A phenomenological theory of retardation of two-dimensional magnetic solitons

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We calculate the retardation of a two-dimensional magnetic soliton with small radius in the exchange approximation based on the generalized phenomenological theory of magnetic relaxation. The soliton's viscosity coefficient is found to consist of two terms of different origin. The first term corresponds to the direct contribution of exchange relaxation. The second term reflects the variation in the length of the magnetization vector and can be explained by the following mechanism. Near the soliton the local value M of the magnetization vector differs from the equilibrium value M_0. As the soliton moves its energy is expended in causing M to depart from its equilibrium value M_0 in the given region of the magnetic substance, after which the deviation relaxes. [S1063-7761(96)01906-3]

1. Such nonlinear excitations as topological solitons[11] play an important role in describing the physical properties of low-dimensional magnetic materials.[2–4] In particular, allowing for magnetic vortices (in the case of easy-plane anisotropy) and localized 2D-solitons is important when dealing with two-dimensional (2D) magnetic materials (see the reviews by Bar'yakhtar and Ivanov,[3] and Ivanov and Kolezhuk.[4] An important parameter in soliton dynamics is the viscous friction coefficient η, which determines the friction force $\mathbf{f}$ acting on a soliton whose velocity is $\mathbf{v}$, i.e., $\mathbf{f}=-\eta\mathbf{v}$. The coefficient enters into the formula for the soliton contribution to the correlation functions and determines the width of the central peak in the inelastic neutron scattering cross section.[2–4]

Calculations of η for a domain wall that are based on the Landau–Lifshitz equation[5] with the usual relaxation term of the Hilbert type lead to a number of contradictions with the experimental data.[6] Unphysical divergent expressions emerge for 2D-magnetic solitons (see a discussion of this aspect in Ref. 7). Bar'yakhtar[8,9] suggested a generalized phenomenological theory of relaxation in ferromagnets based on allowing for the real dynamic symmetry of magnetic materials. He introduced relaxation terms of different origin (exchange and relativistic), which resulted in correct expressions for the dependence of the magnon damping constant on the wave vector and made it possible to describe several experiments in the dynamics of magnetic inhomogeneities. Bokov *et al.*[10] noted that this theory allows for a quantitative description of the results of their measurements of the dependence of the viscosity coefficient of a domain wall on a magnetic field perpendicular to the easy axis (a specific calculation can be found in Ref. 11). Exchange relaxation, considered in Ref. 12, made it possible to explain the results of the experiments[13] on retardation of a singular soliton, a Bloch point in a yttrium iron garnet. The material is known to have an extremely low coefficient of relaxation of spin waves and a low Gilbert damping constant. These experiments produced an unexpected result: the friction coefficient for a Bloch point proved to be extremely large—so large that the response of the Bloch point to a short pulse of the driving force was purely damped rather than oscillatory, whereas the ordinary relaxation theory[6] predicts a well-defined oscillatory dynamics of the Bloch point in such an experiment.

Here we calculate the viscous friction coefficient for a topological soliton and show that the application of the Bar'yakhtar generalized theory[8] to non-one-dimensional solitons exhibits features not yet discussed in the literature.

2. The Landau–Lifshitz equation for magnetization vector $\mathbf{M}$ of a ferromagnet with an exchange relaxation term[8] can be written as

$$\frac{\partial \mathbf{M}}{\partial E}=-g[\mathbf{MF}]-\gamma\nabla^2\mathbf{F}, \tag{1}$$

where $g=2|\mu_0|/\hbar$ is the gyromagnetic ratio, γ is a constant characterizing the intensity of exchange relaxation, and the vector $\mathbf{F}$ represents the effective field of the ferromagnet, which is determined by the variational derivative of the ferromagnet energy functional $W\{\mathbf{M}\}$ with respect to the magnetization: $F=-\delta W\{\mathbf{M}\}/\delta\mathbf{M}$.

