is emitted, which essentially depends on the ratio between the density-modulation length and the wavelength of the plasma wave excited by the pulse. For large-scale density modulation, the emission is directed forward in a certain cone of angles, and its intensity peaks when the angle is close to zero. For small-scale density modulation, the emission appears at all angles, and its intensity peaks at an angle of $\theta\simeq2\pi/3$. The emission intensity increases under resonance conditions, when the density-modulation period coincides with the wavelength of the plasma wave. However, this case requires special consideration, which obviously goes beyond standard perturbation theory, where we neglect the effect that density modulation of the plasma has on the propagation of the waves emitted by the pulse. The neglect of this effect for waves emitted at angles close to $\pi/2$ may be wrong. In particular, this relates to emission at a frequency close to ω_p under resonance conditions ($k_p=k_r$).

Estimates have shown that energy lost by the pulse through the wave emission even under optimum conditions has no substantial effect on the energy of the pulse for reasonable propagation lengths.

The effect that we have considered can be used as a new method for diagnosing powerful, short laser pulses in a plasma, as well as for creating smoothly tunable IR sources. In particular, radiation at frequencies close to the plasma frequency were recently observed in experiment.[8]

In conclusion, we express our appreciation to N. E. Andreev and V. P. Silin for comments, and also to G. A. Askar'yan, who pointed out Ref. 6 to us. This work was carried out with the financial support of the Russian Fund for Fundamental Research (Grants No. 94-02-03847 and No. 94-02-03843).

[1]T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979); L. M. Gorbunov and V. I. Kirsanov, Zh. Éksp. Teor. Fiz. **93**, 509 (1987) [Sov. Phys. JETP **66**, 290 (1987)].
[2]K. B. Dysthe, E. Mjothus, and J. Trulson, J. Geophys. Res. **83**, 1985 (1978).
[3]V. K. Tripathi and C. S. Liu, Phys. Plasmas **1**, 990 (1994).
[4]G. A. Askar'yan, Zh. Éksp. Teor. Fiz. **42**, 1360 (1962) [Sov. Phys. JETP **15**, 943 (1962)].
[5]D. H. Auston, K. P. Cheung, G. A. Valdmanis, and D. A. Kleinman, Phys. Rev. Lett. **53**, 1555 (1984).
[6]E. T. J. Nibbering, P. F. Curley, G. Grillon *et al.*, Opt. Lett. **21**, 62 (1996).
[7]K. R. Chen and J. M. Dawson, Phys. Rev. Lett. **68**, 29 (1992); Zhi-Min Dai and Lei Xu, Phys. Rev. E **49**, 745 (1994).
[8]H. Hamster, A. Sullivan, S. Gordon *et al.*, Phys. Rev. Lett. **71**, 2725 (1993).

Translated by W. J. Manthey