New types of nuclear magnons induced by a nonmagnetic coating on the surface of a magnetic crystal

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(Submitted 29 December 1995; resubmitted 8 April 1996)
Zh. Éksp. Teor. Fiz. **110**, 1411–1432 (October 1996)

The example of a cubic antiferromagnet is used to show that consistently taking into account the effect of the phonon subsystem on Suhl–Nakamura exchange in the subsystem of nuclear spins of a thin magnetic film with a nonmagnetic coating makes possible the formation of qualitatively new types of propagating bulk, surface, and bulk–surface nuclear spin-wave excitations, with no analog either in the case of an unbounded magnetic material or in the case of a separate magnetic film. © *1996 American Institute of Physics.* [S1063-7761(96)01810-0]

1. INTRODUCTION

Recently it has been shown[1] that while the NMR frequency ω_n and the wave vector $\mathbf{k}$ of spin waves propagating in an unbounded magnetic material satisfies the elastostatic criterion (s is the minimum phase velocity of the elastic waves)

$$\omega_n \ll sk, \tag{1}$$

even for a semibounded magnetic material or a thin magnetic film the indirect interaction of the elastic and nuclear subsystems of the magnetic material via electron spin waves leads to the formation of qualitatively new types of both surface and bulk nuclear spin-wave excitations. The dispersive properties and the conditions for localization of such excitations are determined primarily not by the inhomogeneous exchange interaction in the electron spin subsystems, as they are in the case of an unbounded magnetic material,[2–4] but by dynamical magnetoelastic interaction (i.e., they can be called, by analogy with magnetostatic spin waves,[5] nonexchange magnetic excitations). The physical mechanism responsible for the formation of this class of magnetic excitations is the indirect spin–spin exchange via the long-range field of quasistatic elastic strain. Following Ref. 1, we will use the term "elastostatic nuclear magnetic spins" for this type of nonexchange nuclear magnons. As for the traditional types of nuclear spin waves,[2–4] we will call them exchange nuclear spin waves. We can then expect that the presence on the surface of a thin magnetic film with a one- or two-sided nonmagnetic coating that is in acoustic contact with the surface of a magnetically ordered crystal has a strong effect on the Suhl–Nakamura exchange in the subsystem of nuclear spins and hence on the structure of the spectrum of nuclear spin-wave excitations formed because of the interaction of this type. Up till now, however, there have been no studies of the effect of the phonon subsystem on the nuclear spin dynamics of a bounded magnetic material with a nonmagnetic coating.

2. STATEMENT OF THE PROBLEM

In view of what has been said above, the goal of the present work is to study the phonon mechanism of formation of the anomalies in the spectrum of one-particle nuclear spin excitations in a thin magnetic film (of thickness d) caused by a one- or two-sided nonmagnetic coating (of thickness t or thicknesses t and l). Since the cubic antiferromagnets $KMnF_3$ and $RbMnF_3$ constitute real magnetic objects in which exchange nuclear spin waves have been experimentally observed and theoretically studied,[6,7] for an example of a magnetic medium we take the two-sublattice model (where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetizations of the sublattices of the electron spin subsystem) of a cubic antiferromagnet (AFM) with allowance for the first (K_1) and second (K_2) magnetic anisotropy constants. In addition, Ref. 1 also studied the elastostatic nuclear spin waves in a model of a uniaxial AFM crystal, whereas cubic magnetic materials belong to the class of multiaxial crystals, which also has an effect on the conditions of formation and the dispersive properties of these waves (this is demonstrated below). The energy density W in this model of a cubic AFM, a model that allows for the interaction of the three subsystems of a real crystal, the electron-spin, nuclear-spin, and lattice subsystems, can be represented in terms of the ferromagnetism vectors $\mathbf{M}$ and $\mathbf{m}$ and antiferromagnetism vectors $\mathbf{L}$ and $\mathbf{l}$ for the electron $(\mathbf{M},\mathbf{L})$ and nuclear $(\mathbf{m},\mathbf{l})$ spin systems as follows ($|\mathbf{M}|\ll|\mathbf{L}|$ and $|\mathbf{m}|\ll|\mathbf{l}|$):

$$\begin{gathered}
W = W_m + W_{me} + W_e, \\
W_m = M_0^2\left[\frac{1}{2}\delta\mathbf{M}^2 + K_1(L_x^2L_y^2 + L_x^2L_z^2 + L_z^2L_y^2) + K_2L_x^2L_y^2L_z^2 + A(\mathbf{L}\cdot\mathbf{l} + \mathbf{M}\cdot\mathbf{m}) + \frac{\alpha}{2}\left(\frac{\partial\mathbf{L}}{\partial x_i}\right)^2 - 2\mathbf{M}\cdot\mathbf{h}\right], \\
W_{me} = M_0^2[B_1(L_i^2u_{ii}) + 2B_2(L_iL_ku_{ik})(1-\delta_{ik})m], \\
i,k = x,y,z, \\
W_e = c_{11}u_{ii}^2 + (c_{12}u_{ii}u_{kk} + c_{44}u_{ik}^2)(1-\delta_{ik}), \\
2M_0\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2, \quad 2M_0\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2, \\
2m_0\mathbf{m} = \mathbf{m}_1 + \mathbf{m}_2, \quad 2m_0\mathbf{l} = \mathbf{m}_1 - \mathbf{m}_2,
\end{gathered} \tag{2}$$

where δ, α, A, and $K_{1,2}$ are the constants of, respectively, homogeneous exchange, inhomogeneous exchange, hyperfine coupling, and anisotropy; $\widetilde{B}$ and $\widetilde{C}$ are the magnetostriction and elastic constants; u_{ik} is the elastic stress tensor; and

$\mathbf{h}=\mathbf{H}/M_0$ is the external magnetic field. To simplify calculations we assume that the elastic properties of the nonmagnetic coating are isotropic and can be characterized by the Lamé coefficients λ and μ. Following Ref. 8, we examine the magnetoelastic dynamics of the magnetic material by using a system of coupled dynamical equations consisting of the Landau–Lifshitz equations in terms of the vectors $\mathbf{M},\mathbf{m}$ and $\mathbf{L},\mathbf{l}$ and the fundamental equation of the theory of elasticity for the lattice displacement vector $\mathbf{u}$. When the external magnetic field H is negligible compared to the exchange and hyperfine fields (accordingly, $|\mathbf{M}|\ll|\mathbf{L}|$ and $|\mathbf{m}|\ll|\mathbf{l}|$), it is convenient to describe the dynamics of the multilayered structure in terms of the Lagrangian $\mathscr{L}$ (see Ref. 9). If ξ is the coordinate along the normal $\mathbf{n}$ to the surface of the magnetic field, the structure of the density of the Lagrangian $\mathscr{L}$ for a magnetic film with a one- or two-sided nonmagnetic coating can be represented in the following form (ρ_1 and ρ_2 are the densities of the magnetic and nonmagnetic media):

$$\mathscr{L}=\begin{cases} L_2, & d<\xi<t+d, \\ L_1, & 0<\xi<d, \\ L_2, & -l<\xi<0, \end{cases} \tag{3}$$

$$\begin{aligned} L_1=2M_0^2\bigg[&\frac{\alpha}{2c^2}\left(\frac{\partial \mathbf{L}}{\partial t}\right)^2+\frac{1}{2g_n^2AM_0^2}\left(\frac{\partial \mathbf{l}}{\partial t}\right)^2 \\ &-K_1(L_x^2L_y^2+L_x^2L_z^2+L_z^2L_y^2)-K_2L_x^2L_y^2L_z^2-A\mathbf{L}\cdot\mathbf{l} \\ &-\frac{\alpha}{2}\left(\frac{\partial \mathbf{L}}{\partial x_i}\right)^2-\frac{4}{g_e\delta M_0}[h(\mathbf{L}\cdot\dot{\mathbf{L}})]-\frac{2}{\delta}(\mathbf{L}\cdot\mathbf{h})^2-W_{me}\bigg] \\ &+\frac{\rho_1\dot{\mathbf{u}}^2}{2}-W_e, \quad L_2=\frac{\rho_2\dot{\mathbf{u}}^2}{2}-\frac{\lambda}{2}u_{ii}^2-\mu u_{ik}^2, \end{aligned} \tag{4}$$

where g_e and g_n are the gyromagnetic ratios for the electron and nuclear spin systems, and $c^2=(g_eM_0)^2\delta\alpha$ is the phase velocity of the electron spin wave in an unbounded AFM. In deriving (4) we allowed for the fact that since the nuclear spin system of the crystal is in a paramagnetic state, at frequencies that are low compared to the exchange frequencies and in weak magnetic fields the ferromagnetism vectors of not only the electron spin system of the crystal ($\mathbf{M}$) but also of the nuclear spin system ($\mathbf{m}$) are small, with the result that the term $A\mathbf{M}\cdot\mathbf{m}$ in (2) can be ignored in comparison to $A\mathbf{L}\cdot\mathbf{l}$. Thus, the conditions of applicability of the Lagrangian (4) for describing the interaction of the electron-spin, nuclear-spin, and elastic subsystems in the model of a two-sublattice AFM practically coincide with those stated in Ref. 9. Assuming that the outer surfaces of the acoustically continuous three-layered structure considered here is free from strain and following the reasoning of Ref. 10, we can easily show that the dynamical system of equations of the given problem determined by the variations of (3) and (4) with allowance for the paramagnetic state of the nuclear spins must be augmented by the following system of boundary conditions:

$$\sigma_{ik}^{(1)}n_k=\sigma_{ik}^{(2)}n_k, \quad u_i^{(1)}=u_i^{(2)},$$

$$\frac{\partial \mathbf{L}}{\partial \xi}=0 \quad \text{at } \xi=d \quad \text{or } \xi=0; \tag{5}$$

$$\sigma_{ik}^{(2)}n_k=0 \quad \text{at } \xi=t+d \quad \text{or } \xi=-l. \tag{6}$$