To describe solitons whose radius R is much smaller than Δ_0, a quantity of the order of the domain-wall thickness, we write the ferromagnet energy in the exchange approximation:

$$W=\int\left\{f(M)+\frac{\alpha}{2}\left(\frac{\partial \mathbf{M}}{\partial x_i}\right)^2\right\}d\mathbf{r}. \tag{2}$$

Here α is the inhomogeneous exchange constant, and the function $f(M)$ determines how the ferromagnet energy depends on the absolute value of magnetization, $M=|\mathbf{M}|$, and has a sharp peak at $M=M_0$, where M_0 is the equilibrium magnetization value (for details see Ref. 5). The energy relaxation rate is determined by the dissipation function of the ferromagnet, $dW/dt=-2Q$. The dissipation function corresponding to Eq. (1) can be written as[8]

$$Q=\frac{1}{2}\gamma\int(\nabla\mathbf{F})^2d\mathbf{r}. \tag{3}$$

This expression is inconvenient for analyzing soliton retardation, since the dissipation function Q is defined in terms of the effective field **F** rather than in terms of the time derivative of magnetization. To calculate the rate of soliton energy dissipation we must express **F** in terms of the magnetization **M** and its derivatives. Thus, calculating the friction force for a moving soliton, $f=2Q/v$, where $f=|\mathbf{f}|$ and $v=|\mathbf{v}|$, or the viscosity coefficient $\eta=Q/v^2$ requires determining the effective field **F**. Since in the static case $\mathbf{F}=0$, one should expect that at low soliton velocities $|\mathbf{F}| \propto v$ and $Q \propto v^2$ hold, and that η remains finite as $v \to 0$.

If the dissipation constant γ is small (more precisely, the dimensionless exchange relaxation constant λ introduced below is small), Eq. (1) yields for the effective field to lowest order in γ the following:

$$\mathbf{F}=\mathbf{F}_\perp+\mathbf{F}_\parallel, \quad \mathbf{F}_\perp=\frac{1}{g}\left[\mathbf{m}\frac{\partial \mathbf{m}}{\partial t}\right], \quad \mathbf{F}_\parallel=F\mathbf{m}, \tag{4}$$

where $\mathbf{m}=\mathbf{M}/M$ is the normalized magnetization vector. In the nondissipative approximation the projection of **F** on **m** remains indeterminate. This is because, while in the nondissipative approximation ($\gamma=0$) Eq. (1) has an integral of motion $\mathbf{M}^2=M_0^2=\text{const}$ and can be written as an equation for the unit vector **m**, when the exchange dissipation term is taken into account we must also allow for variations in $|\mathbf{M}|$. Note that this fact sets our approach, based on the complete system of equations of Ref. 8, apart from the approaches developed say, in Refs. 7 and 14, which employ models for the dynamics of the unit vector.

Now we derive an equation for F. To this end we multiply both sides of Eq. (1) by **m** and use the explicit form of $\mathbf{F}_\perp$. The result is

$$-\lambda a^2\nabla^2F+\lambda a^2(\nabla\mathbf{m})^2F = -\frac{1}{gM_0}\frac{\partial M}{\partial t}+\frac{\lambda a^2}{j}\left(\mathbf{m}\nabla^2\left[\frac{\partial \mathbf{m}}{\partial t}\mathbf{m}\right]\right). \tag{5}$$

Here we have introduced the dimensionless exchange relaxation constant λ:

$$\lambda a^2 g M_0=\gamma,$$

where a is the lattice constant (see Refs. 13, 15, and 16).

