Analysis of the distribution of axially symmetric and spherically symmetric heat pulses in helium II in the absence of superfluid turbulence

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(Submitted 27 July 1995)
Zh. Éksp. Teor. Fiz. **109**, 194–203 (January 1996)

The applicability of equations expanded to third order in the temperature deviation and the counterflow velocity in the treatment of the propagation of single large-amplitude heat pulses in stationary helium II not having preliminary excitation in the form of superfluid turbulence is substantiated. A quadratic dependence of the rate of propagation of a simple wave and a nonlinear relationship between the temperature deviation and the counterflow velocity in it are derived. Exact linear solutions that describe axially and spherically symmetric pulses excited by rectangular heat fluxes are obtained. A quadratic solution describing a shock wave is obtained under axial symmetry. A comparison of the nonlinear solution with previous experiments on the propagation of axially symmetric short large-amplitude pulses reveals good agreement for the results on both the evolution of their duration and on the variation of the temperature deviation. A systematic error is found in the measurement of the temperature in some experiments due to the use of only a linear relation between the temperature deviation and the heat flux density. [S1063-7761(96)01201-X]

1. INTRODUCTION

Interesting experiments on the propagation of axially symmetric large-amplitude heat pulses (under conditions in which superfluid turbulence can be neglected) excited in helium II by brief rectangular heat fluxes from a cylindrical heater were described in Refs. 1–4. This question was also partially discussed in Ref. 5. An approximate theoretical treatment of these experiments was performed in Ref. 2. The analogous spherically symmetric problem was considered in Ref. 6.

This paper describes a more rigorous theoretical treatment of this problem on the basis of the equations of two-fluid hydrodynamics.[7,8] A comparison of the nonlinear results obtained with the experimental data[1,2,4] reveals good quantitative agreement.

2. THEORY

The nondissipative equations of two-fluid hydrodynamics[7,8] (with neglect of the dependence of the density of helium ρ on the temperature T and of the entropy and the density of the normal-fluid component on the pressure P) lead to equations describing the propagation of heat pulses in stationary helium II ($\rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s = 0$), that have the following form in the cubic approximation in the absence of superfluid turbulence:

$$\frac{\partial}{\partial y}\left(\tau + \frac{\rho_s k}{2}\tau^2 + \frac{q}{2}\mathbf{u}^2 + \frac{\rho_s k_1}{2}\tau\mathbf{u}^2 + \frac{\rho_s^2 k_2}{6}\tau^3\right)$$
$$+\nabla\left(\mathbf{u} + (\rho_s - \rho_n q)\tau\mathbf{u}\frac{\rho_s^2 k - \rho_s\rho_n k_1 - 2\rho_s\rho_n q}{2}\tau^2\mathbf{u}\right.$$
$$\left.+\frac{\rho_s q - \rho_n q_1}{2}\mathbf{u}^3\right) = 0, \qquad (1)$$

$$\frac{\partial}{\partial y}\left(\mathbf{u} + \rho_s q\tau\mathbf{u} + \frac{\rho_s^2 k_1}{2}\tau^2\mathbf{u} + \frac{\rho_s q_1}{2}\mathbf{u}^3\right) + \nabla\left(\tau + \frac{\rho_s}{2}\tau^2\right.$$
$$\left.+\frac{3\rho_s}{2}\mathbf{u}^2 + \frac{\rho_s^2 k}{6}\tau^3 + \frac{3\rho_s q(\rho_s - \rho_n)}{2}\tau\mathbf{u}^2\right)$$
$$+\{\rho_s + \rho_s q(\rho_s - \rho_n)\tau\}\{\mathbf{u}\nabla\mathbf{u} - (\mathbf{u}\nabla)\mathbf{u}\}$$
$$+\rho_s q(1 - 3\rho_n)\mathbf{u}\times(\mathbf{u}\times\nabla\tau = 0.^{1)} \qquad (2)$$

