Theory of the galvanomagnetic properties of two-dimensional two-component systems

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It is shown in the isotropic case that the transition to a complex representation makes it possible to impart a fairly simple and geometrically visualizable form to the Dykhne method [A. M. Dykhne, Sov. Phys. JETP **32**, 348 (1971)]. Under such an approach Dykhne's symmetry transformation [A. M. Dykhne, Sov. Phys. JETP **32**, 348 (1971)] corresponds to a piecewise linear mapping in the complex conductivity plane. The mappings to which the concrete transformations used in several papers [A. M. Dykhne, Sov. Phys. JETP **32**, 348 (1971); B. I. Shklovskiĭ, Sov. Phys. JETP **45**, 152 (1977); B. Ya. Balagurov, Sov. Phys. Solid State **20**, 1922 (1978); B. Ya. Balagurov, Sov. Phys. JETP **54**, 355 (1981); A. M. Dykhne and I. M. Ruzin, Phys. Rev. B **50**, 2369 (1994); B. Ya. Balagurov, Sov. Phys. JETP **55**, 774 (1982); B. Ya. Balagurov, Sov. Phys. JETP **58**, 331 (1983)] correspond are determined. Linear and quadratic effective characteristics, viz., the mean values of the electric field strength and its square in each of the components, are also considered. General expressions which are valid for an arbitrary concentration and in any magnetic field **H** are found for these parameters. The behavior of the quadratic characteristics (as a function of **H**) in the critical region is investigated for systems with a metal–insulator phase transition.

1. INTRODUCTION

The galvanomagnetic properties of two-dimensional two-component systems have been examined in numerous studies, among which we note the following. Dykhne[1] (see also Refs. 2–4) found the effective conductivity tensor $\hat{\sigma}_e$ in an arbitrary magnetic field **H** in an explicit form for the special case of equal component concentrations (a randomly inhomogeneous system with the critical composition). One of the important consequences of the results in Ref. 1 is the prediction of the existence of an anomalous conductivity. Two other effects[1] are directly related to this phenomenon: saturation of the effective Hall parameter and a linear ($\propto H$) increase in the field and current fluctuations as $H\rightarrow\infty$. A general expression relating the diagonal (ohmic) component σ_{xe} and the Hall component σ_{ae} of $\hat{\sigma}_e$, which is valid for arbitrary concentrations and any **H**, was also obtained in Ref. 1. The use of this relation made it possible for Shklovskiĭ[2] (see also Ref. 3) to find the behavior of the effective Hall coefficient in a weak magnetic field over the entire range of variation of the concentration. Dykhne's general relation has also found application in the theory of the quantum Hall effect (see Ref. 5).

The isomorphism (mutual one-to-one correspondence) of the problems of the galvanomagnetic properties (in any **H**) and conductivity (when **H**$=0$) of arbitrary two-dimensional two-component systems was established in Refs. 6 and 7. The relations found in 6 and 7 made it possible to express the components of $\hat{\sigma}_e$ in terms of the galvanomagnetic characteristics of the components and in terms of the dimensionless effective electrical conductivity of the system f (when **H**$=0$). If f as a function of two arguments (the concentration p and the ratio between the conductivities of the components $h=\sigma_2/\sigma_1$) is known over their entire range of variation for a certain isotropic system, the expressions for σ_{xe} and σ_{ae} obtained in Refs. 6 and 7 give a complete solution of the problem of the galvanomagnetic properties of such a two-dimensional system. For example, the form of the function $f=f(p,h)$ is known in the context of the similarity hypothesis;[8,9] this makes it possible to investigate the galvanomagnetic properties of two-dimensional systems with a metal–insulator phase transition over the entire critical region (see Refs. 6 and 7). Naturally, all the basic results in Refs. 1–4 can be obtained as consequences of the general expressions for σ_{xe} and σ_{xe} found in Refs. 6 and 7.

It should be noted, however, that the methods employed in Refs. 1–7 have a highly formal character. This circumstance together with the fairly cumbersome mathematical operations make it difficult to understand the transformations which can be performed. Therefore, there is an urgent need to simplify the presentation of the basic method (the symmetry transformation) in Refs. 1–7 and to make it as visualizable as is possible. In addition, some other effective characteristics, for example, the mean square of the electric field strength (and current) in each of the components, were not considered in Refs. 1–7. At the same time, as was shown in Refs. 10–12, the study of mean-square values provides useful information on the properties of a system already when **H**$=0$. Thus, the investigation of these and some other effective characteristics in the presence of a magnetic field would certainly be of interest.

In this paper it is shown (for isotropic systems) that the symmetry-transformation method[1–7] takes on a fairly simple and geometrically visualizable form when we move over to the complex representation. The relationship between the complex fields in the original and "primed" systems takes on the form of a linear scalar relation. At the same time, the corresponding relationship for the conductivity is given by a

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piecewise linear transformation in the complex plane. Under such an approach two points ζ_1 and ζ_2 on the complex plane of the conductivity ζ correspond to a two-component system. In this case Dykhne's transformation[1] corresponds to a piecewise linear mapping of ζ_1 and ζ_2 to two other points ζ_1' and ζ_2', which correspond to the primed system. Selection of the mutual or transposed system as the latter (see Sec. 2) leads to the relation obtained by Dykhne[1] (see also Refs. 2–7). If the null system (with a scalar conductivity, see Sec. 3) is taken as the primed system, we arrive at the isomorphism relations found in Refs. 6 and 7.

