Nonlinear waves in metals

V. G. Skobov and A. S. Chernov

International Institute of Physics, 125040 Moscow, Russia
(Submitted 24 April 1995)
Zh. Éksp. Teor. Fiz. **109**, 992–1005 (March 1996)

The effect of the capture of carriers by the field of a large-amplitude rf wave on the electromagnetic properties of metals is studied theoretically. It is shown that carrier capture suppresses collisionless absorption and makes possible the propagation of helicon waves in noble metals in a geometry in which there are open trajectories and helicon waves are not observed in the linear regime. It is further shown that the decrease in collisionless cyclotron absorption at magnetic fields below the helicon threshold in the nonlinear regime can be so significant that the propagation of a new wave not having analogs in the linear regime becomes possible. The frequency of the wave is proportional to the fourth power of the wave vector and inversely proportional to the constant magnetic field, and its damping is inversely proportional to the mean free path of the carriers and the square root of the amplitude of the exciting rf field. The conditions under which the excitation of such waves in copper is possible are indicated. The possibility of the observation of nonlinear waves in aluminum and indium is discussed. [S1063-7761(96)02403-2]

1. INTRODUCTION

It is known that circularly polarized rf waves, i.e., helicon waves, can propagate in strong magnetic fields in metals with unequal concentrations of electrons and holes. The helicon spectrum is specified by the Hall conductivity, i.e., is shaped by all the carriers. The damping of helicon waves is caused by collisions of the carriers. In an alkali metal with a spherical Fermi surface, a helicon wave exists in a range of fields in which its wavelength exceeds the maximal displacement of an electron during a cyclotron period. Therefore, the region where it exists has a weak-field threshold. Below the threshold there is strong cyclotron absorption of the wave by carriers, whose displacement equals the rf wavelength in the metal. This absorption renders the propagation of helicon waves in weaker magnetic fields impossible.

In noble metals there is another type of collisionless absorption. In certain orientations of the constant magnetic field **H** relative to the crystallographic axes, there are open orbits on the Fermi surface of these metals, which correspond to infinite motion of the carriers in the transverse plane. Then the carriers which move in phase with a wave cause collisionless absorption resembling Landau damping. Although the fraction of carriers with open orbits is small, this absorption is very effective. For example, in copper in the $\mathbf{H}\|[110]$ geometry the concentration of such carriers amounts to 0.04 of the electron concentration; however, because of the absorption just mentioned the helicon mode becomes strongly damping and is scarcely observed.[1]

This raises the question of whether the collisionless absorption can be reduced somehow and more favorable conditions for wave propagation can thereby be created. The answer is yes. It was shown theoretically and experimentally in Ref. 2 that electron capture by the field of a strong rf wave weakens the cyclotron damping of a hole doppleron in cadmium and increases the amplitude of the doppleron oscillations of the impedance of the metal slab several fold. In the present paper we analyze the influence of the capture of the carriers responsible for collisionless absorption on the wave properties of metals with unequal electron and hole concentrations. In Sec. 2 the properties of an alkali metal are considered in the case of strong nonlinearity in the range of fields below the helicon threshold. The properties of copper in the $\mathbf{H}\|\mathbf{k}\|[110]$ geometry, where **k** is the wave propagation vector, are analyzed in Secs. 3 and 4. It is shown in Sec. 3 that the capture of carriers with open orbits can suppress the collisionless absorption so much that helicon-wave propagation becomes impossible in that geometry. The wave properties for the opposite circular polarization of the rf field are considered in Sec. 4. We find that the capture of the holes responsible for cyclotron absorption and of the carriers with open orbits responsible for Landau damping creates a possibility for the propagation of a new wave, which does not exist in the linear regime. The wave properties of aluminum and indium in the $\mathbf{H}\|\mathbf{k}\|[100]$ geometry are discussed in Sec. 5.

2. NONLINEAR MODE IN AN ALKALI METAL

Let us consider a metal with a spherical Fermi surface in a constant magnetic field $\mathbf{H}\|\mathbf{k}\|z$. In this case the Fourier transform of the nonlocal conductivity for circular polarizations of a wave field is given by (see, for example, Ref. 3):

$$\sigma_\pm(q)\equiv\sigma_{xx}\pm\sigma_{yx}=\frac{ne^2}{m[\nu-i(\omega\pm\omega_c)]}F_\pm(q), \tag{1}$$

$$F_\pm(q)=\frac{3}{2q^2}\left(1-\frac{1-q^2}{2q}\ln\frac{1+q}{1-q}\right), \tag{2}$$

where

$$q=\frac{kv_F}{\mp\omega_c-\omega-i\nu},\quad \omega_c=\frac{eH}{mc},$$

 1063-7761/96/030535-07$10.00

ω is the angular frequency of the wave, e is the charge of the electron, m is its mass, v_F is the Fermi velocity, n is the concentration, and ν is the collision frequency. (The expressions (1) and (2) are a generalization of the Reuter–Sondheimer equation,[4] which was derived for $H=0$.) We are interested in the case in which $\omega \ll \nu \ll \omega_c$ and $q=\mp k v_F/\omega_c$ is the ratio between the displacement of the electrons at the reference point on the Fermi surface ($v_z=v_F$) during a cyclotron period and the wavelength. The dispersion relation for the wave

$$k^2c^2=4\pi i\omega\sigma_\pm \tag{3}$$

can be written in the form

$$q^2=\mp\frac{2}{3}\xi F_\pm(q), \quad \xi=\frac{6\pi\omega n m^2 v_F^2 c}{eH^3}. \tag{4}$$

