Upper critical fields of the superconducting state of a superconductor-antiferromagnetic metal superlattice

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The formation of the superconducting phase in layered superconductor-antiferromagnetic metal proximity-effect structures is investigated on the basis of a generalized system of Usadel equations for antiferromagnetic superconductors, in which electrons of the same type are carriers of the magnetic and superconducting properties of the system, and the dispersion law of the electrons and holes has the property of nesting. The dependence of the superconducting transition temperature and the longitudinal and transverse upper critical fields on the period of the structure and the state of the interface between the layers is analyzed. The conditions for the appearance of the superconducting phase in proximity-effect superlattices consisting of a superconductor with a ferromagnetic metal and with an antiferromagnetic metal are compared.
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1. INTRODUCTION

The technology of the layer-by-layer spray-deposition of superlattices has achieved impressive results.[1,2] Among the artificial superstructures,[3] the structures formed by alternating layers of superconducting and magnetic metals are ideal objects for investigating the interaction of superconductivity and magnetism. Numerous experimental and theoretical studies devoted to the properties of superlattices consisting of layers of superconducting and ferromagnetic metals (SC/FM superlattices) have already been conducted.[3–10] Some unusual properties, such as the coexistence of bulk superconductivity and ferromagnetic order in the normal layers, a nonlinear dependence of the critical field on the temperature, oscillations of the superconducting transition temperature as a function of the thickness of the FM layer, states with a nontrivial phase difference between the layers, etc., have been predicted and, in some cases, established experimentally for these systems.[5–9]

Alternated superconductor-antiferromagnetic metal (SC/AF) structures have been investigated to a considerably smaller extent; Refs. 4, 11–13 apparently exhaust the list of publications on this subject. At the same time, for example, most high-T_c superconducting materials are obtained by doping layered antiferroamagnets, and the strong antiferromagnetic correlations of copper spins in CuO planes are their characteristic feature (see, for example, the recent reviews in Refs. 14 and 15). The question of how closely related are the magnetic and superconducting properties of high-T_c superconductors still remains open. It is also known (see, for example, Ref. 16) that antiferromagnetic order is more favorable for the appearance of superconductivity than is antiferromagnetic order. It is therefore natural to expect that the superconducting phase of an SC/AF superlattice will have a larger region of existence on the $H-T$ phase diagram than will that of an SC/FM structure. This difference is important for practical applications of such objects.

Bearing in mind high-T_c superconducting materials, as well as the superconducting antiferromagnetic metals based on chromium,[17,18] in Sec. 2 of this paper we give a generalization of the quasiclassical equations of superconductivity[19,20] to the case of antiferromagnetic superconductors, in which electrons of the same type are the carriers of the magnetic and superconducting properties of the systems, and the dispersion law of the electrons and holes for a sufficiently large number of vectors lying near the Fermi surface has the property of nesting. We then use the generalization of the Usadel equations obtained for such systems to investigate the conditions for the formation of the superconducting phase in proximity-effect SC/AF superlattices as a function of the parameters of the system. The basic equations which describe the superconducting state in SC/AF superlattices are obtained in Sec. 3. Numerical calculations and a discussion of these equations are presented in Sec. 4. The conditions for the existence of proximity-induced superconductivity in SC/AF and SC/FM superlattices are also compared in that section. It is also shown that although antiferromagnetic order of the normal metal is more favorable for the appearance of the superconducting phase than is ferromagnetic order, the state of the interlayer boundary and the characteristics of the materials comprising the layers can greatly offset these advantages. The final portion of Sec. 4 is devoted to a comparison of the results of the theory with available experimental data. Some possible generalizations of the model to more general physical systems are discussed in the Conclusions, where the main implications of this work are also given.

2. QUASICLASSICAL EQUATIONS FOR THE SUPERCONDUCTIVITY OF A BAND ANTIFERROMAGNET

Metals in which Cooper pairing and antiferromagnetism of collectivized electrons (or, in the general case, a spin-density wave) coexist at low temperatures have recently attracted heightened interest. Electrons of the same type are responsible for the magnetic and superconducting properties of the system in the chromium alloys $Cr_{1-x}Re_x$, $Cr_{1-x}Ru_x$, etc.,[17,18] in the compounds $LaRh_2Si_2$, YRh_2Si_2, etc.,[21] and possibly in high-T_c superconducting systems with certain concentrations of the charge carriers.[14,15] The superconducting state usually forms on top of the already existing long-range magnetic order of the band electrons. We note that such systems differ significantly from the antiferromagnetic superconductors based on ternary rare-earth compounds (like $SmRh_4B_4$, $NdRh_4B_4$, etc.), where the ordering of the local magnetic moments of the rare-earth ions takes place in the superconducting state of the collectivized electrons. (A review of the theoretical and experimental work in this area can be found in Ref. 16).

The magnetic properties of almost all known band antiferromagnets are closely related to the features of their band structure.[22,18] More specifically, the Fermi surface of such metals has the property of nesting, i.e., it consists of electron and hole parts which nearly coincide following translation by a certain wave vector **Q**. The transition to the magnetically ordered state is accompanied by the appearance of a spin-triplet dielectric gap on the Fermi surface, which influences not only the normal, but also the superconducting properties of the system. The theory of the thermodynamic and transport properties of systems which exhibit the coexistence of superconductivity and spin-density waves has been developed intensively in recent years (see, for example, the monograph by Moskalenko, Kon, and Palistrant[23] and the reviews in Refs. 14 and 15). On the other hand, the generalization of the Eilenberger[19] or Usadel[20] equations to such systems apparently has not yet been discussed in the literature. In this section, following the general scheme for devising the theory of dirty superconductors,[24] we obtain semiclassical equations of the superconductivity of a band antiferromagnet based on consideration of the quasiclassical character of the motion of Cooper pairs.

