An electrodynamic model of the time-dependent Josephson effect in a small bridge

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A model is used to show that the size of a bridge with Josephson properties is limited by the size of the vortex superconducting current, i.e., the magnetic-field penetration depth in the superconductor. Correspondingly, the size of a low-T_c bridge is limited by that of an Abrikosov vortex, and the size of a high-T_c bridge by that of a hypervortex. The microscopic theory of the effect in a low-T_c bridge smaller than the coherence length is developed with the necessary mathematical detail, and the physical meaning of the approximations and the results of solution are explained. © *1996 American Institute of Physics.* [S1063-7761(96)02108-7]

1. INTRODUCTION

Current steps induced by microwave irradiation have been observed on the current–voltage characteristic in experiments with high-T_c bridges with sizes ($\sim 100\,\mu$m) considerably greater than the London penetration depth for a magnetic field, λ_L, and the coherence length ξ_0 (see Ref. 1). The vanishing of the step amplitude, which oscillates as the microwave-radiation power varies, suggests that the current–phase dependence is sinusoidal. With low-T_c bridges such behavior has been observed only for bridge sizes comparable to ξ_0 (see Refs. 2 and 3). For a high-T_c ceramic interpreted as a system of superconducting granules joined by weak links, some researchers (see, e.g., Ref. 4) assume that only one intergranular Josephson link operates in a bridge. Amatuni *et al.*[4] relate the appearance of current steps to the synchronization of vortex motion in the bridge by external microwave radiation. The electrodynamic model developed in this paper explains the difference in the Josephson properties of low- and high-T_c bridge structures by the special way in which a magnetic field penetrates a high-T_c superconductor. The model is used to construct a theory of the nonstationary Josephson effect in a low-T_c bridge structure smaller than ξ_0.

2. MICROSCOPIC THEORY

The theory is based on the well-established and experimentally corroborated ideas about penetration by a magnetic field of a superconductor in the form of domains or Abrikosov vortices,[5] i.e., the normal (N) and superconducting (S) phases of the electron Fermi liquid are spatially separated. At current densities higher than the critical density j_c the superconducting current begins to oscillate in the bridge, which means that inside the bridge the current cannot be closed by the induced magnetic field, i.e., for bridges smaller than a vortex. Consequently, the size of a bridge with Josephson properties is limited by the characteristic size of a vortex. In low-T_c superconductors the vortex diameter is determined by the values of λ_L and ξ_0. When the Bose-condensate velocity reaches its critical value at the center of a bridge whose size is smaller than ξ_0, the order parameter drops to zero, as it does at the core of an Abrikosov vortex. Because of energy losses, the current flowing through the region with $\Delta=0$ creates a drop in electric voltage, V, which determines the variable Josephson current through the normal region. Note that in a high-T_c bridge whose size is comparable to the diameter of a hypervortex the normal region can be multilayered and multiply connected and can exist in the absence of a current due to the inhomogeneity of the material. Although the electrons inside the normal region are unpaired, the phase coherence of the Ψ function of the superconducting condensate of the different sides of the bridge is retained, thanks to collisionless electron transfer through the normal region similar to tunneling through the I-layer in a SIS junction. Analogous electrodynamic ideas were developed in Ref. 6, where $|\Delta|$ varies on the scale of the bridge size.

The model is used to formulate and solve the boundary-value problem for a bridge whose size is smaller than ξ_0 by means of the mathematical tools of the microscopic superconductivity theory. To simplify the reasoning we consider the bridge to be a circular hole of diameter d in an opaque flat screen between the two sides of a superconductor ($k=1,2$). The region of voltage drop ($|\mathbf{r}|\leq|\mathbf{r}_k|$) is limited by the surface determined from the condition $|j(\mathbf{r}_k)|=j_c$, where $\mathbf{r}$ is measured from the center of the bridge. Because of free exchange across the bridge the electron Fermi liquid, being a thermodynamic system, is characterized by a single chemical potential[7] $\mu(\mathbf{r})=p_F^2/2m=\text{const}$ and an order parameter

$$\Delta(\mathbf{r},t)=|\lambda F(\mathbf{r},t;\mathbf{r},t)|\exp\left\{i\int eV(-1)^k dt+i\varphi_k(\mathbf{r})\right\}. \tag{1}$$

