Quantum modulation of beams of channeled electrons and positrons by a transverse electromagnetic wave

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We use the solutions of the Dirac equation to examine the possibility of quantum modulation of beams of channeled electrons and positrons by a transverse electromagnetic wave in the short-wave range. We also find the conditions required to generate modulated beams. © *1996 American Institute of Physics.* [S1063-7761(96)00504-5]

1. It is known that channeling occurs if a charged particle enters a crystal at an angle to a crystallographic axis or plane smaller than a certain angle $\theta_L=\sqrt{2u_0/\varepsilon}$, where u_0 is the depth of a transverse potential well, and ε is the particle's energy.[1] The discovery[2,3] that channeled particles emit radiation opened possibilities for manufacturing sources of short-wave radiation since due to their large Doppler shift and the high oscillation frequency in the channel, $\Omega \sim 10^{14}-10^{16}\ \mathrm{s}^{-1}$ for particle energies $\varepsilon\sim 10$ GeV–10 MeV, channeled particles emit primarily in the x-ray and gamma ranges with an intensity much higher than the intensity of other types of radiation. But while the spontaneous emission of channeled particles has been thoroughly studied both theoretically and experimentally (see, e.g., Refs. 1, 4, and 5), stimulated channeling[6–12] is far from being realized in practice. One reason is that because of the short lifetime of a particle on transverse-motion levels the length of the coherent interaction of the channeled particle with the electromagnetic wave is quite short (e.g., of the order of one micrometer at positron energies amounting to 10 MeV) compared to the interaction length in other types of free-electron lasers (such as the undulator and Cherenkov lasers). Another is the difficulty of controlling the overpopulation of the channeled particles.[13] If the short interaction length is balanced by an increase in the particle number density, generation of stimulated emission of radiation by the channeled particles would require unrealistically high current densities $\sim 10^6\ \mathrm{A/cm}^{-1}$ (see Ref. 1). Under such currents the crystal used for channeling the particles would change shape and finally disintegrate.

Another method of radiation enhancement based on stimulated photon scattering by channeled particles has also been studied.[12] Since absorption of a photon by a channeled particle is a resonant process, the cross section of photon scattering by a channeled particle is 10^4 times larger than the free-electron Compton scattering cross section.[11] Nevertheless, the gain of the electromagnetic wave in stimulated Compton scattering on channeled particles does not exceed the gain of stimulated emission in the channel from initially inversely populated states.

One way of increasing the gain of stimulated emission of radiation by channeled particles is to employ the klystron interaction scheme. This scheme is applicable in the classical interaction mode. The way in which the stimulated process of emission or absorption of external-field photons proceeds is determined by the relationship between the phase of the transverse particle oscillations and the phase of the wave in which the particle finds itself when it enters the crystal.[14] As a result, in the first interaction region the momentum of the particles in the beam becomes modulated and, simultaneously, the particles in the beam begin to be weakly bunched. In the free drift region, particle bunching develops further, and in the second interaction region the particle bunches, gathering in the retarding phases of the wave, effectively amplify the wave.[15] Thus, in the klystron scheme the short interaction length may be balanced by the suppression of stimulated absorption due to particle bunching in the respective phases in the free-drift region. However, employing this scheme for channeled particles is problematic since the effective bunching length (the drift space length) $\sim\lambda v/\Delta v$ (λ is the bunching wavelength, v is the particle velocity, and Δv is the velocity modulation amplitude) amounts to a few microns or is even smaller, which poses obvious difficulties in maintaining the accuracy of the parameters of the scheme, in view of which the particle bunches are spread out. We also note that the klystron scheme is extremely sensitive to beam jitter.

Here we suggest another two-component scheme for ensuring stimulated interaction of channeled particles with the electromagnetic wave. This scheme is applicable in the quantum mode of interaction. The quantum modulated state of the particles, which leads to modulation of the beam density after the interaction at the frequency of the stimulating wave and its harmonics, is formed in the first interaction region.[16] The density-modulated particle beam can be used to generate spontaneous superradiation (see, e.g., Ref. 17).

