Equilibrium of a rotating ellipsoidal bunch of charged particles in its own field

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A self-consistent model of an ellipsoidal bunch interacting with its own nonstationary field is constructed. The conditions under which the dimensions of the bunch do not change in time are determined. © *1996 American Institute of Physics.* [S1063-7761(96)01304-2]

The problem of the accumulation of large charges over a prolonged time is still the subject of both theoretical and experimental studies. This problem is an important part of several scientific and technical efforts, above all accelerator technology and the physics of charged-particle beams.

When fluxes of electrons with high particle density are obtained, the forces of repulsion that arise because of the charges can be balanced by charges of opposite sign (positively charged ions), but in the absence of such neutralization the motion of the electrons is determined not only by the external forces but also by the self-field forces, which are often comparable with the forces produced by the external electrodes. The self-field forces greatly restrict the possibilities of obtaining high densities in bunches of particles. To balance the self-field forces that arise when intense beams of charged particles are created, it is possible to apply to them external fields of various kinds, for example, quadrupole focusing fields and longitudinal magnetic fields.[1,2] When a beam undergoes sufficiently rapid longitudinal acceleration, an effective force that confines the particles in the transverse direction arises.[3] In all cases, the confining fields are produced by systems of electrodes of various kinds. Many studies have been devoted to different methods of confining the beam particles in the transverse direction. We mention, for example, the studies of laminar beams[4] and also Ref. 5, in which a study was made of beams with thermal spread of the transverse velocities. However, the effect of the variable retarded self-field on the bunch dynamics does not appear to have been studied sufficiently (we mention in this connection Ref. 6).

In this paper, we study the possibility of confining electric charges in the self-field when a bunch of ellipsoidal (asymmetric) shape rotates. When an asymmetric bunch rotates suffciently rapidly in vacuum, the strength of the wave fields sufficies to keep the dimensions of the bunch constant in time.

1. We shall study the behavior of a bunch of charged particles (electrons) in the form of a strongly prolate triaxial ellipsoid, making the assumption that one axis is much greater than the other two: $R_z \gg R_x, R_y$. We also assume that there is no motion of the particles along the z axis: $\dot{z} \equiv 0$ for all particles of the beam. We assume that the beam rotates about the z axis, and that the forces exerted by the electric field in the x, y system attached to the principal axes have the form

$$F_x = -\omega_1^2 x, \quad F_y = \omega_2^2 y. \tag{1}$$

The expressions (1) describe the situation in which a compressive force acts along the x axis, while a force that pushes the particles apart acts along the y axis; moreover, the latter exceeds the former ($\omega_2 > \omega_1$). The possibility of fulfillment of the relations (1) will be demonstrated below.

The equations of motion in a system rotating with constant angular velocity $\dot{\theta}$ have the form

$$\ddot{x} - x\dot{\theta}^2 - 2\dot{y}\dot{\theta} = -\omega_1^2 x, \quad \ddot{y} - y\dot{\theta}^2 + 2\dot{x}\dot{\theta} = \omega_2^2 y. \tag{2}$$

Here $\theta(t) = \dot{\theta} t$ is the angle of rotation of the bunch relative to a fixed coordinate system.

For the formulation of the problem, it is important to note that we study the behavior of a collisionless bunch; Eqs. (2) represent the linear approximation, and, for example, no allowance is made for the effect of the axial magnetic self-field on the motion of the particles (see also Ref. 6).

We assume that the dependence of the coordinates x and y on the time is periodic: $x(t) \sim \text{const}\, e^{i\Omega t}$ and $y(t) \sim \text{const}\, e^{i\Omega t}$.