In the strong-field region, where $\xi<1$, the roots of Eq. (4) are less than unity. Also, the logarithm in (2) and, therefore, the root of the dispersion equation for negative polarization are real. This root also defines a helicon wave, which exists in a field $H>H_L$, where H_L is the field strength corresponding to the condition $\xi=1$. In the range $H<H_L$ we have $q^2>1$, and the logarithm in (2) has an imaginary part equal to $-\pi i$ Sgn q, which describes cyclotron absorption. In the range of fields where $\xi \gg 1$ $(q^2\gg 1)$, we have

$$F_-(q)\simeq\frac{3\pi i}{q}\,\mathrm{Sgn}\,q+\frac{3}{q^2}, \tag{5}$$

the imaginary part of this function is large, and an anomalous skin effect appears. Such is the situation in the linear regime. In the nonlinear regime the magnetic field of the wave "captures" the electrons with $v_z=\omega_c/k$ (Ref. 2), which are responsible for cyclotron absorption. The velocity of these electrons is modulated with a frequency

$$\omega_0=\frac{e}{mc}\left|H\mathscr{H}\left(\frac{|S|}{\pi}\right)^{1/2}\frac{\partial^2 S}{\partial p_z^2}\frac{\partial S}{\partial p_z}\right|^{1/2}_{p_z=p_z^0}, \tag{6}$$

where $\mathscr{H}$ is the amplitude of the magnetic field of the wave in the metal, $S(p_z)$ is the area of the cross section of the Fermi surface formed by the $p_z=$const plane, and p_z^0 is the value of p_z for which $kv_z(p_z)=\omega_c(p_z)$. For a spherical Fermi surface we have $p_z^0=m\omega_c/k$, and Eq. (6) takes the form

$$\omega_0=(kv_\perp\Omega)^{1/2}=(q\omega_c\Omega)^{1/2}, \quad \Omega=e\mathscr{H}/mc, \tag{7}$$

where $v_\perp$ is the transverse velocity of the electrons with $v_z=\omega_c/k$. If the modulation frequency ω_0 exceeds the collision frequency ν, cyclotron absorption decreases by a factor ω_0/ν, and the function (4) is consequently replaced by

$$F_{nl}\simeq\frac{3\pi i\nu}{4q\omega_0(q)}+\frac{3}{q^2}. \tag{8}$$

In the case of strong nonlinearity, in which $\omega_0\gg\nu$, the second term in (8), which did not play a significant role in the linear regime, becomes the main term, and the roots of the dispersion equation with a positive imaginary part have the form

$$q_1\simeq(2\xi)^{1/4}\left[1+i\frac{3\pi\nu}{16\sqrt{\omega_c\Omega}}(2\xi)^{1/8}\right], \tag{9}$$

$$q_2\simeq iq_1. \tag{10}$$

These relations are valid in the range of fields below the helicon threshold ($\xi\gg 1$), in which the imaginary term in the square brackets is small in comparison with the real term, i.e., in which the following inequalities hold:

$$\frac{\nu}{\sqrt{\omega_c\Omega}}\left(\frac{H_L}{H}\right)^{3/8}\ll 1\ll\left(\frac{H_L}{H}\right)^{3/4}. \tag{11}$$

In this region the root q_2 characterizes a damping mode, and q_1 characterizes a propagating mode. The spectrum of this nonlinear wave is described by the equation

$$\omega=\frac{c^2v_F^2}{3\omega_p^2\omega_c}k^4, \tag{12}$$

where $\omega_p=(4\pi ne^2/m)^{1/2}$ is the plasma frequency of the electron gas. The damping of the wave is

$$\kappa\equiv\mathrm{Im}\,k_1=\frac{3\pi}{8}\left(\frac{H_L}{2H}\right)^{9/8}\left(\frac{2H}{\mathscr{H}}\right)^{1/2}\frac{1}{l}, \tag{13}$$

where $l=v_F/\nu$ is the electron mean free path. Unfortunately, in alkali metals l is small (of the order of 0.01 cm), and it does not seem possible to make κ so small that the passage of a nonlinear wave through a metal slab would be observed.

Noble metals (Cu, Ag, and Au), as well as the group-III metals aluminum and indium, are more suitable for the observation of nonlinear effects. There are two reasons for this. First, the electron and hole mean free paths can be one or two orders greater in these metals than in alkali metals, so that the nonlinear regime ($\nu\ll\omega_0$) can be realized at much smaller values of $\mathscr{H}$. Second, the Fermi surfaces of these metals are such that in certain orientations of the constant magnetic field **H** relative to the crystallographic axes the bulk of the carriers do not contribute to cyclotron absorption, i.e., cyclotron absorption is already weakened in them as a consequence of the features of their Fermi surfaces. Such a situation is observed in noble metals in the **H**‖**k**‖[110] geometry, as well as in aluminum and indium in the **H**‖**k**‖[100] geometry. This is also responsible for the fact that nonlinear effects should be displayed especially strongly in these metals.