Quantum interaction is achieved if the indeterminacy in the particle path caused by the interaction with the electromagnetic wave (or the initial de Broglie wavelength of the particle) becomes of the order of the characteristic size of the path:

$$\frac{\hbar}{\Delta p_{\|}}\gtrsim\frac{c}{\omega},\quad \frac{\hbar}{\Delta p_{\perp}}\gtrsim\frac{v_{\perp}}{\Omega}, \tag{1}$$

or if $d\lesssim\lambda_D$, where d is the interplanar or interaxial spacing, $\lambda_D=\hbar c/\sqrt{2\varepsilon u_0}$ is the transverse de Broglie wavelength of

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the particle, $\Delta p_\parallel$ and $\Delta p_\perp$ are the longitudinal and transverse variations of the particle momentum in the electromagnetic field, $v_\perp = c\sqrt{2u_0/\varepsilon}$ and Ω are the amplitude and frequency of the oscillation ("wiggle") of the transverse velocity of the particle in the channel, c is the speed of light, and $\hbar$ is Planck's constant. The conditions (1) are equivalent to the requirement that the amplitude of the energy exchange between particle and electromagnetic wave be low, $\Delta\varepsilon \lesssim \hbar\omega$. In the quantum mode the particle is not localized in a definite phase of the wave: states of photon absorption and emission exist simultaneously, with the superposition of such states leading to the formation of a quantum modulated state of the particle after interaction. Since this a state is stationary, the resulting quantum modulation of the beam density can remain after interaction for an indefinitely long time (at least theoretically). This constitutes an important advantage of a quantum modulated beam over a classically grouped beam, which, as noted earlier, spreads even over short distances.

The advantage of this scheme over that of induced emission in the channel of initially inversely populated states is that solving the difficult problem of inversely populating the transverse energy levels in creating a quantum modulated state becomes unnecessary.

To display the possibility of forming a quantum modulated particle beam in the short-wave range in the process of interaction of the channeled particles and the electromagnetic wave we find the wave function of a positron in planar channeling in the field of an external transverse electromagnetic wave and the wave function of an electron in axial channeling. The electromagnetic wave is assumed fairly weak ($\Delta\varepsilon \ll \hbar\omega$), so that the stimulated processes have an essentially quantum nature and are one-photon. Accordingly, the wave functions of a particle are found in an approximation that is linear in the field of the electromagnetic wave.

2. Suppose that a positron with an initial momentum $\mathbf{p}_0$ enters a single crystal at an angle to the crystallographic axis that is smaller than the critical channeling angle. The subbarrier levels of the transverse states are those that are first to be populated,[13] and the wave function of the positron before the interaction with the electromagnetic wave in planar channeling with the effective potential

$$u(x) = 4u_0 x^2/d^2 \tag{2}$$

has the form

$$\psi_i^{(0)} = \sum_n a_n \hat{B}_{s_0} \varphi_n(x) \exp\left\{\frac{i}{\hbar}(p_n z - \varepsilon_0 t)\right\}, \tag{3}$$

where $\hat{B}_{s_0}$ is a constant bispinor,

$$\hat{B}_{s_0} = \sqrt{\frac{\varepsilon_0 + mc^2}{2\varepsilon_0}} \begin{pmatrix} u_{s_0} \\ \dfrac{c(\boldsymbol{\sigma}\mathbf{p})}{\varepsilon_0 + mc^2} u_{s_0} \end{pmatrix},$$

s_0 is the spin quantum number,

$$\varphi_n(x) = \frac{1}{\pi^{1/4}} \sqrt{\frac{\kappa}{2^n n!}} \exp\left\{-\frac{\kappa^2 x^2}{2}\right\} \mathcal{H}_n(\kappa x), \quad \kappa^2 = \frac{\varepsilon_0 \Omega_0}{\hbar c^2},$$

$$\varepsilon_0 = \varepsilon_n^\parallel + \hbar\Omega_0\left(n + \frac{1}{2}\right), \quad \varepsilon_n^\parallel = c\sqrt{p_n^2 + m^2c^2},$$

$$\Omega_0 = \frac{c}{d}\sqrt{\frac{8u_0}{\varepsilon_0}},$$

and the $\mathcal{H}_n(z)$ are Hermite polynomials. The squares of the coefficients $|a_n|$ in Eq. (3) determine the probabilities of the initial populations of the transverse levels,

$$a_n = \int \exp\left\{\frac{i}{\hbar} P_{0x} x\right\} \varphi_n^*(x)\,dx.$$

In the field of a transverse electromagnetic wave with the vector potential

$$\mathbf{A} = \mathbf{A}_0(\varphi)\cos\varphi, \quad \varphi = \omega t - \mathbf{kr}, \tag{4}$$

