Free oscillations and field structure of bush resonances in open electromagnetic cavities with periodic inclusions

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The field structure of bush cavities and the conditions under which they exist are studied. The nature of bush resonances is determined. It is shown that their field has a complex structure. The interaction of the components of the resonance field of bushes is accompanied by the onset of the phenomenon of dynamic nondispersion (a soliton). The paper discusses questions of the stability of the states of the bush resonance field. An analysis is carried out of the connection of the dynamic nondispersion with the physical phenomena on which the formation of a Mandel'shtam–Brillouin doublet is based. © *1996 American Institute of Physics.* [S1063-7761(96)00808-6]

1. INTRODUCTION

The phenomenon of bush resonance can be studied by introducing perturbations in the form of anharmonic terms into the Hamiltonian of Ref. 1. By definition, a bush (of modes) is a multifrequency resonance of a self-consistent (closed) collection of vibrational modes in a state associated with a force type of interaction or a parametric type of interaction. Formations of similar bushes (Wien graphs, intertype oscillations) were detected much earlier than Ref. 1 in the process of studying the interaction of the oscillations (modes) in closed[2,3] and open[4,5] electromagnetic cavities. The chief feature of open systems is the presence of connection channels with the surrounding space, which makes it necessary to study the spectrum of nonself-adjoint operators, which is equivalent to the spectrum of the existing spectral boundary-value problem.[6] The study of such a spectrum led to the detection of so-called Morse critical points in it. As it turned out, they are singularities of the resonant field of open systems. The singularities are formed during the thickening (compression) of the spectrum of free oscillations. Such a compressed region of the spectrum is a self-consistent set of interacting free oscillations, equivalent to a bush of vibrational modes.[1] A singularity in the resonance field of the bush of an open system is a mapping (trace) of the surrounding unbounded space into a set of interacting components of this field.

In the proposed work, theory is employed to analyze experimental studies of the structure of free oscillations and bush resonances when they interact with an electron flux. The electron flux is an extremely essential detail; its presence significantly increases the resolving power of the measurements and makes it possible to study the fine structure of the resonant field under dynamic conditions.

2. THE PHENOMENON OF ANOMALOUS DISPERSION AND THICKENING OF THE SPECTRUM OF FREE OSCILLATIONS IN PERIODIC CAVITIES

Let us consider wave dispersion in the periodic structures that will be used in the experiment. Figure 1(b) shows a schematic drawing of one such structure—a diffraction grating that simultaneously fulfills the function of a periodic inclusion in an open cavity and a delay system. The dispersion equation of a diffraction grating at a distance L from a plane has the form[7]

$$\chi h \tan \chi h=\frac{l}{d}\tan\left(\gamma h\frac{L}{h}\right),$$
$$\chi=\sqrt{k^2-(\pi/s)^2},\quad \gamma=\sqrt{\beta^2-k^2},\tag{1}$$

where l is the period of the diffraction grating; s is the width of the diffraction grating; d is the width of a slit of the grating; h is the depth of the grating; $\tanh(\gamma hL/h)\to 1$ for $L\gg h$; and Eq. (1), reduced to a form convenient for calculation, is written as

$$\frac{c}{v_{\rm ph}}=\sqrt{\frac{d^2}{l^2}\left(1-\frac{\lambda_0^2}{\lambda_{\rm cr}^2}\right)\tan^2\left(\frac{2\pi h}{\lambda_0}\sqrt{1-\frac{\lambda_0^2}{\lambda_{\rm cr}^2}}+1\right)},\tag{2}$$

where c is the speed of light, $v_{\rm ph}$ is the phase velocity, λ_0 is the wavelength, and $\lambda_{\rm cr}=2s$ is the critical wavelength.

The solution of Eq. (2) is the region of existence of waveguide waves (in this case, surface waves) of a diffraction grating of the given geometry (in an experiment in the 2-mm range, $l=0.25$ mm, $d=0.09$ mm, $h=0.4$ mm, $s=7$ mm)—see Fig. 2. Here $\varphi=2\pi$ and $\varphi=\pi$ are curves of equal phase angles of the surface waves λ_w on a period of the diffraction grating.