The superscripts 1 and 2 refer to the magnetic and nonmagnetic media, respectively. If we restrict our analysis to the case of spatially homogeneous states, we can easily show that, depending on the ratio of the anisotropy constants K_{12}, at $H=0$ there can be one of the following energy-nonequivalent equilibrium orientations of the antiferromagnetism vector $\mathbf{L}$: $\mathbf{L}\|\mathbf{l}\|[001]$, $\mathbf{L}\|\mathbf{l}\|[110]$, or $\mathbf{L}\|\mathbf{l}\|[111]$. Solving the given boundary-value problem, we can easily obtain a dispersion equation determining the magnetoelastic dynamics of the interacting electron-spin and nuclear-spin systems, with an arbitrary value of the dimensionless magnetoelastic coupling constant $0<\xi_{me}\leq 1$ (see Ref. 8) in each of the above phases. However, since the respective expression is extremely lengthy, we do not write it explicitly; instead we restrict our analysis of this expression in each phase to separate physically interesting particular cases. Since in many magnetically ordered crystals the condition $\omega_e\gg\omega_n$ is quite accurate (ω_e is the frequency of an AFM resonance in the spectrum of the electron spin waves),[2-4] for convenience in examining the nuclear dynamics under conditions (1) we introduce an effective frequency ω_*, which is related to the real excitation frequency ω of spin waves by

$$\omega_*^2=\frac{\omega^2\omega_T^2}{\omega_n^2-\omega^2}, \quad \omega_T^2=\omega_E g_e A l, \tag{7}$$

where ω_T is the dynamic shift of the NMR frequency,[2-4] $\omega_E=g_e\delta/2M_0$ is the exchange frequency, and $\omega_n=g_nA/2M_0$ is the NMR frequency.

Since, as noted earlier, the goal of this research is to study the effect of a nonmagnetic coating on the formation and the dispersive properties of nuclear spin waves, the following hierarchy of approximations is useful in further analysis of dispersion relations. In the first stage we use (1) to study only the conditions of formation and the dispersive properties if the spectrum of nuclear spin-wave excitations of the elastostatic type. This requires only analyzing the general dispersion relations to lowest order in the parameters $\omega_*/sk_\perp$ and $ck_\perp/\omega_*$ under the assumption that these parameters are small. Clearly, this is possible for $ck_\perp\ll\omega_*\ll sk_\perp$. The condition $\omega_*/sk_\perp\rightarrow 0$ corresponds to the elastostatic approximation[11] in the magnetoelastic dynamics of a bounded magnetic material,[1] while the condition $ck_\perp/\omega_*\rightarrow 0$ corresponds to the nonexchange approximation widely used in the physics of magnetostatic vibrations.[5] In the second stage, in the nonexchange approximation ($ck_\perp/\omega_*\rightarrow 0$) we examine how discarding the condition $\omega_*/sk_\perp\rightarrow 0$, which corresponds to allowing for the effect of acoustic lag, affects the dispersive properties of the elastostatic nuclear spin waves found at the first stage. The goal of the third stage is to establish the effect of the inhomogeneous exchange interaction on the conditions of propagation and the dispersive properties of the nonexchange types of elastostatic nuclear spin waves (examined at the first stage) in the case where $\omega_* / sk_\perp \to 0$, which corresponds to the region of fairly small wavelengths.

3. THE NONEXCHANGE APPROXIMATION

An additional condition used in the calculations that follow is the restriction on the relative orientation of the displacement vectors **u** of the lattice, the equilibrium antiferromagnetism vector **L** (**l**), and the direction of propagation of spin waves (the wave vector **k**). Below we assume that either $\mathbf{k}\perp\mathbf{u}\perp\mathbf{L}$ or $\mathbf{u}\|\mathbf{L}\perp\mathbf{k}$, which means a considerable simplification in the analytical calculations, since only in this case is the elastostatic dynamics of nonexchange nuclear spin-wave excitations reduced to a boundary value problem of an elliptic partial differential equation for one of the components of the vector **u** of elastic displacements of the lattice, both in the magnetic and nonmagnetic media. Then the corresponding equation in the most general case can be formally written as

$$\mu_\| \frac{\partial^2 u}{\partial \xi^2} + \mu_s \frac{\partial^2 u}{\partial \xi \partial \eta} + \mu_\perp \frac{\partial^2 u}{\partial \eta^2} = 0. \tag{8}$$

Here $\boldsymbol{\eta}$ is the coordinate in the plane of the layer ($\mathbf{u}\perp \boldsymbol{\eta}$), and $\mu_\|$, μ_s, and $\mu_\perp$ are the effective elastic moduli calculated on the basis of (4) in the nonexchange approximation with allowance for the dynamical magnetoelastic interaction and dependent on the relative orientation of **L**, **u**, and **n** (for a specified equilibrium orientation of the antiferromagnetism vector **L**). In a magnetic medium we have $\mu_\| = \mu_\|(\omega)$, $\mu_s = \mu_s(\omega)$, and $\mu_\perp = \mu_\perp(\omega)$, while in the nonmagnetic medium $\mu_\| = \mu_\perp = \text{const}$ and $\mu_s = 0$. For waves propagating in the plane of the layer in the direction specified by $\boldsymbol{\eta}$ [$u \propto \exp(i\omega t - ik_\perp \eta)$] the general solution of Eq. (8) with the structure (3) has the form

$$\begin{aligned} &u(\xi) = A \exp(i\kappa_+ k_\perp \xi) + B \exp(i\kappa_- k_\perp \xi), \quad d<\xi<t+d, \\ &u(\xi) = C \exp(k_\perp \xi) + D \exp(-k_\perp \xi), \quad 0<\xi<d, \\ &u(\xi) = E \exp(k_\perp \xi) + F \exp(-k_\perp \xi), \quad -l<\xi<0, \\ &\kappa_\pm = \frac{\mu_s}{\mu_\|} \pm \sqrt{\left(\frac{\mu_s}{\mu_\|}\right)^2 - \frac{\mu_\perp}{\mu_\|}}. \end{aligned} \tag{9}$$

Thus, the analysis of (9) shows that, depending on the nature of the spatial localization of the amplitude of $u(\xi)$ near the boundary between the magnetic and nonmagnetic media, these elastostatic nuclear spin waves can in principle be divided into surface waves ($\mathrm{Re}\,\kappa_\pm = 0$), bulk waves ($\mathrm{Im}\,\kappa_\pm = 0$), and bulk–surface waves ($\mathrm{Re}\,\kappa_\pm \neq 0$ and $\mathrm{Re}\,\kappa_\pm \neq 0$). But to realize a particular elastostatic nuclear spin wave in practice while satisfying the corresponding inequality we must be sure that the expressions (9) satisfy the system of boundary conditions (5) and (6). Below we specify the conditions for formation of elastostatic nuclear spin waves in a thin AFM film with a nonmagnetic coating for the three magnetic phases mentioned above. If, following Ref. 12, we introduce the notation $K = K_1 + B_1^2(c_{11} - c_{12}) - B_2^2/2c_{44}$, we can easily show that for $K>0$ the equilibrium antiferromagnetism vector **L** (**l**) is directed along one of the energy-equivalent axes coinciding with the cube edges. Without loss of generality we can consider a single case: $\mathbf{L}\|[001]$. Analysis shows that $\omega_*^2 \gg c^2 k_\perp^2$ holds even at $H=0$ in the elastostatic approximation (1), disregarding the inhomogeneous exchange in the electron spin subsystem (here $\kappa_\perp$ is the wave vector of spin vibrations in the plane of the film). The presence of a nonmagnetic coating with $\mathbf{n}\perp[001]\|\mathbf{u}\|\mathbf{L}\perp\mathbf{k}$ gives rise to surface elastostatic nuclear spin waves induced by the Suhl–Nakamura interaction of nuclear spins, with the phonon subsystem of not only the magnetic film but also of the nonmagnetic coating participating in the process. The spectrum of such surface nuclear spin-wave excitations for an arbitrary orientation of the vector **n** normal to the surface of the layers in the XY plane of propagation of the elastostatic nuclear spin waves can be written explicitly both in the case of a one-sided coating ($t\neq 0$ and $l=0$) and in the case of a two-sided coating ($t\neq 0$ and $l\neq 0$):

$$\omega_*^2 = \omega_0^2 + \omega_{me}^2 \frac{\tanh(k_\perp t)\coth(k_\perp d)}{c_{44}/\mu + \tanh(k_\perp t)\coth(k_\perp d)}, \quad t\neq l, \quad l=0, \tag{10}$$

$$\omega_{*\pm}^2 = \omega_0^2 + \omega_{me}^2 \frac{R_\pm}{c_{44}/\mu + R_\pm}, \quad t\neq l, \quad l\neq 0, \tag{11}$$

$$R_\pm = [\tanh(k_\perp t) + \tanh(k_\perp l)]\coth(k_\perp d) \pm \left\{ \frac{[\tanh(k_\perp t) + \tanh(k_\perp l)]^2 \coth^2(\kappa_\perp d)}{4} - \tanh(k_\perp t)\tanh(k_\perp l) \right\}^{1/2},$$

$$\omega_{me}^2 = \frac{\omega_E g_e B_2^2}{c_{44} M_0}, \quad \omega_0^2 = \frac{\omega_E g_e 2K}{M_0}.$$