($\mathbf{A}_0(\varphi)$ is the slowly varying envelope of the electromagnetic wave) the dynamics of a planar channeled positron can be found by solving the Dirac equation

$$i\hbar\frac{\partial\psi}{\partial t} = (\hat{H}_0 + \hat{v})\psi,$$

$$\hat{H}_0 = c(\boldsymbol{\alpha}\hat{\mathbf{p}}) + \beta mc^2 + u(x), \tag{5}$$

$$\hat{v} = -e\boldsymbol{\alpha}\mathbf{A},$$

where $\boldsymbol{\alpha}$ and β and the Dirac matrices, and e is the positron charge. Perturbation theory techniques are used to allow for the field of the electromagnetic wave in the solution of Eq. (5). If the terms of order u_0/ε are ignored, the unperturbed positron wave function is given by (3).

Expanding the wave function $\psi^{(1)}$ perturbed by the electromagnetic field in the complete set of unperturbed functions,

$$\psi^{(1)} = \sum_\alpha c_\alpha(t)\psi_\alpha^{(0)}, \quad \alpha = \{p_z, m, s\},$$
$$i\hbar\dot{c}_\alpha(t) = \langle\psi_\alpha^{(0)}|\hat{v}|\psi_i^{(0)}\rangle, \tag{6}$$

and allowing for $\hbar\omega \ll \varepsilon$ and the fact that the particles are ultrarelativistic ($\varepsilon, \varepsilon_0 \gg mc^2$), we arrive at the following expansion:

$$\psi^{(1)} = -\frac{ec}{2\hbar}\sum_{nms} a_n \times \left\{ \frac{(\mathbf{A}_0\mathbf{F}_{mn}^{(+)})(u_s^+ u_{s_0}) - i(u_s^+ \boldsymbol{\sigma} u_{s_0})[\mathbf{G}_{mn}^{(+)}\mathbf{A}_0]}{-\omega' + (n-m)\Omega_0 + (\partial\Omega/\partial\varepsilon)_0 \hbar k_z v_0 (m+1/2)} \times \exp\{i\omega t - ik_z z\} + \frac{(\mathbf{A}_0\mathbf{F}_{mn}^{(-)})(u_s^+ u_{s_0}) - i(u_s^+ \boldsymbol{\sigma} u_{s_0})[\mathbf{G}_{mn}^{(-)}\mathbf{A}_0]}{\omega' + (n-m)\Omega_0 - (\partial\Omega/\partial\varepsilon)_0 \hbar k_z v_0 (m+1/2)} \times \exp\{-i\omega t + ik_z z\} \right\} \exp\left\{\frac{i}{\hbar}(p_n z - \varepsilon_0 t)\right\} \hat{B}_s \varphi_m(x), \tag{7}$$

where

$$F^{(\pm)}_{zmn}=\frac{1}{2\varepsilon}\left[(\varepsilon+\varepsilon_0)\frac{v_0}{c^2}\mp\hbar k_z\right]I^{(\pm)(1)}_{mn},$$

$$I^{(\pm)(1)}_{mn}=\int\exp\{\mp ik_zx\}\varphi_n\varphi^*_m dx,\quad v_0\equiv\frac{c^2p_{z0}}{\varepsilon_0},$$

$$G^{(\pm)}_{zmn}=\pm\frac{\hbar\omega}{2c\varepsilon}I^{(\pm)(1)}_{mn}\left(\nu_z-\frac{v_0}{c}\right),$$

$$\omega'\equiv\omega-k_zv_0,\quad \mathbf{k}=\boldsymbol{\nu}\frac{\omega}{c},\quad \nu^2=1,$$

$$F^{(\pm)}_{xmn}=\frac{1}{2}(I^{(\pm)(2)}_{xmn}-I^{(\pm)(3)}_{xmn}),\quad I^{(\pm)(2)}_{xmn}=\frac{\hbar}{i\varepsilon_0}$$