The eigenfrequencies Ω_i can be determined from the dispersion relation, and we can obtain

$$\Omega_{1,2}^2 = \dot{\theta}^2 - \frac{\omega_2^2 - \omega_1^2}{2} \pm \sqrt{\left(\frac{\omega_2^2 + \omega_1^2}{2}\right)^2 - 2\dot{\theta}^2(\omega_2^2 - \omega_1^2)}. \tag{3}$$

Both solutions are positive, $\Omega_{1,2}^2 > 0$, if

$$\frac{(\omega_2^2 + \omega_1^2)^2}{8(\omega_2^2 - \omega_1^2)} > \dot{\theta}^2 > \omega_1^2 \quad \text{and} \quad 3\omega_1^2 > \omega_2^2.$$

Equations (2) have two invariants, and they have the form

$$I_1 = (A_2 x + 2\dot{\theta}\dot{y})^2 + \frac{1}{\Omega_1^2}(2\dot{\theta} y(\dot{\theta}^2 + \omega_2^2) - A_1 \dot{x})^2, \tag{4}$$

$$I_2 = (A_1 y - 2\dot{\theta}\dot{x})^2 + \frac{1}{\Omega_2^2}(2\dot{\theta} x(\dot{\theta}^2 - \omega_1^2) + A_2 \dot{y})^2, \tag{5}$$

where

$$A_1 = \Omega_1^2 + \dot{\theta}^2 + \omega_2^2, \quad A_2 = \Omega_2^2 + \dot{\theta}^2 - \omega_1^2.$$

For the self-consistent description of the bunch, we take an invariant in the form of a linear combination of I_1 and I_2:

$$I = \alpha_1 I_1 + \alpha_2 I_2,$$

where $\alpha_1 > 0$, $\alpha_2 > 0$. We represent the invariant in the form

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$$I=A\dot{x}^2+B\dot{y}^2+2C_1\dot{x}y+2C_2x\dot{y}+Dx^2+Ey^2,$$

$$A=\alpha_1\frac{A_1^2}{\Omega_1^2}+4\alpha_2\dot{\theta}^2,\quad B=4\alpha_1\dot{\theta}^2+\alpha_2\frac{A_2^2}{\Omega_2^2},$$

$$C_1=-2\dot{\theta}A_1\left(\alpha_1\frac{\dot{\theta}^2+\omega_2^2}{\Omega_1^2}+\alpha_2\right),$$

$$C_2=2\dot{\theta}A_2\left(\alpha_1+\alpha_2\frac{\dot{\theta}^2-\omega_1^2}{\Omega_2^2}\right),$$

$$D=\alpha_1A_2^2+4\alpha_2\frac{\dot{\theta}^2}{\Omega_2^2}(\dot{\theta}^2-\omega_1^2)^2,$$

$$E=4\alpha_1\frac{\dot{\theta}^2}{\Omega_1^2}(\dot{\theta}^2+\omega_1^2)^2+\alpha_2A_1.$$

For the distribution function of the particles, we set

$$f=\varkappa\delta(I-1),$$

where $\varkappa$ is a normalization constant, and $\delta(x)$ is the Dirac delta function. This is a model distribution—it does not take into account the actual spread of the particles with respect to I. It yields a constant value of the density within the bunch, and this, in its turn, determines the linear nature of the coordinate dependence of the self-field forces. Other more physical distribution functions do not lead to significant qualitative differences—additional forces that depend nonlinearly on the coordinates appear. In the present problem, the nonlinear forces are negligibly small. We note also that a δ function as distribution in the phase space is very widely used to describe the most varied situations in which it is important to take into account self-field forces.[1]

Use of the expression for I leads to the relation

$$f=\varkappa\delta\left\{A\left(\dot{x}+\frac{C_1y}{A}\right)^2+B\left(\dot{y}+\frac{C_2x}{B}\right)^2+x^2\frac{BD-C_2^2}{B}+y^2\frac{AE-C_1^2}{A}-1\right\}. \tag{6}$$

It follows from (6) that the density of the particles is constant within the ellipse $x^2/R_x^2+y^2/R_y^2=1$ and equal to zero outside it, where the semiaxes are determined by the equations

$$R_x^2=\frac{\Omega_2^2\left(4\alpha_1\dot{\theta}^2+\alpha_2\frac{A_2^2}{\Omega_2^2}\right)}{2\alpha_1\alpha_2A_2^2(\Omega_2^2-\Omega_1^2)^2},$$

$$R_y^2=\frac{\Omega_1^2\left(\alpha_1\frac{A_1^2}{\Omega_1^2}+4\alpha_2\dot{\theta}^2\right)}{2\alpha_1\alpha_2A_1^2(\Omega_1^2-\Omega_2^2)^2}. \tag{7}$$

