Effective Lagrangian in finite-temperature QED in an external magnetic field and the static limit of the polarization operator

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The low-temperature corrections to the one-loop effective QED Lagrangian in an external magnetic field, with finite fermion density, are calculated as the sum over a finite number of Landau excited levels, and are valid over a wide range of parameters. The simple form of the expansion allows one to find the components of the one-loop polarization operator in the static limit and obtain expressions for the Hall conductivity and the screening in an electron gas. Anisotropic Debye screening in a strong magnetic field is demonstrated. [S1063-7761(96)00801-6]

1. INTRODUCTION

In the vicinity of a number of cosmic objects the fermion density and the magnetic field strength take very large values[1,2] and it becomes necessary to take quantum corrections into account.[3,4] Information about many properties of a relativistic electron–positron plasma in an external magnetic field can be obtained within the framework of $(3+1)$-dimensional quantum electrodynamics (QED) with a chemical potential μ and temperature T in a constant and homogeneous external magnetic field B.[5–15] At the same time, in many cases the result has an extremely complicated form[9–12] and even a comparison of the same calculations carried out in different ways is extremely difficult (see the Introduction to Ref. 4). In this regard, it would be most desirable to work out an easily reproducible procedure that would allow one to obtain in a form suitable for analysis at least a few quantities in the physically interesting parameter region.

The expression for the one-loop effective action (we will consider the Lagrangian $\mathscr{L}^{\text{eff}}(B,\mu,T)$) of QED with a finite fermion density and temperature in an external magnetic field is well known.[8,13,14] Nevertheless, for specific values of the parameters μ, T, and B the Lagrangian $\mathscr{L}^{\text{eff}}(B,\mu,T)$ can be calculated only by numerical methods. An analytic expression for the plasma (i.e., μ- and T-dependent) part of the effective Lagrangian $\tilde{\mathscr{L}}^{\text{eff}}(B,\mu,T)$ has been obtained only in the limits μ or $B\to\infty$ (Ref. 14) and $T\to\infty$ (Ref. 15), while the calculation of $\tilde{\mathscr{L}}^{\text{eff}}(B,\mu,T)$ at intermediate values of the parameters is fraught with difficulty.[14] In a recent preprint[16] we showed that at low temperatures it is possible to expand the one-loop effective action for finite values of μ and B. The low-temperature corrections (as well as the μ-dependent part at $T=0$) in this case are expressed in terms of elementary functions as finite sums over the Landau excitation levels. Such a representation, in turn, allows one to make some progress in the calculation of the polarization operator and an examination of its static limit.[16]

The situation with the one-loop polarization operator $\Pi_{\mu\nu}(p)$ is more complicated than the situation with the effective action. The full covariant expression for the one-loop polarization operator for $B,\mu,T\neq 0$ was obtained in Refs. 10 and 11, where it was represented in the form of an expansion over six transverse covariant tensor structures and the corresponding extremely complicated scalar coefficients were calculated. Although for some applications it is sufficient to know some of the components of the polarization operator in the static limit $p_0=0$, $\mathbf{p}\to\mathbf{0}$, even just separating this limit from the total expression is not a simple task (Refs. 10 and 11 present an exhaustive analysis of the analytic properties of $\Pi_{\mu\nu}(p)$ and the polarization, but specific limits were not calculated). At the same time, some components of the polarization operator can be obtained from the expression for the effective action as derivatives with respect to μ and B. Consequently, in those cases where the expression for the effective action can be obtained in simple form, we can also easily calculate certain elements of the polarization operator. This holds in the low-temperature limit, when the components Π_{00}, $\Pi_{01}=\Pi_{10}^*$ and $\Pi_{02}=\Pi_{20}^*$ in the static limit can be represented in terms of elementary functions as finite sums over the Landau excited levels[16] (these calculations were confirmed in Ref. 17, where the aforementioned components in the static limit at $T=0$ were calculated as the corresponding one-loop integrals with zero external momentum).

