Collective effects in the quantum scattering of radiation

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Quantum effects in the collective scattering of radiation by relativistic electrons are calculated under the assumption that the photon energy is much less than the electron rest energy. The ratio of the photon energy to the mean thermal kinetic energy of the electrons is assumed to be arbitrary. Conditions under which the quantum effects can suppress the classical collective effects in the scattering are established. © *1996 American Institute of Physics.* [S1063-7761(96)00502-9]

1. INTRODUCTION

The quantum scattering of radiation by free electrons can be described by the classical Klein–Nishina formula.[1] For nonrelativistic electrons, the quantum correction to the Thomson scattering cross section $\sigma_T = 8\pi e^4/3m_e^2c^4$ is $\sigma = \sigma_T + \delta\sigma_{KN}$, $\delta\sigma_{KN} = -2\sigma_T\hbar\omega/m_ec^2$ (Ref. 1). In many practical applications, one is interested in the scattering of radiation, not by individual electrons, but by a collection of them having a definite density n_e, when collective effects can be important. In the classical limit, collective effects arise at wavelengths greater than the Debye electron radius $\lambda_e = (T_e/4\pi n_e e^2)^{1/2}$ (T_e is the electron temperature, and n_e is their density); in this case, the scattering cross section is not equal to the Thomson cross section and depends strongly on the collective parameter $\delta_e = \lambda^2/8\pi^2\lambda_e^2$, decreasing sharply when it increases.

Although the general expressions of the classical (nonquantum) collective scattering are fairly well known, no simple analytic expression exists in the literature. It can be obtained using dispersion relations for the permittivity and has the form:

$$\sigma_{cl} = \sigma_T\left\{1 - \delta_c + \frac{3}{8}\,\delta_e^2\left[(2+2\delta_e+\delta_e^2)\times\ln\left(\frac{2+\delta_e}{\delta_e}\right) - 2 - 2\delta_e\right]\right\} \tag{1}$$

for simplicity, we have set $\omega \gg \omega_{pe}$, where ω_{pe} is the electron plasma frequency; the condition $\lambda_e \ll \lambda \ll 2\pi c/\omega_{pe}$ corresponds to the fairly wide range of frequencies $\omega_{pe} \ll \omega \ll \omega_{pe}c/v_{Te}$.

Hitherto, quantum effects have not been calculated in collective scattering, and not even the first quantum corrections in the collective scattering regime have been investigated. The present paper is devoted to quantum effects in collective scattering in the case when the parameter $\hbar\omega/m_ec^2$, which occurs in the noncollective corrections, is small. This modest problem encounters considerable difficulties. The present brief exposition of the results of investigations that have been made is devoted to overcoming the mathematical and physical difficulties in the solution of this problem and to possible applications to the Comptonization of radiation and radiative transfer in the interior of the sun. A useful result is not only the modification of the quantum corrections of order $\hbar\omega/m_ec^2$ by the collective effects but also the appearance of additional quantum effects in collective interactions; these can be significant even if $\hbar\omega/m_ec^2$ is assumed to be small.

We note that the Klein–Nishina corrections can be written (if we divide and multiply by T_e) in the form

$$\delta\sigma_{KN} = -2z(v_{Te}^2/c^2)\sigma_T, \tag{2}$$

where $v_{T_e} = (T_e/m_e)^{1/2}$ is the mean thermal velocity of the electrons, and

$$z = \hbar\omega/T_e \tag{3}$$

is the ratio of the photon energy to the mean thermal energy of the electrons. The expression for the Thomson cross section (the cross section without allowance for collective effects), like (1) (when allowance is made for collective effects), is valid if one can ignore the relativistic corrections, which in order of magnitude are equal to $(v_{Te}^2/c^2)\sigma_T$. Therefore, the Klein–Nishina corrections are greater than the relativistic corrections when $z \gg 1$. The limit of applicability of the relation (2) can be expressed in terms of the parameter z in the form $z \ll c^2/v_{Te}^2$.