We use the "excitonic insulator" model proposed by Keldysh and Kopaev[25] to select the Hamiltonian of an antiferromagnetic superconductor. Interacting electrons and holes, whose dispersion law has the property of "nesting," are considered in this model. The theory of band antiferromagnetism with a nesting Fermi surface was previously developed to a high level and was presented in detail in the reviews in Refs. 22 and 18. Allowing the superconducting pairing of electrons, in the self-consistent field approximation we have

$$H=\int d\mathbf{r}\Bigg\{\sum_{\sigma}\left[\psi_\sigma^+(\mathbf{r})\hat{\xi}_1\psi_\sigma(\mathbf{r})+\varphi_\sigma^+(\mathbf{r})\hat{\xi}_2\psi_\sigma(r)\right] +[\Delta_1(\mathbf{r})\psi_\uparrow^+(\mathbf{r})\psi_\downarrow^+(\mathbf{r})+\Delta_2(\mathbf{r})\varphi_\uparrow^+(\mathbf{r})\varphi_\downarrow^+(\mathbf{r})+\text{h.c.}] +H_{\text{exc}}\sum_{\sigma}\left[\psi_\sigma^+(\mathbf{r})\varphi_{-\sigma}(\mathbf{r})\right]+\text{h.c.}\Bigg\}. \quad (1)$$

Here $\psi_\sigma^+(\mathbf{r})$, $\psi_\sigma(\mathbf{r})$ and $\varphi_\sigma^+(\mathbf{r})$, $\varphi_\sigma(\mathbf{r})$ are the creation and annihilation operators of electrons and holes in the first and second bands, respectively, whose dispersion law satisfies the condition (for nesting)

$$\xi_1(\mathbf{p})=\xi_{\mathbf{p}}-\mu,\quad \xi_2(\mathbf{p}-\mathbf{Q})=-\xi_{\mathbf{p}}-\mu.$$

Here for the case of an antiferromagnetic (doubled) cell we take $\mathbf{Q}=\pm\mathbf{K}/2$, where $\mathbf{K}$ is the vector of the reciprocal lattice of the crystal; the operator $\hat{\xi}_{\mathbf{p}}$ is understood to be $\hat{\mathbf{p}}/2m-E_F$ if there is no external magnetic field, and $(\hat{\mathbf{p}}-e\mathbf{A})^2/2m-E_F$ if there is such a field; $\Delta_1(\mathbf{r})$ and $\Delta_2(\mathbf{r})$ are the band superconducting order parameters; the magnetic order parameter H_{exc} (the Néel molecular field) can be assumed to be real;[26,27] $E_F=p_0^2/2m$ is the Fermi energy; μ is the chemical potential; and in this section we set $\hbar=1$. For simplicity, the superconducting pairing of electrons from different energy bands is neglected.

We introduce the Green's functions $G_{11}(1,2)=-\langle\hat{T}_\tau\psi_\uparrow(1)\psi_\uparrow^+(2)\rangle$ and $F_{11}(1,2)=\langle\hat{T}_\tau\psi_\downarrow^+(1)\psi_\uparrow^+(2)\rangle$, where $\hat{T}_\tau$ is the τ-ordering operator, and $1=(\mathbf{r}_1,\tau_1)$, and $2=(\mathbf{r}_2,\tau_2)$. It is easy to see that the system of Gor'kov equations for electrons in the first band is distinguished from the standard system (see, for example, Ref. 24) by the presence of the additional function $G_{21}(1,2)=-\langle\hat{T}_\tau\varphi_\downarrow(1)\psi_\uparrow^+(2)\rangle$, which describes the antiferromagnetic correlations of electrons from different bands. As usual, to expand in gradients of the order parameter, it is convenient to switch to a mixed representation by introducing the coordinates of the center of inertia of the pair **R** and the coordinates of the relative motion **r**. After finding the Fourier transform with respect to **r**, the expansion can be found for the Green's function near the Fermi surface by identifying the main terms and discarding the terms which are smaller than main terms by T_c/E_F. The simplified equations for the first point of the Green's functions in the mixed representation then take the forms

$$\left(i\omega-\xi+\frac{i}{2}v_0\mathbf{n}\cdot\nabla_{\mathbf{R}}-p_0\mathbf{n}\cdot\mathbf{v}_s\right)G_{11}+\Delta_1F_{11}-H_{\text{exc}}G_{21}=1,$$

$$\left(i\omega+\xi-\frac{i}{2}v_0\mathbf{n}\cdot\nabla_{\mathbf{R}}-p_0\mathbf{n}\cdot\mathbf{v}_s\right)F_{11}+\Delta_1^+G_{11}=0, \quad (2)$$

$$\left(i\omega+\xi-\frac{i}{2}v_0\mathbf{n}\cdot\nabla_{\mathbf{R}}+p_0\mathbf{n}\cdot\mathbf{v}_s\right)G_{21}-H_{\text{exc}}G_{11}=0.$$

Here $\xi=(p^2-p_0^2)/2mT_c$, $p_0=mv_0$ and v_s is the superfluid velocity of the condensate.

Simpler expansions can be obtained for the Green's functions

$$\tilde{G}_{11}(1,2)=-\langle\hat{T}_\tau\psi_\downarrow^+(1)\psi_\downarrow(2)\rangle,$$

$$\tilde{F}_{11}(1,2)=-\langle\hat{T}_\tau\psi_\uparrow(1)\psi_\downarrow(2)\rangle,$$

$$\tilde{G}_{21}(1,2)=-\langle\hat{T}_\tau\varphi_\uparrow^+(1)\psi_\uparrow(2)\rangle,$$

as well as for electrons in the second band. Now, however, that instead of (2), we obtain

$$\left(i\omega+\xi-\frac{i}{2}v_0\mathbf{n}\cdot\nabla_{\mathbf{R}}+p_0\mathbf{n}\cdot\mathbf{v}_s\right)G_{22}-\Delta_2F_{22}-H_{\text{exc}}G_{12}=1,$$

$$\left(i\omega-\xi+\frac{i}{2}v_0\mathbf{n}\cdot\nabla_{\mathbf{R}}-p_0\mathbf{n}\cdot\mathbf{v}_s\right)F_{22}+\Delta_2^+G_{22}=0, \tag{3}$$

$$\left(i\omega-\xi+\frac{i}{2}v_0\mathbf{n}\cdot\nabla_{\mathbf{R}}-p_0\mathbf{n}\cdot\mathbf{v}_s\right)G_{12}-H_{\text{exc}}G_{22}=0.$$

We note that due to the antiferromagnetic type of interaction between the electrons in different bands, the Green's functions of the second band appear with a reversed sign for the spin, i.e.,

$$G_{22}(1,2)=-\langle\hat{T}_\tau\varphi_1(1)\varphi_\downarrow^+(2)\rangle,$$

$$F_{22}(1,2)=\langle\hat{T}_\tau\varphi_\uparrow^+(1)\varphi_\downarrow^+(2)\rangle,$$

$$G_{12}(1,2)=-\langle\hat{T}_\tau\psi_\uparrow(1)\varphi_\downarrow^+(2)\rangle,$$

etc.