In the present paper we show that in QED with finite fermion density in an external magnetic field, a low-temperature expansion can be performed that is valid over a wider range of the parameters. In this case, the plasma part of the effective Lagrangian can be represented in terms of elementary functions in the form of a finite sum over partly filled Landau levels. The fermion density, magnetization, Hall conductivity, and some components of the polarization operator can be written in the same form. Here we will show that at finite temperatures, the singularities present at $T=0$ (Ref. 16) vanish. We will also show that using the general properties of the polarization operator and knowing the effective Lagrangian, one can obtain the information needed to investigate static screening in an electron (positron) gas in an external magnetic field. As an example, we calculate the potential created by some simple charge configurations and demonstrate the anisotropy of Debye screening in an external magnetic field.

The present paper is organized as follows: Sec. 2. calculates the low-temperature expansion of the one-loop effective Lagrangian for the case in which the edge of the upper Landau excited level does not coincide with the Fermi surface. In Sec. 3. expressions are obtained for the magnetization, fermion density, and components of the polarization operator. In Sec. 4. the approach used in Sec. 2. is developed for the case where the edge of one of the Landau levels coincides with the Fermi surface. Section 5. is dedicated to a study of Debye screening in an electron gas in an external magnetic field.

2. LOW-TEMPERATURE QED EXPANSION IN AN EXTERNAL MAGNETIC FIELD

We consider QED with a finite fermion density in an external magnetic field, described by the Lagrangian:[1)]

$$\mathscr{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\bar{\psi}(i\partial\!\!\!/-eA\!\!\!/-\gamma_0\mu-m)\psi. \qquad (1)$$

At $T,\mu,B\neq 0$, the one-loop effective Lagrangian $\mathscr{L}^{\rm eff}(B,\mu,T)$, which obtains after integrating over the fermion fields, has the form[8,13,14]:

$$\mathscr{L}^{\rm eff}(B,\mu,T)=\mathscr{L}^{\rm eff}(B)+\tilde{\mathscr{L}}^{\rm eff}(B,\mu,T), \qquad (2)$$

where

$$\tilde{\mathscr{L}}^{\rm eff}(B,\mu,T)=\frac{1}{\beta}\frac{eB}{(2\pi)^2}\sum_{k=0}^{\infty}b_k\int_{-\infty}^{\infty}dp_\|\{\ln[1+\exp(-\beta(\varepsilon_k(p_\|)-\mu))]+\ln[1+\exp(-\beta(\varepsilon_k(p_\|)+\mu))]\} \qquad (3)$$

is the contribution of the medium ($\beta=1/T$, $p_\|$ denotes the projection of the momentum on the direction of the magnetic field, $\varepsilon_k(p_\|)=\sqrt{m^2+2eBk+p_\|^2}$, $b_k\equiv 2-\delta_{n,0}$), and

$$\mathscr{L}^{\rm eff}(B)=-\frac{1}{8\pi^2}\int_0^\infty\frac{ds}{s^3}\left[eBs\coth(eBs)-1-\frac{1}{3}(eBs)^2\right]\exp(-m^2s) \qquad (4)$$

is the effective Heisenberg–Euler Lagrangian.[18]

Integrating expression (3) by parts, we have

$$\tilde{\mathscr{L}}^{\rm eff}(B,\mu,T)=\frac{eB}{(2\pi)^2}\sum_{n=0}^{\infty}b_n\int_{-\infty}^{\infty}dp_\|\frac{p_\|^2}{\varepsilon_n(p_\|)}(f_+(T)+f_-(T)), \qquad (5)$$

where $f_\pm(T)$ denotes the Fermi distribution:

$$f_\pm(T)=\frac{1}{1+e^{\beta(\varepsilon\mp\mu)}}. \qquad (6)$$

Using representation (as $T\rightarrow 0$ the Fermi distribution goes over to the Heaviside step function, $\lim_{T\rightarrow 0}f_\pm=\theta(\pm\mu-\varepsilon)$, and expression (5) takes the form[15]