Averaging over the velocities by means of (6) also leads to the equations

$$\bar{\dot{x}}=-\frac{C_1}{A}y,\quad \bar{\dot{y}}=-\frac{C_2}{B}x,\quad \overline{(\dot{x}-\bar{\dot{x}})^2}=\overline{\Delta\dot{x}^2}=\frac{1}{2A},\quad \overline{\Delta\dot{y}^2}=\frac{1}{2B}. \tag{8}$$

Here $\overline{\Delta\dot{x}^2},\overline{\Delta\dot{y}^2}$ are the mean random (thermal) velocities. The most interesting range of parameter values is determined by the relations

$$\alpha_1=\alpha_2,\quad \delta_0=\omega_2^2-\omega_1^2\ll\omega_1^2,\quad \dot{\theta}^2\simeq\omega_1^4/2\delta_0,$$

$$\Omega_1^2\simeq\dot{\theta}^2+\omega_1\sqrt{\delta_0},\quad \Omega_2^2\simeq\dot{\theta}^2-\omega_1\sqrt{\delta_0},\quad A_1\simeq\frac{\omega_1^4}{\delta_0}+\omega_1^2,$$

$$A_2\simeq\frac{\omega_1^4}{\delta_0}-\omega_1^2. \tag{9}$$

If these relations are satisfied, then for C_1/A and C_2/B we obtain

$$\frac{C_2}{2B\dot{\theta}}\simeq\frac{1}{2}\left(1-\frac{2\delta_0}{\omega_1^2}\right),\quad -\frac{C_1}{2\dot{\theta}A}\simeq\frac{1}{2}\left(1+\frac{2\delta_0}{\omega_1^2}\right), \tag{10}$$

and for the semiaxis ratio R_x/R_y we have

$$\frac{R_x}{R_y}\simeq1+\frac{\delta_0}{\omega_1^2}. \tag{11}$$

The bunch in question consists of nonrelativistic particles. In this connection, it is important to calculate the mean velocities of the particles in the coordinate system at rest. Since the rotating coordinates x,y are related to the fixed coordinates x_1,y_1 by

$$x=x_1\cos\theta+y_1\sin\theta,\quad y=-x_1\sin\theta+y_1\cos\theta,$$

we can obtain for the velocities $\dot{x}_1$ and $\dot{y}_1$

$$\dot{x}_1=(\dot{x}-\dot{\theta}y)\cos\theta-(\dot{y}+\dot{\theta}x)\sin\theta,$$

$$\dot{y}_1=(\dot{x}-\dot{\theta}y)\sin\theta+(\dot{y}+\dot{\theta}x)\cos\theta. \tag{12}$$

The relations (12) express the velocities in the laboratory system in terms of the velocities and coordinates in the moving system, and also in terms of the angular velocity and the rotation angle θ.

Averaging over the velocities using Eqs. (8) and (10) leads to the relations

$$\bar{\dot{x}}_1=\frac{\delta_0}{\omega_1^2}\dot{\theta}(y\cos\theta-x\sin\theta),$$

$$\bar{\dot{y}}_1=\frac{\delta_0}{\omega_1^2}\dot{\theta}(y\sin\theta+x\cos\theta). \tag{13}$$

The maximum value of the velocities is attained at $x\sim y\sim R_\perp$. In order of magnitude

$$v_{1\,\max}\sim\frac{\delta_0}{\omega_1^2}\dot{\theta}R_\perp,\quad R_\perp=\frac{1}{2}(R_x+R_y).$$

It is important to note that there also exists a region of nonrelativistic velocities with $\dot{\theta}R_\perp/c>1$, since the condition $v_{1\,\max}<c$ means that $\dot{\theta}R_\perp/c<\omega_1^2/\delta_0$, where $\delta_0\ll\omega_1^2$.