Let us trace the sign change of the increment $\Delta\lambda$ as the phase velocity increases monotonically, $v_{\rm ph}\to c$. The wavelength of the retarded wave varies in accordance with the equation $\lambda_w=\lambda v_{\rm ph}/c$ as $v_{\rm ph}\to c$. Consequently, $v_{\rm ph}\to c$ is accompanied by an increase of λ_w and a corresponding variation of the phase increment on a period of the diffraction grating. At point θ ($\varphi=\pi$), it becomes critical: small variations $\delta\lambda_w$ result in a sign change of the increment $\pm\Delta\lambda$. The connections between the group velocity $v_{\rm gr}$ and the phase velocity $v_{\rm ph}$ of the waves and the sign of the in-

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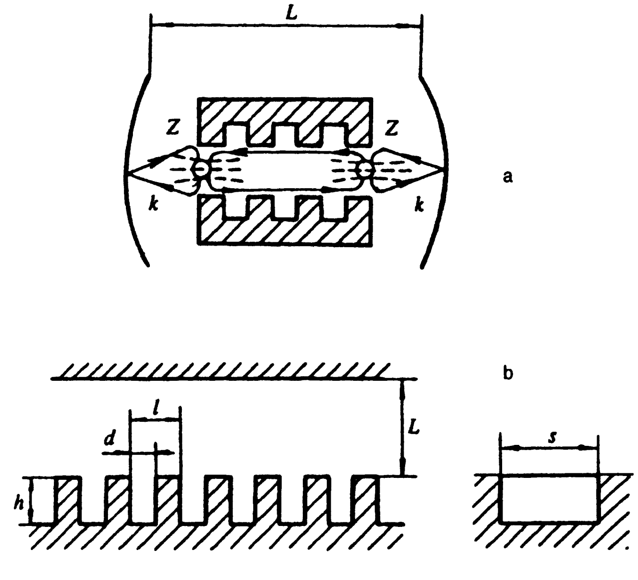

FIG. 1. (a) Open cavity with a periodic inclusion in the form of a segment of open periodic waveguide. (b) Diffraction grating located under the plane.

crement $\Delta\lambda$ for normal and anomalous dispersions on a diffraction grating can be written by means of the Rayleigh formula as

$$v_{\mathrm{gr}}=v_{\mathrm{ph}}-\lambda\frac{dv_{\mathrm{ph}}}{d\lambda},$$
$$v_{\mathrm{gr}}=v_{\mathrm{ph}}+\lambda\frac{dv_{\mathrm{ph}}}{d\lambda}. \tag{3}$$

Thus, critical phase states exist on a diffraction grating, in which small frequency variations change the field structure (the types of waves). This case shows up in Fig. 2 as a transition from the normal to the anomalous type of dispersion close to the transmission boundary (in an experiment for a 2-mm range of waves, $\lambda_{\mathrm{gr}}=1.6$ mm).

Consequently, perturbations caused by $\lambda_w\to\lambda_0$ lead to "reflection" of the waves, which manifests itself as a change in the type of dispersion. Moreover, this reflection occurs not from the boundary itself but from a band whose center is the boundary ($\lambda_{\mathrm{gr}}=1.6$ mm). A similar phenomenon was observed at the boundaries of the absorption band of light by sodium vapor in the Kundt–Rozhdestvenskiĭ experiments.[8] The bent spectral bands in these experiments describe curves similar to Wien graphs[2,3] and are caused by the phenomenon of anomalous dispersion. The perturbing band in the Kundt–Rozhdestvenskiĭ experiments (the absorption band) is determined by the excited sodium atoms, which change the state of the macroscopic system (the sodium vapor). As a result, the electromagnetic field can have critical phase states, by passing through which it can propagate in the macroscopic system in the form of waves with normal or anomalous dispersion. In the case under consideration, the function of the macroscopic system is fulfilled by the diffraction grating, while the change of the states is implemented in the process $v_{\mathrm{ph}}\to c$.