We see that these magnetic excitations have no analog in an unbounded magnetic material and that their dispersive properties are dictated entirely by the Suhl–Nakamura interaction of the crystal phonon subsystem, since the range over which this type of nuclear spin-wave excitations can exist is solely determined by the size of the magnetic gap in the spectrum ω_{me} of a homogeneous AFM resonance. More than that, another factor that is important for the existence of the spectrum (10) and (11) is the presence of a nonmagnetic coating, since in the limit in which no such coating exists ($\mu \to 0$) the frequencies of the surface elastostatic nuclear spin waves transform into the frequencies of a homogeneous NMR of a thin free magnetic film whose thickness d satisfies the elastostatic criterion (1). Even in the case $t\neq 0$ and $l=0$ the relative thicknesses of the nonmagnetic coating t/d and l/d can have a strong effect on the nature of the dispersion curve for the propagating surface elastostatic nuclear spin waves. In particular, for $1/d \ll k_\perp \ll 1/t$ the dispersion of the surface elastostatic nuclear spin wave (10) corresponds to a forward wave ($\partial\omega/\partial k_\perp > 0$) and a backward wave ($\partial\omega/\partial k_\perp < 0$) for $1/t \ll k_\perp \ll 1/d$. Equations (10) and (11) suggest that in comparison to a one-side nonmagnetic coating a two-sided coating of a nonmagnetic doubles the number of branches in the spectrum of surface nuclear spin excitations. If under conditions (1) and in the same geometry of the problem ($\mathbf{n}\perp[001]\|\mathbf{u}\|\mathbf{L}\perp\mathbf{k}_\perp$) we examine the condition for formation of bulk elastostatic nuclear spin waves, we can easily see that for $\mathbf{k}\in xy$ and $u_x=u_y=0$ and irrespective of whether there is a nonmagnetic coating the spectrum of these waves in a film of a cubic AFM with a one- or two-sided nonmagnetic coating [the criterion (1)], for an arbitrary orientation of $\mathbf{n}$ in the (001) plane, has no dispersion in the current approximation and consists of two frequency-nondegenerate branches (even though the external magnetic field is zero[1)]):

$$\omega_*^2=\omega_0^2, \quad \omega_*^2=\omega_0^2+\omega_{me}^2. \tag{12}$$

To demonstrate that a nonmagnetic coating on the thin magnetic film may have a strong effect on the conditions of formation, not only of surface elastostatic nuclear spin waves but also of bulk elastostatic nuclear spin waves, we analyze the relative position of the vectors $\mathbf{u}$, $\mathbf{n}$, $\mathbf{k}_\perp$ in which even for the case of a separate film of a cubic AFM there can be both a forward bulk wave ($\partial\omega/\partial k_\perp>0$) and a backward bulk wave ($\partial\omega/\partial k_\perp<0$). To this end we examine the second variant of the main geometry of the relative position of the vectors $\mathbf{u}$, $\mathbf{k}_\perp$, and $\mathbf{L}$ analyzed within this problem: $\mathbf{k}_\perp\perp\mathbf{u}\perp\mathbf{L}$ ($\mathbf{L}\|[001]$). As a result, in the case of a one- or two-sided nonmagnetic coating the dispersion relations for bulk nuclear spin-wave excitations with $\mathbf{k}_\perp,\mathbf{n}\in xy$ can be written accordingly in the following form (here ϕ is the angle between $\mathbf{L}$ and $\mathbf{n}$):

$$\mu_\|\kappa+\tanh(k_\perp t)\coth(\kappa k_\perp d)=0, \quad t\neq 0, \quad l=0, \tag{13}$$

$$\mu_\|^2\kappa^2+\mu_\|\kappa[\tanh(k_\perp t)+\tanh(k_\perp l)]\coth(\kappa k_\perp d)$$
$$+\tanh(k_\perp t)\tanh(k_\perp l)=0, \quad t\neq 0, \quad l\neq 0, \tag{14}$$

where

$$\kappa^2=\frac{\mu_\perp\mu_\|-\mu_s^2}{\mu_\|^2},$$

$$\mu_\|=\frac{c_{44}}{\mu}\left[1-\frac{\omega_{me}^2\cos^2\phi}{\omega_{me}^2+\omega_0^2-\omega_*^2}\right],$$

$$\mu_\perp=\frac{c_{44}}{\mu}\left[1-\frac{\omega_{me}^2\sin^2\phi}{\omega_{me}^2+\omega_0^2-\omega_*^2}\right],$$

$$\mu_s=\frac{c_{44}}{\mu}\frac{\omega_{me}^2}{\omega_{me}^2+\omega_0^2-\omega_*^2}\sin 2\phi.$$

Thus, Eqs. (13) and (14) imply that there is a region in which this type of elastostatic nuclear spin waves exists for an arbitrary value of $k_\perp$ with $\omega_a^2\leqslant\omega_*^2\leqslant\omega_b^2$, where ω_a and ω_b are the roots of the equation $\kappa=0$. We can easily show that here the bulk excitations ($\kappa^2<0$) are formed with participation of only one type of polarization of the spin vibrations: $\mathbf{L}$, $\mathbf{l}\perp\mathbf{k}_\perp$. The case of a separate film in (13) and (14) corresponds to the limit $\mu\to 0$. The modes of the propagating bulk elastostatic nuclear spin waves form an infinite denumerable set. Analysis shows that the most significant changes introduced by a nonmagnetic coating in comparison with the case of a separate magnetic film with the same external parameters involve the mode of the spectrum of bulk elastostatic nuclear spin waves for which the component of the wave vector normal to the surface, k_n, obeys the following condition:

$$k_n d\ll 1. \tag{15}$$

This mode in the spectrum of bulk elastostatic nuclear spin waves and the vibrational mode with the same polarization but not satisfying (15) are of opposite types.[2)] For instance, when $\mathbf{n}\|\mathbf{L}$ holds, the mode that is quasihomogeneous in film thickness and belongs to the spectrum of bulk excitations is a wave of the backward type, while the dispersion laws of the other modes of the spectrum of nonexchange bulk elastostatic nuclear spin waves belong to waves of the forward type. The situation is opposite to that in the case $\mathbf{n}\perp\|[001]$. In both cases considered here the spectrum of the propagating bulk elastostatic nuclear spin waves not satisfying the condition (15) has one condensation point as $k_\perp\to 0$ and one condensation point as $k_\perp\to\infty$, which coincide, respectively, with ω_a and ω_b for $\mathbf{L}\|\mathbf{n}$ or with ω_b and ω_a for $\mathbf{L}\perp\mathbf{n}$. But if the relative orientation of the vectors $\mathbf{n}$ and $\mathbf{L}$ coincides with neither of the above particular cases, then, as Eqs. (13) and (14) imply, the spectrum of nonexchange bulk elastostatic nuclear spin waves in the situation considered here consists of two frequency bands having which, when the condition (15) is violated, have a common condensation point in the limit $k\to 0$ that lies between ω_a and ω_b and two different condensation point (ω_a and ω_b) as $k\to 0$. In this case the dispersive properties of the modes of the bulk elastostatic nuclear spin waves that belong to the upper band do not differ qualitatively from those discussed above for the case $\mathbf{n}\|\mathbf{L}$, while the distribution of wave types among the modes of bulk excitations for the lower band coincides with that discussed for the case where $\mathbf{n}\perp\mathbf{L}\|[001]$. The presence of a nonmagnetic coating on the surface of the film leads to a situation in which, in contrast to an isolated AFM film under the conditions (15), the upper mode of elastostatic nuclear spin waves is a backward wave, while the lower mode is a forward wave. The combined analysis of (13)–(15) suggests that for $\mu_s\neq 0$ or $\mu_s=0$, $\mu_\perp(\omega)\neq\text{const}$, the frequency ranges in which the mode that is quasihomogeneous in the thickness of the magnetic film and belongs to bulk elastostatic nuclear spin waves exists in the multilayered structure strongly depend on the ratio of the thicknesses of the magnetic film proper and its nonmagnetic coating. The corresponding frequency limit for the mode in the spectrum for the case of a two-sided nonmagnetic coating of an AFM film in the limit $\kappa_\perp\to 0$ and with allowance for (13) and (14) is specified by the following expression:

$$\mu_s^2-\mu_\perp\mu_\|+\frac{\mu_\|(t+l)}{d}=0. \tag{16}$$

For $\mathbf{n}\|\mathbf{z}$ and $\mathbf{k}\in xz(\mu_s=0$ and $\mu_\omega=\text{const})$, the frequency interval in which there exists a similar mode of the spectrum of bulk elastostatic nuclear spin waves [Eqs. (13) and (14)] is independent of the presence of a nonmagnetic coating for all modes of the spectrum:

$$\mu_s^2-\mu_\perp\mu_\|=0. \tag{17}$$

Now let us see how the structure of the spectrum of propagating surface and bulk elastostatic nuclear spin waves changes for the same types of relative geometry of the various vectors **u**, **L**, $\mathbf{k}_\perp$ ($\mathbf{u}\|\mathbf{L}\perp\mathbf{k}_\perp$ and $\mathbf{u}\perp\mathbf{L}\|\mathbf{k}_\perp$) when the equilibrium state of the antiferromagnetism vector in the multilayered structure corresponds to the phase with $\mathbf{L}\|[110]$ ($-K_2<K\leqslant 0$). As for the case $\mathbf{u}\perp\mathbf{L}\|\mathbf{k}_\perp$, we can easily show that in this situation and when the condition (1) is satisfied, only bulk excitations can propagate in the $(1\hat{1}0)$ plane ($\mathbf{u}\|[1\bar{1}0]\perp\mathbf{k}_\perp$).[3] Here the structure of the excitation spectrum differs little from that of the bulk elastostatic nuclear spin waves studied above in the same geometry for the phase $\mathbf{l}\|[001]$. As a result Eqs. (10)–(14) remain valid with allowance for the fact that now we have (here ϕ is the angle between **L** and [001])

$$\mu_\|=A_\xi\left(1-\frac{\omega_{me\xi}^2}{a_{\xi\xi}}\right)\cos^2\phi+A_\eta\sin^2\phi,$$

$$\mu_\perp=A_\xi\left(1-\frac{\omega_{me\xi}^2}{a_{\xi\xi}}\right)\sin^2\phi+A_\eta\cos^2\phi,$$

$$\mu_s=\left[-A_\xi\left(1-\frac{\omega_{me\xi}^2}{a_{\xi\xi}}\right)+A_\eta\right]\sin 2\phi, \qquad (18)$$

where

$$a_{\xi\xi}=\omega_{me\xi}^2+\omega_{0\xi}^2-\omega_*^2,\quad a_{\eta\eta}=\omega_{me\eta}^2+\omega_{0\eta}^2-\omega_*^2,$$