For the mean square velocities, we have

$$\overline{\dot{x}_1^2}+\overline{\dot{y}_1^2}=\frac{1}{2}\left(\frac{1}{A}+\frac{1}{B}\right)+\dot{\theta}^2\frac{\delta_0^2}{\omega_1^4}(x^2+y^2).$$

The term $\sim(1/A+1/B)$ is relatively small in the case of sufficiently large values of the constants $\alpha_1=\alpha_2$, and then the total velocity of the particles differs only slightly from the mean velocity.

2. We calculate the self-field forces that act on the particles of the bunch, including the forces exerted by the retarded field.

The components of the potential in the laboratory system are determined by the expressions

$$\Phi(\mathbf{r}_1,t)=\int \rho\left(\mathbf{r}',t-\frac{|\mathbf{r}_1-\mathbf{r}'|}{c}\right)\frac{d\mathbf{r}'}{|\mathbf{r}_1-\mathbf{r}'|},$$

$$A(\mathbf{r}_1,t)=\frac{1}{c}\int \mathbf{j}\left(\mathbf{r}',t-\frac{|\mathbf{r}_1-\mathbf{r}'|}{c}\right)\frac{d\mathbf{r}'}{|\mathbf{r}_1-\mathbf{r}'|}. \quad (14)$$

We represent the total potential in the form $\Phi=\Phi^{(0)}+\Phi^{(r)}$, where

$$\Phi^{(0)}(\mathbf{r}_1,t)=\int \rho(\mathbf{r}',t)\frac{d\mathbf{r}'}{|\mathbf{r}_1-\mathbf{r}'|},$$

and $\Phi^{(r)}$ can be represented in the form of a series in powers of $1/c$:

$$\Phi^{(r)}(\mathbf{r}_1,t)=\sum_{k=1}^{\infty}\frac{(-1)^k}{c^k}\frac{\partial^k}{\partial t^k}\int \rho(\mathbf{r}',t)|\mathbf{r}_1-\mathbf{r}'|^{k-1}d\mathbf{r}', \quad (15)$$

while the vector potential can also be represented in the form of a series:

$$\mathbf{A}(\mathbf{r}_1,t)=\frac{1}{c}\sum_{k=1}^{\infty}\frac{(-1)^k}{c^k}\frac{\partial^k}{\partial t^k}\int \mathbf{j}(\mathbf{r}',t)|\mathbf{r}_1-\mathbf{r}'|^{k-1}d\mathbf{r}'. \quad (16)$$

In Eqs. (14)–(16), ρ and $\mathbf{j}$ are, respectively, the charge density and the current density. The charge density is assumed to be constant and can be expressed in terms of the total number N of particles in the bunch:

$$\rho=\frac{3eN}{4\pi R_x R_y R_z}.$$

The current density $\mathbf{j}$ has the nonvanishing component $j_\theta=\dot{\theta}\rho r'$. In Cartesian space coordinates $\mathbf{r}'$, we have

$$j_{x'}=-y'\dot{\theta}\rho, \quad j_{y'}=x'\dot{\theta}\rho.$$

To calculate the series (15) and (16), we use an expansion of $|\mathbf{r}_1-\mathbf{r}'|^{k-1}$ in powers of r_1/r'.

At the same time, we note that only the terms that determine the force that depends linearly on the coordinates are calculated. In (15), this corresponds to allowance for the term proportional to $(\mathbf{r}_1\cdot\mathbf{r}')^2$, while in (16) it is necessary to take into account only the term $(\sim\mathbf{r}_1\cdot\mathbf{r}')$. Significant simplifications arise from the fact that the angular velocity and the dimensions of the bunch are constant. After replacement of the coordinates x' and y' according to

$$x'=x_2\cos\theta-y_2\sin\theta, \quad y'=x_2\sin\theta+y_2\cos\theta$$

(i.e., after the introduction of variables of integration associated with the principal axes of the elliptic section) and allowance for the constancy of $\dot{\theta},R_x,R_y$, we obtain for

$$E_{x_1}^{(r)}=-\frac{\partial\Phi^{(r)}}{\partial x_1}-\frac{1}{c}\frac{\partial A_{x_1}}{\partial t}$$