The phenomenon of anomalous dispersion is not restricted to the cases shown above. Similar processes are

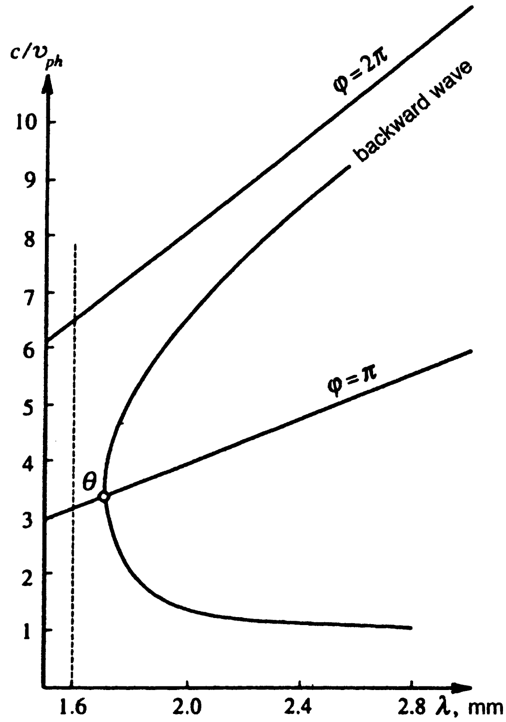

FIG. 2. Dispersion characteristic of waveguide types of waves of a diffraction grating.

implemented when free oscillations interact in electromagnetic cavities. For an open cavity, the spectrum of free oscillations is equivalent to the set of eigenvalues of the characteristic determinant[5,6]

$$\mathscr{F}(\kappa)\equiv\det\{I-A(\kappa)\}, \tag{4}$$

where κ is a dimensionless frequency, I is the unit operator, and $A(\kappa)$ is the operator function of the spectral boundary-value problem.

In contrast with closed cavities, the eigenfunctions κ in open electromagnetic cavities are complex-valued. In this way, i.e., by using nonself-adjoint operators, we take into account the effect of the surrounding space on the spectrum of free oscillations of the cavity. An investigation of the eigenvalue spectrum of the characteristic determinant leads to the concept of the so-called Morse critical points,[4,5] in the neighborhood of which the free oscillations of cavities interact. Determination of the Morse critical points reduces to a search for the resolvent $R(\kappa)=\{I-A(\kappa)\}^{-1}$ of the operator. For $\kappa\to 0$, the norm $\|A(\kappa)\|<1$, and the point spectrum of the operator is restricted to some neighborhood with its center at zero (a Morse critical point). In this case, there exists an empty neighborhood of a Morse critical point (a forbidden band), where there are no free oscillations of the cavity. Consequently, in this neighborhood, the oscillations either damp out or are "pushed out" by an interaction (a collision); this is characteristic of free linear oscillations, which by definition have a finite undamped supply of energy (a stationary state). It is essential that the operator–function method of Ref. 6 makes it possible to study the spectral characteristics of open systems while taking into account nonspectral parameters. In this case, the dispersion equation has the form

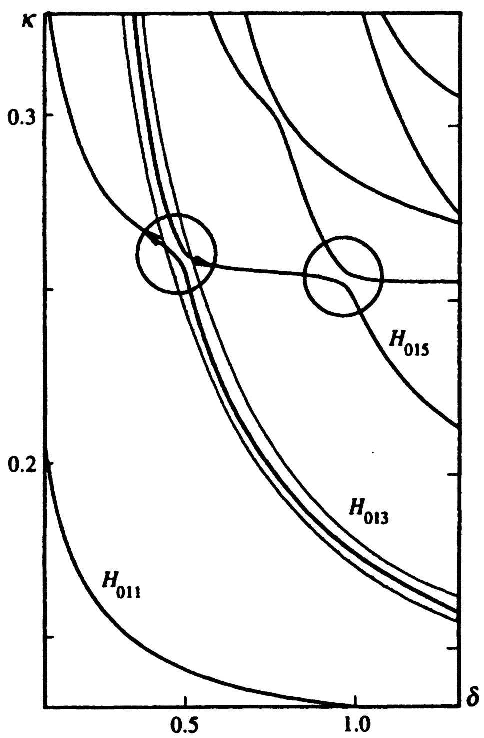

FIG. 3. Dispersion curves of types of waves of an open periodic waveguide structure (the result of a numerical experiment). The circles enclose the Wien graphs.

$$\mathscr{F}(\kappa,\delta)=\det\{I-A(\kappa,\delta)\}=0,$$

where δ is one of the nonspectral parameters of the problem.