Here the dimensionless densities of the superfluid (ρ_s) and normal-fluid (ρ_n) components ($\rho_s + \rho_n = 1$), the dimensionless temperature deviation $\tau = T'(\partial S/\partial T/\rho_s S$, where $T' - T - T_0$, the dimensionless counterflow velocity vector of the normal-fluid and superfluid components $\mathbf{u} = (\mathbf{v}_n - \mathbf{v}_s)/C_2 = \mathbf{w}/C_2$, and the dimensionless time $y = tC_2/R$ were introduced, the spatial coordinates were normalized to the scale factor R, and expansions of the entropy $S(\tau, \mathbf{u}^2)$ to third order and of $\rho_n(\tau, \mathbf{u}_2)$ to second order were used. In addition, the following notation was introduced:

$$k = \frac{\partial \log(\partial S/\partial T)}{\partial \log S(T)} \sim 1, \quad q = 1 - \frac{\partial \log S_n(T)}{\partial \log S(T)},$$

$$S_n = \frac{S}{\rho_n}, q_1 - q\left(1 - 2\frac{\partial S_n}{\partial w^2} \Big/ \frac{\partial \log \rho_n}{\partial T}\right),$$

$$k_1 = k - \frac{\partial \log S_n(T)}{\partial \log S(T)}\left(2 - \frac{\partial \log\{S_n^2/(\partial S_n/\partial T)\}}{\partial \log S(T)}\right),$$

$$k_2 = k\frac{\partial \log(\partial^2 S/\partial T^2)}{\partial \log S(T)},$$

and C_2 for the absolute value of the velocity of second sound. In the normalizations and coefficients all the parameters correspond to "unperturbed" helium II (at $T = T_0$). In Eqs. (1) and (2) the second-order terms with the coefficient $\rho_s \partial \log \rho(T)/\partial \log S(T) < 2\times 10^{-3}$ were neglected, and the excitation of first sound by second sound as a consequence of quadratic coupling[12,13] was likewise not taken into account, since it would produce third-order terms with a coefficient smaller than C_2/C_1 in the equations. It is correct to take into account the cubic terms only when $|\mathbf{u}|\gg 2\partial \log \rho(T)/\partial \log S(T)$, which corresponds to the condition $Q\gg 2\rho_n C_2^3 T\partial\rho/\partial T$ on the heat flux density. This condition was satisfied in the experiments under discussion.

The energy conservation law in the quadratic approximation under the adopted assumptions can be written in the form

$$\frac{\partial}{\partial y}\left(\tau+\frac{(m+k)\rho_s}{2}\tau^2+\frac{q+m\rho_s}{2}\mathbf{u}^2\right)+\nabla(\mathbf{u}+(\rho_s-q\rho_n+m\rho_s)\tau\mathbf{u})=0. \tag{3}$$

Here the total energy density and the energy flux density were normalized to $\rho\rho_s ST$ and $\rho\rho_s STC_2$, respectively, and $m=S/\partial S/\partial T)T\simeq 1/6$.

When the propagation of a simple plane wave is considered, Eqs. (1) and (2) lead to the expression for the velocity of second sound

$$C_2(\mathbf{u})=C_2\left\{1+\frac{6\rho_s-3q-k\rho_s}{2}\left|\mathbf{u}\right|-\left\{3\rho_s\rho_n+\frac{k\rho_s}{4}\left(\rho_s\frac{\partial \log k(T)}{\partial \log S(T)}-\rho_n\right)\right\}\mathbf{u}^2\right\} \tag{4}$$

and the relationship between the temperature and the counterflow velocity

$$\tau=|\mathbf{u}|-\frac{2\rho_s-q+k\rho_s+4q\rho_n}{4}\mathbf{u}^2-\left\{\frac{\rho_n(\rho_s-\rho_n)^2}{4}-\frac{k\rho_s}{12}\left(\rho_s\frac{\partial \log k(T)}{\partial \log S(T)}-\rho_n\right)\right\}|\mathbf{u}|^3. \tag{5}$$

The linear term in (4) was obtained by Khalatnikov[14] and has been firmly corroborated experimentally.[11] The coefficients of $\mathbf{u}^2$ in (4) and $|\mathbf{u}|^3$ in (5) were written under the assumption S_n=const. An analysis of the omitted terms shows that their contribution is less than 5% at $T>1.2$K.