3. NONLINEAR HELICON WAVES IN THE PRESENCE OF OPEN ORBITS

3.1. Model of the Fermi surface and nonlocal conductivity of copper in the H‖k‖[110] geometry

As we know, in this geometry there are three groups of carriers on the Fermi surface of copper. In the range of longitudinal momenta $|p_z|<p_1\equiv 0.2\hbar$ Å^{-1} there is a layer of hole orbits of the "dog bone" type. In the ranges of p_z defined by the inequalities $p_1<|p_z|<p_2\equiv 0.8\hbar$ Å^{-1} there are closed electronic orbits, which form the main group of carriers. Finally, in the two narrow ranges where $p_2<|p_z|<p_3$ there are open orbits. The cross sections separating the electronic orbits from the hole orbits and from the

open orbits pass through saddle points. This means that $\partial S/\partial p_z$ goes to infinity at $p_z=p_1$ and $p_z=p_2$. Hence it follows that the function $\partial S/\partial p_z$ has a minimum in the range (p_1,p_2). This qualitative conclusion is confirmed by the numerical calculations of $S(p_z)$ and $\partial S/\partial p_z$ performed for copper by Powell (as reported from a private communication in Ref. 6).

To describe the properties of copper in the nonlinear regime we use the model proposed in Ref. 5, which was devoted to a study of Doppler-shifted cyclotron resonance and the associated doppleron. In this model the electronic and hole orbits are assumed to be circular, and the dependence of their area on p_z is defined by the expressions

$$S_h(p_z)=-S_1\left[a-\left(1-\frac{p_z^2}{p_1^2}\right)^{1/2}\right],\quad |p_z|<p_1, \tag{14}$$

$$S_e(p_z)=S_0-\pi p(p_2-p_1)\arcsin\left(\frac{2|p_z|-p_1-p_2}{p_2-p_1}\right),$$
$$p_1<|p_z|<p_2, \tag{15}$$

where a, p, S_0, and S_1 are constants. When the parameters have the values

$$a=3.3,\quad p=0.6\hbar\,\text{Å}^{-1},\quad S_0=4S_1=4\hbar^2\text{Å}^{-2},$$

the plots of $S_e(p_z)$ and $S_h(p_z)$ given by Eqs. (14) and (15) are very close to those which were obtained by Powell using numerical calculations. We note that in the range of p_z from 0 to p_1 the function $\partial S_h/\partial p_z$ increases monotonically from zero to infinity, while $|\partial S_e/\partial p_z|$ has a minimum at $p_z=(p_2-p_1)/2$, at which its value is equal to $2\pi p$.

In the case under consideration, the contributions of the electrons and holes to the nonlocal conductivity are given by

$$\sigma_\pm^j(q)=\pm\frac{2i}{(2\pi\hbar)^3}\int dp_z S_j(p_z)\left(1+\frac{kc}{2\pi eH}\frac{\partial S_j}{\partial p_z}\pm i\gamma_j\theta_j\right)^{-1}\frac{ec}{H}, \tag{16}$$

$$\gamma_j=\nu_j/\omega_{cj},\quad j=e,h,\quad \theta_e=1,\quad \theta_h=-1.$$

Apart from $\sigma_\pm^e$ and $\sigma_\pm^h$, the contribution of the carriers with open orbits must also be taken into account. Since the constant magnetic field **H** does not restrict the motion of these carriers along the open trajectories, their contribution to σ_{xx} is essentially the same as for $H=0$. For simplicity we note that the open part of the Fermi surface has the form of a cylinder described by the equation

$$\frac{p_x^2+p_z^2}{2m_0}=\varepsilon_0, \tag{17}$$

where m_0 and ε_0 are the mass and Fermi energy of the carriers with open orbits. Then the calculation of the contribution of these carriers to σ_{xx} in the nonlocal limit gives

$$\sigma_{xx}^0=\frac{2n_0e^2}{m_0v_0|k|},\quad n_0=\frac{m_0\varepsilon_0 p_B}{2\pi^2\hbar^3}, \tag{18}$$

where $v_0=(2\varepsilon_0/m_0)^{1/2}$, n_0 is the carrier concentration, and p_B is the dimension of the Brillouin zone parallel to the p_y axis.

Substituting (14) and (15) into (16) and integrating with respect to p_z from $-p_2$ to p_2, we obtain

$$\sigma_\pm=\sigma_\pm^e+\sigma_\pm^h+\frac{1}{2}\sigma^0=\pm i\frac{ec}{H}\left[\frac{n_e}{1\pm i\gamma_e}F_e\left(\frac{q}{1\pm i\gamma_e}\right)-\frac{n_h}{1\mp i\gamma_h}F_h\left(\frac{\eta q}{1\mp i\gamma_h}\right)\right]+\frac{n_0ec}{H|q|}\frac{p}{m_0v_0}, \tag{19}$$

$$F_e(x)=1+\frac{x^2}{2\sqrt{1-x^2}}\left(\ln\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\mp\pi i\right)=1-\frac{x^2}{\sqrt{x^2-1}}\arctan\frac{1}{\sqrt{x^2-1}}, \tag{20}$$

$$F_h(y)=\frac{1}{1+y^2}\left(1-\frac{\pi}{4a}\right)^{-1}\times\left[1-\frac{\pi}{4a}-\frac{\pi y^2(1\pm iy)}{2(1+y^2)a}+\frac{y^2}{2\sqrt{1+y^2}}\left(\ln\frac{\sqrt{1+y^2}+1}{\sqrt{1+y^2}-1}\pm\pi i\right)\right], \tag{21}$$

where

$$q=\frac{kcp}{eH},\quad \eta=\frac{S_1}{2\pi p_1 p}\simeq 1.3, \tag{22}$$

$$n_e=\frac{4S_0(p_2-p_1)}{(2\pi\hbar)^3}\simeq 4\times 10^{22}\ \text{cm}^{-3},\quad n_h=\frac{4S_1p_1}{(2\pi\hbar)^3}\left(a-\frac{\pi}{4}\right)\simeq 8\times 10^{21}\text{cm}^{-3}. \tag{23}$$