As a result of these simple but somewhat lengthy calculations, we obtain the complete system of 12 equations for the Green's functions of an antiferromagnetic superconductor. It is convenient to write these equations in the form of a single matrix equation. We present it in the so-called "t representation"[24] after taking the Fourier transforms with respect to the variable ξ. We have

$$\left\{i\omega+i\hat{\sigma}_x\left(\frac{d}{dt}+\frac{1}{2}v_0\mathbf{n}\cdot\nabla_{\mathbf{R}}\right)\right.$$
$$-p_0\mathbf{n}\cdot\mathbf{v}_s\left(\mathbf{R}+\frac{1}{2}\mathbf{n}v_0t\right)-\hat{\Delta}\left(\mathbf{R}+\frac{1}{2}\mathbf{n}v_0t\right)$$
$$\left.-H_{\text{exc}}\hat{\gamma}\right\}\hat{G}_\omega(\mathbf{R},\mathbf{n},t)=\delta(t), \tag{4}$$

where the matrices are defined in the following manner:

$$\hat{G}_\omega(\mathbf{R},\mathbf{n},t)=\begin{pmatrix} G_{11} & \tilde{F}_{11} & -G_{12} & 0\\ -F_{11} & -\tilde{G}_{11} & 0 & \tilde{G}_{12}\\ G_{21} & 0 & -G_{22} & -\tilde{F}_{22}\\ 0 & -\tilde{G}_{21} & F_{22} & \tilde{G}_{22}\end{pmatrix}, \tag{5}$$

$$\hat{\sigma}_x=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix},$$

$$\hat{\Delta}=\begin{pmatrix}0&\Delta_1&0&0\\-\Delta_1^*&0&0&0\\0&0&0&\Delta_2\\0&0&-\Delta_2^*&0\end{pmatrix},$$

$$\hat{\gamma}=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix}.$$

[To avoid misunderstandings we note the following. In the present approximation the Green's functions which describe interband superconducting convolutions and are located on the inverse diagonal in (5) were omitted (were set equal to zero). Therefore, when matrix equations like (4) are expanded, the relations corresponding to the elements along the inverse diagonal of the matrix equation should be omitted.]

As usual, Green's functions that are integrated with respect to the energy, i.e., at a zero value of the argument t, appear in the expression for the order parameter. In this sense Eq. (4) contains excess information regarding the Green's functions at any value of t. It can be simplified, if along with (4) we consider another equation, which is obtained in the following manner.[24] In writing the Gor'kov equations, we differentiate the original functions $G_{11}(1,2)$, $F_{11}(1,2)$, $G_{21}(1,2)$, etc. with respect to τ_2, rather than τ_1. Then, repeating the same calculations as in the derivations of relations (2), (3), etc., we obtain a conjugate system of 12 equations for the same functions. Then, subtracting the paired equations for the Green's functions, we can obtain a relation for $\hat{G}_\omega(\mathbf{R},\mathbf{n},t=0)$, which is an analog of the Eilenberger equations.[19] Introducing the function $\hat{\mathscr{G}}_\omega(\mathbf{R},\mathbf{n})=\hat{G}_\omega(\mathbf{R},\mathbf{n})\hat{\sigma}_x$, we have

$$iv_0\mathbf{n}\cdot\nabla_{\mathbf{R}}\hat{\mathscr{G}}_\omega(\mathbf{R},\mathbf{n})+[\hat{\omega},\hat{\mathscr{G}}_\omega(\mathbf{R},\mathbf{n})]=0, \tag{6}$$

where $\hat{\omega}=(i\omega-p_0\mathbf{n}\cdot\mathbf{v}_s)\hat{\sigma}_x-\hat{\Delta}(\mathbf{R})-H_{\text{exc}}\hat{\gamma}$, and $[a,b]=ab-ba$.

The general properties of the Green's function $\hat{\mathscr{G}}_\omega(\mathbf{R},\mathbf{n})$ are a generalization of the properties following from Eq. (6). For example, $\mathrm{Tr}(\hat{\mathscr{G}}_\omega(\mathbf{R},\mathbf{n}))$ does not depend on the projection of $\mathbf{R}$ parallel to $\mathbf{n}$, since

$$iv_0\mathbf{n}\cdot\nabla_{\mathbf{R}}\,(\hat{\mathscr{G}}_\omega(\mathbf{R},\mathbf{n}))=0.$$

It can be shown that $\tilde{G}_{11}(\mathbf{R},\mathbf{n})=G_{11}(\mathbf{R},\mathbf{n})$ and $\tilde{G}_{22}(\mathbf{R},\mathbf{n})=G_{22}(\mathbf{R},\mathbf{n})$ and that the relation $\hat{\mathscr{G}}^2_\omega(\mathbf{R},\mathbf{n})=-1/4$ is valid.

In the dirty limit ($l\ll\xi$, where l is the electron mean free path and ξ is the superconducting correlation length) $\hat{\mathscr{G}}_\omega(\mathbf{R},\mathbf{n})$ can be rendered isotropic with respect to $\mathbf{n}$. The derivation of the closed equation for the isotropic function

$$\hat{\mathscr{G}}_\omega(\mathbf{R})=\int\frac{d\mathbf{n}}{2\pi}\hat{\mathscr{G}}_\omega(\mathbf{R},\mathbf{n})$$

is based on the general properties of Eq. (6) and the properties of $\hat{\mathscr{G}}_\omega(\mathbf{R},\mathbf{n})$ which follow from that equation. Formally repeating all the arguments from Ref. 24, we arrive at an equation, which generalizes the Usadel equation,[20] of the form

$$2D\nabla_{\mathbf{R}}\{\hat{\mathscr{G}}_\omega(\mathbf{R})(\nabla_{\mathbf{R}}\hat{\mathscr{G}}_\omega(\mathbf{R}))\}+[\hat{\omega},\hat{\mathscr{G}}_\omega(\mathbf{R})]=0, \tag{7}$$

where $D=v_0l/3$. The normalization $\hat{\mathscr{G}}^2_\omega(\mathbf{R})=-1/4$ should be added here. As usual, when a magnetic field is present, the vector potential appears only in gradients belonging to F

functions. The explicit form of the system of independent Usadel equations for a dirty antiferromagnetic superconductor is presented below in Sec. 3.

The boundary condition for (7) matches the solutions to the expressions for the Green's function in the bulk of the superconductor:

$$G_{11}(\infty)=-\frac{i\omega}{2}(\omega^2+\Delta_1^2+H_{\rm exc}^2)^{-1/2},$$

$$F_{11}(\infty)=\frac{\Delta_1^*}{2}(\omega^2+\Delta_1^2+H_{\rm exc}^2)^{-1/2},$$

$$G_{21}(\infty)=-\frac{H_{\rm exc}}{2}(\omega^2+\Delta_1^2+H_{\rm exc}^2)^{-1/2},$$

etc. The Usadel equations are usually supplemented by a formula for the order parameter and an expression for the current density. The self-consistent condition for the total (summed) order parameter of the system $\Delta(\mathbf{R})=\Delta_1(\mathbf{R})+\Delta_2(\mathbf{R})$ is

$$\Delta(\mathbf{R})\ln\frac{T_c}{T}=2\pi T\sum_{n\geqslant 0}\left(\frac{\Delta(\mathbf{R})}{\omega_n}+2F_{11}(\mathbf{R})-2F_{22}(\mathbf{R})\right).$$

The expression for the mean current in the general case has the form

$$j(\mathbf{R})=i4\pi eN(0)DT\sum_{\omega}(\tilde{F}_{11}\nabla_{\mathbf{R}}F_{11}+G_{12}\nabla_{\mathbf{R}}G_{21}+F_{22}\nabla_{\mathbf{R}}\tilde{F}_{22}+\tilde{G}_{21}\nabla_{\mathbf{R}}\tilde{G}_{12}-\text{c.c.}),$$

where $N(0)$ is the density of states at the Fermi surface. A discussion of the current-voltage characteristics of superconductors with spin-density waves on the basis of a microscopic treatment can be found in Refs. 26 and 28; the proximity-induced superconductivity of an SC/band antiferromagnet bilayer was considered in Refs. 29 and 30. Here, however, we shall not deal with these questions.