$$\tilde{\mathscr{L}}^{\rm eff}(T=0,B,\mu)=\frac{eB}{(2\pi)^2}\sum_{n=0}^{\left[\frac{\mu^2-m^2}{2eB}\right]}b_n\left\{\mu\sqrt{\mu^2-m^2-2eBn}-(m^2+2eBn)\ln\left(\frac{\mu+\sqrt{\mu^2-m^2-2eBn}}{\sqrt{m^2+2eBn}}\right)\right\}, \qquad (7)$$

where [. . .] denotes the integer part. In order to obtain the low-temperature expansion, we make a substitution of variables and again integrate by parts:

$$\tilde{\mathscr{L}}^{\rm eff}(T,B,\mu)=\frac{eB}{(2\pi)^2}\sum_{n=0}^{\infty}b_n\left\{[\varepsilon\sqrt{\varepsilon^2-m^2-2eBn}-(m^2+2eBn)\ln(\varepsilon+\sqrt{\varepsilon^2-m^2-2eBn})]\times(f_++f_-)\Big|_{\sqrt{m^2+2eBn}}^{\infty}-\int_{\sqrt{m^2+2eBn}}^{\infty}d\varepsilon[\varepsilon\sqrt{\varepsilon^2-m^2-2eBn}-(m^2+2eBn)\ln(\varepsilon+\sqrt{\varepsilon^2-m^2-2eBn})]\left(\frac{\partial f_+}{\partial\varepsilon}+\frac{\partial f_-}{\partial\varepsilon}\right)\right.. \qquad (8)$$

At low temperatures the contributions from the terms with $\partial f_-/\partial\varepsilon$ and f_- and from the lower limit in the first term are exponentially small ($\mu>0$). Carrying out the substitution of variables and rewriting the derivative of the Fermi distribution in the form

$$\frac{\partial f_+}{\partial\varepsilon}=-\frac{1}{4T}\cosh^{-2}\left(\frac{\varepsilon-\mu}{2T}\right),$$

we obtain

$$\tilde{\mathscr{L}}^{\rm eff}(T,B,\mu)=\frac{eB}{(2\pi)^2}\sum_{n=0}^{\infty}b_n\left\{-(m^2+2eBn)\times\ln\sqrt{m^2+2eBn}\,\theta(\mu-\sqrt{m^2+2eBn})+\int_{\sqrt{m^2+2eBn}-\mu}^{\infty}dq\Big[(\mu+q)\sqrt{(\mu+q)^2-m^2-2eBn}-(m^2+2eBn)\ln(\mu+q)+(\sqrt{(\mu+q)^2-m^2-2eBn})\Big]\cosh^{-2}\left(\frac{q}{2T}\right)\right\}. \qquad (9)$$

In the zero-temperature limit, the derivative of the Fermi distribution goes over to a δ-function and in the limit $T\rightarrow 0$ we return to expression (7). In order to obtain the low-temperature correction, we extend the lower integration limit in Eq. (9) to $-\infty$ (at low T the function $(1/4T)\cosh^{-2}(q/2T)$ decays exponentially as one moves away from the point $q=0$). Next, expanding the expression in brackets about the point $q=0$, we obtain the low-temperature correction to the Lagrangian (7):[16]

$$\Delta \mathscr{L}^{\rm eff}(T,B,\mu)=\frac{eBT^2}{6}\sum_{n=0}^{\left[\frac{\mu^2-m^2}{2eB}\right]} b_n\frac{\mu}{(\mu^2-m^2-2eBb)^{1/2}}+O(T^4). \quad (10)$$

Thus, in the low-temperature limit only the excited levels (i.e., those whose edges lie below the Fermi level) contribute to the effective Lagrangian.

The above expansion is valid for

$$\frac{T}{\mu-\sqrt{m^2+2eBn}}\ll 1, \quad (11)$$

i.e., until the distance from the edge of any Landau level $\varepsilon_k(p_\parallel=0)=\sqrt{m^2+2eBk}$ to the Fermi surface μ is much greater than the temperature.