Figure 3 shows the results of a numerical experiment on the study of the interaction of the free oscillations in open periodic resonant structures (gratings)[4] when δ is a geometrical parameter of the diffraction grating (the dimensionless height). It can be seen that there are critical values of δ for which the eigenfrequencies of the free oscillations (the dispersion curves) are pushed apart, describing Wien graphs (surrounded by circles in the figure), with empty neighborhoods being formed with centers of symmetry at the Morse critical points. The spectral line bounded by the band is especially noteworthy; here the discontinuity in the neighborhood of a Morse critical point is accompanied by bends in opposite directions, similar to what occurred in the Kundt–Rozhdestvenskiĭ experiments. The difference is that here, instead of an absorption band, an empty site occurs. This is understandable, since, according to the condition of the problem, free oscillations in space do not damp out. As a result of the "collision" (approach), they exchange momenta, and their frequencies "diverge" in opposite directions. In this case, as in Fig. 2, the "pushing apart" can be accompanied by the phenomenon of anomalous dispersion.

A symmetric collision of two spheres accompanied by elastic impact can serve as a mechanical analog of the formation of a Morse critical point. The trajectories of the spheres in this case describe "Wien graphs," at the center of symmetry of which their imaginary zero-energy point is highlighted (the Morse critical point). If the oscillations damp out when the collision occurs, the empty neighborhood turns into an absorption band.

Thus, a Morse critical point is a center of thickening of the spectrum of free oscillations. One can distinguish a bush of interacting oscillations or can excite such a bush in the neighborhood of a critical point, if the conditions for the existence of such a complex resonant field are fulfilled in a resonant system or in some medium.[1] The thickening of the oscillations is accompanied by interaction via induced currents or captured electrons, as occurs in dielectric cavities.[9]

To obtain a compressed spectrum of the free oscillations, it is expedient to use ring cavities known from the solution of the Sturm–Liouville problem.[10] In an ideal ring, one eigenvalue (frequency) can correspond to an infinite set of field configurations (eigenfunctions). The finiteness of the energy makes it impossible to implement an observable field in this case. In actuality, a small number of resonant-field configurations exist simultaneously in rings. Therefore, to obtain a bush resonant field, one uses a system of rings with an overlap region in which a compressed spectrum arises.

Figure 1 shows a simple version of an open cavity in which it is possible to arrange two rings with an overlap region. It consists of a section of periodic waveguide and an open cavity. One of the rings is formed by waves that flow around a section of the periodic structure and come together in reactive zones of Z communication channels. The second ring is formed by waves that pass through the communication channels and are brought together by the mirrors of the open cavity. The volume of the section is the region where the fields of the rings overlap. In the case of two field configurations (which corresponds to the given experiment), the resonant field of the ring is written as the superposition

$$\varphi_1=C_1'\psi_1'+C_2'\psi_2', \tag{5}$$

where ψ_1' and ψ_2' are the eigenfunctions (or types of waves) corresponding to one eigenvalue (frequency). For these functions, $\lambda_{w_1}\neq\lambda_{w_2}$ in general; they both are stable because the reactive zones Z introduce indeterminacy into the resonance distance and smear its value. As a result, a ring can be stably closed by different λ_w corresponding to the same frequency.

Similar reasoning holds for the second ring:

$$\varphi_2=C_1''\psi_1''+C_2''\psi_2''. \tag{6}$$

The set of φ_1 and φ_2 in the overlap region is a bush of oscillations[1] or an intertype oscillation.[5,6] In Fig. 3, the regions of existence of the bushes (Wien graphs) are surrounded by circles. When the open cavity is tuned (by varying L), their eigenfrequencies approach or diverge from each other; i.e., the spectrum of the resonant field of the bush is compressed or dispersed. If the field configurations coincide in space and time ($\lambda_{w_1}\equiv\lambda_{w_2}$), they become identical, and the field of the bush will be equivalent to the field of a single oscillation. The "collision" of the components of the resonance field of the bush reduces to their approach and mutual perturbation, the degree of which varies in the process of tuning. Under the conditions of the experiment, the perturbations are transmitted to the electron flux, which makes their interaction dynamics observable.