3. FORMATION OF THE SUPERCONDUCTING PHASE IN SC/AF SUPERLATTICES: BASIC EQUATIONS

Let us utilize the formalism developed in the preceding section to investigate the conditions of the transition to the superconducting state in layered superconductor–normal band antiferromagnet structures.

Let both metals, viz., the SC and the AF, be "dirty," let them satisfy the condition $l_{\rm S,AF}\ll\xi_{\rm S,AF}$, where $l_{\rm S(AF)}$ is the mean free path and $\xi_{\rm S(AF)}$ is the correlation length in the SC(AF) layer. As we know,[20,24] it is convenient to use Green's functions that have been integrated with respect to the energy and averaged over the Fermi surface to describe the superconducting state of such metals.

The SC layer. For the SC layer we have the usual system of Usadel equations for $G_{\rm S}(\mathbf{r},\omega)$ and $F_{\rm S}(\mathbf{r},\omega)$.[24] Near the phase transition to the normal state $G_{\rm S}\simeq 1$ holds, and for the anomalous Green's function we have

$$-\frac{D_{\rm S}}{2}\Pi^2F_{\rm S}=\frac{\Delta_{\rm S}}{\hbar}-\omega F_{\rm S}. \tag{8}$$

Here $\Pi=\nabla+i2\pi\mathbf{A}/\Phi_0$ is the gradient-invariant momentum operator, $\mathbf{A}$ is the vector potential, Φ_0 is the flux quantum, $D_{\rm S}$ is the diffusion coefficient, and $\hbar\omega=\pi T(2n+1)$. The self-consistent equation for the order parameter $\Delta_{\rm S}$ and $F_{\rm S}$ has the form

$$\Delta_{\rm S}\ln\left(\frac{T_{c0}}{T}\right)=2\pi T\sum_{\omega}\left(\frac{\Delta_{\rm S}}{\hbar\omega}-F_{\rm S}\right),$$

where T_{c0} is the temperature of the superconducting transition of the bulk SC.

The solution is found in the form

$$F_{\rm S}=\Delta_{\rm S}(\hbar\omega+2\pi T_{c0}\rho(t))^{-1},$$

where $t=T/T_{c0}$. Equation (8) then assumes into the form[31]

$$\Pi^2F_{\rm S}=-k_{\rm S}^2F_{\rm S},\quad k_{\rm S}^2=2\rho(t)/\xi_{\rm S}^2. \tag{9}$$

Here the characteristic correlation length is $\xi_{\rm S}=(\hbar D_{\rm S}/2\pi T_{c0})^{1/2}=2\xi(0)/\pi$, where $\xi(0)$ is the correlation length in the Ginzburg-Landau theory. The parameter $\rho(t)$ describes the depairing effects and determines the superconducting transition temperature through the relation

$$\ln(t)=\Psi(1/2)-\Psi(1/2+p/t), \tag{10}$$

where $\Psi(x)$ is the digamma function.

The AF layer. The metallic contact with the superconductor results in the appearance of proximity-induced superconductivity in a thin layer of the normal metal. In accordance with the results of the preceding section, the induced superconducting state of the *AF* metal is described by the following system of independent equations, which were obtained in accordance with (7)

$$-\frac{D_{\rm N}}{2}\Pi(G_{11}\Pi F_{11}-F_{11}\nabla G_{11})=\frac{\Delta_{1n}}{\hbar}G_{11}-\omega F_{11}, \tag{11a}$$

$$-\frac{D_{\rm N}}{2}\Pi(G_{22}\Pi F_{22}-F_{22}\nabla G_{22})=\frac{\Delta_{2n}}{\hbar}G_{22}-\omega F_{22}, \tag{11b}$$

$$D_{\rm N}\nabla(G_{11}\nabla G_{12}-G_{12}\nabla G_{22})-2\omega G_{12}-i\frac{H_{\rm exc}}{\hbar}(G_{11}+G_{22})=0, \tag{12a}$$

$$D_{\rm N}\nabla(G_{12}\nabla G_{11}-G_{22}\nabla G_{21})-2\omega G_{21}-i\frac{H_{\rm exc}}{\hbar}(G_{11}+G_{22})=0, \tag{12b}$$

$$(\Delta_{1n}+\Delta_{2n})\ln\left(\frac{T_{cn}}{T}\right)=2\pi T\sum_{\omega}\left(\frac{\Delta_{1n}+\Delta_{2n}}{\hbar\omega}-F_{11}-F_{22}\right). \tag{13}$$

We recall that $D_{\rm N}$ is the diffusion coefficient in the AF layer; Δ_{1n} and Δ_{2n} are the band superconducting order parameters; $H_{\rm exc}$ is the antiferromagnetic exchange energy; the anomalous Green's functions $F_{11(22)}$, as usual, describe the condensate of Cooper pairs, and $G_{11(22)}$ and $G_{12(21)}$ describe the normal excitations in the system. In accordance with the generally accepted notation, in this section we switch to Green's functions which are distinguished from the functions in Sec.

2 by normalization: $G_{11}\rightarrow G_{11}/2i$, $F_{11}\rightarrow F_{11}/2i$, $F_{22}\rightarrow -F_{22}/2i$, etc., i.e., $G_{ii}^2+F_{ii}^2+G_{ij}G_{ji}=1$, where $i\neq j=1, 2$ and there is no summation over the repeated indices.