The formula for the dissipation function Q can be written in terms of the unit vector **m** and the longitudinal component F of the effective field as follows:

$$Q=\frac{1}{2}\lambda a^2 g M_0\int d\mathbf{r}\left\{\frac{1}{g^2}\left(\nabla\left[\frac{\partial \mathbf{m}}{\partial t}\mathbf{m}\right]\right)^2+F^2(\nabla\mathbf{m})^2 +(\nabla F)^2+F\frac{2}{g}\left(\mathbf{m}\nabla^2\left[\frac{\partial \mathbf{m}}{\partial t}\mathbf{m}\right]\right)\right\}. \tag{6}$$

No approximations have been introduced, except for using (4) for $\mathbf{F}_\perp$. The only condition here is that $\lambda(a/\Delta)^2$ be much smaller than unity (where Δ is the characteristic scale of inhomogeneities), which is true for almost all magnetic materials. If only this condition is taken into account, F is determined by solving the linear inhomogeneous differential equation (5), whose right-hand side depends not only on the nature of the solution for the unit vector **m** but also on the variation in the absolute value of magnetization. Since the differential operator in (5) is positive definite and the equation has no homogeneous solution decreasing far from the soliton, it is obvious that we have $F=0$ at $\partial\mathbf{M}/\partial t=0$. To leading order in the velocity we can assume $\mathbf{M}=\mathbf{M}(\boldsymbol{\xi})$, with $\boldsymbol{\xi}=\mathbf{r}-\mathbf{v}t$, where $\mathbf{M}(\boldsymbol{\xi})$ has the same form as $\mathbf{M}(\mathbf{r})$ in a soliton at rest. This means that for low velocities the value of F can be calculated by using the static soliton solution obtained with allowance for variation in the length of the magnetization vector.

For Bloch-point solitons, in which the magnetization length varies greatly (say, $M=0$ at the center of a Bloch point), calculating F requires solving the equation for $M=M(\mathbf{r})$ exactly. Only then can F be calculated (see Ref. 12). However, the majority of solitons, which do not contain singularities, have another small parameter, the smallness of the variation of magnetization along the length, $\mu=M-M_0\ll M_0$ (actually, the longitudinal susceptibility $\chi_\parallel$ is the small parameter). Taking all this into account, we can write $\partial M/\partial t$ in terms of vector **m**.

With this simplifying assumption in mind, we write the energy of the homogeneous exchange interaction in the form

$$f(M)-f(M_0)=\frac{1}{2}\frac{d^2f(M_0)}{dM_0^2}(M-M_0)^2 =\frac{1}{2\chi_\parallel}(M-M_0)^2. \tag{7}$$

Here we have allowed for the fact that $df/dM=0$ holds at $M=M_0$ and have introduced the longitudinal susceptibility $\chi_\parallel$ of the ferromagnet. Combining this with (2), we arrive at the following expression for F:

$$F=-\frac{\mu}{\chi_\parallel}+\alpha\nabla^2M-\alpha(\nabla\mathbf{m})^2M. \tag{8}$$

If we now substitute this into Eq. (5), we get the desired equation for determining F that depends only on the unit vector **m**:

$$-\nabla^2F+F(\nabla\mathbf{m})^2=\frac{1}{g}\left(\mathbf{m}\nabla^2\left[\frac{\partial \mathbf{m}}{\partial t}\mathbf{m}\right]\right) +\frac{\alpha\chi_\parallel}{g\lambda a^2}\frac{\partial}{\partial t}[(\nabla\mathbf{m})^2]. \tag{9}$$

Thus, to calculate the retardation of a soliton with $\mu\ll M_0$, it is sufficient to find its structure in the nondissipative approximation on the basis of the equation for the unit vector **m**. Then, solving (9) with allowance for the explicit form of $\mathbf{m}(\mathbf{r}-\mathbf{v}t)$, we determine F and after that calculate the value of the dissipation function. Note that Eq. (9) has the form of a partial linear equation with a right-hand side, and its solution can be expressed as the sum of the general solution of the homogeneous solution and a particular solution of the nonhomogeneous equation. Clearly, the general solution contributes nothing to the dissipation function (6), so that we can use any particular solution of Eq. (9).