Several questions not previously discussed are also considered in this paper. For example, the linear effective characteristics of fields and currents, which are similar to those studied in Ref. 10 when $\mathbf{H}=0$, are investigated. In particular, both the absolute values and the directions of some vector quantities, viz., the mean values of the electric field strength $\mathbf{E}$ in each of the components, are found in explicit form for a system with the critical composition ($p=1/2$). However, the main emphasis in this paper is placed on the study of the effective characteristic ψ_i, i.e., the mean value of the square of the field strength $\mathbf{E}$ in the ith component. General expressions, which are valid at arbitrary concentrations in any $\mathbf{H}$, are found for ψ_i ($i=1$, 2), and their explicit form is determined for $p=1/2$. The dependence of ψ_i on $\mathbf{H}$ in the critical region is ascertained for systems with a metal–insulator phase transition. Such an investigation, together with the results of the study of the critical behavior of the conductivity (see Ref. 6), makes it possible, in particular, to ascertain the region where the effects predicted by Dykhne[1] exist. One more quadratic effective characteristic, which describes the distribution of the Joule heat evolved in a sample among the components when a current flows through it, is considered in this paper.

2. SYMMETRY TRANSFORMATION

1. We consider an isotropic inhomogeneous two-dimensional system in a transverse magnetic field $\mathbf{H}$. In this case Ohm's law has the form

$$\mathbf{j}=\hat{\sigma}\mathbf{E}, \quad \hat{\sigma}=\begin{pmatrix} \sigma_x & \sigma_a \\ -\sigma_a & \sigma_x \end{pmatrix}, \tag{1}$$

where σ_x is the ohmic component and σ_a is the Hall component of the conductivity tensor $\hat{\sigma}(\mathbf{r})$, $\mathbf{E}$ is the electric field strength, and $\mathbf{j}$ is the current density. As Dykhne noted,[1] in the two-dimensional case the transformation of the field $\mathbf{E}$ and the current $\mathbf{j}$ to the "primed" system

$$\mathbf{E}=a\{\mathbf{E}'+b\hat{v}\mathbf{j}'\}, \quad \mathbf{j}=a\{c\mathbf{j}'+d\cdot\hat{v}\mathbf{E}'\}, \tag{2}$$

$$\hat{v}=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{3}$$

with the coordinate-independent coefficients a, b, c, and d leaves the equations of the constant current unchanged. The conductivity tensors in the original $[\hat{\sigma}(r)]$ and primed $[\hat{\sigma}'(r)]$ systems are related by the expression

$$\hat{\sigma}(\mathbf{r})=\{c\hat{\sigma}'(\mathbf{r})+d\hat{v}\}\{1+b\hat{v}\hat{\sigma}'(\mathbf{r})\}^{-1}. \tag{4}$$

The effective conductivity tensors $\hat{\sigma}_e$ and $\hat{\sigma}_e'$ are related to one another by a similar expression.

In the isotropic case under consideration it is convenient to introduce the complex quantities

$$E=E_x-iE_y, \quad j=j_x-ij_y. \tag{5}$$

Then the analog of the transformation (2) is

$$E=a\{E'-ibj'\}, \quad j=a\{cj'-idE'\}. \tag{6}$$

We write Ohm's law in its complex representation in the form

$$j=-i\zeta E, \tag{7}$$

$$\zeta=-\sigma_a+i\sigma_x, \tag{8}$$

so that instead of (4) we obtain

$$\zeta=\frac{c\zeta'+d}{1-b\zeta'}. \tag{9}$$

Thus, under the symmetry transformation defined by (2) and (6) the complex "conductivities" of the original (ζ) and primed (ζ') systems are related to one another by the equality (9). This relation can be regarded as a piecewise linear mapping of the complex variable ζ with the real coefficients b, c, and d. Different values can be assigned to the three real parameters (conductivities) of the primed system by appropriately adjusting these coefficients.

The preceding arguments are valid for two-dimensional isotropic systems with an arbitrary dependence of $\hat{\sigma}(\mathbf{r})$ on the coordinates. Let us now consider a two-component medium, i.e., a system for which the conductivity tensor $\hat{\sigma}(\mathbf{r})$ takes the constant values $\hat{\sigma}_1$ and $\hat{\sigma}_2$ in the first and second components, respectively. In this case the complex conductivity $\zeta_1=-\sigma_{a1}+i\sigma_{x1}$ corresponds to the first component, and $\zeta_2=-\sigma_{a2}+i\sigma_{x2}$ corresponds to the second component. Therefore, in the complex ζ plane the isotropic two-component system is represented by the two points ζ_1 and ζ_2.

2. Let the primed system differ from the original system by an interchange of the conductivities of the components: $\hat{\sigma}_1'=\hat{\sigma}_2$ and $\hat{\sigma}_2'=\hat{\sigma}_1$. These two tensor equalities lead to four conditions ($\sigma_{x1}'=\sigma_{x2}$, $\sigma_{a1}'=\sigma_{a2}$, $\sigma_{x2}'=\sigma_{x1}$, and $\sigma_2'=\sigma_1$) for the three coefficients b, c, and d in the transformation (9), so that the system of equations is redefined. However, the corresponding system of equations can be solved, if the sign of the magnetic field is also reversed in the primed system (see Ref. 1). Thus, as the mutual system we take the primed system for which

$$\hat{\sigma}_1'=\hat{\sigma}_2^T, \quad \hat{\sigma}_2'=\hat{\sigma}_1^T, \tag{10}$$

where the superscript T denotes transposition. In the complex representation the relations (10) correspond to the equalities $\zeta_1'=-\zeta_2^*$ and $\zeta_2'=-\zeta_1^*$, where the asterisk denotes complex conjugation. A geometric interpretation of the transformation from the original system to the mutual system is given in Fig. 1.