As we know, when a step heat pulse is excited in the 0.95–1.9 K temperature range, a shock wave forms at once at the front (see Refs. 7 and 11). Its rate of propagation is

$$C_{2f}=(C_2(\mathbf{u}_f)+C_2)/2,$$

and the relationship between τ_f and $|\mathbf{u}_f|$ in the quadratic approximation, according to (3), corresponds to (5). The trajectory of the shock-wave front in the planar, axially symmetric, and spherically symmetric cases is

$$y_f-y_0=\pm\left\{x_f-x_0-\frac{p}{2}\int_{x_0}^{x_f}\left|\mathbf{u}_f\right|dx+\left(\frac{p^2}{4}-\frac{p_1}{2}\right)\int_{x_0}^{x_f}\mathbf{u}_f^2dx\right\}, \tag{6}$$

where x_0 and y_0 are the initial coordinate and time of the excited pulse, and p and p_1 are the coefficients of the linear and quadratic terms in (4), respectively. Here and in the following, wherever two signs are used for cylindrical and spherical shock waves, the upper sign corresponds to a diverging front, and the lower sign corresponds to a converging front.

Let us consider the problem of the excitation of heat pulses by a cylindrical heater of radius $R(x=1)$, to which a rectangular pulse of duration Δ with heat flux density Q_0 is fed at time $y=1$:

$$Q(1,y)=Q_0\{\theta(y-1)-\theta(y-1-\Delta)\}, \tag{7}$$

where θ is the Heaviside step function $[\theta(0)=1/2]$. Then the boundary condition for u at $x=1$ in the quadratic approximation is

$$u(1,y)\{1+(\rho_s+m\rho_s-q\rho_n)\tau(1,y)\}=\pm\varepsilon\{\theta(y-1)-\theta(y-1-\Delta)\}, \tag{8}$$

where $\varepsilon=Q_0/\rho\rho_s STC_2$ is the cutoff amplitude of the normalized heat flux density. We assume that $\varepsilon\ll 1$.

2.1. Linear problem

The boundary conditions for a step pulse are

$$u(1,y)=\pm\varepsilon\theta(y-1),\quad u_f=\pm r_f\quad \text{when } y-1=\pm(x-1). \tag{9}$$

According to a theorem of S. V. Kovalevskaya,[15] these boundary conditions are sufficient to prove that in the neighborhood of the point $y=x=1$ there is a single solution of the linear part of Eqs. (1) and (2).

The solution can be represented in the form

$$u=\varepsilon u_1\theta(z),\quad \tau=\varepsilon\tau_1\theta(z), \tag{10}$$

where $z=(y-1)\mp(x-1)$,

$$u_1(x,y)-\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n\sum_{k=0}^{n}\frac{a_{nk}}{x^{k+1/2}},$$

$$\tau_1(x,y)=\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n\sum_{k=0}^{n}\frac{b_{nk}}{x^{k+1/2}}, \tag{11}$$

$$a_{n+k,k}=(\mp 1)^k a_{n0}\frac{(2k+1)!!}{4^k k!}\frac{\prod_{m=1}^{k}(2m-3)}{\prod_{m=1}^{k}(n+m)},$$

$$a_{00}=\pm 1,\quad a_{n0}=-\sum_{k=1}^{n}a_{nk},$$

$$b_{nk}=\mp a_{nk}\frac{2k-1}{2k+1},$$

and converges in the region $x\geq 1$, $|y-x|<2$ for a diverging flux and in the region $0<x\leq 1$,$|y+x-2|<2x$ for a converging flux.

We present the first four terms of the series in (11):

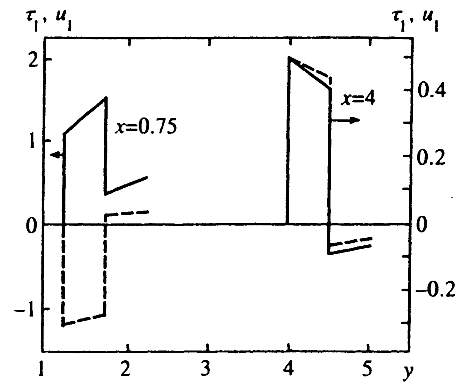

FIG. 1. Converging ($x=0.75$) and diverging ($x=4$) rectangular pulses ($\Delta=0.5$); solid line—τ_1 dashed line —u_1.