For $|\mathbf{r}|>|\mathbf{r}_k|$ the electric potential is $U(\mathbf{r},t)=(-1)^k V/2$, where $V(t)$ is the voltage drop across the bridge slowly varying with time t (on the Δ_0^{-1} scale), p_F is the Fermi momentum, and e is the electron charge.

The system of equations for the temporal causal Green's function of a superconductor in a field with a scalar potential $U(\mathbf{r},t)$ and a vector potential $\mathbf{A}$ (see Refs. 8–11) can be reduced, after plugging in (1), to the following equation:

$$\left\{\left[i\frac{\partial}{\partial t}-eU(\mathbf{r})-\frac{1}{2m}\left(\frac{\partial}{\partial\mathbf{r}}+ie\mathbf{A}-i\nabla\varphi_k\right)^2-\mu+(-1)^k eV\right]\right.$$
$$\times\left[i\frac{\partial}{\partial t}+eU+\frac{1}{2m}\left(\frac{\partial}{\partial\mathbf{r}}-ie\mathbf{A}\right)^2+\mu\right]$$

$$-|\Delta(\mathbf{r})|^2\Big\} G(\mathbf{r},t;\mathbf{r}',t')$$

$$\simeq -i\left[i\frac{\partial}{\partial t} - eU(\mathbf{r}) - \frac{1}{2m}\left(\frac{\partial}{\partial \mathbf{r}} + ie\mathbf{A} - i\nabla\varphi_k\right)^2 - \mu + (-1)^k eV\right]\delta^3(\mathbf{r}-\mathbf{r}')\delta(t-t')$$

$$+\frac{1}{m}\left(\frac{\partial}{\partial \mathbf{r}}\Delta\right)\frac{\partial}{\partial \mathbf{r}}F^+(\mathbf{r},t;\mathbf{r}',t'). \tag{2}$$

A similar equation can be written for

$$F^+(\mathbf{r},t;\mathbf{r}',t') = \langle T(\psi^+(\mathbf{r},t)\psi^+(\mathbf{r}',t'))\rangle \times \exp[-i\mu(t-t')].$$

In what follows we ignore the magnetic field $\mathbf{A}$ induced by the current and contributing a term $\propto d^2/\lambda_L^2$ to the current. By introducing the factor

$$\exp\{-i\mu(t-t') + ip(\mathbf{r}-\mathbf{r}')\}$$

into the Green's function (a Fourier transformation) we exclude rapid spatial oscillations, and the remaining left-hand side of Eq. (2) in the $d \gg p_F^{-1}$ approximation is a second-order linear equation in the spatial variable $\mathbf{r}$. The rapid variation (on the scale of the bridge size) of the order parameter caused by the decrease in the condensate velocity in the course of condensate flow ($j \propto d^2/r^2$) is replaced by a discontinuity:

$$|\Delta(\mathbf{r})| = \Delta_0\theta((\mathbf{r}-\mathbf{r}_k)\mathbf{r}_k), \quad \nabla\varphi_k = 0, \tag{3}$$

where $\theta(x)$ is the Heaviside function. The second term on the right-hand side of Eq. (2) contains the gradient of the order parameter, $\partial\Delta(\mathbf{r})/\partial\mathbf{r}$, varying over the size of the narrow, and determines for $d < \xi_0$ the superconducting component of the field across the bridge. The equation is solved by an approximation method developed for layered boundary-value problems. The kernel of the inverse operator acting on the right-hand side of Eq. (2) (the second term) is the boundary Green's function $G_b(\mathbf{r}-\mathbf{r}_k)$, which is the solution of the corresponding homogeneous equation in the variable $\mathbf{r}-\mathbf{r}_k$ that becomes a linearly independent equation at the NS-boundary $\mathbf{r}-\mathbf{r}_k = 0$, where the coefficient of the equation, $|\Delta(\mathbf{r})|$ changes abruptly. The first derivative of the Green's function $G_b(\omega,\mathbf{r}-\mathbf{r}_k)$ experiences a jump at the NS-boundary equal to