Here n_e and n_h are the electron and hole concentrations, and q is the ratio of the minimal displacement of the electrons during a cyclotron period to the rf wavelength. It is convenient to use the first expression in (20) for $q^2<1$ and the second for $q^2>1$. Hence it is seen that F_e is real when $q^2>1$ and $\gamma_e\to 0$. The fact that it does not have an imaginary part means that cyclotron absorption vanishes when $q^2>1$. This is a consequence of the fact that $|\partial S_e/\partial p_z|$ has a minimum and that among the electrons there are none which would satisfy the condition for cyclotron absorption $kv_z=\omega_c$ when $q^2>1$.

3.2. Properties of helicon waves and impedance of a metal slab

Let us consider the properties of a mode whose field rotates in the same sense as do electrons in a magnetic field **H**. The properties of this mode at large values of H can be described by the asymptotic expressions for F_e and F_h at small q, which have the form

$$F_e\simeq 1+i\frac{\pi}{2}q^2,\quad F_h\simeq 1-i\frac{\pi}{2}\left(1-\frac{\pi}{4a}\right)^{-1}\eta^2q^2. \tag{24}$$

The imaginary parts of these functions characterize the cyclotron absorption of the wave by electrons, and holes, respectively. With consideration of (19) and (24), we write the dispersion equation (3) in the form

$$q^2=\xi\left[1+2i\left(\lambda q^2+\frac{\lambda_0}{|q|}\right)\right], \tag{25}$$

where

$$\xi=\frac{4\pi\omega(n_e-n_h)p^2c}{eH^3}\equiv\left(\frac{H_1}{H}\right)^3, \tag{26}$$

$$\lambda\equiv\frac{\pi}{4}\frac{n_e+\eta^2 n_h}{n_e-n_h}\simeq 1.3, \quad \lambda_0\equiv\frac{n_0}{2m_0v_0}\frac{p}{n_e-n_h}\simeq 0.05, \tag{27}$$

the value presented for λ_0 corresponds to $n_0=0.04n_e$ and $m_0v_0=0.3\hbar$ Å^{-1}. At strong fields the real and imaginary parts of the root of Eq. (25) are

$$q'\equiv \operatorname{Re} q\simeq\sqrt{\xi}, \quad q''\equiv \operatorname{Im} q\simeq\lambda\xi^{3/2}+\lambda_0. \tag{28}$$

The expressions (28) are valid in the range of fields where the conditions $\xi\ll 1$ and $q''<q'$ hold. These conditions can be represented in the form

$$\lambda_0^2<(H_1/H)^3\ll 1. \tag{29}$$

In this region there is a helicon wave, whose wave vector and damping are given by

$$k_H\equiv\frac{eH}{pc}q'=k_a\left(\frac{H_1}{H}\right)^{1/2}, \tag{30}$$

$$\kappa_l\equiv\frac{eH}{pc}q''=\frac{eH}{pc}\left[\lambda\left(\frac{H_1}{H}\right)^{9/2}+\lambda_0\right], \tag{31}$$

$$k_a\equiv\frac{eH_1}{pc}=\left[\frac{4\pi\omega(n_e-n_h)e^2}{pc^2}\right]^{1/3}, \tag{32}$$

where k_a^{-1} is of the order of the thickness of an anomalous skin layer.

The value of κ_l depends nonmonotonically on H. The first term in (31), which represents cyclotron absorption by electrons and holes, decreases rapidly with increasing H. The second term, which represents the contribution of carriers with open orbits (Landau damping), increases linearly with H. Therefore, $\kappa_l(H)$ has a minimum at the field strength

$$H_m=H_1\left(\frac{7\lambda}{2\lambda_0}\right)^{2/9}\simeq 3H_1. \tag{33}$$

The corresponding value of κ_l is $\kappa_m\simeq 0.2k_a$. Thus, the damping κ_l is of the order of k_H, i.e., the helicon mode is strongly damped. When the frequency of the exciting field is 1 MHz and the thickness d of the copper slab equals 0.02 cm, $\kappa_m d\simeq 9$. It is impossible to observe the signal of a helicon wave passing through such a slab, because it is weakened 6000-fold. This corresponds to experiment: helicon waves are not observed in the $\mathbf{H}\|\mathbf{k}\|[110]$ geometry.[1]

We now show that the situation is radically altered in the nonlinear regime. We obtain an expression for the nonlinear conductivity by means of a qualitative treatment based on simple physical arguments. Let us consider the motion of an electron with an open orbit and a constant magnetic field **H** in the field of an rf wave. We write the equation of motion in the system moving with the wave in which the electric field is absent and the magnetic field is stationary. In this system the equation has the form

$$\dot{\mathbf{p}}=-\frac{e}{c}[\mathbf{v}(\mathbf{H}+\mathcal{H}(z))], \tag{34}$$

where $\mathcal{H}(z)=\{\mathcal{H}\cos kz, \mathcal{H}\sin kz, 0\}$ is the magnetic field vector of the wave, and the dot denotes differentiation with respect to time. Since we have $v_y=\partial\varepsilon/\partial p_y\equiv 0$ for the dispersion law (17), Eq. (34) reduces to two equations for $\dot{v}_x$ and $\dot{v}_z$, which do not contain H:

$$\dot{v}_x=\Omega_0 v_z\sin kz, \quad \dot{v}_z=-\Omega_0 v_x \sin kz, \tag{35}$$

where $\Omega_0=e\mathcal{H}m_0c$. Landau damping is governed by carriers with a velocity v_z equal to the phase velocity of the wave in the metal ω/k. Since the latter is smaller than the Fermi velocity of the carriers by several orders of magnitude, the difference between the transverse velocity of the effective carriers and v_0 can be disregarded. Setting $v_x\simeq v_0$ in the second equality in (35), we obtain the following equation of motion of a particle along the z axis:

$$\ddot{z}=-\frac{\omega_0^2}{k}\sin kz, \tag{36}$$

where

$$\omega_0^2=kv_0\Omega_0. \tag{37}$$

The first integral of Eq. (36) has the form

$$\dot{z}^2=v_{z0}^2+\frac{2\omega_0^2}{k^2}\cos kz. \tag{38}$$

It follows from (38) that particles for which $v_{z0}>w\equiv\sqrt{2}\omega_0/k$ perform unbounded motion along the z axis and that carriers for which $v_{z0}<w$ are captured by the field of the wave and undergo oscillatory motion with a frequency of the order of ω_0. The former are called transiting carriers, and the latter are called trapped carriers. The damping of the wave depends on ω_0/ν_0, where ν_0 is the collision frequency of the carriers. At small wave field amplitudes, at which $\omega_0\ll\nu_0$, no carriers are captured, and the linear regime is realized. At large field amplitudes, at which $\omega_0\gg\nu_0$, a fraction of the carriers equal to w/v_0 is captured by the magnetic field of the wave. These carriers make the main contribution to the Landau damping. Equation (35) yields the relation $v_x\dot{v}_x=-v_z\dot{v}_z$, which means that the oscillations of each particle along the z axis are accompanied by the modulation of v_x. This oscillating addition to v_x has a $\pi/2$ phase shift relative to $\mathcal{H}_y(z)$. The situation is similar to what occurs for the oscillations of a trapped particle in a high-frequency ($\omega\tau\gg 1$) electric field. If no collisions occur ($\nu\to 0$), the particle does not absorb energy from the field. Absorption is associated with damping of the oscillations, which is caused by the scattering of the particles. Here the absorption is weakened by a factor of $(\omega/\nu)^2$ in comparison with the case of a low-frequency field. Similarly, the absorp-

tion of the wave energy by trapped carriers in the nonlinear regime should decrease by a factor of $(\omega_0/\nu_0)^2$. Therefore, an expression for the nonlinear conductivity can be obtained, if the local conductivity $\sigma_0=n_0e^2/m_0\nu_0$ is multiplied by the fraction of trapped carriers w/v_0 and by the factor ν_0^2/ω_0^2, which describes the decrease in the efficiency of absorption by the trapped particles. Thus, to within a factor of order unity, the contribution of the carriers with open orbits and the conductivity in the case of strong nonlinearity $(\omega_0 \gg \nu_0)$ is given by

$$\sigma_{xx}^n \simeq \frac{\sqrt{2} n_0 e^2 \nu_0}{m_0 k v_0 \omega_0}. \tag{39}$$

A comparison of (39) with (18) reveals that in the nonlinear regime the absorption decreases in proportion to $\nu_0/\sqrt{2}\omega_0$. This result is similar to the results for the nonlinear magnetic Landau damping of helicon waves in metals,[7] for the nonlinear cyclotron damping of hole dopplerons in cadmium,[2] and for the nonlinear anomalous skin effect.[8]

Hence it follows that the decrease in the efficiency of the absorption of energy by trapped conduction electrons can be described, if the following replacements are made in the second formula in (28) and in (31):

$$\lambda \to \lambda_n, \quad \lambda_0 \to \lambda_n^0. \tag{40}$$

Here

$$\lambda_n = \frac{\pi}{4\sqrt{2}(n_e-n_h)} \left(n_e \frac{\nu_e}{\omega_{0e}} + n_h \eta^2 \frac{\nu_h}{\omega_{0h}} \right), \tag{41}$$

$$\lambda_n^0 = \lambda_0 \frac{\nu_0}{\sqrt{2}\omega_0} = 0.05 \frac{m_0 \nu_0 c}{e\sqrt{H_1 \mathscr{H}}} \left(\frac{H}{H_1} \right)^{1/4}, \tag{42}$$

where ω_{0e} and ω_{0h} are the oscillation frequencies of the trapped electrons and holes, respectively. To find ω_{0e} and ω_{0h}, (15) and (14) must be plugged into (6), and the value of p_{zj}^0 must be found from the conditions for cyclotron absorption by electrons and holes, which have the form

$$\frac{q}{2\pi p} \left| \frac{\partial S_j(p_z)}{\partial p_z} \right| = 1, \quad j=e,h.$$

In strong fields, where $q \ll 1$, this leads to the following expression:

$$\omega_{0j} \simeq 1.7 \frac{e\sqrt{H\mathscr{H}}}{cm_j|q|} = 1.7 \frac{e\sqrt{H_1\mathscr{H}}}{cm_j} \left(\frac{H}{H_1} \right)^2, \quad j=e,h. \tag{43}$$