The band antiferromagnets based on chromium have fairly high Néel temperatures: $T_N\sim 200-300$ K. (Extensive experimental data on the magnetic properties of Cr and its compounds have been assembled in the review in Ref. 18.) Taking this circumstance into account, we restrict ourselves to a treatment of the situation of greatest current interest, in which the magnetic ordering temperature T_N significantly exceeds the superconducting transition temperature of the SC/AF superlattice $T_c\leq T_{c0}$. This actually means that we shall neglect the temperature-induced changes in the magnetic properties of the system when we investigate the conditions for the formation of the superconducting phase. In addition, it is assumed that the upper critical fields of the superconducting state of the SC/AF superlattice do not exceed the fields for the transition of the AF layer from an antiferromagnetic phase to a spin-flop phase or a ferromagnetic phase. Then the gradients of the Green's functions G_{11}, G_{22}, G_{12}, and G_{21} can be neglected near the phase transition to the superconducting state. It can be shown that this approximation corresponds to the neglect of terms of second order in the order parameter $\Delta(r)$. Equations (12), which describe antiferromagnetic correlations, are solved directly and give

$$G_{\rm N}(\mathbf{r},\omega)=G_{11}(\mathbf{r},\omega)=G_{22}(\mathbf{r},\omega)=\hbar\omega\ \mathrm{Sgn}(\omega)((\hbar\omega)^2+H_{\rm exc}^2)^{-1/2},$$

$$G_{12}(\mathbf{r},\omega)=-G_{21}(\mathbf{r},\omega)=iH_{\rm exc}\ \mathrm{Sgn}(\omega)((\hbar\omega)^2+H_{\rm exc}^2)^{-1/2}. \tag{14}$$

Next, it is convenient to pass from the band variables to the "total" variables $F_{\rm N}(\mathbf{r},\omega)=-F_{11}(\mathbf{r},\omega)+F_{22}(\mathbf{r},\omega)$ and $\Delta_{\rm N}(\mathbf{r})=\Delta_{1n}(\mathbf{r})+\Delta_{2n}(\mathbf{r})$. In accordance with (11), the linearized equation for the anomalous Green's function $F_{\rm N}$ takes on the form

$$\frac{D_{\rm N}}{2}G_{\rm N}\Pi^2F_{\rm N}=\omega F_{\rm N}-\frac{\Delta_{\rm N}}{\hbar}G_{\rm N}. \tag{15}$$

Equation (15) formally coincides with the equation for the anomalous Green's function of a nonmagnetic or a ferromagnetic metal. However, while the Green's function of the normal excitations is a constant ($G_{\rm N}\simeq 1$) in the latter cases in the vicinity of the phase transition to the superconducting state, the function $G_{\rm N}$ for an AF metal (14) is strongly dependent on the frequency and the exchange field.

Since the superconducting state in the AF layer appears only as a result of the proximity effect, we seek a solution in a form analogous to $F_{\rm S}$, but with $\rho(t)=-1$ (Ref. 31):

$$F_{\rm N}=\Delta_{\rm N}(G_{\rm N}\hbar\omega-\pi T)^{-1}.$$

Then Eq. (15) is written in the standard form:

$$\Pi F_{\rm N}=k_{\rm AF}^2F_{\rm N},\quad k_{\rm AF}^2=2\pi T(\hbar D_{\rm N}G_{\rm N})^{-1}. \tag{16}$$

Taking into account that the characteristic functions satisfy $\hbar\omega\sim T_c\ll T_N$, from (14) we have $G_N\simeq\hbar\omega/H_{\rm exc}$, and the expression for $k_{\rm AF}$ is

$$k_{\rm AF}=2/\xi_{\rm AF},\quad \xi_{\rm AF}=[2\hbar D_{\rm N}(2n+1)/H_{\rm exc}]^{1/2}. \tag{17}$$

Here we introduced the characteristic damping distance of the superconducting correlations in the AF layer $\xi_{\rm AF}$. We note that the destruction of Cooper pairs is considerably more efficient in the AF layer than in a normal nonmagnetic layer of the same thickness, i.e., $\xi_{\rm AF}$ is much smaller than the corresponding damping distance $\xi_{\rm N}=(\hbar D_{\rm N}/2\pi T)^{1/2}$ in a normal metal with the same diffusion coefficient. In this respect the action of an antiferromagnetic exchange field coincides with that of a ferromagnetic exchange field. The difference lies in the fact that the equation like (16) for an FM layer contains a complex parameter, i.e., the characteristic wave vector for an FM metal is[6–8]

$$k_{\rm FM}=2(1+i)/\xi_{\rm FM},\quad \xi_{\rm FM}=(4\hbar D_{\rm N}/H_{\rm exc})^{1/2} \tag{18}$$

(here $H_{\rm exc}$ is the ferromagnetic exchange field). This causes not only the oscillatory behavior of the damping of the wave function in an FM layer, but also the complex character of the depairing parameter $\rho(t)$ through the boundary conditions. Since the latter parameter determines the $H-T$ diagram of the superconducting state through Eq. (10), in a final analysis it produces the difference between the properties of SC/FM and SC/AF superlattices.

Equations (9) and (16) should be supplemented by boundary conditions on the interface between the layers. We use the relations[32,33]

$$F_{\rm S}(\mathbf{r},\omega)=F_{\rm N}(\mathbf{r},\omega),\quad \frac{d}{d\mathbf{r}}F_{\rm S}(\mathbf{r},\omega)=\eta\frac{d}{d\mathbf{r}}F_{\rm N}(\mathbf{r},\omega). \tag{19}$$

The phenomenological parameter η depends on the character of the electron scattering on the boundary and the properties of the materials comprising the layers; its value is unknown in the general case. When there is mirror reflection on the boundary we have $\eta=\sigma_{\rm N}/\sigma_{\rm S}$, where $\sigma_{\rm N}(\sigma_{\rm S})$ is the conductivity of the AF(SC) layer in the normal state. As we know,[33] the relations (19) take into account the effect of one interface, i.e., it is assumed that the dimensions of the layer of the normal metal are greater than the corresponding correlation length. For superlattices of thin normal layers, such a single-mode approximation can lead to the loss of some details of the behavior of the system that are caused by the overlapping of the wave functions of the condensate from the next-nearest neighboring layers.[34,8] At the same time, the condition (19) is fully applicable to many superlattices of interest, including superlattices with normal layers based on high-T_c superconducting materials. The latter are known to have an especially short coherence length even in the superconducting state.

The rest of the derivation of the expressions for the superconducting transition temperature T_c, the upper critical field normal to the layers $H_{c2\perp}$ and the upper critical field parallel to the layers $H_{c2\parallel}$ is similar to the procedure discussed in detail during the treatment of SC/FM and SC/N lattices.[5–8,31,35] Therefore, we present the final expressions at once, retaining the notations in Ref. 7 to be specific. Lattices with fairly thick AF layers, i.e., $d_{\rm AF}\gg\xi_{\rm AF}$, are assumed below.