$$\omega_{0\xi}^2=\frac{\omega_E g_e}{M_0}\left(K+\frac{K_2}{2}\right),\quad \omega_{0\eta}^2=\frac{\omega_E g_e}{M_0}(-2K),$$

$$\omega_{me\xi}^2=\frac{\omega_E g_e B_2^2}{M_0 c_{44}},\quad \omega_{me\eta}^2=\frac{2\omega_E g_e B_1^2}{M_0(c_{11}-c_{12})},$$

$$A_\xi=\frac{c_{44}}{\mu},\quad A_\eta=\frac{c_{11}-c_{12}}{2\mu}.$$

As for the case $\mathbf{u}\|\mathbf{L}\perp\mathbf{k}_\perp$, with allowance for (18) the corresponding dispersion equations for all ω and $k_\perp$ satisfying (1) and for the vector **n** lying in the (001) plane have the form

$$\mu_\|\kappa+\tanh(k_\perp t)\coth(\kappa k_\perp d)=0,\quad t\neq 0,\quad l=0, \qquad (19)$$

$$\mu_\|^2\kappa^2+\mu_\|\kappa[\tanh(k_\perp t)+\tanh(k_\perp l)]\coth(\kappa k_\perp d)$$
$$+\tanh(k_\perp t)\tanh(k_\perp l)=0,\quad t\neq 0,\quad l\neq 0, \qquad (20)$$

where

$$\mu_\|=A_\xi\left(1-\frac{\omega_{me\xi}^2}{a_{\xi\xi}}\right)\cos^2\phi+A_\eta\left(1-\frac{\omega_{me\eta}^2}{a_{\eta\eta}}\right)\sin^2\phi,$$

$$\mu_\perp=A_\xi\left(1-\frac{\omega_{me\xi}^2}{a_{\xi\xi}}\right)\sin^2\phi+A_\eta\left(1-\frac{\omega_{me\eta}^2}{a_{\eta\eta}}\right)\cos^2\phi,$$

$$\mu_s=\left[-A_\xi\left(1-\frac{\omega_{me\xi}^2}{a_{\xi\xi}}\right)+A_\eta\left(1-\frac{\omega_{me\eta}^2}{a_{\eta\eta}}\right)\right]\sin 2\phi.$$

Thus, the combined analysis of (18)–(20) demonstrates that in the phase $\mathbf{L}\|[110]$ for $\kappa^2>0$ there can be surface elastostatic nuclear spin waves similar to those found for the case $\mathbf{L}\|[001]$, but the range of existence of these surface waves depends significantly on the relative orientation of **n** and the direction [001] in the (110) plane. In particular, for certain values of the magnetoelastic and elastic parameters such surface elastostatic nuclear spin waves may be entirely absent for a given relative orientation of **n** and [001] ($\mu_\|\geqslant 0$). Still greater changes in the phase $\mathbf{L}\|[110]$ in comparison to (10)–(12) are inflicted on the spectrum of propagating bulk elastostatic nuclear spin waves ($\mathbf{u}\|\mathbf{L}\perp\mathbf{k}_\perp$). Comparison of (13) and (14) with (18)–(20) shows that here, in contrast to the case with $\mathbf{L}\|[001]$, the equilibrium orientation of the antiferromagnetism vector **L** (**l**) along the diagonal of a cube face stimulates the formation of an additional rhombic anisotropy for waves with $\mathbf{u}\|\mathbf{L}\perp\mathbf{k}_\perp$. In view of this, this Suhl–Nakamura phonon exchange mechanism under conditions (1) and $\mathbf{L}\|[110]$ allows propagation of an entirely new class of bulk anisotropic elastostatic nuclear spin waves with dispersion, in contrast to the bulk excitation of the phase $\mathbf{L}\|[001]$ [Eq. (12)] discussed above and having the same relative orientation of the vectors **u**, **L**, and $\mathbf{k}_\perp$. The spectrum of this class of anisotropic bulk elastostatic nuclear spin waves for an arbitrary value of $|\mathbf{k}_\perp|$ is localized in the $\omega k_\perp$ plane in the form of two nonoverlapping bands: the low-frequency band $\omega_a^2<\omega_*^2<\omega_b^2$ and the high-frequency band $\omega_c^2<\omega_*^2<\omega_d^2(\omega_b^2<\omega_c^2$ and $\omega_{a,b,c,d}$ are the roots of the equation $\kappa^2=0$). The types of waves for the propagating bulk modes with the same label ν but belonging to, respectively, the low- and high-frequency bands, are opposite. Moreover, because of the presence of a nonmagnetic coating on the surface of the magnetic film, the high- and low-frequency modes that are quasihomogeneous in the thickness of the magnetic film, satisfy the condition (15), and belong to the spectrum of bulk elastostatic nuclear spin waves are of the type opposite to that of the other modes belonging to the same frequency band of the class of waves in question. If the collinear or perpendicular orientation of the vectors **n** of **L** is realized in this structure, each of the above bands of the spectrum of anisotropic bulk elastostatic nuclear spin waves contains one condensation point for $k_\perp\to 0$ and one condensation point for $k_\perp\to\infty$. Depending on the relative orientation of **n** and **L**, these points coincide with $\omega_{a,b,c,d}$. In what follows we label the quantities belonging to the high- and low-frequency bands of the spectrum by plus or minus signs, respectively. Then, for instance, in the case $\mathbf{n}\|[001]$ the dispersion laws for the high- and low-frequency modes of the spectrum of bulk elastostatic nuclear spin waves satisfying the condition (15) refer to the forward or backward waves if $\omega_{0\xi}^2<\omega_{0\eta}^2$ and $\omega_{me\xi}^2+\omega_{0\xi}^2>\omega_{me\eta}^2+\omega_{o\eta}^2$ hold simultaneously. Here for the modes of anisotropic elastostatic nuclear spin waves that do not satisfy condition (15) we have $\omega_+\to\omega_c$ and $\omega_-\to\omega_b$ in the limit $k\to 0$, while $\omega_+\to\omega_d$ and $\omega_-\to\omega_a$ in the limit $k_\perp\to\infty$. Another situation is possible in principle in the same geometry $\mathbf{n}\|[001]$: if condition (15) is not satisfied, the dispersion curves both in the upper and lower bands of the spectrum of bulk anisotropic elastostatic nuclear spin waves correspond to forward waves. Naturally, the modes that are quasihomogeneous as a function of the depth of the magnetic film and belong to the spectrum of bulk elastostatic nuclear spin waves are backard waves. The necessary condition here is that $\omega_{0\eta}<\omega_{0\xi}$ and $\omega_{me\xi}^2+\omega_{0\xi}^2$

$<\omega_{me\xi}^2+\omega_{0\eta}^2$ simultaneously. But if the angle between **n** and [001] is a right angle, then, with the same external parameters, for all the modes of the spectrum of bulk anisotropic elastostatic nuclear spin waves described by (18)–(20) the type of wave changes to the opposite in comparison to the above case of **n**‖[001]. An important feature of the spectrum of quasihomogeneous bulk anisotropic nuclear spin waves in a thin AFM film with a two-sided nonmagnetic coating (**L**‖[110]) is that for $k_\perp \neq 0$ the dispersion curves smoothly transform into the dispersion curve of surface excitations, which, as shown above, have the same orientation of the vectors **u**, **L**, and $\mathbf{k}_\perp$.

Especially interesting is the case where for $\mathbf{u}\|\mathbf{L}\perp\mathbf{k}_\perp$ the direction of **n** coincides neither with [001] nor with [110] ($\mu_s \neq 0$). In this case the nature of the spatial distribution of the long-range field of quasistatic magnetoelastic strains that realize the type of spin-wave excitations in question inside the magnetic field is described, with allowance for (18), by the following formulas:

$$u=A_+\exp[i(\kappa_+k_\perp\xi+\omega t)]+A_+\exp[i(\kappa_-k_\perp\xi+\omega t)],$$

$$\kappa_\pm=\frac{\mu_s}{\mu_\|}\pm\sqrt{\left(\frac{\mu_s}{\mu_\|}\right)^2-\frac{\mu_\perp}{\mu_\|}}. \qquad (21)$$

As a result, for $k_\perp \to 0$ the condensation points of the spectrum of anisotropic bulk elastostatic nuclear spin waves not satisfying the condition (15) in each of the two bands mentioned earlier do not coincide with the edges of the bands either for **n**‖[001] or for **n**⊥[001], while for $k_\perp \to \infty$ the condensation points of the spectrum of this type of elastostatic nuclear spin-wave bulk excitations coincide with $\omega_{a,b,c,d}$. Hence we can conclude that for $\phi \neq 0$ the presence of a nonmagnetic coating allows a smooth transition at $k_\perp \neq 0$ of the dispersion curve of a bulk elastostatic nuclear spin wave ($\kappa^2<0$) that is quasihomogeneous in film thickness, into the dispersion curve of a surface elastostatic nuclear spin wave [$(\kappa_+-\kappa_-)^2<0$ and $\mu_\|<0$]. Thus, if we still use the terminology adopted in the physics of magnetostatic waves, such nuclear spin-wave excitations must be called surface–bulk anisotropic elastostatic nuclear spin waves. An analysis of (18)–(21) shows that the presence of a nonmagnetic coating for $\phi \neq 0$ and $\pi/2$ gives rise to dispersion for all quasihomogeneous modes of the spectrum of bulk anisotropic elastostatic nuclear spin waves, which for $\phi=0$ or $\phi=\pi/2$ constitute two, and for $0<\phi<\pi/2$ four, nondispersive levels determined by the equation $\kappa^2=0$. The range of existence of these types of nuclear spin-wave excitations for fixed external parameters and $k_\perp \to 0$ depends on the relative thickness of the magnetic and nonmagnetic layers. The corresponding frequency limits are determined by the system of Eq. (16) and (18). When $(\kappa_+-\kappa_-)^2<0$ holds, nonexchange surface elastostatic nuclear spin waves similar to those mentioned earlier can propagate in the system, a fact that has been already noted. The corresponding dispersion relations generalizing (10) and (11) to the case of a rhombic anisotropy in the phase **L**‖[110] can easily be obtained via (18)–(21), provided that $(\kappa_+-\kappa_-)^2<0$. An important feature of the given spectrum of nuclear spin-wave excitations is that in the elastostatic limit (1) considered here, for a two-sided nonmagnetic coating, both the high- and low-frequency modes that are quasihomogeneous in the thickness d of the magnetic film and belong to the spectrum of bulk nuclear spin-wave excitations smoothly transform at $k_\perp = k_{*\pm}$ into the high- and low-frequency modes of the spectrum of surface nuclear spin-wave excitations of the hybrid structure considered here. But if the film of a cubic AFM has a nonmagnetic coating only on one side, then, as in the the case with **L**‖[001], there is only one branch of surface nuclear spin waves of the elastostatic type, and for $k_\perp d \ll 1$ the dispersion curve of this branch is smoothly transformed (at $k_\perp = k_*$) into the dispersion curve corresponding to a low-frequency mode of the bulk nuclear spin-wave excitations of the elastostatic type considered above.