The equality between the numerical coefficients in the expressions (43) for electrons and holes is accidental. Substituting (40)–(43) into (31), taking into account the relation between n_e, n_h, and n_0, and setting

$$m_e \nu_e = m_h \nu_h = m_0 \nu_0 \equiv m\nu,$$

we write the expression for the damping κ of a helicon wave in the nonlinear regime in the form

$$\kappa \simeq \frac{m\nu}{p} \sqrt{\frac{H_1}{\mathscr{H}}} \left[0.6 \left(\frac{H_1}{H} \right)^{11/2} + 0.05 \left(\frac{H}{H_1} \right)^{5/4} \right]. \tag{44}$$

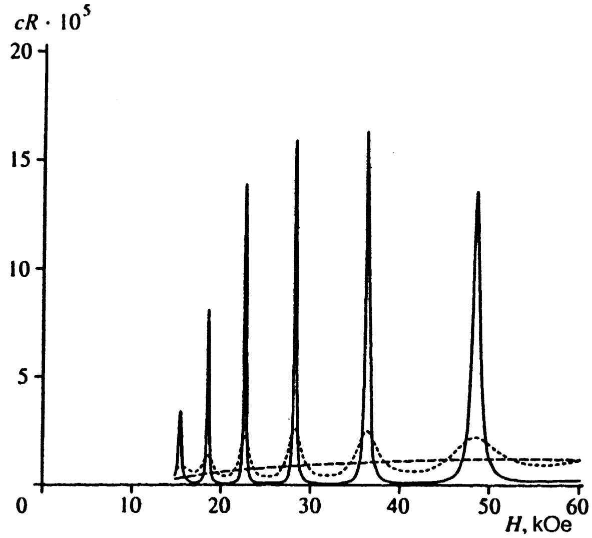

FIG. 1. Calculated plots of $R(H)$ for a copper slab and various amplitudes of the exciting field: solid curve—200 Oe, short dashes—4 Oe, long dashes— 0.1 Oe.

The damping κ as a function H has a minimum at $H_m \simeq 1.8 H_1$. When the frequency of the wave field equals 5 MHz, $H_1 \simeq 15$ kOe, and the damping minimum is located at a field of 27 kOe. For a wave amplitude $\mathscr{H}=200$ Oe and a collision frequency $\nu=3\times10^8$ s^{-1}, the damping at the minimum amounts to about 6 cm^{-1}.

The incidence of a large-amplitude radio wave to a semi-infinite metal occupying the half-space $z>0$ excites a helicon wave, whose field has the form

$$\mathscr{H} \exp{(ik_H - \kappa)z},$$

where k_H and κ are defined by Eqs. (30) and (44), respectively. The character of this distribution is the same as in the linear regime. The role of the nonlinearity reduces to the fact that the Landau damping caused by the carriers in open orbits is drastically reduced, and helicon-wave propagation becomes possible. The helicon spectrum and the harmonic character of the distribution of the wave field at the values of $\mathscr{H}$ achievable in metals remain the same as in the linear regime. Therefore, the field in a metal slab will be the result of the superposition of harmonic waves that have been reflected repeatedly from its surfaces. In other words, when there is only one mode in a metal slab, its impedance under antisymmetric excitation is given by the expression [see, for example, Eq. (4.26) in Ref. 3]

$$Z = \frac{8\pi\omega}{c^2 k} \frac{1-\exp(ikd)}{1+\exp(ikd)}, \tag{45}$$

where $k=k_H+i\kappa$ and d is the thickness of the slab. When $\kappa d \ll 1$ and $k_H d = \pi(2n+1)$, where $n=1,2,3,\ldots$, the function $R(H)=\mathrm{Re}Z$ has sharp peaks, which are caused by the excitation of standing helicon waves in the slab. The results of the calculation of R for a frequency of 5 MHz, a collision frequency $\nu=3\times10^8$ s^{-1}, and a slab thickness $d=0.02$ cm are presented in Fig. 1. The curve depicted by the long dashes corresponds to the linear regime.

4. NONLINEAR NONLOCAL WAVE

Let us now examine the wave properties of copper for the opposite circular polarization. It is known that a doppleron, i.e., a wave caused by the Doppler-shifted cyclotron resonance of electrons undergoing minimal displacement during a cyclotron period, exists in this polarization over a fairly broad vicinity of the helicon threshold. In strong fields doppleron damping is caused by the collisions of resonant electrons, and in weak fields it is caused by the collisionless absorption of the wave by holes and carriers with open orbits. The latter increases with decreasing H and prevents doppleron propagation in weak fields. We shall consider just this range of magnetic fields in this section. Since ξ is much greater than unity at small H, we can use the asymptotic expressions for F_e and F_h at large q, which have the form

$$F_e(q) \simeq -\frac{2}{3q^2}, \quad F_h(\eta q) \simeq \frac{i}{q} + \frac{1}{q^2}. \tag{46}$$

The coefficients of the first and second terms in the expression for $F_h(\eta q)$ in (46) in the model under consideration differ from unity by several percent. However, since this model correctly describes the properties of copper only on a qualitative level, but not on a quantitative level, we neglect these differences.

There is no imaginary term containing $-i/q$ in the expression for F_e. This is because the displacement of the electrons during a cyclotron period, which is proportional to $\partial S_e(p_z)/\partial p_z$, has a minimum. Therefore, at short wavelengths ($q>1$) the cyclotron absorption condition is not satisfied for electrons. As a result, F_e is a real function. In addition, it is important that F_e changes sign and becomes negative when $q>1$. In fact, the quantity in the denominator in the integrand in (16), which can be written in the form

$$1 + \frac{q}{2\pi p}\frac{\partial S_e(p_z)}{\partial p_z},$$

is negative when $q>1$. Physically, this means that in the short-wavelength range the Doppler frequency shift for all electrons exceeds their cyclotron frequency ω_c. As a result, the contribution of electrons to the nonlocal Hall conductivity changes sign.