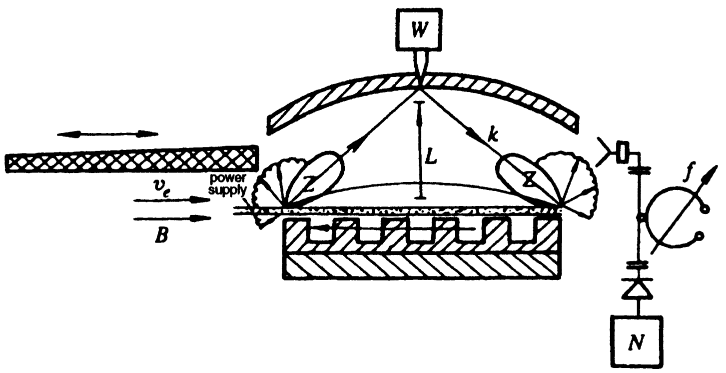

FIG. 4. Schematic drawing of an experimental apparatus (generator) with measurement devices.

3. EXPERIMENT

The experimental setup is shown in Fig. 4. The object of interest is the resonant system of the generator of the diffraction radiation.[11] It is an open cavity with a periodic inclusion in the form of a diffraction grating (see Fig. 1(b), which here simultaneously fulfills the function of a slow-wave structure. As a result of scattering in the reactive zones Z and re-reflection from the spherical mirror, the surface waves of the diffraction grating form a ring. The path of such waves is shown by arrows in Fig. 4. The second ring is formed by the closure of the surface waves of the diffraction grating in the communication channels Z. The overlap region of the resonant fields of the rings here is the waveguide diffraction grating, where they interact with each other and with the electron flux that excites them. As usual, the electron flux is focused by a longitudinal constant magnetic field.

The systems under consideration, shown in Fig. 4 and Fig. 1(a), are physically equivalent. In the process of tuning the open cavity of the diffraction radiation, the power W and the frequency are measured, which makes it possible to continuously monitor the existence of the annular oscillations and the variation in the structure of their fields.

Figure 5 shows how the frequency $f(L)$ and the power $P(L)$ vary in the process of tuning the rings. The curves shown in Fig. 5(a) were recorded with the following values of the accelerating voltage U and current I_l of the electron flux: $U_1=3434$ V=const, $I_{1e}=70$ mA=const; the plots of $f_2(L)$ and $P_2(L)$ correspond to $U_2=U_1=$ const, but to another value of the current of the electron flux: $I_{2e}=80$ mA =const. It can be seen that the dispersion curves $f_1(L)$ and $f_2(L)$ are straight lines; as the current of the electron flux varies, they are simply displaced parallel to themselves. Such lines are characteristic of single oscillations. Consequently, we are observing the case in which the fields of the rings are identical.

Figure 5(b) shows plots for another frequency range. For this case, $U=4330$ V=const, $I_l=70$ mA=const. Here already, instead of a line showing identity of the fields (a single oscillation), band χ appears, showing that the resonant field of the rings has a complex structure. In the interval