From the standpoint of analytical calculations, the most complicated phase is **L**‖[111] ($-K \geq K_2/3$). Employing the restrictions on the relative orientation of the vectors **u**, **L**, and $\mathbf{k}_\perp$ adopted earlier, we can easily show that of the two cases considered here, $\mathbf{k}_\perp \perp \mathbf{u}_\perp$ and $\mathbf{u}\|\mathbf{L}\perp\mathbf{k}_\perp$, only the first is realized in the given phase. Here, as in the phases with **L**‖[001] or **L**‖[110], only bulk ($\kappa^2<0$) or only surface ($\kappa^2>1$ and $\mu_\|<0$) elastostatic nuclear spin waves can be realized. Now, however, for **L**‖**n** or **L**⊥**n** the corresponding dispersion equation has the form (19) or (20) with

$$\mu_\|=\left(c_{44}-\frac{\omega_E B_2^2}{3M_0\Delta}\right)\frac{1}{\mu}, \quad \Delta=\omega_{me}^2+\omega_0^2-\omega_*^2,$$

$$\mu_\perp=\left(c_{11}-c_{12}-\frac{\omega_E 2B_1^2}{3M_0\Delta}\right)\frac{1}{\mu}, \quad \mu_s=-\frac{2\sqrt{2}\omega_E B_1 B_2}{3\mu M_0\Delta}, \qquad (22)$$

$$\omega_0^2=\omega_E g_e\left(-\frac{4K_*}{3}\right)M_0, \quad K_*=K+\frac{K_2}{3},$$

$$\omega_{me}^2=\omega_E g_e\left(\frac{4B_1^2}{c_{11}-c_{12}}+\frac{B_2^2}{c_{44}}\right)\frac{1}{3M_0}.$$

Thus, in contrast to the earlier cases of **L**‖[001] or **L**‖[1$\bar{1}$0], for a strictly collinear or perpendicular relative orientation of the vectors **L** and **n** and a fixed value of the wave number $|\mathbf{k}_\perp|$ there can be a surface ($\kappa^2<0$) or bulk ($\kappa>0$) elastostatic nuclear spin wave of the forward or backward type propagating in the film of a cubic AFM in the phase **L**‖[111] for the same external parameters ($\mathbf{k}_\perp \perp \mathbf{u} \perp \mathbf{L}$).

Now let us see what an effect an external magnetic field H that does not alter the spatial orientation of the equilibrium order parameter **L**‖[001] will have on the elastostatic spin dynamics of a film of a cubic AFM with a nonmagnetic coating (**H**⊥[001] or **H**‖[001], and $H<H_{EA}$, where H_{EA} is the field strength inducing a spin-flip transition). It is well known that the case **H**⊥**L** is reduced to the formation in the AFM crystal of additional magnetic anisotropy in the plane with the normal along the equilibrium vector **L** (see Ref. 12). As a result, if we ignore demagnetization effects (in AFM crystals they are weakened by exchange effects[5]), then, from the mathematical standpoint, with **H**⊥**L** the dispersion relations for a propagating elastostatic nuclear spin wave of the bulk or surface type already in the phase **L**‖[001] do not differ structurally from those studied earlier in the phase

$\mathbf{L}\|[110]$ with $H=0$ [Eqs. (19) and (20)]. But now the relationships (18) assume the form (ϕ is the angle between $\mathbf{H}$ and [001])

$$\mu_{\|}=\frac{c_{44}}{\mu}\left(1-\frac{\omega_{me}^2 a_{\eta\eta}}{\Delta}\cos^2\phi-\frac{\omega_{me}^2 a_{\xi\xi}}{\Delta}\sin^2\phi+\frac{\omega_{me}^2 a_{\xi\eta}}{\Delta}\sin 2\phi\right),$$

$$\mu_{\perp}=\frac{c_{44}}{\mu}\left(1-\frac{\omega_{me}^2 a_{\eta\eta}}{\Delta}\sin^2\phi-\frac{\omega_{me}^2 a_{\xi\xi}}{\Delta}\cos^2\phi-\frac{\omega_{me}^2 a_{\xi\eta}}{\Delta}\sin 2\phi\right),$$

$$\mu_s=\frac{c_{44}}{\mu\Delta}[(a_{\eta\eta}-a_{\xi\xi})\omega_{me}^2\sin 2\phi+\omega_{me}^2 a_{\xi\eta}\cos 2\phi], \tag{23}$$

$$a_{\xi\eta}=\omega_{H_\xi}\omega_{H_\eta},\quad a_{\eta\eta}=\omega_{me}^2+\omega_0^2+\omega_{H_\eta}^2-\omega_*^2,$$

$$\omega_{H_i}\equiv g_e H_i,\quad \Delta=a_{\eta\eta}a_{\xi\xi}-a_{\xi\eta}^2,$$

$$a_{\xi\xi}=\omega_{me}^2+\omega_0^2+\omega_{H_\xi}^2-\omega_*^2.$$

Thus, when an external magnetic field $\mathbf{H}\perp\mathbf{L}$ is introduced into the picture, even in the $\mathbf{L}\|[001]$ phase of a thin film of a cubic AFM there can be bulk and surface–bulk anisotropic elastostatic nuclear spin waves, which were found earlier in the phase $\mathbf{L}\|[110]$. The spectrum of these waves, as in the case of $\mathbf{L}\|[110]$ ($\mathbf{u}\|\mathbf{L}\perp\mathbf{k}_\perp$), has a two-band structure. However, in the phase $\mathbf{L}\|[110]$ the formation of such waves was related to the symmetry properties of the phase, i.e., was spontaneous, to use the terminology common in the physics of spin-flip phase transitions.[13] In the present case ($\mathbf{H}\perp\mathbf{L}\|[001]$), however, the formation of the surface, bulk, and surface–bulk types of nonexchange anisotropic elastostatic nuclear spin waves is induced by the external magnetic field. As a result such anisotropic elastostatic nuclear spin waves can referred to as induced spin-wave excitations. The presence in the phase $\mathbf{L}\|[001]$ of a magnetic field $\mathbf{H}\|[001]$ leads to the following dispersion equation for $\mathbf{L}\|\mathbf{u}\perp\mathbf{k}_\perp\in xy$, which in the nonexchange approximation determines the spectrum of propagating elastostatic surface nuclear spin waves ($\mathbf{p}\equiv\mathbf{k}_\perp/|\mathbf{k}_\perp|$):

$$\mu_{\|}^2+\mu_{\|}\tanh(k_\perp t)\coth(k_\perp d)+p\mu_*\tanh(k_\perp t)-\mu_*^2=0,\quad t\neq 0,\quad l=0, \tag{24}$$

$$\mu_{\|}^2+\mu_{\|}[\tanh(k_\perp t)+\tanh(k_\perp l)]\coth(k_\perp t)+\tanh(k_\perp t)\tanh(k_\perp l)-p\mu_*[\tanh(k_\perp t)-\tanh(k_\perp l)]-\mu_*^2=0,\quad t\neq 0,\quad l\neq 0, \tag{25}$$

$$\mu_{\|}=\frac{c_{44}}{\mu}\frac{(\omega_m^2-\omega_*^2)(\omega_m^2-\omega_*^2-\omega_{me}^2)-4\omega_*^2\omega_H^2}{(\omega_m^2-\omega_*^2)^2-4\omega_*^2\omega_H^2},$$

$$\mu_*=\frac{c_{44}}{\mu}\frac{\omega_{me}^2\omega_*\omega_H}{(\omega_m^2-\omega_*^2)^2-4\omega_*^2\omega_H^2},\quad \omega_m^2=\omega_{me}^2+\omega_H^2+\omega_0^2. \tag{26}$$

Analysis of (24)–(26) shows that in comparison to the case $H=0$ considered earlier [Eqs. (10) and (11)], introducing a magnetic field $\mathbf{H}$ parallel to the z axis gives rise to the following additional features in the spectrum of surface nuclear spin-wave excitations of a bounded magnetic material with a nonmagnetic coating: (1) nonreciprocity in the spin-wave spectrum for $t\neq l$: $\omega_*(k_\perp)\neq\omega_*(-k_\perp)$; (2) doubling of the number of branches [in comparison to (11)] in the spectrum of surface nuclear spin waves in the case of a one-sided nonmagnetic coating; and (3) the appearance on the dispersion curves of sections with $\partial\omega_*/\partial k_\perp=0$ for $k_\perp\neq 0$. Note that because of the gyrotropic effect $\mu_a\neq 0$ the four branches of surface elastostatic nuclear spin waves also occur in a separate AFM film, but without allowance for a nonmagnetic coating this type of nonexchange elastostatic spin-wave excitations is nondispersive. If the frequencies of these levels are denoted in increasing order as $\omega_{1,2,3,4}$ (a separate magnetic film) and $\omega_{A,B,C,D}$ (a magnetic film with a nonmagnetic coating), we can easily show that the following chain of inequalities holds:

$$\omega_1^2<\omega_A^2<\omega_B^2<\omega_2^2<\omega_3^2<\omega_C^2<\omega_D^2<\omega_4^2. \tag{27}$$