3. FERMION DENSITY AND THE POLARIZATION OPERATOR

Having a simple representation for the low-temperature corrections to the effective Lagrangian at our disposal, we can also calculate the fermion density, magnetization, Hall effect, and certain components of the polarization operator in the static limit $p_0=0$, $\mathbf{p}\to\mathbf{0}$.

Using expressions (7) and (10) and the definitions of the fermion density and the magnetization $\rho=\partial\mathscr{L}^{\rm eff}/\partial B$, we obtain for the density

$$\rho(B,\mu,T)=\frac{eB}{2\pi^2}\sum_{n=0}^{\left[\frac{\mu^2-m^2}{2eB}\right]} b_n\sqrt{\mu^2-m^2-2eBn}\left\{1-\frac{T^2\pi^2}{6}\frac{m^2+2eBn}{(\mu^2-m^2-2eBn)^2}\right\}+O(T^4) \quad (12)$$

and for the plasma part of the magnetization $\tilde{M}$

$$\tilde{M}(B,\mu,T)=\frac{e}{4\pi^2}\sum_{n=0}^{\left[\frac{\mu^2-m^2}{2eB}\right]} b_n\left\{\mu\sqrt{\mu^2-m^2-2eBn}-(m^2+4eBn)\ln\left(\frac{\mu+\sqrt{\mu^2-m^2-2eBn}}{\sqrt{m^2+2eBn}}\right)+\frac{\pi^2T^2\mu(\mu^2-m^2-eBn)}{(\mu^2-m^2-2eBb)^{3/2}}\right\}+O(T^4). \quad (13)$$

The temperature corrections diminish the fermion density (at fixed chemical potential, increasing the temperature leads to "evaporation" of the electron (positron) gas, and raises the minima of the oscillating magnetization (compare the de Haas–van Alphen effect at zero and finite temperature.)[14,15]

Below we calculate the five components of the one-loop polarization operator in the static limit $p_0=0$, $\mathbf{p}\to\mathbf{0}$. The component Π_{00} in the static limit can be expressed as the derivative of the density with respect to the chemical potential:[19]

$$\Pi_{00}(p_0=0,\mathbf{p}\to 0)=e^2\frac{\partial\rho}{\partial\mu},$$

$$\Pi_{00}(p_0=0,\mathbf{p}\to 0)=e^2\frac{eB\mu}{2\pi^2}\sum_{n=0}^{\left[\frac{\mu^2-m^2}{2eB}\right]} b_n\left\{(\mu^2-m^2-2eBn)^{-1/2}+T^2\pi^2\frac{m^2+2eBn}{(\mu^2-m^2-2eBn)^{5/2}}\right\}. \quad (14)$$

For $B=0$ the component $\Pi_{00}(p_0=0,\mathbf{p}\to\mathbf{0}$ completely determines the screening of static charge at large distances (the Debye radius) $r_D^{-2}=\Pi_{00}(p_0=0,\mathbf{p}\to\mathbf{0})$ (Ref. 19), but for $\mu,B\neq 0$ this is no longer the case. The tensor structure of the polarization operator in this case is much more complicated (in this connection, consider the example from QED $_{2+1}$ (Ref. 20)), and the scalar coefficients are functions of two variables: $p_\perp^2$ (the projection of the momentum in the direction of the field) and $p_\parallel^2$ (the projection of the momentum perpendicular to the field). The question of screening will be considered in detail in Sec. 5.

The components $\Pi_{01}=\Pi_{10}^*$ and $\Pi_{02}=\Pi_{20}^*$ in the static limit can be expressed in terms of the derivative of the fermion density with respect to the magnetic field strength:

$$\Pi_{0j}(p\to 0)=ie\varepsilon_{ij}p_i\frac{\partial\rho}{\partial B},\quad i,j=1,2, \quad (15)$$

This follows from the definition of the polarization operator

$$\Pi_{\mu\nu}(x,x')=i\frac{\delta\langle j_\mu(x)\rangle}{\delta A_\nu(x')}. \quad (16)$$