Using (9), from the correspondence between the points of the original and mutual systems we find

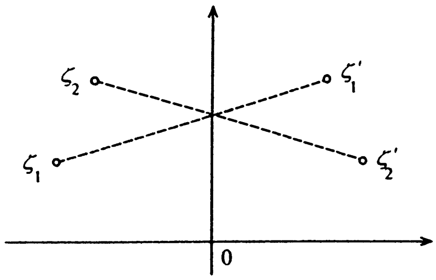

FIG. 1. Transformation to the mutual system: $\zeta_1' = -\zeta_2^*$, $\zeta_2' = -\zeta_1^*$.

$$c' = 1, \quad b' = \frac{\sigma_{x1} - \sigma_{x2}}{\sigma_{x1}\sigma_{a2} - \sigma_{x2}\sigma_{a1}},$$

$$d' = \frac{\sigma_{x2}(\sigma_{x1}^2 + \sigma_{a1}^2) - \sigma_{x1}(\sigma_{x2}^2 + \sigma_{a2}^2)}{\sigma_{x1}\sigma_{a2} - \sigma_{x2}\sigma_{a1}}. \tag{11}$$

(We marked the transformation coefficients with a prime; when comparisons are made with Ref. 4, the difference in notation must be taken into account, i.e., b' and d' must be interchanged). The substitution of (11)_ into the equality relating $\hat{\sigma}_e$ and $\hat{\sigma}_e'$ [of form (4) or (9)] leads to the reciprocity relations (see Ref. 1, as well as Refs. 3 and 4).

We now take the system which we call the transposed system as the primed system. This system differs from the original system only with respect to the sign of the magnetic field (see Ref. 1):

$$\hat{\sigma}_1' = \hat{\sigma}_1^T, \quad \hat{\sigma}_2' = \hat{\sigma}_2^T. \tag{12}$$

In the complex representation the relations (12) correspond to the equalities $\zeta_1' = -\zeta_1^*$ and $\zeta_2' = -\zeta_2^*$. Thus, as is seen from Fig. 2, the transition to the transformed system is not (when $\mathbf{H} \neq 0$) identical. We note that the system of equations for the coefficients can be solved in this case, too. Using (9), from the correspondence between the points we find

$$c'' = 1, \quad b'' = 2\frac{\sigma_{a1} - \sigma_{a2}}{(\sigma_{x1}^2 + \sigma_{a1}^2) - (\sigma_{x2}^2 + \sigma_{a2}^2)},$$

$$d'' = 2\frac{\sigma_{a1}(\sigma_{x2}^2 + \sigma_{a2}^2) - \sigma_{a2}(\sigma_{x1}^2 + \sigma_{a1}^2)}{(\sigma_{x1}^2 + \sigma_{a1}^2) - (\sigma_{x2}^2 + \sigma_{a2}^2)}. \tag{13}$$

(In this case the coefficients are marked with a double prime; when comparisons are made with Ref. 4, the following differences in notation must be taken into account: $b'' \to 2d$, $d'' \to 2b$.) From the equality like (4) or (9) relating the effective characteristics $\hat{\sigma}_e$ and $\hat{\sigma}_e'$ we find the general Dykhne relation (see Ref. 1, as well as Refs. 3 and 4):

$$(\sigma_{xe}^2 + \sigma_{ae}^2 b'' - 2\sigma_{xe} - d'' = 0 \tag{14}$$

with the coefficients b'' and d'' from (13). The results (13) and (14) were also recently published in Ref. 5.

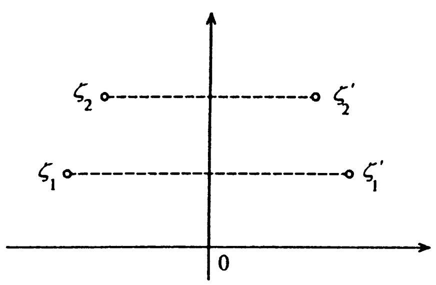

FIG. 2. Transformation to the transposed system: $\zeta_1' = -\zeta_1^*$, $\zeta_2' = -\zeta_2^*$.

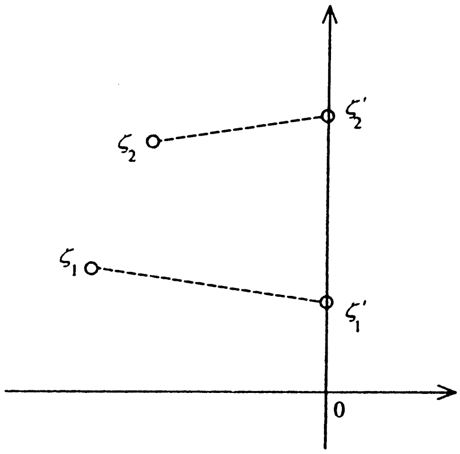

FIG. 3. Transformation to the null system: $\zeta_1' = i\lambda_1$, $\zeta_2' = i\lambda_2$.

3. ISOMORPHISM RELATIONS

We note that the transition to the so-call null system[6,7] is simplest (from the standpoint of solvability of the system of equations for the transformation coefficients). The null system is nearly identical to the original system when $\mathbf{H}=0$, but it has the altered conductivities λ_1 and λ_2 for the components. Therefore, in this case

$$\hat{\sigma}_1' = \lambda_1 \cdot \hat{1}, \quad \hat{\sigma}_2' = \lambda_2 \cdot \hat{1}, \tag{15}$$

where $\hat{1}$ is a diagonal unit matrix. In the complex representation relations (15) correspond to the equalities $\zeta_1' = i\lambda_1$ and $\zeta_2' = i\lambda_2$. The transition to the null system in the complex plane is depicted graphically in Fig. 3.