Substituting (46) into (19) and neglecting the small terms of the order of γ, we write the nonlocal conductivity for plus polarization and $q \gg 1$ in the form

$$\sigma_+(q) \simeq \frac{ec}{Hq^2}\left[-i\left(\frac{2}{3}n_e + n_h\right) + (n_h + 2n_0)q \right]. \tag{47}$$

Despite its numerical smallness (n_h and n_0 are small in comparison with n_e), the real term in (47) is not small in comparison with the imaginary term. Therefore, the roots of the dispersion equation (3) are essentially complex, i.e., there are no propagating modes in the $q \gg 1$ region in the linear regime. The capture of holes and carriers with open orbits by the magnetic field of a large-amplitude rf wave, as in the case of helicon-wave damping, results in a decrease in collisionless absorption, and σ_+ consequently becomes almost purely imaginary. An expression for the conductivity in the case of strong nonlinearity ($\omega_{0h} \gg \nu_h$, $\omega_0 \gg \nu_0$) can be obtained by making the following replacement in (47):

$$(n_h + 2n_0) \rightarrow n_h \frac{\nu_h}{\sqrt{2}\,\omega_{0h}(q)} + 2n_0 \frac{\nu_0}{\sqrt{2}\,\omega_0(q)}. \tag{48}$$

The calculation of $\omega_{0h}(q)$ for $q \gg 1$ gives

$$\omega_{0h} \simeq 2.6\,\frac{e\sqrt{H\mathscr{H}q}}{cm_j}. \tag{49}$$

When (48), (49), and (37) are taken into account, the expression obtained for the nonlinear conductivity is

$$\sigma_n(q) \simeq \frac{n_f ec}{Hq^2}\left(-i + 0.16\gamma\sqrt{q\frac{H}{\mathscr{H}}} \right), \tag{50}$$

where

$$n_f = \frac{2}{3}n_e + n_h = 0.86 n_e, \quad \gamma = cm\nu/eH,$$

and, as before, we set $m_h\nu_h = m_0\nu_0 = m\nu$.

Substituting (50) into the dispersion equation (3), we rewrite it in the form

$$q^4 = \xi_f\left(1 + 0.16i\gamma\sqrt{q\frac{H}{\mathscr{H}}} \right), \tag{51}$$

where ξ_f is defined by an expression which is obtained from (26) when $(n_e - n_h)$ is replaced by n_f. Henceforth we shall disregard the difference between ξ_f and ξ.

In the range of magnetic fields in which the inequalities

$$0.04\,\frac{\nu mc}{e\sqrt{H\mathscr{H}}}\left(\frac{H_1}{H}\right)^{3/8} \ll 1 \ll \left(\frac{H_1}{H}\right)^{3/4}, \tag{52}$$

hold, the roots of Eq. (51) with a positive imaginary part have the form

$$q_n \simeq \xi^{1/4}\left(1 + 0.04i\frac{\nu mc}{e\sqrt{H\mathscr{H}}}\xi^{1/8} \right), \quad q_{2n} = iq_n. \tag{53}$$

The root q_n, which has a small imaginary part, corresponds to a nonlinear nonlocal wave similar to the mode (9) in an alkali metal. However, unlike the latter, the nonlinear wave in copper has much weaker damping and a much broader range of magnetic fields (because of the small numerical coefficient and the small value of ν, the imaginary term in the parentheses in (53) is two to three orders of magnitude smaller than the corresponding quantity in the case of an alkali metal). The spectrum of this wave, which does not have analogs in the linear regime, is determined by all the carriers, and its field rotates in the sense opposite to the direction of rotation of the helicon wave field. The wave vector and the damping (the reciprocal damping distance) in the region (52) are given by

$$k_n = k_a\left(\frac{H}{H_1}\right)^{1/4}, \tag{54}$$

$$\kappa_n \simeq \frac{0.04}{l}\left(\frac{H_1}{\mathscr{H}}\right)^{1/2}\left(\frac{H_1}{H}\right)^{5/8}, \tag{55}$$

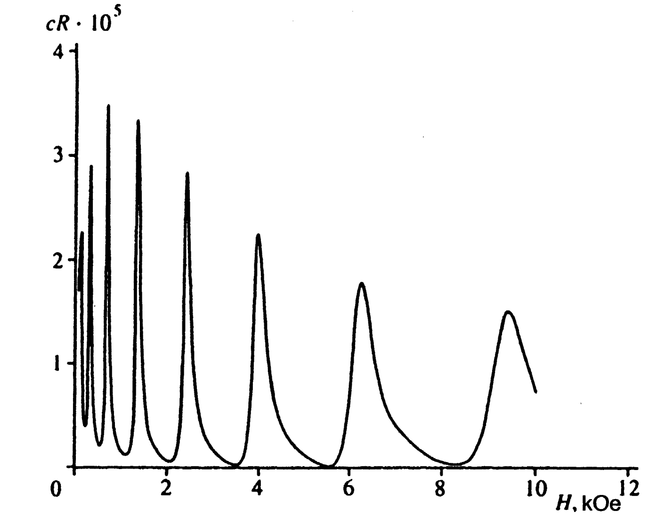

FIG. 2.

where $l=p/m\nu$ is a quantity of the order of the electron mean free path. In fields $H \sim H_1$ the nonlinear wave transforms into a familiar doppleron.[5] Also, nonlinearity does not influence the spectrum of the doppleron, but diminishes its damping.