$$u_1(x,y)=\pm\frac{1}{\sqrt{x}}\left\{1\mp\frac{3z}{8}\left(1-\frac{1}{x}\right)+\frac{3z^2}{256}\left(11-\frac{6}{x}-\frac{5}{x^2}\right)\right.$$
$$\left.\mp\frac{z^3}{2048}\left(83-\frac{33}{x}-\frac{15}{x^2}-\frac{35}{x^3}\right)\right\}, \tag{12}$$

$$\tau_1(x,y)=\frac{1}{\sqrt{x}}\left\{1\mp\frac{z}{8}\left(3+\frac{1}{x}\right)+\frac{3z^2}{256}\left(11+\frac{2}{x}+\frac{3}{x^2}\right)\right.$$
$$\left.\mp\frac{z^3}{2048}\left(83+\frac{11}{x}+\frac{9}{x^2}+\frac{25}{x^3}\right)\right\}, \tag{13}$$

which provide 1% relative accuracy at $-0.5<z<1$. It follows from (13) that at the surface of the heater τ_1 decreases with time in the case of diverging flux and increases in the case of converging flux. In contrast to (5), τ and u differ to first order behind the front (see Fig. 1)

$$u_1\mp\tau_1=\frac{z}{2x^{3/2}}\left\{1\mp\frac{3z}{16}\left(1+\frac{1}{x}\right)+\frac{z^2}{256}\left(11+\frac{6}{x}+\frac{15}{x^2}\right)\right\}. \tag{14}$$

It can be seen from (12) and (13) that the approximate treatment for a diverging flux in Ref. 2 is erroneous.[2)]

The solution of the analogous spherically symmetric problem for a step pulse has the form

$$u=\pm\varepsilon\left\{\frac{1}{x^2}+\left(\frac{1}{x}-\frac{1}{x^2}\right)e^{\mp}\right\}\theta(z),\quad \tau=\varepsilon\frac{1}{x}e^{\mp}\theta(z). \tag{15}$$

This solution differs fundamentally from the solution obtained in Ref. 6 in a treatment of a similar problem.[3)]

2.2. Nonlinear problem

According to (5) and (8)–(10), the boundary conditions in the quadratic approximation can be written in the form

$$u(1,y)\{1+\varepsilon(\rho_s+m\rho_s-q\rho_n)\tau_1(1,y)\}=\pm\varepsilon\,\theta(y-1), \tag{16}$$

where $\tau_1(1,y)=1-(y-1)/2+3(y-1)^2/16-y-1)^3$, and at the front

$$\tau_f\mp u_f=-\varepsilon^2(2\rho_s-q+k\rho_s+4q\rho_n)/4x. \tag{17}$$

We write the solution of Eqs. (1) and (2) in the quadratic approximation in the region $y>y_f$, $0<\pm(x-1)<\pm(x_f-1)$ in the form

$$u=\varepsilon u_1+\varepsilon^2u_2,\quad \tau=\varepsilon\tau_1+\varepsilon^2\tau_2. \tag{18}$$

It is convenient to write the equations for u_2 and τ_2 in the form

$$\frac{\partial\tau_2}{\partial y}+\frac{\partial u_2}{\partial x}+\frac{u_2}{x}=\frac{(2-k)\rho_s-q(1+2\rho_n)}{2}\frac{\partial_1^2}{\partial y}$$
$$+\frac{(1-k)\rho_s-q\rho_n}{2}\frac{\partial(\tau_1^2-u_1^2)}{\partial y}, \tag{19}$$

$$\frac{\partial\tau_2}{\partial x}+\frac{\partial u_2}{\partial y}=-(2-q)\rho_s\frac{\partial u_1^2}{\partial x}+(q-1)\rho_s\frac{u_1^2}{x}$$
$$+\frac{(q-1)\rho_s}{2}\frac{\partial(\tau_1^2-u_1^2)}{\partial x}. \tag{20}$$

According to (16) and (9), the boundary conditions have the form

$$u_2(1,y)=\mp(\rho_s+m\rho_s-q\rho_n)\tau_1(1,y), \tag{21}$$

and, according to (17), (12), (13), and (6), at the front we have

$$\tau_{2f}=\pm u_{2f}-\frac{(2-q)\rho_s}{x}+\frac{6\rho_s-3q-k\rho_s}{4x^{3/2}}. \tag{22}$$