Using (9), we find the transformation coefficients from the correspondence between the points (compare with Refs. 6 and 7):

$$b = B/\lambda_1, \quad B = (\sigma_{a1} - \sigma_{a2})/(\sigma_{x1} - \lambda\sigma_{x2}),$$

$$c = (\sigma_{x1} + \sigma_{a1}B)/\lambda_1, \quad d = -\sigma_{a1} + \sigma_{x1}B. \tag{16}$$

The parameter $\lambda = \lambda_2/\lambda_1$ is determined from the quadratic equation

$$(\lambda\sigma_{x1} - \sigma_{x2})(\sigma_{x1} - \lambda\sigma_{x2}) + \lambda(\sigma_{a1} - \sigma_{a2})^2 = 0.$$

Let the conductivity of the first component σ_1 in the original system for $\mathbf{H}=0$ be greater than the conductivity of the second component: $\sigma_1 > \sigma_2$. Then, in order for the limiting transition $\lambda \to \sigma_2/\sigma_1$ to occur when $\mathbf{H} \to 0$, the following root should be chosen (see Refs. 6 and 7)

$$\lambda = \frac{1}{4\sigma_{x1}\sigma_{x2}}\{[(\sigma_{x1} + \sigma_{x2})^2 + (\sigma_{a1} - \sigma_{a2})^2]^{1/2}$$

$$-[(\sigma_{x1} - \sigma_{x2})^2 + (\sigma_{a1} - \sigma_{a2})^2]^{1/2}\}^2. \tag{17}$$

The parameter λ_1, as well as the coefficient a, remain undetermined.

We use $\mathbf{E}_i=\mathbf{E}_i(\mathbf{r})$ and $\mathbf{E}_i^0=\mathbf{E}_i^0(\mathbf{r})$ to denote the electric field strength in the ith component ($i=1, 2$) of the original and null systems, respectively. Then from (2), (6), and (16) we find the relationship between $\mathbf{E}_i$ with $\mathbf{E}_i^0$ (compare with Ref. 6):

$$\begin{cases} E_{x1}=a(E_{x1}^0-BE_{y1}^0) \\ E_{y1}=a(BE_{x1}^0+E_{y1}^0) \end{cases} \quad \begin{cases} E_{x2}=a(E_{x2}^0-\lambda BE_{y2}^0) \\ E_{y2}=a(\lambda BE_{x2}^0+E_{y2}^0) \end{cases}. \tag{18}$$

Let the electric field strength (as a function of the coordinates) in the original system be known for $\mathbf{H}=0$ and an arbitrary ratio between the conductivities of the components $h=\sigma_2/\sigma_1$. Then, after the replacement $h\rightarrow\lambda$ with λ from (17), we find the field in the null system, and using (18), we determine the electric field strength $\mathbf{E}(\mathbf{r})$ in the original system for $\mathbf{H}\neq 0$. Relations (2), (4), and (16)–(18) establish the isomorphisms in the problems concerning the conductivity (when $\mathbf{H}=0$) of an isotropic two-dimensional two-component system and its galvanomagnetic properties (see Refs. 6 and 7).

We assume that the effective conductivity σ_e of the original system is known when $\mathbf{H}=0$:

$$\sigma_e=\sigma_e(p;\sigma_1,\sigma_2)\equiv\sigma_1 f(p,h), \quad h=\sigma_2/\sigma_1, \tag{19}$$

where p is the concentration (the fraction of the area occupied) of the first component. The effective conductivity of the null system $\sigma_\lambda=\lambda_1 f(p,\lambda)$ is thereby also known. Then, from the relation like (4) or (9) relating $\hat{\sigma}_e$ and $\hat{\sigma}_e'$ we find the expressions for the components of the effective conductivity tensor (see Refs. 6 and 7):

$$\sigma_{xe}=\sigma_{x1}\sigma_{x2}(1-\lambda^2)f/D, \tag{20}$$

$$\sigma_{ae}=\sigma_{a1}-(\sigma_{a1}-\sigma_{a2})\sigma_{x1}\lambda(1-f^2)/D, \tag{21}$$

where

$$D=\lambda(1-f^2)\sigma_{x1}+(f^2-\lambda^2)\sigma_{x2}. \tag{22}$$

In (20)–(22) $f(p,\lambda)$ should be taken with λ from (17) as the function f.

For a randomly inhomogeneous two-component two-dimensional system with the critical composition ($p=1/2$) we have[10]

$$f(1/2,\lambda)=\lambda^{1/2}. \tag{23}$$

In this case it follows from (20)–(22) for $p=1/2$ that[1] (see also Refs. 3, 4, and 6)

$$\sigma_{xe}=(\sigma_{x1}\sigma_{x2})^{1/2}\left[1+\left(\frac{\sigma_{a1}-\sigma_{a2}}{\sigma_{x1}+\sigma_{x2}}\right)^2\right]^{1/2},$$

$$\sigma_{ae}=\frac{\sigma_{a1}\sigma_{x2}+\sigma_{a2}\sigma_{x1}}{\sigma_{x1}+\sigma_{x2}} \tag{24}$$

one of the most interesting consequences of (24) is the modification of the dependence of σ_{xe} on $\mathbf{H}$ in a strong magnetic field (the anomalous conductivity[1]). It is not difficult to see that if $\sigma_{xi}\propto H^{-2}$ and $\sigma_{ai}\propto H^{-1}$ as $H\rightarrow\infty$, we would have $\sigma_{xe}\propto H^{-1}$ in a strong magnetic field. As was noted in Ref. 6 (see also Sec. 6 of this paper), when $p\neq 1/2$, the anomalous conductivity can exist only in a narrow vicinity of the critical concentration and in a finite range of magnetic fields. In this case, the "normal" asymptote $\sigma_{xe}\propto H^{-2}$ is restored at sufficiently large $\mathbf{H}$.