Let us now consider the excitation of a nonlinear wave in copper. We assume that the excitation of a metal slab is antisymmetric with respect to the electric field. The formula for the impedance Z of a slab in the case in which the field in the metal is the result of the superposition of two eigenmodes has been given, for example, in Ref. 3 (Eq. (3.11)). Taking into account the second relation in (53), the inequality $q_n \gg 1$, and the fact that the mode q_{2n} is strongly damped, we can bring the expression for the impedance of a slab when there is diffuse reflection of the carriers from its surfaces into the form

$$Z_+ = \frac{8\pi\omega p}{ceH}\left[1 - i\left(\frac{2}{q_n} + \frac{1+e^{ikd}}{1-e^{ikd}}\right)\right]\left[1 + \left(2q_n - \frac{i}{q_n}\right)\frac{1+e^{ikd}}{1-e^{ikd}}\right]^{-1}, \tag{56}$$

where $k=k_n+i\kappa_n$. The results of a calculation of $R=\mathrm{Re}\,Z_+$ for a frequency of 5 MHz, a slab thickness of 0.2 mm, and $\nu=3\times10^8$ s are presented in Fig. 2. Instead of the smooth curve in the linear regime, the plot of $R(H)$ consists of a series of sharp peaks, which correspond to the excitation of standing waves with the spectrum (54). Although the wave damping given by (55) is a decreasing function of H, the height of the peaks has a maximum at a fairly low field strength.

5. NONLINEAR WAVES IN ALUMINUM AND INDIUM

In conclusion, we discuss the possibility of the existence of waves with the spectrum (54) in group-III metals. Unlike the noble metals, in these metals there are no open orbits, and the derivative $\partial S/\partial p_z$ does not tend to infinity at any p_z. The hole Fermi surfaces of aluminum and indium are such that in the $\mathbf{H}\|[100]$ geometry, the areas of their cross sections are maximal for $p_z=0$ and decrease monotonically with increasing p_z. Also, $|\partial S/\partial p_z|$ varies in a complicated nonmonotonic manner. It increases very sharply near the central cross section, reaches a maximum, then decreases a little, has a broad minimum, increases a little again, reaches a second maximum, which essentially coincides with the first in height, and thereafter decreases monotonically. According to this behavior of $|\partial S/\partial p_z|$, all the holes (the majority carriers in group-III metals) can be divided into three different groups. The first group corresponds to cross sections near the central one, for which $|\partial S/\partial p_z|$ increases from zero to its maximum value. Although the areas of the corresponding cross sections of the Fermi surface are large, the range of p_z in which they exist is very small. Therefore, the relative concentration of holes from this group is small and does not exceed 0.2. The main group is the second group of holes, for which $|\partial S/\partial p_z|$ varies from the first maximum to the second. This group of holes plays a role similar to the role of electrons in noble metals: they do not contribute to cyclotron absorption when $q\gg1$ (weak fields), and their contribution to the nondissipative part of the nonlocal conductivity is proportional to $1/q^2$. The holes of the first and third groups mediate cyclotron absorption when $q\gg1$; however, because of their relatively small concentration, this absorption, as in the noble metals, is weakened in comparison with the alkali metals. In addition, the oscillation frequencies of the "trapped" holes ω_{01} and ω_{03}, which are given by expressions similar to (6), are large. In the first group S and $\partial^2 S/\partial p_z^2$ are large, and in the third group the cyclotron mass m is small, since the areas of the corresponding cross sections are several times smaller than that of the central cross section. Therefore, it would be expected that nonlinear nonlocal waves should also be observed in aluminum and indium at fields below the helicon threshold.

This research was performed with partial financial support from the International Science Foundation (Grant No. N3F300).

[1] J. R. Merrill, Phys. Rev. **166**, 716 (1968).
[2] I. F. Voloshin, G. A. Vugal'ter, V. Ya. Demikhovskiĭ *et al.*, Zh. Éksp. Teor. Fiz. **73**, 1503 (1977) [Sov. Phys. JETP **46**, 790 (1977)].
[3] A. S. Chernov and V. G. Skobov, Phys. Rep. **244**, 1 (1994).
[4] G. E. H. Reuter and E. H. Sondheimer, Proc. R. Soc. London, Ser. A **195**, 336 (1948).
[5] V. V. Lavrova, C. V. Medvedev, V. G. Skobov *et al.*, Zh. Éksp. Teor. Fiz. **66**, 700 (1974) [Sov. Phys. JETP **39**, 338 (1974)].
[6] B. Perrin, G. Weisbuch, and A. Libchaber, Phys. Rev. B **1**, 1501 (1970).
[7] G. A. Vugal'ter and V. Ya. Demikhovskiĭ, Zh. Éksp. Teor. Fiz. **70**, 1419 (1976) [Sov. Phys. JETP **43**, 739 (1976)].
[8] O. I. Lyubimov, N. M. Makarov, and V. A. Yampol'skiĭ, Zh. Éksp. Teor. Fiz. **85**, 2159 (1983) [Sov. Phys. JETP **58**, 1253 (1983)].

Translated by P. Shelnitz