$$(\sqrt{\omega^2 - 0^2} + \sqrt{\omega^2 - \Delta_0^2})G_b(\omega,0)\frac{m}{\mathbf{p}\mathbf{n}_k} = 1,$$

where $\mathbf{n}_k$ is the normal to the NS-boundary. As a result, for $|\mathbf{r}| < |\mathbf{r}_k|$, the solution for the component of the causal Green's function related to $\nabla\Delta(\mathbf{r})$ has the form

$$G_2(\mathbf{r},t;\mathbf{r}',t') = \frac{1}{2}\sum_{s=1}^{2} G_{2s}(\mathbf{r},t;\mathbf{r}',t'),$$

$$G_{2s}(\mathbf{r},t;\mathbf{r}',t') = \frac{mp_F}{i(2\pi)^4}\sum_{k,m=1}^{2}\int d\omega\, d\xi\, d\frac{\mathbf{p}}{|\mathbf{p}|} \times \left(d\mathbf{r}_k\frac{\partial}{\partial \mathbf{r}_k}\Delta(\mathbf{r}_k,t)\right)\frac{(-1)^s F^+_{\omega \mathbf{p}s}(\mathbf{r}_k;\mathbf{r}')}{\omega_k + (-1)^m\sqrt{\omega_k^2 - \Delta_0^2}}$$

$$\times\exp\left[-i\omega(t-t') + i\mathbf{p}(\mathbf{r}'-\mathbf{r}) - \frac{i\xi_k m(\mathbf{r}_k-\mathbf{r})\mathbf{n}_k}{\mathbf{p}\mathbf{n}_k} + i \times (-1)^s\omega_k\left|\frac{m(\mathbf{r}_k-\mathbf{r})\mathbf{n}_k}{\mathbf{p}\mathbf{n}_k}\right|\right], \tag{4}$$

where $\xi = p^2/2m - \mu$, $\xi_k = \xi + (-1)^k eV/2$, $\omega_k = \omega + i\delta(-1)^m + (-1)^k eV/2$, and $i\delta$ is a small imaginary addition to ω that defines the way in which the poles of the integrand are passed along the integration contour. The Fourier transform of the Gor'kov function $F^+_{\omega\mathbf{p}}(\mathbf{r}_k;\mathbf{r}')$ can be expressed in terms of $G_{\omega\mathbf{p}}(\mathbf{r}_l;\mathbf{r}_k)$ by the same method of solving the equation for $F^+(\mathbf{r}_k,t;\mathbf{r}',t)$ in the variables $\mathbf{r}'$ and t. The arguments of $G_{\omega\mathbf{p}}$ lie within the normal region. The solution of the equation for the Green's function of the normal state of electrons ($\Delta = 0$) has the form

$$G_n(\mathbf{r}_l,t';\mathbf{r}_k,t) = \frac{1}{i(2\pi)^4}\sum_{m=1}^{2}\int d^3p\, d\omega\, G_n(\omega,\xi) \times\exp\left\{ i\left[\mathbf{p}(\mathbf{r}_l-\mathbf{r}_k) - \omega(t'-t) + \int_{-\infty}^{t'} eU(\mathbf{r}_l - \mathbf{p}(t'-\tau),\tau)d\tau - \int_{-\infty}^{t} eU \times(\mathbf{r}_k - \mathbf{p}(t-\tau),\tau)d\tau\right]\right\}, \tag{5}$$