$$\times\int\exp\{\mp ik_xx\}\varphi^*_m\frac{d\varphi_n}{dx}dx,\tag{8}$$

$$G^{(\pm)}_{xmn}=\frac{1}{2}(I^{(\pm)(2)}_{xmn}+I^{(\pm)(3)}_{xmn}),$$

$$F^{(\pm)}_{ynm}=-J^{(\pm)}_{ynm}=\mp\frac{\hbar k_y}{2\varepsilon}I^{(\pm)(1)}_{mn},$$

$$I^{(\pm)(3)}_{xmn}=\frac{\hbar}{i\varepsilon}\int\exp\{\mp ik_xx\}\varphi_n\frac{d\varphi^*_m}{dx}dx,$$

which describes stimulated one-photon emission and absorption by a channeled positron.

Let us assume that the electromagnetic wave is in resonance with the transverse oscillator levels of the positron in the channel $\omega'=\Omega$. After the interaction only the resonant terms with $m=n\pm1$ contribute to the sum (7): transitions take place from the initial level n to the neighboring levels. In real situations, where the channel potential differs from the parabolic law (2), the positron energy levels are nonequidistant. Bearing in mind, however, that the equidistant nature of the energy spectrum of planar channeled positrons is violated only slightly, $\delta(\omega_{mn-1})\ll\omega_{nn-1}$ ($\hbar\omega_{nn-1}=\varepsilon_{\perp n}-\varepsilon_{\perp n-1}$, where $\varepsilon_{\perp n}$ are the transverse-energy levels), we can assume that resonance with the neighboring levels is achieved thanks to the spectral width stimulated by the electromagnetic wave.

To study quantum modulation we calculate the variation of the particle probability density after interaction with the electromagnetic field:

$$\rho^{(1)}=2\,\mathrm{Re}\,\psi^{(0)+}\psi^{(1)}=-\frac{ec}{4\hbar}\mathrm{Re}\sum_{nml}a^*_la_n\varphi^*_l\varphi_m$$

$$\times\left\{\frac{\exp\{i(\omega t-k_zz)\}}{(n-m)\Omega_0+\Omega'_0(m+1/2)\hbar k_zv_0-\omega'}\right.$$

$$\times\left[\frac{v_0}{c^2}(\mathbf{A}_0\mathbf{v})G^{(+)}_{zmn}+\left(1+\frac{v_0v_z}{c^2}\right)(\mathbf{A}_0\mathbf{F}^{(+)}_{mn}\right.$$

$$\left.-i\boldsymbol{\kappa}[\mathbf{G}^{(+)}_{mn}\mathbf{A}_0])\right]+\left[\frac{v_0}{c^2}(\mathbf{A}_0\mathbf{v})G^{(-)}_{zmn}+\left(1+\frac{v_0v_z}{c^2}\right)\right.$$

$$\left.\times(\mathbf{A}_0\mathbf{F}^{(-)}_{mn}-i\boldsymbol{\kappa}[\mathbf{G}^{(-)}_{mn}\mathbf{A}_0])\right]$$

$$\left.\times\frac{\exp\{-i(\omega t-k_zz)\}}{(n-m)\Omega_0-\Omega'_0(m+1/2)\hbar k_zv_0+\omega'}\right\}$$

$$\times\exp\left\{\frac{i}{\hbar}(p_n-p_l)z\right\},\tag{9}$$

where $\boldsymbol{\kappa}=u^+_{s_0}\boldsymbol{\sigma}u_{s_0}$. Calculations of the nondipole matrix elements $I^{(\pm)(1,2,3)}_{nm}$ of the transitions between the levels of transverse motion lead to the following expression for the beam density:

$$\rho^{(1)}=\frac{ec}{4\hbar}\mathrm{Re}\sum_{\substack{nm\\pl}}c_{mp}J^{(1)}_{np}\varphi_l(x,\varepsilon_0)a_na^*_l\exp\left\{\frac{i}{\hbar}(p_n-p_l)z\right\}$$