Having calculated the components $\Pi_{01}=\Pi_{10}^*$ and $\Pi_{02}=\Pi_{20}^*$, we can solve the more general problem of determining one of the scalar coefficients in the expansion of the polarization operator in tensor structures.[16,21] As we have already noted, for $T,\mu\neq 0$ the polarization operator in QED$_{3+1}$ with an external magnetic field can be expanded in terms of six transverse tensor structures[10] (we use a somewhat modified representation):

$$\begin{aligned}\Pi_{\mu\nu}(p|T,\mu,B)=&\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\mathscr{A}+\left(\frac{p_\mu p_\nu}{p^2}-\frac{p_\mu u_\nu+u_\mu p_\nu}{(pu)}+\frac{u_\mu u_\nu}{(pu)^2}p^2\right)\mathscr{B}\\&+F_{\mu\alpha}F^{\alpha\beta}p_\beta F_{\nu\phi}F^{\phi\rho}p_\rho\mathscr{C}\\&+F_{\mu\lambda}p^\lambda F_{\nu\phi}p^\phi\mathscr{D}+i(p_\mu F_{\nu\lambda}p^\lambda\\&-p_\nu F_{\mu\lambda}p^\lambda+p^2F_{\mu\nu})\mathscr{E}\\&+i(u_\mu F_{\nu\lambda}p^\lambda-u_\nu F_{\mu\lambda}p^\lambda\\&+(pu)F_{\mu\nu})\mathscr{F}.\end{aligned} \quad (17)$$

The scalars $\mathscr{A}$, $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$, $\mathscr{E}$, and $\mathscr{F}$ are functions of p_0^2, $p_3^2=p_\parallel^2$, $p_1^2+p_2^2=p_\perp^2$, and B (u^μ is the 4-velocity of the medium,[19] $u^\mu=(1,0,0,0)$), and, consequently, the last term in expansion (17) makes the leading contribution to Π_{0j} and we can determine the coefficient $\mathcal{F}$ in the static limit:[2)]

$$\mathcal{F}(p_0=0, \mathbf{p}\to 0)=\frac{e}{B}\frac{\partial\rho}{\partial B}. \tag{18}$$

It is easy to verify that the components Π_{0j} describe the conductivity in the plane perpendicular to the magnetic field, which in our case (a nondissipative medium) has a purely Hall-like character:[16]

$$\sigma_{ij}=\left.\frac{\partial j_i}{\partial E_j}\right|_{E\to 0}=i\left.\frac{\partial \Pi_{0i}(p)}{\partial p_j}\right|_{p\to 0}=e\varepsilon_{ij}\frac{\partial\rho}{\partial B}, \quad i,j=1,2. \tag{19}$$

Substituting the function describing the fermion density into expression (19), we have

$$\Pi_{0j}(p_0, \mathbf{p}\to 0)$$
$$=\frac{ie^2\varepsilon_{ij}p_i}{2\pi^2}\sum_{n=0}^{\left[\frac{\mu^0-m^2}{2eB}\right]} b_n\left\{\frac{\mu^2-m^2-3eBn}{(\mu^2-m^2-2eBn)^{1/2}}\right.$$
$$\left.-\frac{T^2\pi^2}{3}\frac{(\mu^2-m^2-2eBn)(m^2+2eBn)+3eBn\mu^2}{(\mu^2-m^2-2eBn)^{5/2}}\right\}. \tag{20}$$

From the foregoing expression it is clear that the Hall conductivity in QED_{3+1} is an oscillating function of the chemical potential and magnetic field strength, and in the limit $T\to\infty$ has an inverse-square-root singularity (recall, in this regard, that the polarization operator in QED_{3+1} with an external magnetic field has exactly the same kind of singularities at the thresholds of pair-formation;[11] also compare with QED_{2+1}, Ref. 21). The above-mentioned oscillations are of the same nature as the "giant oscillations" of solid-state physics[22] and resonance effects in QED[3,6,7] and semiconductors.[23,24] In the following section we will calculate the temperature corrections at the points $\mu\to\sqrt{m^2+2eBk}$ and show that the singularities disappear.