where $G_n(\omega,\xi) = 2f((-1)^{m+1}\omega)[\omega + i\delta(-1)^m - \xi]^{-1}$, and $f(\omega) = [1+\exp(\omega/T)]^{-1}$ is the Fermi distribution function. The causal Green's function can be expressed as a linear combination of the advanced and retarded Green's functions, which are analytic in the lower and upper half-planes of the complex frequency ω [see Eq. (17.22) in Ref. 8]. Electron scattering in the bridge is taken into account in the Fourier spectrum

$$G_n(\omega,\mathbf{p}) = 2f((-1)^{m+1}\omega)[\omega - \xi + i(-1)^m p_F/ml]^{-1},$$

which in the $(\mathbf{r},t)$-representation reduces [see formula (39.9) in Ref. 8] to multiplying the Green's function by $\exp(-|\mathbf{r}_l - \mathbf{r}_k|/2l)$, where l is the mean free path of electrons in the bridge. If we employ the above method of solving Eq. (2), we can express the component $F_2^+(\mathbf{r},t;\mathbf{r}',t')$ in terms of the function $F^+(\mathbf{r}_l,t';\mathbf{r}_k,t)$, which vanishes under the assumption $\Delta = 0$ holds in the voltage-drop region. Hence the assumption $\Delta(0) = 0$ contradicts nothing in the solution of the resulting boundary-value problem. In the zeroth approximation in the small parameter $2|\mathbf{r}_k|/\xi_0$, the solution at the center of the bridge is independent of the position $\mathbf{r}_k$ of the jump in $|\Delta(\mathbf{r})|$, i.e., is independent of the size of the voltage-drop region. The error introduced by the fact that the continuous variation of $|\Delta(\mathbf{r})|$ is replaced by a jump (3) in the left-hand side of Eq. (2) contributes nothing to the amplitude of the superconducting current even in the first approximation in d/ξ_0. The condition $\Delta(0)=0$ follows from the physics of the problem. The gap in the electron energy spectrum, $|\Delta(\mathbf{r})|$, vanishes at the critical condensate velocity,

$$\frac{1}{m}\left(\frac{1}{2}\nabla\varphi_k-e\mathbf{A}\right)$$

[see Eq. (2)], and is determined by the finite binding energy of an electron pair in the superconductor. The current density decreases as the distance from the center of the bridge grows, $j(\mathbf{r})\propto d^2/r^2$, and $|\Delta(\mathbf{r})|$ varies accordingly over a distance of order d. The function $|\Delta(\mathbf{r})|$ is approximately described by the equation $\nabla^2\Delta(\mathbf{r})=0$ (see Ref. 6). Note that the electroneutrality condition in the metal is satisfied exactly since $G_2(\mathbf{r},t;\mathbf{r},t+0)=0$.

By definition, the current density at the center of the bridge is

$$j(t)=\left\{\frac{ie}{2m}\left(\frac{\partial}{\partial\mathbf{r}'}-\frac{\partial}{\partial\mathbf{r}}\right)_{\mathbf{r}'\to\mathbf{r}}-\frac{e^2}{m}\mathbf{A}\right\}G(\mathbf{r},t;\mathbf{r}',t+0).$$

The final expression has the form

$$j(t)=j_n(V)+j_i(V)+\mathrm{Im}\left\{j_s(V)\exp\left(i\int 2eV\,dt-\frac{|\mathbf{r}_1-\mathbf{r}_2|}{l}\right)\right\}. \tag{6}$$