It can easily be shown that for linear spin waves the right-hand side of Eq. (9) and, hence, F are proportional to $m_\perp^2$, where $m_\perp$ is the spin wave amplitude. Hence the contribution of the terms with F to the dissipation function (6) is proportional to $m_\perp^4$ and must be taken into account, and for the magnon decay constant we arrive at the same expression as in Ref. 8. For nonlinear waves with a sizable amplitude (topological solitons), the contribution of F can be substantial. From Eq. (9) it immediately follows that F consists of two terms: one is proportional to $\chi_\parallel$ and tends to zero as $\chi_\parallel \to 0$, and the other contains no small parameters of the theory. Soliton relaxation based on Eqs. (6) and (9) was analyzed earlier for periodic nonlinear magnetization waves,[16] domain walls,[15,16] and Bloch points.[12] In all these cases the first term on the right-hand side of Eq. (9) vanishes, and only the contribution of F proportional to $\chi_\parallel$ remains. Thus, in these examples we can formally assume $\chi_\parallel \to 0$ and, neglecting the contribution of F, we can calculate Q with allowance for only the first term in the dissipation function (6). Below we show that for 2D-solitons with a small radius the right-hand side of Eq. (9) and F contain both terms, including one without the small parameters of the theory, so that the approximation $F=0$ is never valid, even when $\chi_\parallel = 0$.

3. In the nondissipative approximation, Eq. (1) has a well-known solution of the two-dimensional soliton type, which in terms of the angular variables for **m**, i.e., $m_x + i m_y = \sin\theta \exp(i\varphi)$ and $m_z = \cos\theta$, corresponds to

$$\theta = \theta(r), \quad \varphi = \nu\chi + \varphi_0. \tag{10}$$

Here r and χ are the polar coordinates in the plane of the two-dimensional magnetic material, $\nu = \pm 1, \pm 2, \ldots$ is the topological charge of the soliton, $\varphi_0 = \text{const}$, and the function $\theta(r)$ is determined by the type of magnetic material (for more details see Refs. 1, 3, and 4). We assume that the soliton is moving with a constant velocity **v** and that $\partial\theta/\partial t = -(\mathbf{v}\nabla\theta)$ and $\partial\varphi/\partial t = -(\mathbf{v}\nabla\varphi)$. If the soliton velocity is low, we can use the explicit form of solution (10) and write

$$(\mathbf{v}\nabla\theta) = (\mathbf{v}\mathbf{e}_r)(d\theta(r)/dr), \quad (\mathbf{v}\nabla\varphi) = (\nu/r)(\mathbf{v}\mathbf{e}_\chi),$$

where $\mathbf{e}_r$ and $\mathbf{e}_\chi$ are the unit vectors of the polar coordinate system. Now we can easily write the right-hand side of Eq. (9) explicitly:

$$-(\mathbf{v}\mathbf{e}_\chi)\frac{\nu}{gr}\left[\left(\nabla^2\theta - \frac{4\nu\theta'}{r}\right)\sin\theta + \left(2(\theta')^2 + \left(\frac{\nu}{r}\right)^2 \sin^2\theta\right)\cos\theta\right] + (\mathbf{v}\mathbf{e}_r)\frac{\alpha\chi_\parallel}{\lambda g a^2}\left[(\theta')^2 + \left(\frac{\nu}{r}\right)^2 \sin^2\theta\right]', \tag{11}$$

where the prime stands for the derivative with respect to r. This implies that

$$F = (\mathbf{v}\mathbf{e}_r) f_1(r) + (\mathbf{v}\mathbf{e}_\chi) f_2(r).$$

We were able to find the explicit form of the functions $f_1(r)$ and $f_2(r)$ only for the case of a localized soliton with a small radius $R \ll \Delta_0$, where Δ_0 is the domain wall thickness (note that only small-radius solitons are stable under collapse; see Refs. 13 and 14). In this important case the soliton structure is described by the well-known Belavin–Polyakov solution[17] (see also Refs. 1, 3, and 4),

$$\tan\frac{\theta}{2} = \left(\frac{R}{r}\right)^{|\nu|},$$

where R is the soliton radius, and $\nu = \pm 1, \pm 2, \ldots$ is the topological charge. Since the soliton energy $E = 4\pi\alpha M_0^2 |\nu|$ increases with ν, we restrict our discussion to $\nu = \pm 1$. If we allow for the explicit form of $\theta(r)$, formula (11) simplifies considerably and becomes

$$\frac{4}{r^3 g}\sin^3\theta(1+\cos\theta)\left[-(\mathbf{v}\mathbf{e}_\chi) + \frac{\chi_\parallel \alpha}{\lambda a^2}(\mathbf{v}\mathbf{e}_r)\right].$$