The present investigation is devoted to quantum collective scattering under the restrictions

$$1 \ll z \ll c^2/v_{T_e}^2, \tag{4}$$

when the quantum corrections are dominant compared with the relativistic corrections. At the same time, if the quantum corrections are small, it is convenient to separate the factor $v_{T_e}^2/c^2$ and write the expression for the cross section in the form

$$\sigma = \sigma_{cl} - T(z,\delta_e)(v_{Te}^2/c^2)\sigma_T, \tag{5}$$

where σ_{cl} is given by the expression (1). In this case, the problem is to determine the factor $T(z,\delta_e)$, which, in general, can be a more complicated function of z in the interval (4) than the function that corresponds to the expression (2), but in the limit $\delta_e \ll 1$ it must go over to $T(z,\delta_e) = 2z$ in accordance with (2).

The treatment given below does not presuppose that the quantum corrections are small, but it is assumed that the parameter zv_{Te}^2/c^2 is small, i.e., that the quantum corrections corresponding to the noncollective scattering regime are small. Therefore, in the general case we shall obtain a result that does not presuppose smallness of the quantum contributions, and then the expression (5) will not be used. In the collective regime, there arises the new parameter $z^2 v_{Te}^2/c^2$, which may be either small or large [in the latter case, we have $c/v_{Te} \ll z \ll c^2/v_{Te}^2$, which does not contradict the condition (4) when $v_{Te}/c \ll 1$]. When $z \gg c/v_{Te}$ holds, the quantum corrections need not be small, and they may even completely suppress the collective effects in the scattering. However, it is true that this range of frequencies is not large, and for applications the case in which the quantum corrections are small has the main interest.

2. SOME GENERAL RELATIONS FOR QUANTUM SCATTERING

In the general case, the scattering probability can be represented in terms of the square of the absolute value of the scattering matrix element M multiplied by a δ function that expresses the quantum law of conservation of energy and momentum in the scattering. In the conservation law, the first quantum corrections in the parameter $2\hbar\omega/m_e c^2$ are very readily found:

$$\omega-\omega'-(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}-\hbar\frac{(\mathbf{k}-\mathbf{k}')^2}{2m_e}=0, \tag{6}$$

where $\{\mathbf{k},\omega\}$ is the 4-vector of the incident wave, and $\{\mathbf{k}',\omega'\}$ is the 4-vector of the scattered wave. The following term in the expansion has an additional factor of order $\hbar\omega/m_e c^2$ compared with the final term in (6). In the relation (6), besides the quantum corrections it is sufficient to take into account the first Doppler corrections [the second term in Eq. (7)]:

$$\omega'=\omega\left[1-\frac{2v_{Te}}{c}\,y\sqrt{1-x}-z\,\frac{v_{Te}^2}{c^2}\,(1-x)\right],$$
$$y=\frac{(\mathbf{k}-\mathbf{k}')\cdot\mathbf{v}}{|\mathbf{k}-\mathbf{k}'|v_{Te}\sqrt{2}}, \tag{7}$$

where x is the cosine of the scattering angle. It is necessary to include the first Doppler corrections because they are odd in y and we are concerned with the problem of determining all the quantum corrections containing the factor v_{Te}^2/c^2. At the same time, the quantum corrections in collective scattering can enter in the first power in v_{Te}/c and have a different parity with respect to y. Indeed, for the parameter s that determines the classical collective effects in the scattering (see Ref. 2), we obtain from the relation (6) with allowance for the quantum corrections

$$s\equiv\frac{\omega-\omega'}{|\mathbf{k}-\mathbf{k}'|v_{Te}\sqrt{2}}\cong y+\frac{v_{Te}}{2c}\,z\sqrt{1-x}-\frac{v_{Te}^2}{2c^2}\,zy(1-x). \tag{8}$$

The second term on the right-hand side has opposite y parity to the first, and the third is obtained with allowance for the first Doppler corrections. Thus, in the quantum corrections in (8) effects that are both even and odd in y are taken into account. In accordance with the inequality (4), we ignore the square of the linear Doppler corrections and the quadratic Doppler corrections, retaining only the quantum corrections.