the equation

$$\begin{aligned}E_{x_1}^{(r)}=&-\frac{1}{2}\int\frac{d\mathbf{r}_2}{r_2^5}\rho(\mathbf{r}_2)(x_2^2-y_2^2)\{x_1[3(\cos 2\dot{\theta}(t-\tau)\\&-\cos 2\dot{\theta}t)-6\dot{\theta}\tau\sin 2\dot{\theta}(t-\tau)-6\dot{\theta}^2\tau^2\cos 2\dot{\theta}(t-\tau)\\&+4\dot{\theta}^3\tau^3\sin 2\dot{\theta}(t-\tau)]+y_1[3(\sin 2\dot{\theta}(t-\tau)-\sin 2\dot{\theta}t)\\&+6\dot{\theta}\tau\cos 2\dot{\theta}(t-\tau)-6\dot{\theta}^2\tau^2\sin 2\dot{\theta}(t-\tau)\\&-4\dot{\theta}^3\tau^3\cos 2\dot{\theta}(t-\tau)]\}, \quad \tau=r_2/c. \end{aligned}\quad (17)$$

Note that (17) is obtained by summing series in powers of $1/c$, i.e., we have taken into account the retardation in all orders.

The component $E_{y_1}^{(r)}$ of the retarded field can be readily obtained from the conditions

$$\frac{\partial E_{x_1}^{(r)}}{\partial x_1}+\frac{\partial E_{y_1}^{(r)}}{\partial y_1}=0, \quad \frac{\partial E_{x_1}^{(r)}}{\partial y_1}=\frac{\partial E_{y_1}^{(r)}}{\partial x_1}.$$

The density ρ has the form $\rho=3eN/4\pi R_\perp^2 R_z$. Here $R_x\simeq R_y\simeq R_\perp$.

The nonlinear terms are in order of magnitude $\sim r_1^2/R_\perp^2$, i.e., for small values of x and y they are small. For what follows, the most interesting range of parameters is that for which $\dot{\theta}R_\perp/c>1$, i.e., the case when the terms proportional to $\dot{\theta}^3\tau^3$ make the largest contribution to the integral for the field. From (17), we obtain

$$\begin{aligned}E_{x_1}^{(r)}\simeq&\frac{4\dot{\theta}^3}{5c^3}\frac{eN\delta}{R_\perp}\left(-x_1\sin 2\dot{\theta}\left(t-\frac{R_\perp}{c}\right)\right.\\&\left.+y_1\cos 2\dot{\theta}\left(t-\frac{R_\perp}{c}\right)\right),\\E_{y_1}^{(r)}\simeq&\frac{4\dot{\theta}^3}{5c^3}\frac{eN\delta}{R_\perp}\left(x_1\cos 2\dot{\theta}\left(t-\frac{R_\perp}{c}\right)\right.\\&\left.+y_1\sin 2\dot{\theta}\left(t-\frac{R_\perp}{c}\right)\right).\end{aligned}\quad (18)$$

Further, in (18) we express the coordinates x,y in terms of x_1,y_2 by means of the relations $x_1=x\cos\theta-y\sin\theta$, $y_1=x\sin\theta+y\cos\theta$. Note that the fixed coordinate system is used in the entire treatment. The introduction of the variables x and y in place of x_1 and y_1 can be regarded as a method of obtaining a more convenient form of expression of the equations. For nonrelativistic values of the velocities ($\dot{\theta}R_\perp\ll c$), this transformation is a transition to a rotating system attached to the principal axes of the elliptic section of the bunch.

In the variables x and y, the relations (18) have the form

$$E_x^{(r)}\simeq\frac{4\dot{\theta}^3}{5c^3}\frac{eN\delta}{R_\perp}\left(x\sin\frac{2\dot{\theta}R_\perp}{c}+y\cos\frac{2\dot{\theta}R_\perp}{c}\right),$$

$$E_y^{(r)}\simeq\frac{4\dot{\theta}^3}{5c^3}\frac{eN\delta}{R_\perp}\left(x\cos\frac{2\dot{\theta}R_\perp}{c}-y\sin\frac{2\dot{\theta}R_\perp}{c}\right). \quad (19)$$

Further, we take into account the zeroth term in the retardation of the electric field:

$$E_x^{(0)}=\frac{3eN}{2R_\perp^2R_z}x,\quad E_y^{(0)}=\frac{3eN}{2R_\perp^2R_z}y. \tag{20}$$

To obtain the situation described by the relations (1), we must set

$$\frac{2\dot{\theta}R_\perp}{c}=2\pi k+\frac{3\pi}{2}, \tag{21}$$

where $k=0,1,2,...$ is a nonnegative integer.