4. LOW-TEMPERATURE EXPANSION IN THE LIMIT $\mu\to(M^2+2eBk)^{1/2}$

In this section we show how the approach developed in Sec. 2. can be extended to the case $\mu\to(m^2+2eBk)^{1/2}$. At $T=0$, not all the functions represented in the foregoing section are smooth at this point, and not even the components of the polarization operator have a continuous limit. By way of an example, let us consider the Hall conductivity $e\partial\rho/\partial B$.

Differentiating the effective Lagrangian (3) with respect to the chemical potential in order to obtain an expression for the fermion density at arbitrary temperature, and then calculating the derivative with respect to the magnetic field strength B (below we assume that the temperature is low, so that the part that depends on f_- can be dropped), we write the conductivity σ in the form

$$\sigma=\sigma_{(1)}+\sigma_{(2)}=\frac{e}{(2\pi)^2}\sum_{k=0}^{\infty} b_k\int dp_\parallel f_+ + \frac{eB}{(2\pi)^2}\sum_{k=0}^{\infty} b_k\int dp_\parallel\frac{\partial f_+}{\partial\varepsilon_k}\frac{\partial\varepsilon_k}{\partial B}. \tag{21}$$

First consider the term $\sigma_{(2)}$. Transforming variables, we obtain

$$\sigma_{(2)}=\frac{e^2B}{\pi^2}\sum_{k=1}^{\infty} b_k ek\int_{\sqrt{m^2+2eBk}-\mu}^{\infty} dz \times\frac{1}{\sqrt{(z+\mu)^2-m^2-2eBk}}\left(-\frac{1}{4T}\right)\cosh^{-2}\left(\frac{z}{2T}\right). \tag{22}$$

Let $\mu\to(m^2+2eBk)^{1/2}$. We isolate the term in the sum (22) corresponding to k_0 (the contribution of the other terms in $\sigma_{(2)}$ can be calculated by the method described in Sec. 2.):

$$\lim_{\mu\to\sqrt{m^2+2eBk_0}}\sigma_{(2)}^{k_0}=-\frac{eB}{4\pi^2T}ek_0\int_0^\infty dz\frac{z^{-1/2}}{\sqrt{z+2\mu}} \times\cosh^{-2}\left(\frac{z}{2T}\right). \tag{23}$$

Expanding the square root in (23) at $z=0$, we finally obtain

$$\sigma_{(2)}^{k_0}=-\frac{e^2(2^{3/2}-1)\zeta(3/2)}{8\pi^{5/2}}\frac{\mu^2-m^2}{\sqrt{2T\mu}}\propto\frac{1}{\sqrt{T}}. \tag{24}$$

Calculating $\sigma_{(1)}$ in an analogous way, we can verify that $\sigma_{(1)}\propto\sqrt{T}$. Thus, at finite temperature, the inverse-square-root singularity in the expression for the Hall conductivity (as well as for the components of the polarization operator calculated above) disappears. In the low-temperature limit the conductivity is everywhere finite, and in the limit $\mu\to(m^2+2eBk)^{1/2}$ the leading contribution is proportional to $T^{-1/2}$.

This same procedure can be applied to the magnetization and the fermion density. Here one can easily verify that the temperature, as expected, smooths the corresponding functions (cf. Refs. 14 and 15).

5. DEBYE SCREENING AND THE EFFECTIVE LAGRANGIAN

An important aspect of temperature-dependent field theory (in particular, non-Abelian field theory)[25,26] is the question of static screening. To treat this question, it is necessary to know the Green's function of the gauge field $\mathcal{D}_{\mu\nu}$ with corresponding quantum corrections:

$$\mathcal{D}_{\mu\nu}=(D_{\mu\nu}^{-1}-\Pi_{\mu\nu})^{-1} \tag{25}$$

(here $D_{\mu\nu}$ is the bare photon propagator).

Formally, nothing stands in the way of calculating $\mathcal{D}_{\mu\nu}$ by inverting the corresponding matrix, but in our case, in which the polarization operator $\Pi_{\mu\nu}$ has an extremely complex form (17), this is in general impossible in a practical sense.