4. LINEAR EFFECTIVE CHARACTERISTICS

Let us find the mean values of the electric field strength in each of the components $\langle\mathbf{E}\rangle^{(1)}$ and $\langle\mathbf{E}\rangle^{(2)}$. Here

$$\langle(\ldots)\rangle^{(i)}=\frac{1}{V}\int_{V_i}(\ldots)d\mathbf{r} \tag{25}$$

is the mean over the volume of the ith component V_i; the mean over the entire volume V is given by the sum of the $\langle(\ldots)\rangle^{(i)}$. (In the two-dimensional case V and V_i should be construed as the corresponding areas.) According to these definitions we have the identity

$$\langle\mathbf{E}\rangle^{(1)}+\langle\mathbf{E}\rangle^{(2)}=\langle\mathbf{E}\rangle. \tag{26}$$

Next, the effective conductivity tensor is determined using the relation $\langle\mathbf{j}\rangle=\hat{\sigma}_e\langle\mathbf{E}\rangle$, whence follows the equality

$$\hat{\sigma}_1\langle\mathbf{E}\rangle^{(1)}+\hat{\sigma}_2\langle\mathbf{E}\rangle^{(2)}=\hat{\sigma}_e\langle\mathbf{E}\rangle. \tag{27}$$

From (26) and (27) we find the expressions sought

$$\langle\mathbf{E}\rangle^{(1)}=(\sigma_1-\sigma_2)^{-1}(\hat{\sigma}_e-\hat{\sigma}_2)\langle\mathbf{E}\rangle,$$

$$\langle\mathbf{E}\rangle^{(2)}=(\hat{\sigma}_1-\hat{\sigma}_2)^{-1}(\hat{\sigma}_1-\hat{\sigma}_e)\langle\mathbf{E}\rangle. \tag{28}$$

Relations (28), which are valid in both the two-dimensional and three-dimensional cases, transform into the equations obtained in Ref. 11 when $\mathbf{H}\rightarrow 0$.

In the two-dimensional case the expressions (28) take an especially simple form in the complex representation. If the complex electric field strength E as defined in (5) is introduced, instead of (28) we obtain

$$\langle E\rangle^{(1)}=\frac{\zeta_e-\zeta_2}{\zeta_1-\zeta_2}\langle E\rangle, \quad \langle E\rangle^{(2)}=\frac{\zeta_1-\zeta_e}{\zeta_1-\zeta_2}\langle E\rangle. \tag{29}$$

The complex conductivity ζ is defined in (7). The expressions (20)–(22) should be taken for the components of $\hat{\sigma}_e$ in Eqs. (28) and (29) in the two-dimensional case under consideration.

Calculating $(\langle\mathbf{E}\rangle^{(i)})^2=\langle E\rangle^{(i)}\langle E^+\rangle^{(i)}$ for a two-dimensional system with the critical composition ($p=1/2$), from (29) and (24) we find

$$(\langle\mathbf{E}\rangle^{(1)})^2=\frac{\sigma_{xe}\sigma_{x2}}{\sigma_{xe}(\sigma_{x1}+\sigma_{x2})+2\sigma_{x1}\sigma_{x2})}(\langle\mathbf{E}\rangle)^2,$$

$$(\langle\mathbf{E}\rangle^{(2)})^2=\frac{\sigma_{x1}}{\sigma_{x2}}(\langle\mathbf{E}\rangle^{(1)})^2. \tag{30}$$

Thus, the absolute values of the means of the fields at the critical concentration ($p=1/2$) satisfy the relation

$$|\langle\mathbf{E}\rangle^{(1)}|=(\sigma_{x2}/\sigma_{x1})^{1/2}|\langle\mathbf{E}\rangle^{(2)}|. \tag{31}$$

When $\mathbf{H}\rightarrow 0$, the results obtained by Dykhne[10] follow from (30) and (31).

Raising both sides of the equality (26) to the second power and taking into account (30), we find the cosine of the angle θ between the vectors $\langle\mathbf{E}\rangle^{(1)}$ and $\langle\mathbf{E}\rangle^{(2)}$ for $p=1/2$

$$\cos\theta=(\sigma_{x1}\sigma_{x2})^{1/2}/\sigma_{xe}. \quad (32)$$

When $\mathbf{H}\to 0$, we have $\sigma_{xe}\simeq\sigma_e=(\sigma_1\sigma_2)^{1/2}$ (see Ref. 10), and it follows from (32) that $\theta=0$, i.e., when $\mathbf{H}=0$, $\langle\mathbf{E}\rangle^{(1)}$ and $\langle\mathbf{E}\rangle^{(2)}$ are parallel. In a strong magnetic field we have $\sigma_{xi}\propto H^{-2}$ and $\sigma_{xe}\propto H^{-1}$ (an anomalous conductivity), so that $\theta\simeq\pi/2$. Thus, when $p=1/2$ and $H\to\infty$, the vectors $\langle\mathbf{E}\rangle^{(1)}$ and $\langle\mathbf{E}\rangle^{(2)}$ are perpendicular.