$$\times\left\{\frac{\exp\{i(\omega t-k_zz)\}\varphi_m(x,\varepsilon_0-\hbar k_zv_0)(-1)^{p-n}}{\omega'-(n-m)\Omega_0-\Omega'_0(m+1/2)\hbar k_zv_0}\right.$$

$$\times\left[\frac{2\varepsilon_0-\hbar\omega}{\varepsilon_0-\hbar\omega}\left(A_{0z}\frac{v_0}{c^2}+A_{0x}\frac{\hbar k_x}{2\varepsilon_0}+A_{0x}\frac{\omega_{np}}{k_xc^2}\right)\right.$$

$$-\frac{i\kappa\hbar\omega}{2c(\varepsilon_0-\hbar\omega)}\left([\boldsymbol{\nu},\mathbf{A}_0]_z-cA_{0y}\frac{\omega_{np}}{k_xc^2}-cA_{0y}\frac{\hbar k_x}{2\varepsilon_0}\right)$$

$$\left.+\frac{v_0^2}{c^2}\frac{\hbar\omega\varepsilon_0A_{0z}}{(\varepsilon_0-\hbar\omega)^2}\left(\nu-\frac{v_0}{c}\right)\right]$$

$$-\frac{\exp\{-i(\omega t-k_zz)\}\varphi_m(x,\varepsilon_0+\hbar k_zv_0)}{\omega'+(n-m)\Omega_0-\Omega'_0(m+1/2)\hbar k_zv_0}$$

$$\times\left[\frac{2\varepsilon_0+\hbar\omega}{\varepsilon_0+\hbar\omega}\left(A_{0z}\frac{v_0}{c^2}-A_{0x}\frac{\hbar k_x}{2\varepsilon_0}-A_{0x}\frac{\omega_{np}}{k_xc^2}\right)\right.$$

$$+\frac{i\kappa\hbar\omega}{2c(\varepsilon_0+\hbar\omega)}\left([\boldsymbol{\nu},\mathbf{A}_0]_z+cA_{0y}\frac{\omega_{np}}{k_xc^2}+cA_{0y}\frac{\hbar k_x}{2\varepsilon_0}\right)$$

$$\left.-\frac{v_0^2}{c^2}\frac{\hbar\omega\varepsilon_0A_{0z}(\nu_z-v_0/c)}{(\varepsilon_0+\hbar\omega)^2}\right],\tag{10}$$

where $\kappa=|u^+_{s_0}\boldsymbol{\sigma}u_{s_0}|$ is the average value of the particle spin prior to interaction,

$$c_{mp}=\begin{cases}\sqrt{\dfrac{\mu!}{M!}}\chi P^{|m-p|/2}_{|m+p|/2}(\chi), & m-p=2k;\\ \mu=\min\{m,p\} & \\ 0, & m-p=2k+1;\\ M=\max\{m,p\}, & k\text{ an integer,}\end{cases}$$

$$J^{(1)}_{np}=\sqrt{\frac{\mu_0!}{M_0!}}(-2\xi)^{(M_0-\mu_0)/2}e^{-\xi}L^{M_0-m_0}_{M_0}$$

$$\times(2\xi)(\mathrm{sign}k_x)^{M_0-m_0},$$

$$\xi=k_x^2d2^{-7/2}(u_0\varepsilon)^{-1/2}\hbar c,$$

$$\chi=\frac{2}{\varepsilon_0^{1/4}\varepsilon^{1/4}(\varepsilon_0^{-1/2}+\varepsilon^{-1/2})},\quad M_0=\max\{n,p\},$$

$$\mu_0=\min\{n,p\},\quad \hbar\omega_{np}=\varepsilon_{\perp n}-\varepsilon_{\perp p},$$

and $P^\nu_\mu(\chi)$ and $L^n_m(\xi)$ are, respectively, the Legendre and Laguerre polynomials.

In the dipole approximation $kd \ll 1$ Eq. (10) simplifies considerably. Here, ignoring the nonresonant terms in the wave function and averaging over the transverse coordinate, we arrive at the following expression for the positron probability variation:

$$\rho^{(1)}=\rho_0 \mathrm{Re}\left\{\Gamma \exp\left(i\omega t - ik_z z + i\frac{\Omega_0}{c}z\right)\right\},$$
$$\Gamma=\frac{1}{2}eA_{0x}\sqrt{\frac{\Omega_0}{2\hbar\varepsilon_0}}\sum_n\left[\sqrt{n}a_n a_{n-1}^* + \sqrt{n+1}a_n^* a_{n+1}\right]\Delta t_n, \tag{11}$$

where ρ_0 is the initial beam density, and Δt_n is the positron's lifetime on the level n.