3. SCATTERING MATRIX ELEMENTS

The greatest difficulties arise in the calculation of the scattering matrix element, which consists of two components, namely, the matrix element for scattering by an individual ("bare") charge M_{ind} and the matrix element for collective scattering (by the screening Debye "cloud") M_{coll}:

$$M=M_{\text{ind}}+M_{\text{coll}}. \tag{9}$$

Of course, interference (partial canceling of the matrix elements) makes it impossible to calculate the two effects independently, and in the classical limit the interference leads to the rapid decay of the scattering cross section noted previously [described by the expression (1)].

As can be seen from (7), the parameter zv_{Te}/c can arise in the limit $zv_{Te}^2/c^2 \ll 1$ in the problem of the collective quantum corrections. Leaving this parameter arbitrary for the time being, we first expand only in the parameter zv_{Te}^2/c^2. Subsequently, we shall also expand with respect to the parameter zv_{Te}/c to terms of second order in order to take into account all the contributions containing v_{Te}^2/c^2.

In the general case, the matrix element M_{ind} corresponds to the Klein–Nishina formula (or, more precisely, the square of its absolute value leads to the Klein–Nishina formula). Using the standard expressions of quantum electrodynamics, we can verify that the first quantum corrections to the Thomson matrix element of the classical scattering are strictly equal to zero, while the corrections (2) are solely due to the recoil effect in the scattering described by the final term in the conservation law (6). The matrix element M_{ind} contains only the small parameter zv_{Te}^2/c^2 and not the parameter zv_{Te}/c. Thus, the matrix element M_{ind} can be set equal to the classical Thomson matrix element.

The situation with regard to the collective matrix element is more complicated. Usually, it is obtained from the general theory of fluctuations. For the quantum relativistic case this is an extremely complicated problem, and hitherto such calculations have not been made. Our tactic is to solve for the square of this matrix element and then find its sign from the correspondence principle (by considering the classical limit). As is well known (see Refs. 23 and 3), for nonrelativistic particles the matrix element M_{coll} is proportional to the coefficient of the nonlinear density of the charge excited in the plasma by two waves—the incident and the scattered—and is inversely proportional to the longitudinal permittivity $\varepsilon_{\omega-\omega',\mathbf{k}-\mathbf{k}'}$ at the beat frequency. The probability of purely collective scattering (with neglect of the ordinary Thomson scattering) can be found from the equation for the quasilinear diffusion of particles by the scalar potential generated by the nonlinear charge density of the two waves at their beat frequency.[2] A general relativistic quantum quasilinear equation is found in Ref. 4. Here, we shall use this circumstance to find the quantum corrections. The relativistic quantum quasilinear equation [see Eq. (1.122) in Ref. 4] can be used here by expanding its coefficients with respect to the parameter $\hbar\omega/m_e c^2$. We find that the first quantum correction of order $\hbar\omega/m_e c^2$ in these coefficients is strictly equal to zero. Thus, it is shown that the matrix element $M_{\rm coll}$ can be expressed in terms of the nonlinear coefficient of the quadratic charge density and in terms of the linear permittivity by the same expression (Ref. 2, see also Ref. 3) as in the classical description except that the nonlinear coefficient of the charge density and the linear permittivity must be described by the quantum expressions.

To find these last, we use the perturbation theory developed in Ref. 5 in the equation for the relativistic quantum density matrix. For the longitudinal permittivity we obtain the result found earlier in Ref. 6, and for the coefficient of the quadratic charge density we find a rather cumbersome expression containing squares of energy denominators with virtual pair production. Rather lengthy calculations show that in the limit $\hbar \to 0$ both expressions go over into the well-known classical expressions. Further, ignoring the quadratic terms in the parameter $\hbar\omega/m_e c^2$, we can write the linear electron permittivity at the beat frequency, which occurs in the denominator of the collective scattering matrix element, in the form

$$\varepsilon_- \equiv \varepsilon_{\omega-\omega',\mathbf{k}-\mathbf{k}'} = 1 + \frac{4\pi e^2}{(\mathbf{k}-\mathbf{k}')^2} \int \frac{d\mathbf{p}}{(2\pi)^3} \times \frac{\Phi_{\mathbf{p}} - \Phi_{\mathbf{p}-\hbar\mathbf{k}+\hbar\mathbf{k}'}}{\omega-\omega'+\varepsilon_{\mathbf{p}-\hbar\mathbf{k}+\hbar\mathbf{k}'}-\varepsilon_{\mathbf{p}}+i0}, \tag{10}$$

where $\Phi_{\mathbf{p}}$ is the electron distribution function (the electron occupancies). Finally, rather lengthy calculations for the coefficient of the quadratic charge density show that to zeroth order in the parameter v_{Te}^2/c^2 and neglecting the quadratic corrections in the parameter $\hbar\omega/m_e/c^2$, the coefficient can be expressed in terms of the approximate quantum relation (10) for the electron permittivity by the same expression as in the classical limit (see Ref. 2).