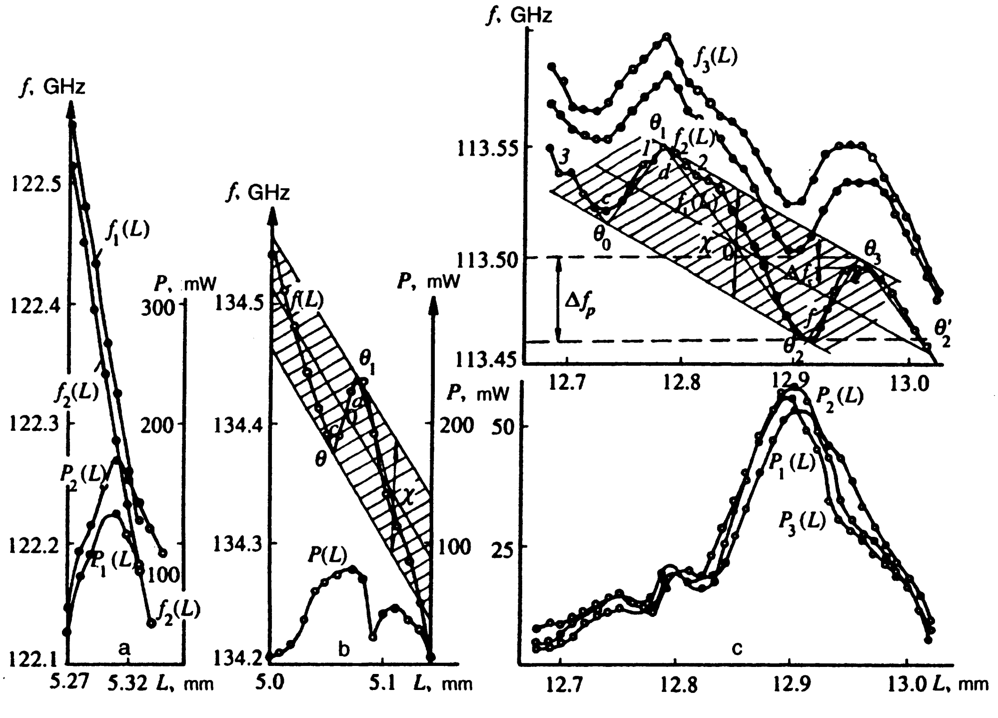

FIG. 5. Character of the variation of frequency f and power P in the process of tuning annular oscillations: (a) dispersion curves of a single oscillation (case in which the resonant-field configurations of the bush are identical); (b) dispersion band χ of the bush resonance; (c) dispersion band of the bush resonance for a resonance system (optical cavity) with an additional inclusion in the form of a wedge-shaped quartz plate.

$\theta_0-\theta_1$, a field configuration is observed that determines the existence of an anomalous type of dispersion at the diffraction grating. In regions c and d, the sign of the increment $\Delta\lambda$ reverses, and the types of dispersion change. Here they can be expected to exist simultaneously. However, these regions contain little information on such a state; this is associated with the weakly expressed ring formed on the diffraction grating.

To eliminate indeterminacy, an additional ring was "connected" by introducing a quartz plate into the volume of the open cavity (Fig. 4). The quartz plate is made in the shape of a wedge, in order to prevent the generator from being excited in another regime. The plots for this case are shown in Fig. 5(c), where the range of existence of rings is reproduced three times: $f_1(L)$ corresponds to $U_1=3103$ V, $f_2(L)$ corresponds to $U_2=3106$ V, and $f_3(L)$ corresponds to $U_3=3109$ V. The current of the electron flux was identical in all three cases and equalled $I_{1l}=I_{2l}=I_{3l}=70$ mA=const. In this case, the measurement step was $\Delta L=0.01$ mm each, and the wavelength was about $\lambda=2.64$ mm. It can be seen that the character of the $f(L)$ and $P(L)$ variation is stable on all three graphs. In the neighborhood of point e (and of points similar to it), there is a stable section of "length" $\Delta L=0.02$ mm where no dispersion properties of the periodic structure manifest themselves; i.e., $v_{\mathrm{ph}}\equiv v_{\mathrm{gr}}$. This relation can be proven based on the fact that the resonant field of the bush is a combination of two wave functions of type φ_1 and φ_2. As already known, critical phase increments exist at the diffraction grating on the period in which small variations $\delta\lambda_w$ cause the types of dispersion to change. In the process of tuning the resonance field of the bush, the doublet $\lambda_{w_1}=\lambda_{w_2}$ is continuously tuned and by necessity covers the critical region of the phase increments, where the opposite types of dispersion given by Eq. (3) are allowed to exist simultaneously. Equalizing the quantities of energy of the resonance field of the bush stored in these configurations causes, as seen from Eq. (3), the identity

$$\sum_{i=1}^{2}\lambda_i\frac{dv_i}{d\lambda_i}=0 \tag{7}$$

to be satisfied.