As Eq. (11) shows, the positron probability density as a result of the interaction with the electromagnetic field becomes modulated at the wave's frequency (this is true for the one-photon approximation; in the higher orders of perturbation theory we have modulation at the harmonics of the wave's frequency).

Allowing for the fact that the levels are nonequidistant leads only to beats in the modulation percentage,

$$\Gamma=\sum_n \Gamma_n \exp\{i(\delta\omega_{nn-1})t - i(\delta k_{nn-1})z\}, \tag{12}$$

since the detuning of the one-photon transition frequencies from the resonance frequency is fairly small in this case:

$$\delta\omega_{nn-1} \ll \omega, \quad \delta k_{nn-1} \ll k.$$

Estimates of the modulation percentage for an electromagnetic wave propagating at an angle θ to the channel axis lead to the formula

$$\Gamma \sim \frac{eE \cos\theta v_\perp \Delta t}{\hbar\omega}, \tag{13}$$

where E is the electromagnetic field strength. The modulation percentage is equal in order of magnitude to the classical variation of the positron energy[14] divided by the energy of the emitted photon, i.e., the average number of emitted (absorbed) photons. For 10 MeV positrons in planar channeling in a diamond crystal, $\Omega \simeq 8 \times 10^{15}$ c^{-1}, so that resonant interaction with the electromagnetic wave is possible in the optical range ($\omega = 4\times 10^{15}$ s^{-1}) if $\theta \approx \pi$ or in the ultraviolet region ($\omega = 2\times 10^{16}$ s^{-1}) if $\theta \approx 53°$. Since the positron lifetime on transverse levels is determined primarily by the inelastic scattering on the crystal's electrons,[18,19] and since from the classical viewpoint the region near the nucleus is inaccessible for positrons, to estimate Δt we use the approximate formula that was obtained in Ref. 19 and describes the scattering on the valence electrons of the crystal:

$$\Delta t \approx \frac{\hbar^2 cd}{8n_p z_v e^4 \beta_v \langle x^2\rangle \ln(1/k_v^2\langle x^2\rangle)},$$

where $\langle x^2\rangle$ is the mean square distance from a channeled particle to the channeling plane, n_p is the atom number density in the crystal plane, d is the interplanar separation, z_v is the number of valence electrons, β_v is a constant of order unity, and k_v is the characteristic momentum of electrons in the electron plasma of the crystal, $k_v \sim 1/d$. Allowing for the fact that for a diamond crystal $n_p \approx 2.7\times 10^{15}$ cm^{-2} and $d \approx 1.54$ Å and assuming that $\langle x^2\rangle \sim d/2$, we arrive at the following value for the lifetime: 2.5×10^{15} s ($\Delta t\Omega \sim 20$). Note that because of positron scattering in the channel plane the Doppler resonance width is of order $\omega \sin\theta\sqrt{\langle\theta^2\rangle}$ (here $\langle\theta^2\rangle$ is the mean square angle of positron scattering in the channel plane), which cannot exceed the width $1/\Delta t$ caused by the finite lifetime.

Thus, Eq. (13) shows that 10% modulation of a beam of 10 MeV positrons can be achieved in the optical range by applying an electromagnetic wave with $E \sim 10^6$ V cm^{-1}, while in the ultraviolet range this can be done with $E \sim 10^7$ V cm^{-1}.