Equations (24)–(26) imply that if in the case of a one-sided nonmagnetic coating ($t=\infty$ and $l=0$) we express $k_\perp$ as a function of the frequency ω for elastostatic surface nuclear spin waves with $\kappa_\perp>0$ [$k_\perp=k_+(\omega)$] and $\kappa_\perp<0$ [$k_\perp=k_-(\omega)$], respectively, we can easily show that in the entire frequency range of excitations in which both $k_+(\omega)$ and $k_-(\omega)$ are realized simultaneously,

$$k_+(\omega)+k_-(\omega)=k_0(\omega), \tag{28}$$

where $k_0(\omega)=k_\perp(\omega)$ is the solution of the system of equations (24)–(26) when $t=l=\infty$. Let us assume that in addition to $H_z\neq 0$ we also have $H_{x,y}\neq 0$. This, bearing in mind what we have just said, we can express the structure of the spectrum of nonexchange elastostatic nuclear spin waves in the phase $\mathbf{L}\|[001]$ for an arbitrary relative orientation of $\mathbf{n}$ and $\mathbf{L}$ by the following dispersion relation:[4)]

$$\mu_{\|}^2\kappa^2+\mu_{\|}\kappa\tanh(k_\perp t)\coth(\kappa k_\perp d)+p\mu_*\tanh(k_\perp t)-\mu_*^2=0,\quad t\neq 0,\quad l=0, \tag{29}$$

$$\mu_{\|}^2\kappa^2+\mu_{\|}\kappa[\tanh(k_\perp t)+\tanh(k_\perp l)]\coth(k_\perp t)+\tanh(k_\perp t)\tanh(k_\perp l)+p\mu_*[\tanh(k_\perp t)-\tanh(\kappa k_\perp d)]-\mu_*^2=0,\quad t\neq 0,\quad l\neq 0, \tag{30}$$

$$\mu_{\|}=\frac{c_{44}}{\mu}\left(1-\frac{\omega_{me}^2 a_{\eta\eta}}{\Delta}\cos^2\phi-\frac{\omega_{me}^2 a_{\xi\xi}}{\Delta}\sin^2\phi+\frac{\omega_{me}^2 a_{\xi\eta}}{\Delta}\sin 2\phi\right),$$

$$\mu_{\perp}=\frac{c_{44}}{\mu}\left(1-\frac{\omega_{me}^2 a_{\eta\eta}}{\Delta}\sin^2\phi-\frac{\omega_{me}^2 a_{\xi\xi}}{\Delta}\cos^2\phi-\frac{\omega_{me}^2 a_{\xi\eta}}{\Delta}\sin 2\phi\right),$$

$$\mu_s=\frac{c_{44}}{\mu\Delta}[(a_{\eta\eta}-a_{\xi\xi})\omega_{me}^2 \sin 2\phi+\omega_{me}^2 a_{\xi\eta} \cos 2\phi],$$

$$\mu_*=\frac{\omega\omega_{H_z}\omega_{me}^2 c_{44}}{\mu\Delta}, \quad \Delta=a_{\xi\xi}a_{\eta\eta}-4\omega^2\omega_{H_z}^2-a_{\xi\eta}^2,$$

$$a_{\xi\eta}=\omega_{H_\xi}\omega_{H_\eta}, \quad a_{\eta\eta}=\omega_{me}^2+\omega_0^2+\omega_{H_\eta}^2+\omega_{H_z}^2-\omega_*^2,$$

$$a_{\xi\xi}\omega_{me}^2+\omega_0^2+\omega_{H_\xi}^2+\omega_{H_z}^2-\omega_*^2.$$

In addition to the nonreciprocity effect $[\omega_*(k_\perp)\neq\omega_*(-k_\perp)]$, a new feature of anisotropic elastostatic nuclear spin waves, compared to the case of $H_z=0$, is that for $\phi\neq 0$, $\pi/2$ the dispersion laws of all four branches of the spectrum of the mode that is quasihomogeneous in film thickness and belongs to bulk anisotropic waves ($\kappa^2<0$) for $k_\perp\neq 0$ are smoothly transformed into the dispersion curves for surface nuclear spin waves ($\kappa^2>0$ and $\mu_\parallel<0$). Hence in this case all four branches are surface–bulk anisotropic elastostatic nuclear spin waves, i.e., their number for $H_z\neq 0$ doubled. Note that here we examine only those equilibrium orientations of the antiferromagnetism vector **L** that correspond to allowing for only the first and second magnetic anisotropy constants in the thermodynamic potential of a cubic AFM. At the same time, from Ref. 14 it follows that if we allow for higher-order invariants (in particular, those related to the third anisotropy constant), "angular" phases form in a cubic AFM already in a zero magnetic field, just as they do in a cubic ferromagnet. In this case the equilibrium antiferromagnetism vector may lie in planes with the normal directed either along [001] or along $[1\bar{1}0]$, not coinciding with any of the above directions. We can easily show that the results of this analysis of the conditions of existence of elastostatic nuclear spin waves can be generalized in a natural manner to this case, too.

4. ACOUSTIC RETARDATION EFFECTS

Up to this point we have assumed that the elastostatic criterion (1) is valid, which means we ignored acoustic retardation effects. Since analysis shows that the main effects related to allowing for the finiteness of the speed of sound do not change drastically when the equilibrium orientation of the antiferromagnetism vector changes, in this section we examine the manifestation of this effect using the example of the phase $\mathbf{L}\|[001]$ for $|\mathbf{H}|\neq 0$. Calculations show that allowing for acoustic retardation in the magnetic film ($s_1=c_{44}/\rho_1<\infty$) and in the nonmagnetic coating ($s_2=\mu/\rho_2<\infty$) in the nonexchange approximation $ck_\perp/\omega_*\to 0$ with the same values of $\mu_\parallel$, $\mu_\perp$, and μ_s leads to the following generalization of the dispersion equations (29) and (30):

$$\mu_\parallel^2\kappa^2+\mu\kappa\gamma \tanh(\gamma k_\perp t)\coth(\kappa k_\perp d)+p\mu_*\gamma \tanh(\gamma k_\perp t) -\mu_*^2=0, \quad t\neq 0, \quad l\neq 0, \tag{31}$$

$$\mu_\parallel^2\kappa^2+\mu\kappa\gamma[\tanh(\gamma k_\perp t)+\tanh(\gamma k_\perp l)]\coth(\kappa k_\perp d) +p\mu_*\gamma[\tanh(\gamma k_\perp t)-\tanh(\gamma k_\perp l)] +\gamma^2 \tanh(\gamma k_\perp t)\tanh(\gamma k_\perp l)-\mu_*^2\gamma^2=0, \quad t\neq 0, \quad l\neq 0, \tag{32}$$

$$\kappa^2=\frac{\mu_\perp\mu_\parallel-\mu_s^2}{\mu_\parallel^2}-\frac{\omega^2}{s_1^2\mu_\parallel k_\perp^2}, \quad \gamma^2=1-\frac{\omega^2}{s_2^2k_\perp^2}.$$

By analyzing Eqs. (31) and (32) we can study all the main consequences of the effect of the finiteness of the speed of propagation of acoustic vibrations in the magnetic or nonmagnetic medium on the dispersive properties of the above elastostatic types of nuclear spin waves established. Since the region of magnetoacoustic resonance has been thoroughly studied, we assume that the thickness $d\gg a$ (where a is the lattice constant) of the magnetic film is so small that the following criterion (similar to the elastostatic criterion) holds: $\omega_*d\ll s_1$. In this case we employ the following hierarchy of approximations that has proved convenient for analysis. In the first stage we ignore the finiteness of the speed of sound in the nonmagnetic medium ($s_1\to\infty$), since we will see that allowing for the speed $s_1<\infty$ has the most profound effect on the spectrum of the types of elastostatic nuclear spin waves studied above. Analysis shows that from the standpoint of varying the shape of the dispersion curve the presence of acoustic retardation in AFM is most important for the spectrum of the bulk and surface elastostatic nuclear spin waves that are quasihomogeneous in film thickness. The main features of the spectrum of surface and bulk nuclear spin waves related to allowing for acoustic retardation can be formulated as follows.

(1) If the elastostatic spin wave in the range of wave vectors determined by the condition $k_\perp\approx\omega_n/s_1$ corresponds to a backward wave, then the dispersion curve for the surface elastostatic nuclear spin waves or the bulk elastostatic nuclear spin waves satisfying the condition (15) may acquire a peak in the specified range of wave vectors $k_\perp$.

(2) Allowing for acoustic retardation in a magnetic medium gives rise to dispersion in nondispersive levels of surface elastostatic nuclear spin waves even for a separate AFM film in the case $\mathbf{H}\|\mathbf{L}\perp\mathbf{n}$.

(3) If we examine the formal limit $k_\perp d\to\infty$, we can conclude that in this case the dispersive properties of surface elastostatic nuclear spin waves far from the region of nuclear magnetoacoustic resonance are determined by "retarding terms" algebraic in $\omega/s_1k_\perp$, which of course dominate the exponential terms determining the dispersion law (10) and (11) in the elastostatic limit (1).

(4) For the case of the spectrum of bulk elastostatic nuclear spin waves with $\mathbf{L}\|\mathbf{H}\|[001]\perp\mathbf{n}$, Eqs. (31) and (32) imply that allowing for a speed $s_1<\infty$ induces a dispersion in this type of magnetic excitations. As a result the corresponding spectrum of elastostatic nuclear spin waves in a thin magnetic film with a nonmagnetic coating consisting of two separate dispersion-free levels becomes an infinite denumerable set of modes of bulk nuclear spin-wave vibrations, $\omega_{*\nu}=\omega_{*\nu}(k_\perp)$, $\nu=1,2,3\ldots$.