The superconducting transition temperature for $H=0$ is defined as the highest value of t satisfying Eqs. (9), (10), and (16) and the boundary conditions, which can be rewritten in the form

$$\varphi_0 \tan \varphi_0 = d_S(\xi_S \epsilon)^{-1}, \quad (20)$$

where $\varphi_0 = k_S d_S/2$ and $\epsilon = \xi_{AF}(\eta \xi_S)^{-1}$. It is convenient to represent the expression for k_S (9) as a relation defining the depairing parameter

$$\rho(t_c) = 2\varphi_0^2 (d_S/\xi_S)^{-2}. \quad (21)$$

The magnetic field perpendicular to the layers penetrates the lattice in the form of Abrikosov vortices modulated along the field.[31] Assuming that the condition $\Phi_0/\xi_{AF}^2 \gg H_{c2\perp}$ holds, for $\rho(t)$ we obtain the expression

$$\rho(t) = \rho(t_c) + H_{c2\perp}/2H_{c2GL}, \quad (22)$$

where $H_{c2GL} = \Phi_0/2\pi\xi_c^2$ is the upper critical field of a bulk superconductor in the Ginzburg–Landau theory, and $\rho(t_c)$ is defined by (20) and (21). The field $H_{c2\perp}$ is found as the solution of Eq. (10) for a given $t < t_c$ and $\rho(t)$. We note that when $\xi_{AF} \ll \xi$ holds, the condition indicated above for the field strength is equivalent to the condition $H_{c2\perp} < H_{c2GL}$, whose validity is confirmed by the results obtained.

We find the parallel critical field of the superlattice by assuming that Abrikosov vortices do not form in the SC layers, i.e., by assuming that the variation of the order parameter along an SC layer is negligible in comparison with the magnitude of its variation in the transverse direction. Fulfillment of this condition should be expected for structures with thin SC layers that are actually isolated from one another. These questions were previously investigated by experimental and theoretical means for SC/FM lattices.[3,5,36] It was established that due to suppression of the surface superconductivity at the SC/FM boundary, the formation of vortices in SC/FM lattices begins at considerably greater thicknesses of the SC layers than in SC/N structures. There has been no corresponding analysis for SC/AF systems. Nevertheless, the experimental investigations of the proximity effect in layered contacts[4,12] showed that the superconducting order parameter is strongly reduced on the interface between a superconducting film and a film of either a ferromagnetic or antiferromagnetic metal. Deutscher and de Gennes[37] attribute this effect to the destruction of Cooper pairs on the magnetic defects of the interface, rather than the type of magnetic order in the bulk of the layer. Taking this circumstance into account, we assume that the conditions for the formation of Abrikosov vortices in SC/AF lattices are similar to the conditions for SC/FM systems. Then, following Ref. 7 (see also Ref. 3), when $H_{c2\parallel} < \Phi_0/2\pi d_S^2$ holds, for $\rho\langle t\rangle$ we can obtain the expression

$$\rho(t) = \rho(t_c) + \frac{g(\varphi_0)}{24}(H_{c2\parallel}/H_{c2GL})^2 d_S^2/\xi_S^2, \quad (23)$$

where $\rho(t_c)$ is specified by (20) and (21) and the explicit form of the numerical factor $g(\varphi_0)$ coincides with the expression found in Ref. 7:

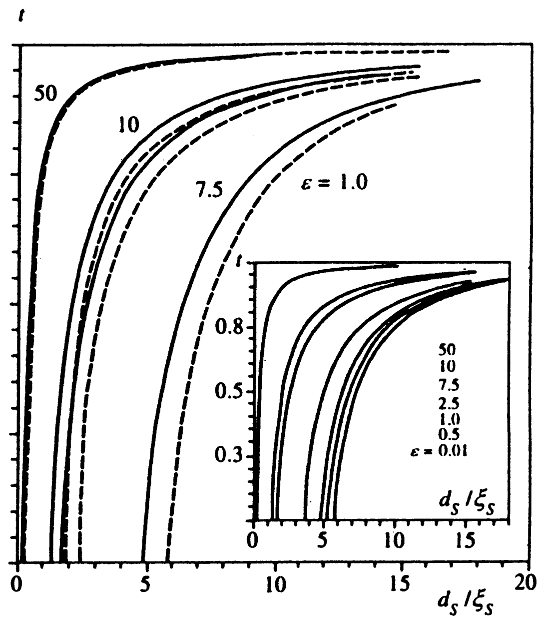

FIG. 1. Dependence of the reduced superconducting transition temperature $t_c = T_c/T_{c0}$ on the reduced thickness of the SC layer d_S/ξ_S and the state of the interface, i.e., the value of ϵ. Here and in Figs. 2–5 the solid lines depict the results for SC/AF superlattices, and the dashed lines depict the analogous results for an SC/FM system calculated from the equations in Ref. 7.

$$g(\varphi_0) = 1 - \frac{3}{2}\varphi_0^{-2} + (3 + 2d_S(\xi_S\epsilon)^{-1})(\varphi_0^2 + d_S(\xi_S\epsilon)^{-1} + d_S^2(\xi_S\epsilon)^{-2})^{-1}.$$

The value of $H_{c2\parallel}$ is found as the solution of Eq. (10) for a given $t < t_c$ and $\rho(t)$ given by (23).

In the general case solutions can be found only numerically. We now move on to a discussion of the results obtained and a comparison of those results with the existing experimental data, as well as the results for *SC/FM* and *SC/N* superlattices.

4. NUMERICAL RESULTS AND DISCUSSION

The solid lines in Fig. 1 show the dependence of the superconducting transition temperature of an SC/AF superlattice on the thickness of the SC layer and ϵ. The dashed lines in this figure are plots of the analogous data for an SC/FM system, which we calculated using the equations in Ref. 7 and which reproduce the corresponding results in that paper (confirming, in particular, the correctness of our numerical methods). As usual, the superconducting state is realized, if the thickness of the SC layer is greater than a certain critical value d_{Sc}. Simple analytical expressions for d_{Sc} can be obtained only in a few limiting cases. For example, for the lattices which we considered with thick AF layers, i.e., $d_{AF} \gg \xi_{AF}$, and a relatively thin SC layer, i.e., $\eta d_S \ll \xi_{AF}$ [two-dimensional (2D) superconductivity], a superconducting state exists if $d_S > d_{Sc} = 14.2487\eta\xi_S^2/\xi_{AF}$. For superconductivity in a similar SC/FM lattice, the thickness of the SC layer must be 50% greater, since in this case[6,7] $d_{Sc} = 20.1505\eta\xi_S^2/\xi_{FM}$. If the SC layer is sufficiently thick, i.e., for $\eta d_S \gg \xi_{AF}$ (3D superconductivity), we have

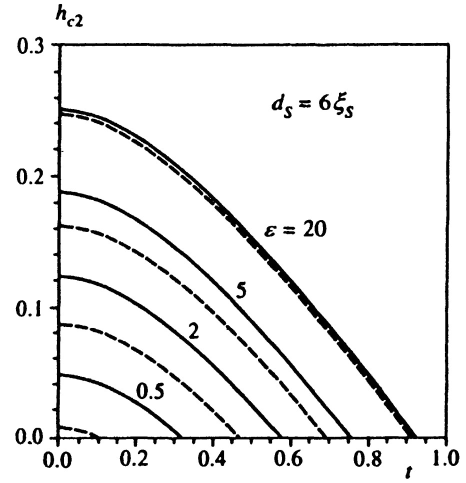

FIG. 2. Dependence of the upper critical field perpendicular to the layers of the lattice, $h_{2c\perp}=H_{c2\perp}/H_{c2GL}$, on the reduced temperature $t=T/T_{c0}$ for various conditions for electron scattering on the interface between the layers, i.e., values of ϵ, and a fixed thickness of the SC layer.