3. Now let us examine the interaction of an electron beam with the electromagnetic field in conditions of axial channeling. In the zeroth approximation, where we ignore the field of the electromagnetic wave, an electron is assumed to move in the field of the atomic chain with the chain's potential approximated by

$$u(r_\perp) = -\alpha/r_\perp, \tag{14}$$

where α is a constant depending on the type of crystal and the geometry of the problem, and $r_\perp$ is the distance from axis of the atomic chain. Here the electron state is characterized by a definite projection of the electron momentum on the axis of crystal, p_z, and because the potential is axisymmetric the projection of the electron angular momentum M on this axis is conserved during motion. In the zeroth approximation the solution of the Dirac equation (5) with the potential (14) has the form (3), where the wave function of transverse motion is

$$\varphi_{nm}(r_\perp)=\sqrt{\frac{r_\perp}{2\pi}}R_{nm}(r_\perp)e^{im\varphi}, \tag{15}$$

$$R_{nm}(r_\perp)=\frac{4(\varepsilon\alpha/\hbar^2c^2)^{3/2}}{n^{|m|+3/2}}\sqrt{\frac{2(n+|m|-1/2)!}{(n-|m|-1/2)!}} \times\left(\frac{4\varepsilon\alpha r_\perp}{\hbar^2c^2}\right)^{|m|-1/2}\exp\left\{-\frac{2\varepsilon\alpha r_\perp}{n\hbar^2c^2}\right\} \times F\left(-n+|m|+\frac{1}{2}, 2|m|+1, \frac{4\varepsilon\alpha r_\perp}{n\hbar^2c^2}\right).$$

The coefficients a_{nm} determining the probabilities of initial population of the transverse levels are

$$a_{nm}=\int \exp\{i\mathbf{p}_{\perp 0}\mathbf{r}_\perp\}\varphi_{nm}(r_\perp)d^2r_\perp.$$

In Eq. (15) $m=0,\pm 1,\ldots$ is the magnetic quantum number, $n=\sqrt{-2\varepsilon\alpha^2/\varepsilon_\perp\hbar^2c^2}$, and $F(x,y,z)$ is the confluent hypergeometric function. Calculating the wave function that describes the induced channeling effect by employing first-order perturbation theory in the field of the electromagnetic wave, we arrive at the following expression for the electron probability density (to avoid cumbersome expression we write the result in the dipole approximation):

$$\rho^{(1)}=\frac{ec}{2\hbar}\mathrm{Re}\left\{\sum_{\substack{nm\\ n_0m_0\\ \nu\mu}}\varphi_{\mu\nu}^{*}\varphi_{nm}a_{\mu\nu}^{*}a_{n_0m_0}\exp\left[\frac{i}{\hbar}(p_{n_0}-p_\nu)z\right]\right.$$

$$\times\left[\frac{\exp(i\omega t-ik_zz)}{\omega'-\omega_{n_0n}}-\frac{\exp(-i\omega t+ik_zz)}{\omega'+\omega_{n_0n}}\right]$$

$$\left.\times(\mathbf{A}_{0\perp}\mathbf{I}_{nn_0}^{*})\right\}, \qquad (16)$$

where $\omega_{n_0n}=(\varepsilon_{n_0}-\varepsilon_n)/\hbar=2\varepsilon\alpha^2(1/n^2-1/n_0^2)/\hbar^3c^2$ is the transition energy, $\mathbf{I}_{nn_0}=\langle n|\mathbf{p}_\perp|n_0\rangle/\varepsilon$ is the transition matrix element, and $\mathbf{A}_{0\perp}$ is the transverse component of the electromagnetic field. Calculations of the transition matrix elements lead to the following expression for the probability density of the modulated beam:

$$\rho^{(1)}=\frac{er_\perp}{8\pi\hbar c}\sum_{\substack{n_0m_0n\\ \nu\mu}}R_{\nu\mu}\mathrm{Re}\{\omega_{nn_0}a_{\nu\mu}^{*}a_{n_0m_0}\exp\{i(m_0-\mu)\varphi\}$$

$$\times\left[\frac{\exp(i\omega t-ik_zz)}{\omega'-\omega_{n_0n}}-\frac{\exp(-i\omega t+ik_zz)}{\omega'+\omega_{n_0n}}\right]$$

$$\times[(iA_{0x}+A_{0y})e^{i\varphi}R_{nm_0+1}D_{nn_0}^{m_0+1\,m_0}$$

$$+(iA_{0x}-A_{0y})e^{-i\varphi}R_{nm_0-1}D_{nn_0}^{m_0-1\,m_0}]\}, \qquad (17)$$

where

$$D_{nn_0}^{mm_0}\int_0^\infty r_\perp^3R_{nm}(r_\perp)R_{n_0m_0}(r_\perp)dr_\perp .$$