The solution has the form

$$u_2=\sum_{n=0}^{\infty}z^n\sum_{k=0}^{n+1}\left(\frac{c_{nk}}{x^k}+\frac{a_{nk}}{x^{k+1/2}}\right),$$

$$\tau_2=\sum_{n=0}^{\infty}z^n\sum_{k=0}^{n+1}\left(\frac{d_{nk}}{x^k}+\frac{\beta_{nk}}{x^{k+1/2}}\right), \tag{23}$$

$$\alpha_{n+k,k}=(\mp1)^k\alpha_{n0}\frac{(2k+1)!!}{8^kk!}\frac{\prod_{m=1}^{k}(2m-3)}{\prod_{m=1}^{k}(n+m)},$$

$$\alpha_{k,k+1}=(\pm1)^k\alpha_{01}\frac{(2k-1)!!(2k+3)!!}{8^kk!(k+1)!},$$

$$\beta_{nk}=\mp\alpha_{nk}\frac{2k-1}{2k+1}.$$

All the c_{nk} and d_{nk}, except for c_{01} and d_{00}, can be determined from Eqs. (19) and (20) in terms of the right-hand sides. From (22) it follows that

$$d_{00}\pm c_{00},\quad c_{01}=\pm d_{01}\pm(2-q)\rho_s,$$

$$\alpha_{01}=\mp36\rho_s-3q-k\rho_s)/16,$$

and from (21) it follows that

$$\alpha_{n0}=\mp\frac{\rho_s+m\rho_s-q\rho_n}{2^n}\sum_{k=0}^{n}b_{nk}-\sum_{k=0}^{n+1}c_{nk}-\sum_{k=0}^{n+1}\alpha_{nk}.$$

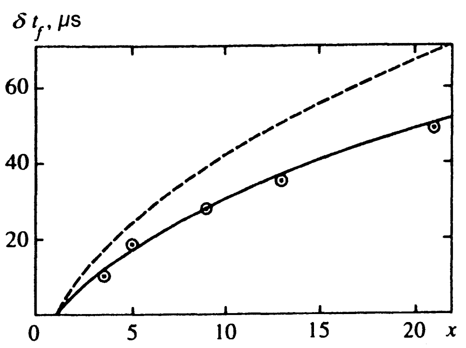

FIG. 2. Dependence of the nonlinear addition to the duration of a rectangular pulse $\delta t_f = R\delta y_f/C_2$ (24) on x ($T_0=1.4$ K, $Q_0=8$ W/cm^2). The dashed curve is a plot of the linear addition, and the points are experimental values from Refs. 1 and 2 (the error is ± 4 μs).

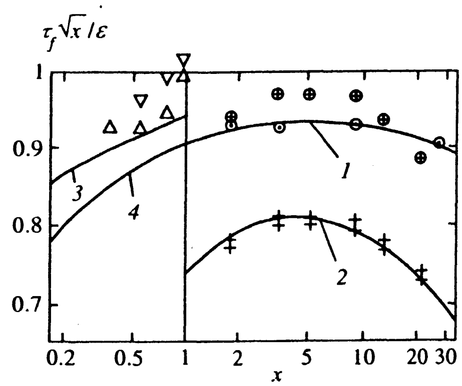

FIG. 3. Nonlinear dependence of $\tau_f\sqrt{x}/\varepsilon$ on x (25) under various experimental conditions (T_0,Q_0)—curves *1–4* and corresponding experimental values from Refs. 1, 2, and 4 [relative errors: $\pm 10^{-2}\sqrt{x}$ at $x>1$, and $\pm 5\times 10^{-2}\sqrt{x}$ when $Q_0=2$ W/cm^2 and $\pm 2\times 10^{-2}\sqrt{x}$ when $Q_0=3$ W/cm^2 at $x<1$ ($T_0=1.4$ K)]: *1* and ⊙—$T_0=1.6$ K, $Q_0=8$ W/cm^2; *2* and ⊕—$T_0=1.4$ K, $Q_0=8$ W/cm^2 (‡—renormalized with the coefficient $n=0.83$); *3* and ∇—$T_0=1.4$ K, $Q_0=2$ W/cm^2; *4* and Δ—$T_0=1.4$ K, $Q_0=3$ W/cm^2.

The domain of applicability of the quadratic solution is $y_f<y<x+(x_p-x)/(x_p-1)$ for $x>1$ and $y_f<y<2-x_c(1-x)/(1-x_c)$ for $x<1$, where $\sqrt{x_p} \ll 1/\varepsilon c_{00}$ and $\sqrt{x_c} \gg \varepsilon d_{01}$, and the solution converges in these domains.