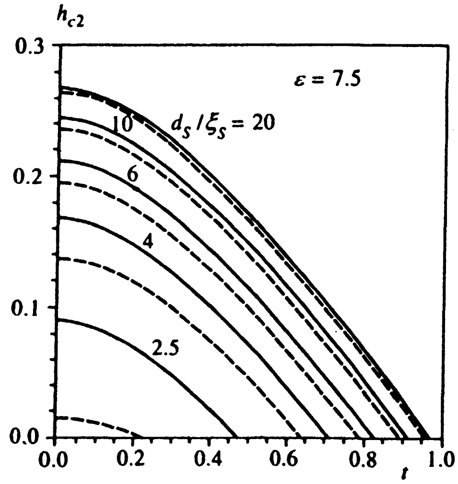

FIG. 3. Dependence of the upper critical field perpendicular to the layers of the lattice $h_{c2\perp}$ on the reduced temperature for various values of the thickness of the SC layer and fixed conditions for electron scattering on the interface between the layers, i.e., a fixed value of ϵ.

$d_{Sc}=5.9293\xi_S$, as in the case of SC/FM systems,[6,7] i.e., the character of the magnetic order in the normal layer is of no consequence here.

As for the dependence on ϵ, the difference in the type of magnetic order in the normal layer is significant at moderate values of this parameter ($1\leqslant\epsilon<10$). In the limiting cases $\epsilon\ll 1$ and $\epsilon\gg 1$ lattices with AF and FM layers exhibit the same behavior. This is because the limit $\epsilon\gg 1$ [where $\rho(t_c)\rightarrow 0$ and $g(\varphi_0)\rightarrow 1$ (Ref. 7)] corresponds to the case of an isolated SC film in a vacuum. In the limit $\epsilon\rightarrow 0$, we have Im $(\rho(t))\rightarrow 0$ (Ref. 7), and the difference between SC/AF and SC/FM lattices once again vanishes. The physics of the phenomenon is that the Cooper pairs are destroyed on the interface so rapidly that the type of magnetic order in the normal metal is no longer of any consequence.

All these laws for the superconducting transition temperature of SC/AF lattices are reproduced by the numerical analysis graphically depicted in Fig. 1. We note that Fig. 1 also clearly displays a transition at $\epsilon\rightarrow 0$ (or $\xi_{AF}\rightarrow 0$) to a proportional dependence of $T_{c0}-T_c$ on $1/d_S^2$, in accordance with the character of the suppression of the transition temperature for an isolated SC film coated by a layer of paramagnetic impurities.[3]

The results of the numerical calculation of the upper critical fields for SC/AF structures are presented in Figs. 2–5 (solid lines). These figures also present the analogous data for SC/FM superlattices (dashed lines), which were obtained from the equations in Ref. 7. Figures 2 and 4 show the dependence of the reduced critical fields $h_{c2\perp}=H_{c2\perp}/H_{c2GL}$ and $h_{c2\parallel}=H_{c2\parallel}/H_{c2GL}$ on the reduced temperature $t=T/T_{c0}$ for various conditions of electron scattering on the interface between the layers, i.e., various values of ϵ, and a fixed thickness of the SC layer. Figures 3 and 5 show the same dependence of the critical fields, but for different values of d_S and a fixed value of ϵ.

When the magnetic field is oriented perpendicular to the layers, its critical value $H_{c2\perp}$ is observed to vary linearly with the temperature in a broad vicinity of T_c, and saturation is achieved at low temperatures (see Figs. 2 and 3). In this respect the behavior of $H_{c2\perp}(t)$ in SC/AF and SC/FM systems is similar and corresponds to the three-dimensional character of the superconducting state of the lattices. We note that the value of $H_{c2\perp}(t=0)$ is significantly dependent on the thickness of the SC layer. Conversely, when the SC layer in superlattices consisting of a superconductor and a nonmagnetic metal is not excessively thin, $H_{c2\perp}(t)$ tends to the same value of $H_{c2\perp}(0)$.[38,9,10] Depairing effects of a magnetic nature, rather than the state of the interface, are decisive for the value of $H_{2c\perp}(0)$. When the field is oriented parallel to the layers (Figs. 4 and 5), the plots of its critical values for sufficiently thick SC layers exhibit an $h_{c2\parallel}\propto 1/d_S$ dependence, and near t_c they are characterized by an $h_{c2\parallel}\propto(1-t)^{1/2}$ law. Such behavior of the parallel critical field corresponds to Tinkham's results[39] for an isolated thin SC film, for which the Ginzburg–Landau theory gives $H_{c2\parallel}(T)\propto\Phi_0/d_S\xi_S(T)$, and to the two-dimensional character of the superconducting state of the superlattices.

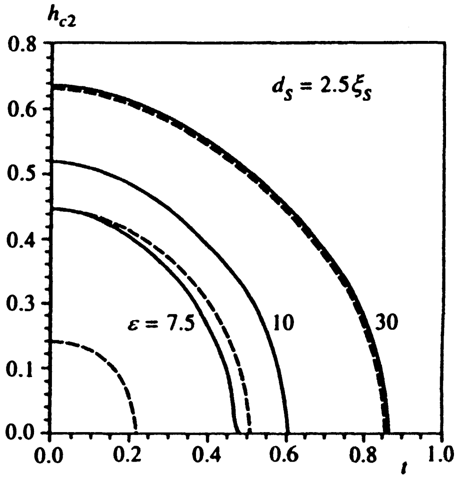

FIG. 4. As in Fig. 2, but for a field oriented parallel to the layers, i.e., for $h_{c2\parallel}=H_{c2\parallel}H_{c2GL}$.

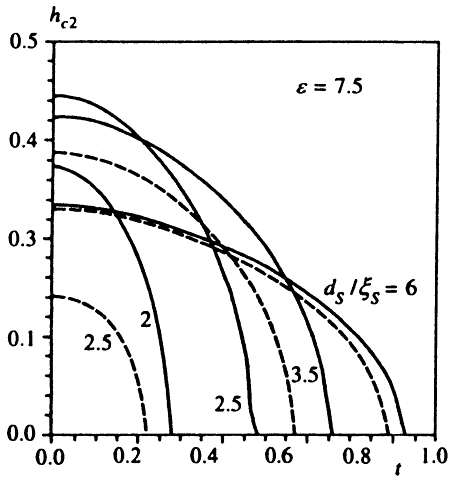

FIG. 5. As in Fig. 3, but for a field oriented parallel to the layers, i.e., for $h_{2c\parallel}$.