Let the electromagnetic wave be in resonance with two adjacent transverse-motion energy levels n_0 and n_0-1, i.e., $\omega_{n_0n_0-1}\equiv\Omega=\omega'$. Then after averaging over the transverse coordinate we have

$$\rho^{(1)}=\rho_0\mathrm{Re}\left\{\Gamma\exp\left(i\omega t-ik_zz+i\frac{\Omega}{c}z\right)\right\}, \qquad (18)$$

$$\Gamma=\frac{eA_{0x}\Omega}{4\hbar c}\sum_{m_0}a_{n_0m_0}\{a_{n_0-1m_0-1}^{*}[D_{n_0n_0-1}^{m_0m_0-1}+D_{n_0-1\,n_0}^{m_0-1\,m_0}]$$

$$+a_{n_0-1\,m_0+1}^{*}[D_{n_0n_0-1}^{m_0m_0+1}+D_{n_0-1\,n_0}^{m_0+1\,m_0}]\}\Delta t_{n_0m_0}.$$

As in the case of planar positron channeling, axial electron channeling leads to quantum modulation of the electron number density at the stimulating-wave frequency. Here, however, in contrast to (11), only the populations of levels n_0 and n_0-1 contribute to the modulation percentage (there is no sum over n_0 in (18)), which may decrease the modulation percentage by a factor of ten.

For the resonant transitions of an electron to the nearest neighboring levels the following estimates can be made: $\omega_{nm_0}\approx 2\varepsilon_\perp/\hbar n_0$ and $D_{nn_0}^{mm_0}\sim\alpha/\varepsilon_\perp$; hence $\omega_{nn_0}D_{nn_0}^{mm_0}\sim v_\perp$. As a result we again arrive at the estimate (13) for the modulation percentage.

In order to investigate the general expression (18) we note that the function $D_{nn_0}^{mm_0}$ can be written as

$$D_{nn_0}^{mm_0}=\frac{2^{|m_0|+|m|}}{n_0^{|m_0|+3/2}n^{|m|+3/2}(2|m_0|)!(2|m|)!\alpha\varepsilon}$$

$$\times\sqrt{\frac{(n_0+|m_0|-1/2)!(n+|m|-1/2)!}{(n_0-|m_0|-1/2)!(n-|m|-1/2)!}}$$

$$\times J_{2|m_0|+1}^{|m|-|m_0|+2,\,2(|m_0|-|m|)}(-n_0+|m_0|$$

$$+1/2,-n+|m|+1/2). \qquad (19)$$

where the functions $J_\gamma^{sp}(\alpha,\alpha')$ are defined in the Appendices in Ref. 20.

When estimating the modulation percentage of an axially channeled electron beam one must bear in mind that the lifetime of an electron on transverse levels in axial channeling is shorter than the lifetime of a positron in planar channeling. For instance, for 3.5 MeV electrons $\Omega\Delta t\sim 6.2$ at best, and this ratio increases with electron energy.[21] (Note that the linewidth Γ of the radiation emitted forward by a particle is related to the lifetime of that particle on the transverse levels by $\Gamma=2\gamma^2\hbar/\Delta t$.) Thus, the field strengths of the electromagnetic wave required for modulating an electron beam on the corresponding resonant transitions is greater by a factor of 10 to 100 than that needed in planar positron channeling.

4. All this suggests that by employing stimulated channeling it is possible, at least in principle, to obtain quantum modulated particle beams. This can easily be achieved for positron beams in planar channeling in the optical range (this requires laser fields of approximately 10^6 V/cm^{-1}) with a fairly broad range of particle energies (10 MeV–1 GeV). Difficulties appear, however, as we move into the short–wave region (say, for the ultraviolet range the laser field must already be roughly 10^7 V/cm^{-1}.

Modulation of an electron beam can be achieved at electron energies $\varepsilon\lesssim 50$ MeV (since at high energies the condition for suppressing the scattering by nuclei is violated and electron channeling becomes very unstable). However, owing to the short coherence length and the low electron population on the respective resonant level, the modulation percentage of a beam of channeled electrons is fairly low.

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