(5) In the anisotropic case $\mathbf{H}\perp\mathbf{L}$ comparison of (10) and (11) shows that in such a geometry a new type of surface nuclear elastic wave, absent in the elastostatic limit (1), may form.[5] The condition for the existence of such a wave is given by the following inequalities:

$$k_\perp^2>\frac{\omega^2}{s_2^2},\quad \mu_\|<0,\quad \mu_\perp>0,\quad |\mu_\|\mu_\perp-\mu_s^2|<\frac{\omega^2|\mu_\||}{(s_1k_\perp)^2}. \quad (33)$$

We can easily show that as $\kappa_\perp t/d\to\infty$ and $k_\perp l/d\to\infty$ the corresponding dispersion relation can be written

$$k_\perp^2=\frac{(\mu_\|/s_1-1/s_2)\omega^2}{\mu_\|\mu_\perp-\mu_s^2-1}. \quad (34)$$

(6) An additional contribution to the general spectrum of magnetoelastic vibrations of this structure, also related to the finiteness of the speed of propagation of elastic vibrations in a magnetic medium, is the formation of a high-lying series of magnetoelastic vibrations caused by the bulk elastic modes of the magnetic layer.

(7) When $s_2<s_1$ holds, allowing for the acoustic retardation effect leads to the formation of a line of finite width for all the above types of elastostatic nuclear spin waves. This line arises because, as the nuclear spin waves propagate along the boundary between the magnetic and nonmagnetic media, they generate a bulk elastic wave in the nonmagnetic layer at the surface of the film. Actually, all the above-mentioned types of elastostatic spin waves that satisfy the condition $\omega_*<s_2k_\perp$ $(s_2<s_1)$, due to the effect of acoustic retardation in the nonmagnetic substrate, are resonant spin-wave states of an elastostatic nuclear spin wave superposed on the continuous spectrum of bulk elastic vibrations in the nonmagnetic coating of the AFM film. If the thickness of this layer is such that the spectrum of its bulk elastic vibrations is discrete, it is natural to assume that the broadening of the line of the travelling elastostatic nuclear spin wave is a nonmonotonic function of the wave number $k_\perp$.

5. EFFECTS OF INHOMOGENEOUS EXCHANGE

Another limit in which we considered the formation of elastostatic types of nuclear spin-wave excitations is $c^2\to 0$, which, as noted earlier, corresponds to ignoring inhomogeneous exchange. In this case, too, the features related to the effect of inhomogeneous exchange on the spectrum can be exhibited by a specific example if the elastostatic nuclear spin wavelength is so small that we can ignore the effect of acoustic retardation on the spin dynamics of the system $(s_{1,2}\to\infty)$. In this example we allow for the inhomogeneous exchange interaction and consider the case of a uniformly magnetized thin AFM film with a two-sided nonmagnetic coating $(t=l)$, assuming that in equilibrium $\mathbf{L}\|[100]$ holds and that the spins on the surface of the magnetic film are free. If we have $\mathbf{k}_\perp\perp\mathbf{u}\perp\mathbf{L}$, then both for $\mathbf{L}\|\mathbf{n}$ and for $\mathbf{L}\perp\mathbf{n}$ the characteristic equation is biquadratic in the component k_n of the wave vector $k_\perp$ of nuclear spin vibrations that is normal to the surface. Using the symmetry of the problem, we can separately write the dispersion equations that characterize the spectrum of symmetric and skew-symmetric one-particle nuclear spin-wave excitations in the thin magnetic film with allowance for inhomogeneous exchange and the indirect spin–spin interaction via the field of virtual elastostatic phonons:

$$\begin{aligned}
&\frac{q_1^2}{a_1}\tan\frac{q_1d}{2}\left[\mu_\|^{(2)}q_2\cot\frac{q_2d}{2}+\tanh(k_\perp t)\right]\\
&\quad=\frac{q_2^2}{a_2}\tan\frac{q_2d}{2}\left[\mu_\|^{(1)}q_1\cot\frac{q_1d}{2}+\tanh(k_\perp t)\right],\\
&\frac{q_1^2}{a_1}\cot\frac{q_1d}{2}\left[\mu_\|^{(2)}q_2\tan\frac{q_2d}{2}+\tanh(k_\perp t)\right]\\
&\quad=\frac{q_2^2}{a_2}\cot\frac{q_2d}{2}\left[\mu_\|^{(1)}q_1\tan\frac{q_1d}{2}+\tanh(k_\perp t)\right],
\end{aligned} \quad (35)$$

$$\mu_\|^{(1)}=\frac{c_{44}(a_1-\omega_{me}^2)}{\mu a_1},\quad \mu_\|^{(2)}=\frac{c_{44}(a_2-\omega_{me}^2)}{\mu a_2},\quad \mathbf{L}\|\mathbf{l}\|\mathbf{n},$$

$$\mu_\|^{(1)}=\mu_\|^{(2)}=\frac{c_{44}}{\mu},\quad \mathbf{L}\|\mathbf{l}\perp\mathbf{n},$$

$$a_i\equiv\omega_{me}^2+\omega_0^2+c^2(k_\perp^2+q_i^2)-\omega_*^2.$$

Here $q_{1,2}^2\equiv k_n^2$ are the roots of the following characteristic equation:

$$q^4+q^2\left(2k_\perp^2-\frac{\omega_*^2-\omega_0^2}{c^2}\right)+k_\perp^2\frac{c^2k_\perp^2-\omega_*^2+\omega_0^2+\omega_{me}^2}{c^2}=0,$$

$\mathbf{L}\|\mathbf{l}\|\mathbf{n}$,

$$q^4+q^2\left(2k_\perp^2-\frac{\omega_*^2-\omega_0^2-\omega_{me}^2}{c^2}\right)+k_\perp^2\frac{c^2k_\perp^2-\omega_*^2+\omega_0^2}{c^2}=0, \quad (36)$$

$\mathbf{L}\|\mathbf{l}\perp\mathbf{n}$.

The two-part structure of the nuclear spin vibrations propagating along the film is related to the simultaneous existence in the film of two mechanisms of spin–spin interaction mentioned earlier, the direct Heisenberg mechanism and the indirect phonon mechanism. The main consequences of the effect of the inhomogeneous exchange interaction in the electron spin subsystem on the spectrum of the nuclear spin-wave excitations of the thin magnetic field are as follows.

(1) For $k_\perp\neq 0$, formation of a minimum on the dispersion curves corresponding to the lowest-order modes of the spectrum of bulk elastoexchange nuclear spin waves propagating along the film. The necessary condition for the existence of such a minimum for a mode of a given order ν $(\eta=1,2,3,4)$ is

$$\left(\frac{\pi\nu}{d}\right)^2<\frac{\omega_{me}^2}{c^2},\quad \mathbf{L}\perp\mathbf{n}. \quad (37)$$

An analysis based on (36) of the partial amplitudes of the vibrations comprising the given wave shows that this type of magnetic vibration is the result of interference of the bulk and surface components of the spin vibrations and therefore is not realized in an unbounded magnetic material. For $d\to\infty$ the dispersion law coincides with the earlier dispersion law for the exchange type of nuclear spin waves.[2–4]

(2) If the lower modes of the spectrum of elasto-exchange bulk nuclear spin waves propagating along the film result from interference of two bulk partial waves in a bounded magnetic material, then from (35) and (36) it follows that a new type of inhomogeneous nuclear spin–spin resonance can occur in an AFM film. This resonance is caused by the possibility of elastostatic and exchange bulk nuclear spin waves propagating simultaneously for a given value of $k_{\perp}$. In the region where these waves interact resonantly, they cannot be separated into elastostatic and exchange bulk nuclear spin waves; rather, a new, elastoexchange, type of propagating bulk electron–nuclear spin wave determined by the equations in (35) and (36) arises.[6)] A forbidden frequency interval is formed in the resonant region, and in this interval the new type of bulk excitation cannot occur. A similar interval in the physics of magnetostatic vibrations is known as the "dipole gap."[17]

(3) There is a certain relationship between the above features of the spectrum of elastoexchange nuclear spin waves and the structure of frequency-equidistant magnon surfaces. The characteristic equation (36) implies that for elastoexchange nuclear spin vibrations in the case $\mathbf{k}\in xy$, $\mathbf{u}\|\mathbf{y}$ one can easily show that the equation of the constant frequency surface can be written in the following form ($k^2=k_x^2+k_z^2$ and $\tan^2\theta=k_x^2/k_z^2$):

$$c^2k^2(\theta)=\omega_*^2-\omega_0^2-\omega_{me}^2\sin^2\theta, \quad \text{for} \quad \omega_*\ll s_1k, \tag{38}$$

while the similar equation for $\omega_*\gg sk$ has the form

$$c^2k^2(\theta)=\omega_*^2-\omega_{me}^2-\omega_0^2. \tag{39}$$

Let us compare the range of existence and the types of nuclear spin-wave modes (38) and (39) with the conditions for existence of a section with negative curvature[18] on the constant-frequency surface (38). We see that if the normal to the surface of the thin magnetic film is positioned so that its direction corresponds to the point on the frequency-equidistant surface with the maximum possible negative curvature, then such a magnetic mode in the spectrum of the thin magnetic film corresponds to a minimum on the dispersion curve of the elasto-exchange nuclear spin wave. But if the direction at the surface of the wave vectors with the maximum possible negative curvature lies in the plane of the magnetic film, the corresponding propagating elasto-exchange nuclear spin wave is direct and the inhomogeneous nuclear spin–spin resonance described above can form for this wave. Thus, by studying the shape of the constant-frequency surface of normal nuclear spin waves in the model of an unbounded crystal one can specify the necessary conditions for observing the above effects in the spectrum of elasto-exchange bulk nuclear spin waves in a thin magnetic film. As the film becomes thicker, the spectrum of such direct bulk elasto-exchange nuclear spin waves becomes transformed into a resonant level superposed on the spectrum of the propagating bulk exchange nuclear spin waves. This effect is the result of interference of nuclear bulk spin waves in a thin magnetic film under conditions such that the bulk nuclear spin wave can undergo multibeam reflection without changing polarization.[19]