For the amplitude of the Josephson current we have

$$j_s(V)=\frac{4\Delta_0^2 i}{eR}\int_{-\infty}^{\infty}d\omega f(-\omega)\left[\left(\omega+\frac{eV}{2}+\sqrt{\left(\omega+i\delta+\frac{eV}{2}\right)^2-\Delta_0^2}\right)\left(\omega-\frac{eV}{2}+\sqrt{\left(\omega+i\delta-\frac{eV}{2}\right)^2-\Delta_0^2}\right)\right]^{-1}, \tag{7}$$

and, in particular, $j_s(0)=8\Delta_0/3eR$ at $T=0$, while for $eV\gg\Delta_0$ we have $j_s(V)\simeq\pi\Delta_0^2/e^2RV$. The l-dependence corresponds to the Chambers relation [see Eq. (5.97) in Ref. 12] for the nonlocal relation between current and field. The curve of j_s vs V for $T=0$ (Fig. 1) has an inflection point at $eV=2\Delta_0$ instead of the Ridel logarithmic singularity. The second term on the right-hand side of the solution (6) (the excess current) can be calculated by the following formula:

$$j_i(V)=\frac{2\Delta_0^2 i\langle\varphi_1-\varphi_2\rangle}{eR}\int_{-\infty}^{\infty}d\omega\times\frac{f(-\omega+eV/2)+f(-\omega-eV/2)}{[\omega+\sqrt{(\omega+i\delta)^2-\Delta_0^2}]^2}\times\exp\left(-\frac{|\mathbf{r}_1-\mathbf{r}_2|}{2l}\right),$$

$$\langle\varphi_1-\varphi_2\rangle=\frac{i}{2\Delta_0^2}\int_{-\infty}^{\infty}(\Delta\nabla\Delta^*-\Delta^*\nabla\Delta)d\mathbf{r}\sim\frac{|\mathbf{r}_1-\mathbf{r}_2|mT_c}{p_F}. \tag{8}$$

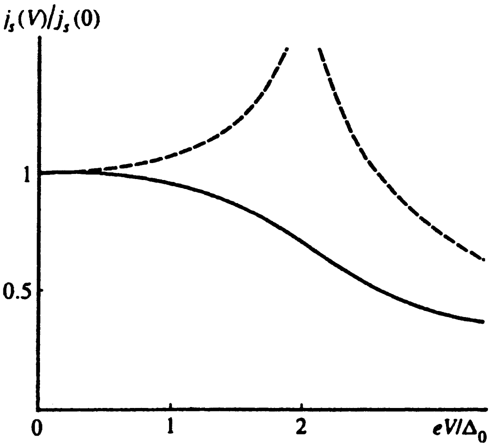

FIG. 1. The dependence of the amplitude of the Josephson current in a microbridge on the voltage, $j_s(V)$, at $T=0$. The dashed curve represents the dependence of Re j_s vs V for a tunnel junction (Ref. 13).

In contrast to the Josephson component, the excess superconducting current is independent of the phase $\int 2eVdt$ for $V(t)\neq 0$. The value of $j_i(V)$ is related (nonlocally) to the variation in the phase of the order parameter in the superconducting sides of the bridge, and this variation cannot be ignored because of the high current density in the bridge. The current j_i is a first-order quantity in d/ξ_0, and the coefficient $\langle\varphi_1-\varphi_2\rangle$ depends only on the size $|\mathbf{r}_1-\mathbf{r}_2|$ of the normal region. The normal component of the current, j_n, is determined by the Green's function component related to the first term on the right-hand side of Eq. (2) and is nonzero (in the present theory) also for $2\Delta_0>eV>0$ only if $\Delta=0$ holds in the bridge. Equation (5) for the function G_n yields $j_n=V/R$, where

$$R=\frac{4\pi^2}{p_F^2e^2}=4\rho l/3$$

is the resistance per unit area in the cross section of the bridge, and ρ is the specific resistance of the metal. The functions $j_s(V)$ and $j_n(V)$ differ from those for tunnel junctions,[13] since a tunneling junction is a system of two independent thermodynamic subsystems with chemical potentials that differ by eV (this independence arises because electron exchange is low in tunnel junctions). To within d^2/ξ_0^2 the current density at the bridge's center, the solution (6), depends on the voltage drop across the bridge, $V(t)$, and does not depend on the way in which the electric potential U depends on $\mathbf{r}$.