3. COMPARISON WITH EXPERIMENT

The solutions lead to two functions of x that are quadratic with respect to ε for the observables in the experiments in Refs. 1–4: the addition δy_f to the pulse duration Δ (at 1.2 K$<T_0<$1.8 K)

$$\delta y_f = \mp\varepsilon p\sqrt{x}-1)\left\{1-\frac{\varepsilon}{2}\left(3p-\frac{2p_1}{p}-(1-k-2m)\rho_s + \rho_n-(3-2\rho_n)\frac{\partial \log S_n(T)}{\partial \log ST}\right)\{1+H(x)\}\right\}, \quad (24)$$

where $|H(x)|<0.05$ at $x<40$, and the temperature jump at the leading edge of the pulse

$$\tau_f = \frac{\varepsilon}{\sqrt{x}}\left\{1-\varepsilon\left(\frac{3p}{8}\sqrt{x}+\left(\frac{13}{8}p-(1-k)\rho_s - 2\rho_s\frac{\partial \log S_n(T)}{\partial \log S(T)}\right)\frac{1}{\sqrt{x}}-\frac{p}{2}+m\rho_s\right)\right\}. \quad (25)$$

The dependence of $\delta tf=\delta yfR/C2$ on x and the corresponding experimental data from Refs. 1 and 2 are presented in Fig. 2. It is seen that (24) agrees well with the experiments. Here and in the following, the accuracy of the experimental points was evaluated as the ratio of the thickness of the lines in the figures in Refs. 1, 2, and 4 to the corresponding measured quantities.

Figure 3 presents the calculated and experimental values of $\tau_f\sqrt{x}/\varepsilon$ corresponding to the different conditions of the experiments in Refs. 1, 2, and 4. The experimental results for $T_0=1.6$ K and $Q_0=8$ W/cm^2 in Ref. 2 agree well with curve *1*, but the results for $T_0=1.4$ K and $Q_0=8$ W/cm^2 in Refs. 1 and 2 differ significantly from the corresponding curve *2*, although the values renormalized with the coefficient $n=0.83$ lie close to this curve. The systematic deviation of the experimental results at $T_0=1.4$ K from the calculation is attributable to the fact that amplitude calibration of the temperature detector only according to the linear relation $T'=Q/\rho C_p C_2$ was employed in those experiments, as was noted in Ref. 1, while here, according to (25), the relationship between T' and Q is significantly nonlinear. The temperature jumps at the leading edge of the pulse in converging fluxes (at $T_0=1.4$ K)[4] obtained under the conditions of weak (when $Q_0=2$ W/cm^2) and strong (when $Q_0=3$ W/cm^2) superfluid turbulence are in qualitative agreement with curves *3* and *4*, respectively. An increase in the temperature jump at the front that is weaker than $1/\sqrt{x}$ was observed in these experiments, in accordance with the calculation, and the slightly increased values of the temperature compared with the calculation ($\sim$10%) can be attributed to dissipation of the energy at the front due to the presence of the initial superfluid turbulence.

Figure 4 presents experimental pulses from Refs. 2 and 4. We note that the characteristic features of the pulse configuration are consistent with the pulses in Fig. 1, which were constructed in the linear approximation. The calculated values of the peaks of the leading and trailing edges of the pulses are also noted here. The calculated results for a di-

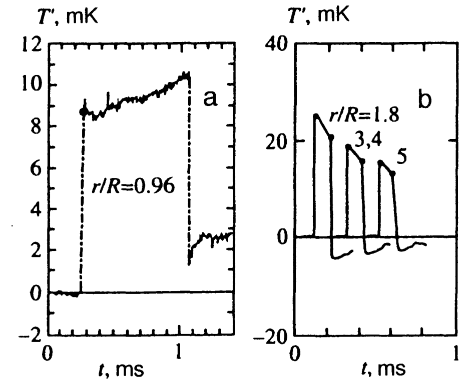

FIG. 4. Experimental converging pulses from Ref. 4(a) and diverging pulses from Ref. 1b) [●—calculated values of the peaks of the leading 1) and trailing (b) (renormalized with the coefficient $1/n=1.2$) edges].

verging flux under the conditions $T_0=1.4$ K and $Q_0=8$ W/cm^2 were renormalized with the coefficient $1/n=1.2$ (Fig. 4b).