This means that if the matrix element of the Thomson scattering is taken as unity and the ionic contribution to the total permittivity is ignored (this is admissible for the scattering cross section averaged over the electron distribution, in which the fraction of electrons with velocities of the order of the ion thermal velocities is small—this, naturally, holds for a thermal electron distribution), then the matrix element of the collective scattering will be equal to $-(\varepsilon_- -1)/\varepsilon_-$, and for the total matrix element we obtain in these units

$$1 - \frac{(\varepsilon_- - 1)}{\varepsilon_-} = \frac{1}{\varepsilon_-}. \tag{11}$$

The canceling of the terms in the matrix elements is very important, but now the role of the collective effects reduces to the appearance of the factor $1/|\varepsilon_-|^2$, in which the quantum effects have so far been taken into account only under the assumptions (4). Using the expansion (7), we can readily establish that under conditions for which the quantum corrections in (10) are small, expanding with respect to them is equivalent to expanding with respect to the parameter zv_{Te}/c, and not with respect to the Klein–Nishina parameter zv_{Te}^2/c^2. Of course, in the case of expansion with respect to the parameter zv_{Te}/c it is necessary to take into account the terms quadratic in this parameter, i.e., the terms of order $z^2v_{Te}^2/c^2$, since the linear terms vanish in the case of averaging over the Maxwellian distribution of the electrons. In addition, we have posed the problem of the calculation of all the quantum corrections (under conditions when the quantum corrections are small) containing the small factor v_{Te}^2/c^2. Under conditions for which the quantum corrections remain small, the term of relative order $z^2v_{Te}^2/c^2$ is admittedly small, but still much greater than the Klein–Nishina term zv_{Te}^2/c^2 by virtue of the condition $z \gg 1$. Thus, in the collective scattering regime the quantum corrections to the scattering can be much greater than those that follow from the Klein–Nishina formula. However, it must be borne in mind that the $z^2v_{Te}^2/c^2$ corrections arise solely as a result of collective effects, and therefore in the range of frequencies in which the collective effects become unimportant the quantum corrections must decrease sharply and tend to the Klein–Nishina corrections of relative order zv_{Te}^2/c^2.

4. SCATTERING CROSS SECTION

Using the results presented above, we obtain the scattering cross section averaged over the electron thermal distribution in the form

$$\sigma = \frac{3}{8}\int_0^\infty d\omega' \int_{-\infty}^\infty dy\, \frac{\exp(-y^2)}{\sqrt{\pi}} \int_{-1}^1 dx(1+x^2) \times \frac{\omega'}{\omega}\frac{1}{|\varepsilon_-|^2}\,\delta\!\left(\omega-\omega'-|\mathbf{k}-\mathbf{k}'|\sqrt{2}v_{Te}y - \hbar\frac{|\mathbf{k}-\mathbf{k}'|^2}{2m_e}\right), \tag{12}$$

where ε_- is given by the expression (10).

The quantum expression for the permittivity that occurs in the scattering cross section can be represented in terms of the well-known function that determines the classical scattering cross section,

$$W(s) = 1 - 2s\exp(-s^2)\int_0^s \exp(t^2)dt + i\sqrt{\pi}\,s \times \exp(-s^2), \tag{13}$$

and in terms of the quantum parameter [below the approximate expression for it is written down by means of the expansion (7)]

$$\kappa = \frac{\hbar|\mathbf{k}-\mathbf{k}'|}{2\sqrt{2}m_e v_{Te}} \cong \frac{z\sqrt{1-x}\,v_{Te}}{2c}\left[1 - \frac{v_{Te}}{c}y\sqrt{1-x} - \frac{v_{Te}^2}{2c^2}(1-x)z\right]. \tag{14}$$

In the first approximation, the parameter κ has the order zv_{Te}/c.