Consequently, when the identity is satisfied, a dispersion doublet forms, and the corresponding phenomenon of dynamic nondispersion $v_{\mathrm{ph}}\equiv v_{\mathrm{gr}}$ (a soliton) occurs. It can be seen from the plots that this event causes a frequency shift Δf_s, i.e., a "pushing apart" of the free oscillations characteristic of Wien plots. In the experiment, half of these graphs are "delineated," and the other half are symmetric to the first half. The "coordinates" of the Morse critical point (θ_3) are highlighted at the center of symmetry of the collision of the free oscillations. At its edges, the $f_0=$ const section undergoes a transition to the $\theta_2-\theta_3$ and $\theta_3-\theta_2'$ branches, with opposite types of dispersion. Here the mutual perturbation of the rings is reduced. As a result, the dispersion doublet breaks down, and the configuration of the resonance field φ in which the larger quantity of energy is stored becomes observable. At point θ_1 and the points analogous to it, no collisions occurred, the oscillations slipped through each other without interaction, and the coordinates of the Morse critical point were not highlighted. It can thus be assumed that the Morse critical point belongs to the spectrum of damped oscillations (to the set of imaginary frequencies); it will then correspond to the coordinates $\mathrm{Re}\,\kappa=0$; $\mathrm{Im}\,\kappa\neq 0$. The state $\mathrm{Im}\,\kappa=0$, $\mathrm{Re}\,\kappa\neq 0$ corresponds to a free undamped oscillation.

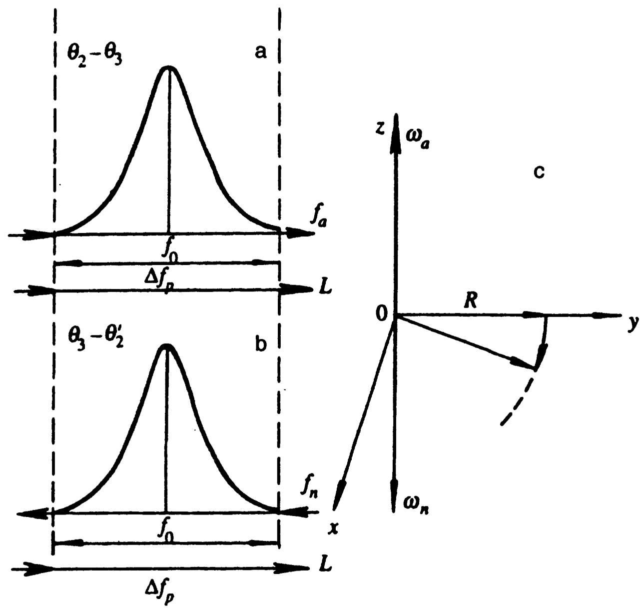

FIG. 6. Allowed absorption bands in the frequency interval Δf_p; (a) anomalous-dispersion case; (b) normal-dispersion case; (c) vector diagrams.

It is noteworthy that the slope of the dispersion lines of the single oscillations (Fig. 5(a)) and of the existence bands χ of the bush resonances (they are shaded in Figs. 5(b) and (c)) are similar. The bush of oscillations is tuned similarly to a single oscillation and is consequently an entire self-consistent set of interacting oscillations (modes).

A. Vector representation of bushes and their spontaneous excitation

It can be seen in the plots of Fig. 5(c) that branches with anomalous ($\theta_2-\theta_3$) and normal ($\theta_3-\theta_2'$) types of dispersion having a common frequency band Δf_p arise on $f_1(L)$. The tuning of certain contours ($Q_1=Q_2$) whose absorption bands completely fit into the band Δf_p (Figs. 6(a) and (b)) is allowable on such branches. On $\theta_2-\theta_3$, the sign of the frequency increment Δf_a coincides with the sign of the increment ΔL. In this case, we shall assume that the vector R (the frequency) rotates clockwise, and that the angular velocity vector ω_a points along the positive z axis (Fig. 6(c)). On $\theta_3-\theta_2'$, Δf_n and ΔL have opposite signs, and the angular velocity vector Δf_n accordingly points along the negative z axis. The section $f_0=$ const under such conditions can be represented as the region where the absorption bands of Fig. 6(a) and (b) are in registration, which corresponds to opposite angular velocity vectors. These vectors, applied to one point, are the positive $+f_a$ and negative $-f_n$ frequencies. Such coalescence of the field vectors in the process of collision determines the stability of the frequency on the $f_0=$

const interval. This state is equivalent to bringing two mid-antipodes into coincidence, causing the formation of opposite frequencies (negative and positive). Under the conditions of the experiment, the existence of frequencies of the antipodes is determined by the components of the resonant field of the bush with $\lambda_{w_1} \neq \lambda_{w_2}$.