Moving $\langle\mathbf{E}\rangle^{(1)}$ to the right-hand side of (26), squaring the equality obtained, and taking into account (30), we find the cosine of the angle θ_1 between the vectors $\langle\mathbf{E}\rangle^{(1)}$ and $\langle\mathbf{E}\rangle$ for $p=1/2$:

$$\cos\theta_1=\frac{\sigma_{xe}+\sigma_{x1}}{\sqrt{\sigma_{xe}}}\left[\frac{\sigma_{x2}}{\sigma_{xe}(\sigma_{x1}+\sigma_{x2})+2\sigma_{x1}\sigma_{x2}}\right]^{1/2}. \quad (33)$$

We can find the cosine of the angle θ_2 between the vectors $\langle\mathbf{E}\rangle^{(2)}$ and $\langle\mathbf{E}\rangle$ in a similar manner; the expression for $\cos\theta_2$ follows from (33) when the following interchange is made: $\sigma_{x1}\leftrightarrows\sigma_{x2}$.

When $\mathbf{H}=0$, it follows from (33) and the expression for $\cos\theta_2$ that $\theta_1=\theta_2=0$, and thus the vectors $\langle\mathbf{E}\rangle^{(1)}$, $\langle\mathbf{E}\rangle^{(2)}$, and $\langle\mathbf{E}\rangle$ are collinear. In a strong magnetic field we can use the model expressions (58) from Sec. 6 (for $\beta_1=\beta_2=\beta$) to obtain the following relations when $H\to\infty$:

$$\cos\theta_1\simeq\left(\frac{\sigma_2}{\sigma_1+\sigma_2}\right)^{1/2},$$

$$\cos\theta_2\simeq\left(\frac{\sigma_1}{\sigma_1+\sigma_2}\right)^{1/2}\quad(p=1/2). \quad (34)$$

For a system with components having sharply different conductivities ($\sigma_2\ll\sigma_1$), it follows from (34) that $\theta_1\simeq\pi/2$ and $\theta_2\simeq 0$. Thus, in this case $\langle\mathbf{E}\rangle^{(1)}$ is perpendicular to $\langle\mathbf{E}\rangle$, and $\langle\mathbf{E}\rangle^{(1)}$ is parallel to $\langle\mathbf{E}\rangle$. We note that it follows from (31) in the case under consideration that $\langle\mathbf{E}\rangle^{(1)}$ is small compared with $\langle\mathbf{E}\rangle^{(2)}\simeq\langle\mathbf{E}\rangle$ at all $\mathbf{H}$. This circumstance is associated with the "displacement" of the electric field from the high-conductivity component when the point of the metal–insulator phase transition is approached (see, for example, Ref. 11).

The mean current density in the kth component $\langle\mathbf{j}\rangle^{(k)}$ can be treated in a similar manner. It is not difficult to see that these characteristics can be expressed in terms of the mean fields. In the real and complex representations we have, respectively,

$$\langle\mathbf{j}\rangle^{(k)}=\hat{\sigma}_k\langle\mathbf{E}\rangle^{(k)},\quad \langle j\rangle^{(k)}=-i\zeta_k\langle E\rangle^{(k)}. \quad (35)$$

For the square of $\langle\mathbf{j}\rangle^{(k)}$ we find

$$(\langle\mathbf{j}\rangle^{(k)})^2=(\sigma_{xk}^2+\sigma_{ak}^2)(\langle\mathbf{E}\rangle^{(k)})^2. \quad (36)$$

When $p=1/2$, it follows from (36) and (31) that the absolute values of the mean currents are related by the expression

$$|\langle\mathbf{j}\rangle^{(1)}|=(\rho_{x2}/\rho_{x1})^{1/2}|\langle\mathbf{j}\rangle^{(2)}|, \quad (37)$$

where $\rho_x=\sigma_x/(\sigma_x^2+\sigma_a^2)$ is the diagonal component of the resistivity tensor $\hat{\rho}=\hat{\sigma}^{-1}$.

5. QUADRATIC EFFECTIVE CHARACTERISTICS

As in the case of $\mathbf{H}=0$ (see Refs. 11 and 12), there is considerable interest in the investigation of the mean squares of the electric field strength in each of the components

$$\psi_i=\langle\mathbf{E}^2\rangle^{(i)}/(\langle\mathbf{E}\rangle)^2. \quad (38)$$

Here $\langle(\dots)\rangle^{(i)}$ has the same meaning as in (25). We note that the fluctuations of the electric field strength (Δ_E^2) and the current density (Δ_j^2) (which were studied in Refs. 1 and 10 for systems with the critical composition)

$$\Delta_E^2=\frac{\langle\mathbf{E}^2\rangle}{(\langle\mathbf{E}\rangle)^2}-1,\quad \Delta_j^2=\frac{\langle\mathbf{j}^2\rangle}{(\langle\mathbf{j}\rangle)^2}-1 \quad (39)$$

can be expressed in terms of the ψ_i:

$$\Delta_E^2=\sum_i\psi_i-1,\quad \Delta_j^2=\frac{1}{\sigma_{xe}^2+\sigma_{ae}^2}\sum_i(\sigma_{xi}^2+\sigma_{ai}^2)\psi_i-1. \quad (40)$$

From the known identity (see, for example, Ref. 10)

$$\langle\mathbf{Ej}\rangle=\langle\mathbf{E}\rangle\langle\mathbf{j}\rangle \quad (41)$$

it follows that σ_{xe} can be expressed in terms of the ψ_i:

$$\sigma_{xe}=\sum_i\sigma_{xi}\psi_i. \quad (42)$$

The equalities (40) and (42) are valid for isotropic n-component systems with $n\geqslant 1$.