The propagator $\mathscr{D}_{\mu\nu}$ can be easily obtained if we make use of the diagonal representation of the polarization operator[27] (that is, by solving the eigenvalue problem $\Pi_{\mu\nu}(p)b^{\nu}=\kappa b_{\mu}$, as was done in Ref. 11):

$$\Pi_{\mu\nu}(p)=\sum_{i=1}^{3}\kappa_i\frac{b_{\mu}^{(i)}b_{\nu}^{(i)*}}{b_{\alpha}^{(i)}b^{\alpha(i)*}}. \tag{26}$$

In this case, we at once have

$$\mathscr{D}_{\mu\nu}(p)=\sum_{i=1}^{3}\frac{1}{p^2-\kappa_i}\frac{b_{\mu}^{(i)*}b_{\nu}^{(i)*}}{b_{\alpha}^{(i)}b^{\alpha(i)*}}, \tag{27}$$

but this does not simplify the calculation of specific components of $\mathscr{D}_{\mu\nu}$.

In order to consider the question of Debye screening, it is sufficient to know only $\mathscr{D}_{00}(p_0=0,\mathbf{p})$. In this special case we performed the corresponding calculations in the Feynman gauge, keeping all the terms in the polarization operator (17). We cannot give the complete expression here and we restrict ourselves to the static limit. It turns out that the coefficients $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$, and $\mathscr{E}$, which are sub-leading in $\mathbf{p}^2$ (see also Ref. 17), can be reduced only to a finite renormalization (of order e^2), which doesn't change the qualitative picture, so that the corresponding terms can be dropped. Finally, the component $\mathscr{D}_{00}(p_0,\mathbf{p})$ can be represented in the following form (we have included the magnetic field strength in the definition of $\tilde{\mathscr{F}}$, $\tilde{\mathscr{F}}=B\mathscr{F}$):

$$\mathscr{D}_{00}=-\frac{\mathbf{p}^2}{(\Pi_{00}+\mathbf{p}^2)\mathbf{p}^2+\tilde{\mathscr{F}}^2p_{\perp}^2}. \tag{28}$$

To leading order, $\mathscr{D}_{00}(p_0,\mathbf{p})$ is defined by just those components of the polarization operator that we can obtain from the effective Lagrangian (it is precisely these components that give the leading contribution in $\mathbf{p}$ to $\Pi_{\mu\nu}$ in the static limit).

The expression for the Coulomb potential

$$A_0(\mathbf{x})=\int d\mathbf{p}e^{-i\mathbf{px}}\mathscr{D}_{00}(p_0=0,\mathbf{p})j_0(\mathbf{p}) \tag{29}$$

in the case of a point charge is complicated, and we were not able to calculate it analytically. To demonstrate qualitative effects, it is sufficient to consider "simpler" charge configurations (i.e., those in which a δ-function removes one of the integrals over the 3-momentum in Eq. (29)). We will calculate, for example, the field produced by an infinitely long thin rod parallel to the z axis:

$$A_0(r)=\rho_c\int_0^{\infty}p_{\perp}dp_{\perp}\int_0^{2\pi}d\phi e^{-ip_{\perp}r\cos\phi}\frac{1}{\Pi_{00}+\tilde{\mathscr{F}}^2+p_{\perp}^2}$$
$$=\rho_cK_0(r\sqrt{\Pi_{00}+\tilde{\mathscr{F}}^2}), \tag{30}$$

where K_0 is the modified Bessel function of the second kind, $\lim_{r\to\infty}K_0(r)\sim\sqrt{\pi/2r}e^{-r}$ [see the discussion of screening in QED_{2+1} (Ref. 28)].

Thus, in a strong magnetic field, Debye screening is described to leading order by two scalar functions, and the assumption made in Ref. 17 that only the Π_{00} component is responsible for screening at $B,\mu\neq 0$, is invalid.