(4) The combined effect of the phonon and Heisenberg mechanisms of spin–spin exchange can lead, even in a bounded magnetic material, not only to resonant states [see item (2)] but also to bound spin-wave states in the spectrum of nuclear spin waves in a thin AFM film. By way of an example we study the effect of inhomogeneous exchange on the condition for formation and the dispersive properties of surface elastostatic nuclear spin waves in a thin AFM film with a two-sided nonmagnetic coating, assuming $\mathbf{L}\|[001]\|\mathbf{H}\perp\mathbf{n}$. Since now the system exhibits a gyrotropic effect ($\mu_*\neq 0$), it is impossible to separate the spectrum of the propagating elastostatic nuclear spin waves into symmetric and skew-symmetric vibrations, which naturally complicates analytical calculations. The effect of inhomogeneous exchange on the spectrum of surface elastostatic nuclear spin waves can be found by analyzing the structure of the partial vibrations comprising the given type of nuclear spin waves. As noted earlier, in this case the characteristic equation is bicubic in the component $k_n\equiv q$ of the wave vector of nuclear spin vibrations normal to the film surface. In the particular case studied here, however, its roots $q_\pm^2$ and q_*^2 can easily be found ($\omega_m^2=\omega_0^2+\omega_{me}^2+c^2k_\perp^2+\omega_H^2$):

$$q_\pm^2=\frac{q_a^2+q_b^2}{2}\pm\sqrt{\frac{(q_a^2-q_b^2)^2}{4}+q_c^4}, \quad q_*^2=-k_\perp^2,$$
$$q_a^2=\frac{\omega_*^2-\omega_m^2}{c^2}, \quad q_b^2=q_a^2+\frac{\omega_{me}^2}{c^2}, \quad q_c^4=\frac{4\omega_*^2\omega_H^2}{c^4}. \tag{40}$$

Analysis of (40) shows that under the conditions (1) one of the partial vibrations (q_*) cannot be a bulk vibration at any frequency ω, which corresponds to the nonexchange surface elastostatic nuclear spin wave studied earlier. As for the other two partial waves comprising the elasto-exchange nuclear spin waves ($q_\pm$), their formation is induced by the presence in this model of the Heisenberg exchange of electron spins, and at least in the range of existence of elastostatic nuclear spin waves one of these partial vibrations can be a bulk vibration. Thus, simultaneously allowing for the effect of the phonon mechanism of indirect spin–spin interaction and the Heisenberg exchange mechanism in the electron spin subsystem on the Suhl–Nakamura exchange in a bounded AFM can lead (for $q_\pm^2>0$) to the formation of a new type of nuclear spin-wave excitations, which [in contrast to the statement under item (2)] are a surface elastostatic nuclear spin wave superposed on the spectrum of exchange nuclear spin waves, or (for $q_\pm^2<0$) to the formation of a bound spin-wave state, a surface elasto-exchange nuclear spin wave.

Up to this point in analyzing the conditions of formation and the dispersive properties of an elastostatic nuclear spin wave we ignored the finite lifetime of a nuclear magnon in a real crystal, which implies that the frequency of a real magnon contains a nonzero imaginary component ($\Delta\omega$) in addition to the real part (ω_*) discussed earlier. This imaginary part ($\Delta\omega\neq 0$) is extremely important in experimental observations of the predicted effects in the nuclear spin dynamics of a thin magnetic field (primarily the multimode nature of the spectrum of bulk elasto-exchange nuclear spin waves and the conditions for the existence of an inhomogeneous nuclear spin–spin resonance). To estimate this effect for an arbitrary value of the wave vector $k_\perp$ in the plane of the field in the case of bulk elasto-exchange nuclear spin waves it is convenient to use Green's functions. This means we must introduce the Green's function

$$G(\xi,\tau)=\begin{cases}\dfrac{\sinh[k_\perp(\xi-\xi_0)]\sinh[k_\perp(\tau-\xi_d)]}{D}, & 0\leqslant\xi<\tau,\\ \dfrac{\sinh[k_\perp(\tau-\xi_0)]\sinh[k_\perp(\xi-\xi_d)]}{D}, & \tau<\xi\leqslant d,\end{cases} \tag{41}$$

$$D=k_\perp\sinh[k_\perp(\xi_d-\xi_0)],$$

$$\coth[k_\perp(d-\xi_d)]=-\frac{\mu}{c_{44}}\tanh(k_\perp t),$$

$$\coth(k_\perp\xi_0)=-\frac{\mu}{c_{44}}\tanh(k_\perp l),$$

which makes it possible to reduce the boundary-value elastic problem (5) and (6) for the elastostatic equations ($\mathbf{u}\perp\mathbf{k}_\perp$) to an appropriate Fredholm equation. Naturally, here we assume the magnetostriction stresses induced by the spin system of the crystal are simply fixed bulk forces. As a result, in the most general case among those considered here, the problem of solving the boundary-value problem for a system of three second-order differential equations in two components of $\mathbf{L}$ and one component of $\mathbf{u}$ and six boundary conditions reduces to solving two integro-differential equation involving only the components of $\mathbf{l}$ and only the exchange boundary conditions in (5). In the nonexchange limit ($c\to0$) these integro-differential equations become a system of two Fredholm equations. Thus, such a system of equations can be interpreted as an integral representation of the time-independent Schrödinger equation, in which the wave function is the two-component antiferromagnetism vector $\mathbf{L}$, while the perturbing potential is nonlocal and is determined by the Green's function of the boundary-value elastostatic problem introduced earlier. Hence all the above types of surface and intrinsic elastostatic nuclear spin waves are bound spin-wave states in a nonlocal potential formed by the long-range field of quasistatic magnetic stresses in the three-layered structure under investigation.

Using the Green's function G, in particular for $\mathbf{L}\|\mathbf{n}$, we see that if we allow for decay, the sufficient condition for experimentally resolving the modes with indices μ and ν belonging to the spectrum of bulk elastoexchange nuclear spin waves [Eqs. (35) and (36)], whose dispersion law satisfies (5) and (6), can be written for an arbitrary value of the wave vector $k_\perp$ and at $t=l$ as follows ($\mathbf{L}\|[001]$ and $\mathbf{u}\perp\mathbf{L}\perp\mathbf{k}_\perp$):

$$\Delta\omega<\frac{1}{\omega_{*\mu}+\omega_{*\nu}}\sqrt{\frac{(\omega_{*\mu}^2-\omega_{*\nu}^2)^2}{4}+R_{\nu\mu}^2\omega_{*\mu}^2\omega_{*\nu}^2},$$

$$\omega_{*\nu}^2=\omega_0^2+\omega_{me}^2\frac{k_\perp^2}{k_\perp^2+(\pi\nu/d)^2}+c^2\left[k_\perp^2+\left(\frac{\pi\nu}{d}\right)^2\right], \tag{42}$$

where $R_{\mu\nu}\sim\mu_1/\mu_2$. The above expression is accurate to within terms of order $(\mu_1/\mu_2)^2$, and the limit $R_{\mu\nu}=0$ corresponds to a thin AFM film on whose surface (which is rigidly fixed) the spins are completely free.

6. CONCLUSION

The results of the present work suggest that for thin AFM films with a nonmagnetic coating, including the effect of magnetoelastic interaction on the Suhl–Nakamura exchange in the nuclear spin subsystem leads to the formation of entirely new features in the nuclear spin dynamics of bounded magnetic materials. The types of spin-wave excitations established in the present paper have analogs neither in the model of unbounded magnetic materials nor in the case of a thin magnetic film of uniaxial AFM crystals. What is important for the formation of such spin-wave excitations is the symmetry properties of the AFM crystal together with the indirect spin–spin interaction via the long-range field of elastostatic phonons in the magnetic film proper and in its nonmagnetic coating. It must be stressed that none of these new types of propagating spin-wave excitations in the nuclear spin system of a bounded magnetic material have analogs in the spectrum of electron spin waves in the crystal. This also significantly distinguishes this class of nuclear spin excitations from the traditionally studied exchange types of nuclear spin waves.[2–4] The characteristic thicknesses of the AFM film d for which elastostatic nuclear spin waves of the types considered here can be observed are determined from the elastostatic condition $d\ll d_*=s/\omega_s$ (for $\omega\approx10^9$ Hz, $s_1\approx10^5$ cm s^{-1}, and $d_*\approx10^{-3}$ cm).

I am grateful to E. P. Stefanovskiĭ, A. L. Sukstanskiĭ, and A. N. Bogdanov for inspiration and fruitful discussions. The work was financially supported by the Ukrainian State Committee for Science and Technology.

[1)]Note that in the case of an unbounded cubic or easy-axis antiferromagnet and $H=0$ the spectrum of the exchange nuclear spin-wave excitation is twofold degenerate in frequency.[2–4]

[2)]Here and in what follows the dispersion curves of all the bulk elastostatic nuclear spin waves not satisfying condition (15) have a point of inflection at $k_\perp=k_*$. Hence when speaking of the type of wave in relation to such spin-wave excitations we always assume, for the sake of definiteness, the region $k_\perp<k_*$.

[3)]Qualitatively, the case $\mathbf{u}\|[001]\perp\mathbf{k}_\perp$ is no different from the one considered here.

[4)]Qualitatively, the structure of the spectrum of nuclear spin waves remains the same for $|\mathbf{H}|\neq0$ and for the case $\mathbf{L}\|[110]\perp\mathbf{n}$.

[5)]This type of nuclear-spin excitations is the magnetoelastic analog of "virtual" polaritons in crystal optics.[15]

[6)]These vibrations are the magnetoelastic analog of the magnetostatic waves of dipole-exchange spin-wave excitations, well-known in physics.[16]

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Translated by Eugene Yankovsky