Here it becomes possible to write the explicit form of the solution of Eq. (9):

$$F = F(r,\chi) = \frac{2rv}{g(r^2+R^2)}\left[\sin\psi - \frac{\alpha\chi_\parallel}{\lambda a^2}\cos\psi\right],$$

where $\cos\psi = (\mathbf{v}\mathbf{e}_r)/v$ and $\sin\psi = (\mathbf{v}\mathbf{E}_\chi)/v$. Thus, $F \to 0$ as $r \to 0$ or $r \to \infty$, but at $r \approx R$ the values of F are considerable even at $\chi_\parallel = 0$.

4. Using the explicit form of $F(r,\chi)$, we can calculate the soliton dissipation function Q. As a result the desired formula for the viscosity coefficient of a small-radius soliton becomes

$$\eta = \frac{8\pi}{3}\frac{aM_0}{gR^2}\left[\lambda a^2 + \frac{\alpha^2\chi_\parallel^2}{\lambda a^2}\right]. \tag{12}$$

We have thus established that in the exchange approximation viscosity coefficient of the soliton contains two terms. The first term is proportional to λa^2 and can be interpreted as the direct contribution of exchange relaxation. However, this contribution differs from that obtained in the simplest version of the generalized phenomenological theory, which does not take into account the contribution of F to the dissipation function (6). The second term is proportional to the parameter $\chi_\parallel^2/\lambda$, that is, it contains the ratio of the small parameters $\chi_\parallel^2$ and λ.

The origin of the second term, containing the factor $\chi_\parallel^2/\lambda$, can be explained in the following way. Near the soliton the local value of M differs from the equilibrium value of the magnetization vector, M_0. Hence as the soliton moves, its energy goes into causing M in the given region of the magnetic material to depart from the equilibrium value M_0, after which this deviation relaxes. Consequently, the relaxation is caused by the variation in the absolute value of the magnetization vector. This is a typical example of relaxation of excitations of a dynamical variable (in our case the unit vector **m**) caused by the interaction with another variable whose dynamics consists of pure damping. Note that this combination of parameters emerges in the description of what is known as slow (or longitudinal) relaxation in magnetic materials with rare-earth ions, a phenomenon well-known in linear excitations, or magnons,[18] and in moving domain walls.[19] With all the differences in the physical processes taking place in these problems, a certain similarity manifests itself. In both cases the relaxation arises because the variation of the dynamical variable (the normalized magnetization vector **m** in our case and the iron-sublattice magnetization vector $\mathbf{M}_{Fe}$ in the case of slow relaxation) displaces from equilibrium the variable with purely damped dynamics (the length M of the magnetization vector and the rare-earth–sublattice magnetization vector $\mathbf{M}_R$, respectively). Here the contribution to the dissipation grows as the corresponding susceptibility $\chi_{\parallel}$ increases and as the relaxation constant λ decreases.

Clearly, these two contributions to the viscosity coefficient of the soliton must have different temperature dependences. Estimating which of the two is dominant requires using specific values for the parameters $\chi_{\parallel}$ and λ. For real quasi-two-dimensional magnetic substances these values are unknown. We note only that according the estimates of Kabanov *et al.*[13] done for materials of the ferrite-garnet type, the terms with $\chi_{\parallel}^2/\lambda$ are greater than the direct contribution of exchange interaction by almost two orders of magnitude. Estimates involving realistic values of $\chi_{\parallel}$ and λ have shown that the contribution of the terms with F and Q is significant, and only by allowing for these terms the experiments of Refs. 10 and 12 could be interpreted.

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