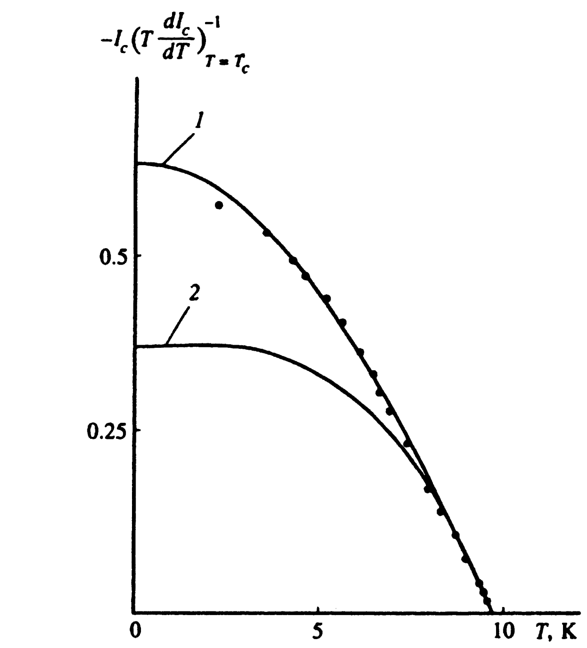

FIG. 2. The theoretical temperature dependence of the critical current $I_c(T)=I_s(T)+I_i(0,T)$ for a microbridge (curve 1) and a tunnel junction (curve 2). The dots designate the experimental data taken from Ref. 14.

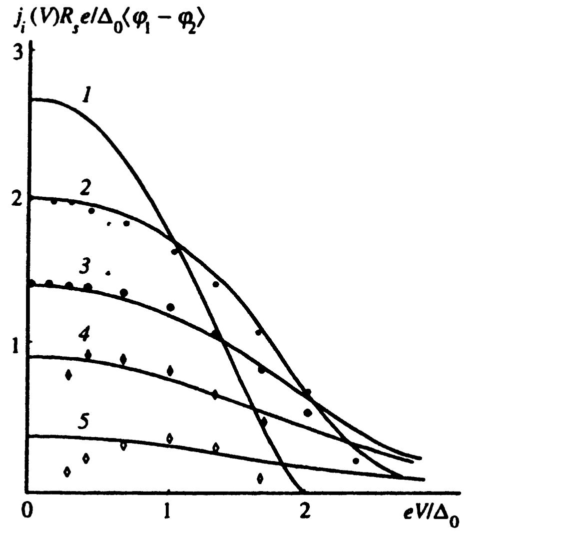

FIG. 3. The dependence of the excess current on voltage, $I_i(V)$ at different reduced temperatures T/T_c: curve 1, 0; curve 2, 0.44; curve 3, 0.64; curve 4, 0.77; and curve 5, 0.91. The solid curves represent the theoretical results, while the ●, ○, ♦, and ◇ designate the experimental data taken from Ref. 14.

3. COMPARISON WITH EXPERIMENT

According to the solution (6), the current–voltage characteristic of a bridge in fixed-current conditions under a constant voltage component $\bar{V}\neq 0$ has the well-known hyperbolic form

$$I(\bar{V})=\{\sqrt{I_s^2(\bar{V})+(\bar{V}/R)^2}+I_i(\bar{V})\}\operatorname{sgn}\bar{V},$$

where $I=jS$, where S is the cross-sectional area of the bridge. A detailed comparison over the entire range of voltages and currents of this theoretical dependence with the experimental current–voltage characteristics of high-resistance point contacts ($R=1-10\ \Omega$)was made in Ref. 14. For $T\ll T_c$ the temperature dependence of the critical current, determined from the solution (6) in the limit $V\to 0$, differs from $I_c(T)$ of a tunnel junction (a comparison with the experimental data of Ref. 14 is done in Fig. 2):

$$I_c(T)=\frac{8}{eR\Delta_0^2}\int_0^{\Delta_0}d\omega\ \omega\sqrt{\Delta_0^2-\omega^2}\tanh\left(\frac{\omega}{2T}\right)\times\left[\exp\left(-\frac{|\mathbf{r}_1-\mathbf{r}_2|}{l}\right)+\langle\varphi_1-\varphi_2\rangle\exp\left(-\frac{|\mathbf{r}_1-\mathbf{r}_2|}{2l}\right)\right].\tag{9}$$