From (10), we obtain

$$\varepsilon_- = 1 + \frac{\omega_{pe}^2}{|\mathbf{k}-\mathbf{k}'|^2 v_{Te}^2}\frac{1}{4\kappa}\left(\frac{1-W(s+\kappa)}{s+\kappa} - \frac{1-W(s-\kappa)}{s-\kappa}\right). \tag{15}$$

Bearing in mind that

$$\frac{\partial}{\partial s}\frac{1-W(s)}{s}=2W(s),$$

we obtain in the limit $\kappa\to 0$ the classical expression for the screening factor in the scattering:

$$\varepsilon_-=1+\frac{\omega_{pe}^2}{|\mathbf{k}-\mathbf{k}'|^2 v_{Te}^2}W(s). \tag{16}$$

5. LIMITING CASES

We consider first the case when the quantum corrections are small, i.e., $zv_{Te}/c\ll 1$, but by virtue of the condition $z\gg 1$ they can exceed the corrections (modified by the collective effects) which are equal in order of magnitude to the Klein–Nishina corrections.

The expansion of (15) with respect to the parameter κ has the form

$$\varepsilon_-=1+\frac{\omega_{pe}^2}{|\mathbf{k}-\mathbf{k}'|^2 v_{Te}^2}\left[W(s)+\frac{\kappa^2}{6}\frac{\partial^2}{\partial s^2}W(s)\right], \tag{17}$$

and with allowance for the expansions (7), (8), and (14)

$$\varepsilon_-=1+\frac{\omega_{pe}^2}{|\mathbf{k}-\mathbf{k}'|^2}\left\{W(y)+\left[\frac{v_{Te}}{2c}z\sqrt{1-x}-\frac{v_{Te}^2}{2c^2}zy(1-x)\right]\frac{\partial W(y)}{\partial y}+\frac{z^2(1-x)v_{Te}^2}{6c^2}\frac{\partial^2 W(y)}{\partial y^2}\right\}, \tag{18}$$

where the final term containing the second derivative of $W(y)$ arises both from (17) and from the expansion of $W(s)$ with respect to the deviations of s from y. The relation (18) does indeed show that the collective effects are determined by the $z^2v_{Te}^2/c^2$ corrections (naturally, the term which is lenear in this parameter will not occur in the final result by virtue of the symmetry of the thermal distribution with respect to y). Expansion of the expression (12) with respect to these two parameters zv_{Te}^2/c^2 and $z^2v_{Te}^2/c^2$ gives the factor $T(z,\delta_e)$ multiplying $\sigma_T v_{Te}^2/c^2$ in the expression for the quantum corrections to the collective scattering [in accordance with the definition (5)]:

$$T(z,\delta_e)=zT_1(\delta_e)+z^2T_2(\delta_e), \tag{19}$$

where

$$T_1(\delta_e)=\frac{3}{16}\frac{1}{\omega^3}\frac{\partial}{\partial\omega}\omega^4\left[\int_{-1}^{1}(1-x)(1+x^2)dx\times\int_{-\infty}^{\infty}y^2\exp(-y^2)\frac{dy}{\sqrt{\pi}}\left|1+\frac{\delta_e}{1-x}W(y)\right|^{-2}\right], \tag{20}$$

and

$$T_2(\delta_e)=\frac{3}{32}\int_{-1}^{1}(1-x)(1+x^2)dx\int_{-\infty}^{\infty}(1-2y^2)\times\exp(-y^2)\frac{dy}{\sqrt{\pi}}\left|1+\frac{\delta_e}{1-x}W(y)\right|^{-2}+\frac{1}{32}\times\int_{-1}^{1}(1-x)(1+x^2)dx\int_{-\infty}^{\infty}\exp(-y^2)\frac{dy}{\sqrt{\pi}}\times\mathrm{Re}\left\{\left[1+\frac{\delta_e}{1-x}W^*(y)\right]\frac{\partial^2 W(y)}{\partial y^2}\frac{\delta_e}{1-x}\right\}\times\left|1+\frac{\delta_e}{1-x}W(y)\right|^{-4}. \tag{21}$$