We note, finally, that the Joule heat Q_i evolved during 1 s per unit volume of the ith component is also expressed in terms of the ψ_i:

$$Q_i=\langle\mathbf{jE}\rangle^{(i)}=\sigma_{xi}\psi_i(\langle\mathbf{E}\rangle)^2. \quad (43)$$

The total heat Q evolved during 1 s per unit volume of the sample is given by the sum of the Q_i from $i=1$ to $i=n$. It is also convenient to introduce the dimensionless characteristic q_i, i.e., the fraction of the Joule heat evolved in the ith component. For $q_i=Q_i/Q$ from (43) we obtain

$$q_i=\frac{\sigma_{xi}}{\sigma_{xe}}\psi_i, \quad (44)$$

where by definition

$$\sum_{i=1}^{n}q_i=1. \quad (45)$$

Substituting expression (44) into equality (45) transforms it with consideration of (42) into an identity.

The functions ψ_i for a two-dimensional two-component system can be found in a general form using the isomorphism relations (see Sec. 3). According to Ref. 11, when $\mathbf{H}=0$, the ψ_i ($i=1,2$) are expressed in terms of the function f, so that for the null system we have

$$\psi_1^0=f-\lambda f',\quad \psi_2^0=f',\quad f'\equiv\partial f(p,\lambda)/\partial\lambda. \quad (46)$$

On the other hand, when $\mathbf{H}\neq 0$, the ψ_i can be expressed in terms of the ψ_i^0 using relations (18). As a result we obtain

$$\psi_1=\frac{\sigma_{xe}}{\sigma_{x1}}\left(1-\lambda\frac{f'}{f}\right),\quad \psi_2=\frac{\sigma_{xe}}{\sigma_{x2}}\lambda\frac{f'}{f} \quad (47)$$

with λ from (17) and σ_{xe} from (20). We note that the substitution of (47) into equality (42) turns it into an identity.

For the mean-square values of the current density we obtain, respectively

$$\langle \mathbf{j}^2\rangle^{(1)}=\frac{\rho_{xe}}{\rho_{x1}}\left(1-\lambda\frac{f'}{f}\right)(\langle \mathbf{j}\rangle)^2,$$

$$\langle \mathbf{j}^2\rangle^{(2)}=\frac{\rho_{xe}}{\rho_{x2}}\lambda\frac{f'}{f}(\langle \mathbf{j}\rangle)^2, \tag{48}$$

where, as above, $\rho_x=\sigma_x/(\sigma_x^2+\sigma_a^2)$ is the diagonal component of $\hat{\rho}=\hat{\sigma}^{-1}$. Finally, for q_i from (44) and (47) we find

$$q_1=1-\lambda\frac{f'}{f},\quad q_2=\lambda\frac{f'}{f}. \tag{49}$$

When the expressions (49) are substituted into the equality (45), it is satisfied identically.

At the critical concentration $p=p_c=1/2$ from (47) and (23) we obtain

$$\psi_1(p_c)=\frac{\sigma_{xe}(p_c)}{2\sigma_{x1}},\quad \psi_2(p_c)=\frac{\sigma_{xe}(p_c)}{2\sigma_{x2}}, \tag{50}$$

where $\sigma_{xe}(p_c)$ was defined in (24). When $p=1/2$, the substitution of (50) into (40) gives

$$\Delta_E^2+1=\Delta_j^2+1=\frac{1}{2}(\sigma_{x1}\sigma_{x2})^{-1/2}$$
$$\times[(\sigma_{x1}+\sigma_{x2})^2+(\sigma_{a1}-\sigma_{a2})^2]^{1/2}, \tag{51}$$

which is identical with accuracy to the notation to the expression obtained in Ref. 3 by another method. In strong magnetic fields, Dykhne's result[1] follows from (51) under the appropriate simplifying assumptions.

Since we have $\sigma_{xi}\propto H^{-2}$ and (for $p=1/2$) $\sigma_{xe}\propto H^{-1}$ as $H\to\infty$, it follows from (50) that in this case the ψ_i increase without bound: $\psi_i\propto H$. Accordingly, **H** and the fluctuations Δ_E^2 and Δ_j^2 increase linearly, in agreement with Ref. 1. These effects are clearly related directly to the anomalous conductivity. Therefore, when $p\neq 1/2$, the linear increase in ψ_i (as well as in Δ_E^2 and Δ_j^2) at large **H**, like the anomalous conductivity (see Ref. 6), occurs only in a small vicinity of the critical concentration and in a finite range of magnetic fields. The dependence of the ψ_i on **H** in the critical region is considered in greater detail in Sec. 6.

It follows from (50) and (43) along with (44) that, as in the case of $\mathbf{H}=0$ (see Ref. 10), the evolution of Joule heat for $p=1/2$ and arbitrary $\mathbf{H}\neq 0$ is identical in each of the components, i.e., $Q_1=Q_2$, so that $q_1=q_2=1/2$. However, the equality $Q_1=Q_2$ holds only for systems with the critical composition. In fact, equating Q_1 and Q_2, from (43) and (47) we obtain the differential equation $2\lambda(\partial f/\partial\lambda)=f$. The solution of this equation (with the obvious condition $f=1$ when $\lambda=1$) is $f=\lambda^{1/2}$. Since f is a monotonic function of λ, in accordance with (23) the equality $Q_1=Q_2$ is possible only when $p=p_c=1/2$.

We note, finally, that a quadratic effective characteristic can also be associated with the Hall component σ_{ae}. We use $\mathbf{E}^{(\nu)}(\mathbf{r})$ to denote the electric field strength in the medium determined for an assigned value of $\langle \mathbf{E}^{(\nu)}\rangle$, where the superscript ν indicates that the mean field is directed along the ν axis. It can be shown that in the two-dimensional case σ_{ae} can be represented in the form

$$\sigma_{ae}=\sigma_{a2}+(\sigma_{a1}-\sigma_{a2})\varphi, \tag{52}$$

where

$$\varphi=\langle[\mathbf{E}^{(x)},\mathbf{E}^{(y)}]_z\rangle^{(i)}/[\langle\mathbf{E}^{(x)}\rangle,\langle\mathbf{E}^{(y)}\rangle]_z \tag{53}$$

is the quadratic effective characteristic mentioned above. In (53) $(\mathbf{a},\mathbf{b})_x=a_xb_y-a_yb_x$, and $\langle(\ldots)\rangle^{(i)}$ has the same meaning as in (25). Expressions (52) and (53), which are valid for arbitrary **H**, coincide in form with the analogous expressions in the case of a weak magnetic field ($\mathbf{H}\to 0$) (compare Ref. 11).