4. DISCUSSION

The experiments under discussion show that when single short large-amplitude heat pulses (v_n up to 300 cm/s) propagate in wide channels under negligibly small initial superfluid turbulence, the superfluidity in helium II is not disturbed, and these experiments are described well by the nondissipative equations of two-fluid hydrodynamics.

The front of a heat flux in helium II propagates with the velocity of second sound $C_2(u_f)$. According to (3), the normalized variation densities of the total energy and the energy flux have linear and quadratic terms. The linear term in the energy flux is determined by the counterflow wave of the "unperturbed" state of helium II (at T_0), and the quadratic term is determined by the temperature perturbation of its state caused by this wave. The linear term in the variation of the total energy reflects the variation of the internal energy associated with the linear variation of the entropy with the temperature, and the quadratic terms reflect the corresponding variation of the total energy, including the quadratic variations of the internal energy with the temperature and the counterflow. A basically linear variation of the internal energy of helium II with the temperature was observed in the experiments under discussion.[1,2,4]

If the diverging flux per unit axial length appearing in response to a step heat flux in the heater under axial symmetry and the total flux appearing under spherical symmetry are considered, the quadratic part of such a flux remains unchanged, but the linear part at the front increases as it propagates in proportion to $\sqrt{x}$ under axial symmetry and in proportion to x under spherical symmetry. The linear part of the internal energy increases accordingly in the layer behind the front. A relative fraction of the additional internal energy equal to $\Delta\sqrt{y}/(y-1)$ in the case of a cylindrical layer and to $(1-e^{-\Delta})$ in the case of a spherical layer is concentrated in the narrow layer behind the front of width $\Delta\leqslant 1$ during the period of operation of the source, $y-1\gg 1$. The corresponding values in the layer of width Δ at the heater are equal to $\Delta/\sqrt{y}(y-1)$ and $\Delta e^{(1-y)}/(y-1)$. This redistribution of the additional entropy in axially and spherically symmetric fluxes in helium II can be explained by analogy to the explanation given by Zel'dovich and Raizer[16] for spherical sound pulses in an ordinary fluid. Although the amplitudes of the energy flux density at the front decrease as $1/\sqrt{x}$ and $1/x$, respectively, the amount of entropy in the flux increases as x and x^2. Thus, the energy flux carries off the additional entropy created by the source toward the front. The picture is reversed in a converging wave ($x<1$). Here the total energy flux at the front decreases, and the additional self-energy accumulates at the heater. Just such a picture is observed in the experiments under discussion, where the temperature pulses, which are proportional to the entropy variations, are basically described by the linear solutions.

5. CONCLUSIONS

A comparison of these results with experimental data on axially symmetric pulses[1,2,4] has shown that the linear solution faithfully describes the characteristic features of the short diverging and converging experimental pulses obtained both in the absence[1,2] and in the presence[4] of superfluid turbulence, and that the nonlinear solution describes these pulses to 1% accuracy in the region where it is applicable both with respect to the temperature deviation and with respect to their variable duration. The evolution of the amplitudes of the leading edge of the pulses obtained under the conditions of developed initial superfluid turbulence[4] is also described well by the nonlinear solution. It has been shown that amplitude calibration of the temperature detectors requires the use of the nonlinear relation between the heat flux density and the temperature. As a result, we maintain that single brief large-amplitude heat pulses in wide channels propagate in helium II as in a superfluid, and are described by the nondissipative equations of two-fluid hydrodynamics.

[1)] We note that in Refs. 5, 9, and 10, where equations similar to (1) and (2) were used in the quadratic approximation in a numerical analysis of experiments, the quadratic terms were taken into account incorrectly. In particular, the equations used there [Eqs. (1) in Ref. 10], do not give the correct,[7] experimentally confirmed [11] formula for $C_2(\mathbf{u})$.

[2)] In Ref. 2 the second terms in (12) and (13) were written, respectively, in the forms $-(y-x)(1-1/x)/2\sqrt{x}$ and $-(y-x)/2\sqrt{x}$ [see Eqs. (14) and (13) in Ref. 2].

[3)] The solution obtained in Ref. 6 is erroneous due to the assumption actually made there that the temperature change at the surface of a spherical heater is proportional to dQ/dt, rather than $Q(t)$ (see Fig. 1 in Ref. 6).

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Translated by P. Shelnitz