The relation (21) and the first term of (20) are obtained without allowance for the corrections described by the second term of (17) by representing the integrand in a form containing the corrections δy to y in the expression $W(s)=W(y+\delta y)$ and the corrections δx to x in the expression for $1-x-\delta x$ when the expansion (7) is used in $\omega_{pe}^2/(\mathbf{k}-\mathbf{k}')^2v_{Te}^2$ and the terms even in y are retained. There is then an integration by parts with respect to y, and the differentiation with respect to x is represented as differentiation with respect to the frequency using the dependence of the parameter δ_e on the frequency ($\delta_e\propto 1/\omega^2$).

The correction (20) goes over into the Klein–Nishina correction in the limit $\delta_e\to 0$ [at the same time $T(\delta_e)\to 2$], whereas the correction (21) vanishes in the limit $\delta_e\to 0$, but for $\delta_e\sim 1$ and $z\gg 1$ the collective correction (21) greatly exceeds the correction (20). Thus, in the collective scattering regime the quantum corrections to the scattering are much greater than those that are described by the noncollective scattering. In the limit $\delta_e\gg 1$, numerical calculation of the integrals gives

$$T_1(\delta_e)\cong\frac{4.96}{\delta_e^2},\quad T_2(\delta_e)\cong\frac{0.31}{\delta_e^2}. \tag{22}$$

We now consider the case $zv_{Te}/c\gg 1$, which, naturally, is compatible with the condition (4) adopted for a plasma at nonrelativistic temperatures. Then we have $\kappa\gg 1$ and $W(\kappa)\cong W(-\kappa)\cong -1/2\kappa^2$, and this leads to the following expression for the screening scattering factor:

$$\varepsilon_-=1+\frac{\delta_e}{1-x}\frac{1}{4\kappa^4}. \tag{23}$$

For $\kappa\sim zv_{Te}/c\ll\delta_{Te}^{1/4}$ (which, naturally, requires fulfillment of the condition $\delta_e\gg 1$) the scattering is suppressed by the collective processes, but the quantum effects significantly reduce it. For $\kappa\gg\delta_e^{1/4}$, the value of the screening factor is close to unity, and therefore the quantum effects suppress the collective reduction of the scattering cross section, and it approaches the Thomson cross section. The range of frequencies in which these last inequalities are satisfied is relatively narrow, but the result itself is of fundamental interest. For applications, the case in which the quantum collective corrections are small is the most interesting one.

6. DISCUSSION OF THE RESULTS

The effects described here must play an important role in radiative transfer and in radiative heat conduction. The value of the effective parameter z is determined by the opacity, which, in its turn, depends on the derivative of the Planck distribution with respect to the temperature. This derivative is proportional to $z^4e^z/(e^z-1)^2$. For the z-independent effects, the effective value of z is determined by the maximum of the last expression and is close to 3.8. For the quantum effects proportional to z^2, the effective value of z will be close to 6, and this makes it possible to obtain estimates by means of the approximation employed here. In the problem of the deficit of solar neutrinos, the solution could consist of reconciliation with a theory of radiative transfer that takes into account collective effects and increases the transparency of the interior of the sun.[7] As zeroth approximation, one may use the estimates of the plasma parameters in the interior of the sun obtained without allowance for the collective effects. Then we have $v_{Te}/c \cong 1/20$, $zv_{Te}/c \cong 0.3$ and $\kappa^2 \cong 0.045$, i.e., one can expect corrections of about 4–5%. This is a value that in the problem of the deficit of solar neutrinos is rather appreciable. An exact result can be obtained only by means of complicated numerical calculations similar to those made in Ref. 7 and taking into account the broadening of the Raman resonance, $1+\delta_e W(y)/(1-x) \simeq 0$.

The quantum corrections are also very important in a different problem, namely the Comptonization of radiation, when for isotropic radiation the main term with the Thomson cross section cancels on account of the balance of the direct and reverse processes, and the entire effect is determined by the relativistic and quantum corrections to the scattering cross section. In the collective scattering regime, they will be determined by the relations described here.

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