Then

$$\omega_1^2=-\frac{3e^2N}{2R_zR_\perp^2m}+\frac{4\dot{\theta}^3}{5c^3}\frac{e^2N\delta}{R_\perp m},\quad \omega_2^2=\frac{3e^2N}{2R_zR_\perp^2m}+\frac{4\dot{\theta}^3}{5c^3}\frac{e^2N\delta}{R_\perp m}, \tag{22}$$

and at the same time

$$\delta=R_x-R_y=\frac{\omega_2^2-\omega_1^2}{\omega_1^2}R_\perp=\frac{\delta_0}{\omega_1^2}R_\perp\ll R_\perp .$$

It follows from (22) that

$$\omega_2^2-\omega_1^2=\frac{3e^2N}{R_zR_\perp^2m}=\delta_0,\quad \omega_1^2=-\frac{3e^2N}{2R_zR_\perp^2m}+\frac{4\dot{\theta}^3}{5c^3}\frac{3e^4N^2}{R_zR_\perp^2m^2}\frac{1}{\omega_1^2}.$$

In the final expression on the right-hand side, the second term must be greater than the first. We obtain

$$\omega_1^2\simeq\frac{2e^2N}{m}\sqrt{\frac{3\dot{\theta}^3}{5c^3}\frac{1}{R_zR_\perp^2}},$$

where $\dot{\theta}$ is determined by the relation (21).

On the other hand, from (9)

$$\dot{\theta}^2\simeq\frac{\omega_1^4}{2\delta_0}=\frac{2}{5}\frac{e^2N}{m}\frac{\dot{\theta}^3}{c^3},$$

from which we obtain the relations

$$\dot{\theta}=\frac{5}{2}\frac{c}{Nr_0}=\frac{2\pi c}{R_\perp}\left(k+\frac{3}{4}\right),\quad r_0=\frac{e^2}{mc^2},$$

$$N=\frac{5R_\perp}{4\pi r_0(k+3/4)}.$$

Finally, for ω_1^2 we obtain

$$\omega_1^2\simeq\frac{c^2}{R_\perp\sqrt{R_\perp R_z}}\sqrt{30\pi(k+3/4)}, \tag{23}$$

and

$$\delta_0=\frac{c^2}{2\pi(k+3/4)R_zR_\perp}\ll\omega_1^2.$$

The larger the value of k, the better the original approximations are satisfied; k is related to the total number of particles by the single-valued expression

$$k=\frac{5R_\perp}{4\pi r_0N}-\frac{3}{4}.$$

As we have already noted, in the framework of the considered linear problem the influence of the axial magnetic self-field is not taken into account. We now establish the conditions under which this approximation is meaningful. For this, we compare the maximum frequency $\omega_H(r)|_{r=R_\perp}$ with the particle gyration frequency $\omega=\dot{\theta}\delta_0/\omega_1^2$.

It is easy to show that

$$\omega_H(R_\perp)=\frac{e^2N}{mc^2R_z}\dot{\theta}.$$

Then the condition under which the effect of the longitudinal field can be ignored has the form

$$\frac{e^2N}{mc^2R_z}\ll\frac{\delta_0}{\omega_1^2},$$

and we can represent it in the form

$$k+\tfrac{3}{4}\ll\frac{3}{10\pi}\frac{R_z}{R_\perp}.$$

The values of k are bounded above by the ratio $R_z/R_\perp$.

Thus, in this paper we have shown that a rotating ellipsoidal bunch with longitudinal dimension greater than the transverse dimensions and interacting with the retarded self-field can have constant transverse dimensions.

For the self-consistent description of ellipsoidal bunches with axes of comparable length, the solution of the three-dimensional problem is of great interest. In this case, the rotational angular velocity is not constant and is the sum of the angular velocities of the precession, rotation proper, and nutation.

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