A comparison of the values of the critical fields for SC/AF and SC/FM lattices reveals the most significant difference for structures with thin SC layers and $1\sim\epsilon\leqslant5$. The results presented in Figs. 3–5 show that for very thin SC layers ($d_S<2.5\xi_S$) the superconducting state is strongly suppressed due to the proximity to the normal neighbors. For example, in the case of a contact with an FM layer, it is completely suppressed for $d_S=2\xi_S$. As either d_S or ϵ increases, the values of the critical fields come closer together. Such behavior is easily understood, if it is taken into account that the limit $d_S\gg1$ or $\eta\ll1$ corresponds to the case of an isolated SC film in a vacuum. As we have already noted, if $\epsilon\to0$, the regions for the existence of the superconducting phase in SC/AF and SC/FM lattices coincide.

The few experimental investigations of SC/AF systems [Pb/Cr (Ref. 4), Pb–Bi/Cr (Ref. 11), V/Cr (Ref. 12), Nd/Cr (Ref. 13)] do not provide a complete picture of their properties and permit only a qualitative comparison of theory and experiment. For example, the rapid decrease in the superconducting transition temperature with increasing thickness of the AF layer when the SC layer is sufficiently thin and the saturation of T_c at large values of d_S were observed already in the first experiments performed to investigate the proximity effect in SC/AF contacts.[4,12] At the same time, while the square-root dependence of $H_{e2\parallel}$ on $T-T_c$ has been firmly established for SC/FM systems,[5,7–10] this question has not yet been raised for SC/AF lattices. Measurements of the temperature dependence of the parallel critical field of Nd/Cr lattices[31] showed that 200-Å Nd/30-Å Cr systems still have three-dimensional superconducting properties and that 55-Å Nd/30-Å Cr structures begin to display a tendency to two-dimensional superconductivity. The ambiguity in the interpretation of the results is due to the large size of the transition layer between the SC and AF metals. Its thickness ($\approx$20 Å) is comparable with the thickness of the Cr layer. In fact, the objects investigated consisted of three alternating layers; this is also indicated by the two-step character of the transition to the superconducting state of such lattices observed by Cheng and Stearns.[13]

The dependence of the upper critical fields of multilayered Pb–Bi/Cr systems on the thickness of the SC layer at a fixed temperature was investigated in Ref. 11. We note that these systems more fully satisfy the requirements of the model, since Cr does not mix with Pb and Bi and the interface between the layers is apparently fairly sharp. Lattices with a thickness of the SC layer in the range $5\xi_S\leqslant d_S\leqslant20\xi_S$ ($\xi_S\approx200$ Å for a Pb–Bi alloy) display an increase in $H_{c2\perp}$ and a decrease in $H_{c2\parallel}/H_{c2\perp}$ as d_S increases. Such behavior of $H_{c2\perp}$ and $H_{c2\parallel}$ as a function of the thickness of the SC layer is also characteristic of the results presented in Figs. 3 and 5. The normal layers in the lattices in Ref. 11 were relatively thin: $d_{AF}=20$ Å and 75 Å. Therefore, in the lower limit the interference effects from nonneighboring interfaces are significant for $d_S\sim5\xi_S$; for $d_S\sim20\xi_S$, the behavior of $H_{c2\perp}$ and $H_{c2\parallel}$ apparently depends on mechanisms within the interior of the SC layer.[11]

5. CONCLUSIONS

As was shown in the present work, the general dependences of the superconducting transition temperature and the upper critical fields on the parameters of the system for SC/AF and SC/FM superlattices are qualitatively the same. In particular, T_c decreases rapidly as the thickness of the normal layer increases when the SC layer is sufficiently thin, and it reaches saturation when the thickness of the SC layer is large. For structures with thin SC layers, the longitudinal critical field $H_{c2\parallel}$ exhibits a nonlinear temperature dependence and a nonmonotonic dependence on the thickness of the SC layer, which are typical of two-dimensional superconductivity. The behavior of the transverse critical field $H_{c2\perp}$ as a function of the temperature and the lattice period corresponds to the three-dimensional character of the superconducting state of lattices with modulation of the Abrikosov vortices along the field.

Nevertheless, the conditions for the existence of the superconducting state in SC/AF superlattices with thin (of the order of a few correlation lengths ξ_S) SC layers can differ significantly from the conditions for SC/FM structures. For example, our numerical comparison showed (Figs. 2–5) that for $d_S\leqslant5\xi_S$ and suitable conditions on the interface between the layers ($1\sim\epsilon<10$) the upper critical fields of SC/AF systems can be several times greater than the corresponding fields for SC/FM lattices. However, the advantages of the antiferromagnetic interaction of the electrons in the formation of Cooper pairs over the ferromagnetic interaction can be lost to a considerable degree due to destruction of the pairs on the interlayer boundary (i.e., due to an inappropriate choice of the materials of the adjacent layers and the state of their interface). Unfortunately, it is most difficult to derive a theoretical description specifically for the boundary effects at the present time.

Let us mention briefly some possible generalizations of the theory and specific results.

In the derivation of the basic equations we neglected the superconducting pairing of electrons from different energy bands. This enabled us to decrease the total number of independent Green's functions and to simplify the corresponding analytical expressions. The generalization of the results in this direction is fairly straightforward, and we assume that it will be accomplished in the future. At the same time, except for some apparently fine details, there is no reason to expect that consideration of the interband Cooper pairing will fundamentally alter the results. In fact, in a quasiclassical treatment of the superconducting state it is natural to describe it in terms of the total (summed) order parameter. The partial superconducting pairing parameters appear only in the intermediate equations. Therefore, it can be expected that although the approximation which we selected determines the form of the original microscopic equations, it is not significant for the final results.

The second approximation used pertained to the magnetic phase of the normal layer and the relation between the critical temperatures of the phase transitions. The relation between the characteristic energy parameters in an artificial superlattice can be arbitrary. For example, high-T_c superconducting materials have fairly high values for the upper critical fields H_{c2}. Therefore, in lattices based on high-T_c superconductors there a transition of the normal layer from the AF phase is possible to a phase induced by an external field (there are several such phases for chromium alloys[18]). A nonmonotonic dependence of the upper critical fields of the superconducting state of the lattice on the temperature should be expected in the vicinity of the magnetic phase transitions. Similar features should be observed for systems with $T_N < T_c$ when the normal layer goes over to a magnetically ordered state. These questions should also be examined further.

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