Let us now consider the potential of two charged planes, parallel and perpendicular to the magnetic field:

$$A_0(x_1)=\rho_c\int dp_1\frac{e^{-ip_1x_1}}{\Pi_{00}+\tilde{\mathscr{F}}^2+p_1^2}$$
$$=\frac{\pi\rho_c}{\sqrt{\Pi_{00}+\tilde{\mathscr{F}}^2}}e^{-x_1\sqrt{\Pi_{00}+\tilde{\mathscr{F}}^2}}, \tag{31}$$

$$A_0(z)=\rho_c\int dp_{\parallel}\frac{e^{-ip_{\parallel}z}}{\Pi_{00}+p_{\parallel}^2}=\frac{\pi\rho_c}{\sqrt{\Pi_{00}}}e^{-z\sqrt{\Pi_{00}}}. \tag{32}$$

It follows from these expressions that Debye screening in the presence of a magnetic field is anisotropic: $r_D^{\parallel}\neq r_D^{\perp}$ (it was shown in Ref. 29 for high temperature and $\mu=0$ that at small distances screening is anisotropic). In dealing with anisotropy, it is necessary to allow for the fact that for realistic parameter values the quantitative effect is small (formally, near the edge of a Landau level Π_{00} and $\tilde{\mathscr{F}}^2$ can be comparable in order of magnitude, but in this case the screening radius is much smaller than the mean distance between the particles and the Debye approximation is therefore invalid). In the examples above, anisotropy results from the appearance of an additional term in the polarization operator, and not from the fact that the squared momenta $p_{\perp}^2$ and $p_{\parallel}^2$ enter into the scalar functions in a different way (as compared with the resonant deflection of electromagnetic waves by a magnetic field[3,4,23,24]).

Assuming that the coefficients $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$, and $\mathscr{E}$, which are sub-leading in $\mathbf{p}$, have no qualitative effect on other components of $\mathscr{D}_{\mu\nu}$, we can rewrite the latter as

$$\mathscr{D}_{\mu\nu}^{-1}=\begin{pmatrix}-(\Pi_{00}+\mathbf{p}^2) & i\tilde{\mathscr{F}}p_2 & -i\tilde{\mathscr{F}}p_1 & 0\\ -i\tilde{\mathscr{F}}p_2 & \mathbf{p}^2 & 0 & 0\\ i\tilde{\mathscr{F}}p_1 & 0 & \mathbf{p}^2 & 0\\ 0 & 0 & 0 & \mathbf{p}^2\end{pmatrix}. \tag{33}$$

It is easy to verify that the antisymmetric and linear (in the momentum) structure in the polarization operator in QED_{3+1}, in contrast to the Chern–Simons term in QED $_{2+1}$ (Ref. 30), does not lead to magnetic screening (we emphasize that when the polarization operator has the form (17), the condition $\Pi_{ii}=0$ (Ref. 25) no longer guarantees that the magnetic mass will be equal to zero).

In the present paper we have demonstrated that it is possible in $(3+1)$-dimensional QED with an external magnetic field to construct a low-temperature expansion of the effective action, where both the effective Lagrangian and the derivative quantities have the same structure as in the limit $T\to 0$, that is, they are finite sums over the Landau excited levels. Using the expressions, we calculated the components of the polarization operator corresponding to the hall conductivity and Debye screening in QED_{3+1} with a finite fermion density in an external magnetic field. We calculated the potential created by some charge configurations and demonstrated the anisotropy of Debye screening in an electron gas in the presence of an external magnetic field.

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[1)] The present paper uses the same notation as in Refs. 15 and 16. We assume that the magnetic field is directed along the z axis, that $F_{12}=-F_{21}$, and that $\mu, eB>0$.

[2)] $\mathscr{C}$, $\mathscr{D}$, $\mathscr{E}$, $\mathscr{F}\equiv 0$ for $B=0$, T and/or $\mu \neq 0$; $\mathscr{E},\mathscr{F}\equiv 0$ for $B,T \neq 0$; $\mu=0$; $\mathscr{B},\mathscr{E},\mathscr{F}\equiv 0$ for $T,\mu=0$, $B \neq 0$; this is explained in detail in Ref. 4.

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