Figure 3 depicts the dependence of the excess current on voltage and temperature, $I_i=I_i(V,T)$. The experimental points, taken from Ref. 14, are marked according to the data of the current–voltage characteristic of a Nb–Nb point contact with a resistance $R_n(11\ \mathrm{K})=7.5\Omega$. The value of the resistance of a single bridge, $R_n=16\rho l/3\pi d^2$, implies that $d<100\ \text{Å}<\xi_0$ holds at $\rho l=4\times 10^{-12}\Omega\ \mathrm{cm}^2$ for Nb. Since $I_i(0)$ is finite at $V=0$, over a broad interval ($\sim 2I_i(0)R$) the current–voltage characteristic of a bridge exhibits[15] (in contrast to that of a SIS junction) a group of current steps of approximately equal height in the maximum-voltage region and at fixed microwave-radiation power. A characteristic feature of the solution (6) is the sinusoidal dependence of the nonstationary Josephson current, in view of which the amplitude of the induced current steps vanishes at certain values of the microwave-radiation power.[15] If we ignore $I_i(0)$, the Nth step vanishes at an amplitude $\tilde{I}$ of the microwave-induced current given by the root of the Bessel function, $J_N(2e\tilde{I}R/\hbar\omega)=0$, where ω_0 is the frequency of the microwave-induced current. The vanishing of the current-step amplitude, which varies with the microwave-radiation power, was observed in experiments involving tin whiskers[3] and Pb- and Nb-film bridges of Dyhem.[2] When the theoretical dependence of the time-dependent current across a bridge of any size or a point contact is not sinusoidal, the current steps oscillate but remain finite in amplitude.[15] A rigorous proof of this assertion based on the fact that the roots of the Bessel function are not multiple can be found in Ref. 16. This criterion is crucial in an experimental verification of the theory.

4. CONCLUSION

Since the surface of high-T_c superconductors is penetrated by a magnetic field along the grain boundaries, a closed superconducting current generates a hypervortex (instead of an Abrikosov vortex in low-T_c superconductors) whose size is determined by the Josephson penetration depth $\lambda_J\propto 1/\sqrt{j_c}$ (see Ref. 17). A high-T_c superconducting junction constitutes a series–parallel electrical circuit consisting of Josephson SIS and SNS junctions with the addition of parallel currents and serial voltages until a quantum magnetic flux induced by the flowing current is trapped, which closes the current circuit inside the junction and leads to a marked reduction in the total Josephson current, just as it does in a tunnel junction with a width exceeding λ_J. Thus, when the high-T_c superconducting junction is smaller than the hypervortex, it has the same characteristics as a single Josephson junction with the total current and voltage. This model explains the experimental data of Ref. 18. Noge *et al.*[19] observed current-step oscillations, accompanying the variation of the microwave-radiation power, in experiments that involved $YBa_2Cu_3O_{7-\delta}$ bridges of micrometer dimensions at 77 K and at current densities 10^4–10^5A cm^{-2}(λ_J=3–1 μm). With larger bridges ($d \gg \lambda_J$) no oscillations were observed.[20] Golovashkin and Lykov[21] explained the presence of current steps by the coherent motion of vortex chains in the bridge.

Thus, the proposed electrodynamic model of the time-dependent Josephson effect, which naturally follows from the ideas about a magnetic field penetrating the surface of a superconductor, explains the observed values of the characteristics of Josephson bridges and point contacts made from low-T_c and high-T_c superconductors and can help in practical work by determining the characteristic size of a Josephson junction.

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