In the Kundt–Rozhdestvenskiĭ experiments, it is allowable to form two states in the macroscopic system (sodium vapor)—the antipodes.

Under the conditions of this experiment, the configurations of the resonance field that interact with the electron flux and take energy from it are the generating elements of the bush. The bush can contain a single generating element. The resonance-field configurations generated by it are "idle," which does not in general prevent a parametric type of interaction from occurring between them and the electron flux. Consequently, in the process of tuning, the generating element can spontaneously generate a bush. When there is a discontinuity in the connections (a force interaction[1]), the idle components break away from the generating element and damp out. In Fig. 5(c), spontaneous generation of a bush is observed on the section of the $f_1(L)$ dispersion curve designated by a *3*; traces of the spontaneous generation of a bush can be seen on sections *1* and *2*. In distinction from "stimulated" production of a bush, in the neighborhood of θ_3, no symptoms of separate existence of opposite types of wave dispersions are observed outside the section $f=$ const, since the bush in this case contains a single generating element with a field configuration that causes a normal type of dispersion, and the other (idle) configurations "disconnect" from the source and damp out when they leave the region of force interaction ($f=$ const).

B. Analysis of the stable and unstable states of the resonance field of a bush

Let us consider the process of forming a dynamic nondispersion from a somewhat different viewpoint, taking into account the interaction of the waves with the electron flux. In the state of generation, the electron flux is grouped and forms a certain charged periodic medium. Since $U=$ const and $I_e=$ const under the conditions of the experiment, the periods of the electronic medium will also be constant, $l_{el}=$ const. Such an electron flux, having been cut by modulation of the charge density, can be considered stationary with respect to the travelling phases of the waves caused by tuning of the rings by variation of L. As a result, the phases will run into the standing (relative to the observer) waves of the electron flux. In conditions under which the dispersion doublet exists, the incoming phases of the waves are opposite (negative and positive frequencies). Consequently, on the $f_0=$ const interval, two opposite longitudinal Doppler effects occur simultaneously, and the resulting frequency shift will equal zero. In such a state, the Stokes and anti-Stokes components of the central frequency are suppressed symmetrically, which is equivalent to the phenomenon of Mandel'shtam–Brillouin compression of the doublet. If there are fluctuations of the quantities of energy stored in the resonance-field configurations that determine the dispersion doublet, the symmetry of the suppression of the Stokes and the anti-Stokes components breaks down, which causes the central frequency to precess. In such cases, random noise is observed on the envelopes of the generator signal.

4. CONCLUSIONS

The studies that have been carried out show that a self-consistent set of interacting oscillations (a bush of modes) in open systems is an entire structure that is tuned in a frequency range like a single oscillation. In the process of tuning, regions of compression of the spectral components of the resonance field of the bush occur, with the formation of singularities (the Morse critical points). Steady-state solutions of Maxwell's equations[4–6] make it possible to study instantaneous (frozen) states of the resonant-field structure of a bush without dealing with the interaction dynamics in the neighborhood of the singularities. Such a property is possessed by nonsteady-state nonlinear equations that are a hybrid of a nonlinear wave equation and the Klein–Gordon equation.[12,13]

In periodic waveguide cavities, the resonance phenomenon can be regarded as a process in which a closed flux of electromagnetic waves travels around a periodic surface. Multiplicity of the configurations of the annular resonance field causes the phenomenon of anomalous dispersion and the dispersion doublet. Such a state of the resonant field is observed in the form of dynamic nondispersion (a soliton), when the waves do not notice the periodic surface around which they flow.

The electromagnetic-wave flux in such a state has properties in common with dissipation-free shock waves, which are well known in dispersive hydrodynamics. Actually, if one assumes the case in which a wave is pressed against the surface of a body, solitons that are formed at the wavefronts of dispersionless shock waves will flow around the body.[14] Soliton waves cause the resistance to the flow to be lost, and the flux will not notice the surface around which it flows.

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