The equalities (18) make it possible to express φ in terms of the corresponding function $\varphi^0(p,\lambda)$ for the null system:

$$\varphi=(\sigma_{xe}/\sigma_{x1})f^{-1}\varphi^0, \tag{54}$$

where $f=f(p,\lambda)$. According to Ref. 11, for φ^0 we have

$$\varphi^0(p,\lambda)=(f^2-\lambda^2)/(1-\lambda^2), \tag{55}$$

so that with consideration of (20) we obtain

$$\varphi=\sigma_{x1}(f^2-\lambda^2)/D \tag{56}$$

with D from (22). The substitution of (56) into (52) leads to expression (21).

6. CRITICAL REGION

The general equations (47) make it possible to thoroughly examine the behavior of the ψ_i in the vicinity of the critical concentration $p_c=1/2$ using the similarity hypothesis.[8,9] Let the conductivities of the components in a zero magnetic field differ sharply ($h=\sigma_2/\sigma_1\ll 1$), so that there is a pronounced metal–insulator phase transition in the system when $\mathbf{H}=0$. According to the ideas underlying the similarity hypothesis, when $h\ll 1$ and $|\tau|\ll 1$[where $\tau=(p-p_c)/p_c$], for $f(p,h)$ we have[8,9] (in the notation of Refs. 11 and 12):

for $\tau>0$, $\Delta_0\ll\tau\ll 1$

$$f\simeq\tau^t\{A_0+A_1(h/\tau^{t/s})+\ldots\}, \tag{57a}$$

for $|\tau|\ll\Delta_0$

$$f\simeq h^s\{a_0+a_1(\tau/h^{s/t})+\ldots\}, \tag{57b}$$

for $\tau<0$, $\Delta_0\ll|\tau|\ll 1$

$$f\simeq\frac{h}{(-\tau)^q}\left\{B_1+B_2\frac{h}{(-\tau)^{t/s}}+\ldots\right\}. \tag{57c}$$

Here $\Delta_0=h^{s/t}$ is the size of the smearing-out region[9] when $\mathbf{H}=0$; the critical indices t, s, and q are related by the expression[9] $t/s=t+q$. According to Ref. 10 [see also Eq. (23) from Sec. 3], in a two-dimensional randomly inhomogeneous system $s=1/2$, so that $q=t$, and the coefficient a_0 from (57b) equals unity.

Below we shall use the same model expressions as in Ref. 6 for σ_{xi} and σ_{ai}:

$$\sigma_{xi}=\sigma_i/(1+\beta_i^2),\quad \sigma_{ai}=\sigma_i\beta_i/(1+\beta_i^2). \tag{58}$$

Here σ_i is the conductivity when $\mathbf{H}=0$; $\beta_i \propto \mu_i H$ is the dimensionless magnetic field; μ_i is the mobility of the carriers. To simplify the treatment we also assume that $\beta_1=\beta_2=\beta$. Now, when $h=\sigma_2/\sigma_1 \ll 1$, we have $\hat{\sigma}_2 \ll \hat{\sigma}_1$ for all $\mathbf{H}$, so that (17) and (20) give

$$\lambda \simeq h/(1+\beta^2), \tag{59}$$

$$\sigma_{xe} \simeq \sigma_1 f/(1+\beta^2 f^2). \tag{60}$$

Then the relations (47) take the form

$$\psi_1 \simeq \frac{(1+\beta^2)f}{1+\beta^2 f^2}\left(1-\frac{h}{1+\beta^2}\frac{f'}{f}\right), \quad \psi_2 \simeq \frac{f'}{1+\beta^2 f^2}, \tag{61}$$

where $f=f(p,\lambda)$ and $f'=\partial f/\partial\lambda$.

According to (59), λ is small at all $\mathbf{H}$, so that the expressions (57) with the replacement $h\rightarrow\lambda$ can be used for $f(p,\lambda)$ when $|\tau|\ll 1$. In this case for the size of the smearing-out region Δ_H (when $\mathbf{H}\neq 0$) we obtain[6]

$$\Delta_H=\Delta_0/(1+\beta^2)^{s/t}, \quad \Delta_0=h^{s/t}, \tag{62}$$

where $s=1/2$ and $t\simeq 1, 3$. Thus, Δ_H is highly dependent on the magnetic field,[6] the size of the smearing-out region decreasing with increasing $\mathbf{H}$, tending to zero as $H\rightarrow\infty$. Therefore, if the system is found within the smearing-out region ($|\tau|\ll\Delta_0$, $\tau\neq 0$) when $\mathbf{H}=0$, a situation in which $|\tau|\gg\Delta_H$ arises, i.e., the system "leaves" this region, in sufficiently large fields $\mathbf{H}$. As was noted in Ref. 6, just this circumstance restricts the range where the anomalous conductivity exists on the high-magnetic-field side. Similar restrictions also appear for ψ_i.

Let us investigate the dependence of ψ_1 and ψ_2 on $\mathbf{H}$ in the critical region. We consider the highly interesting situation in which the system is found within the smearing-out region ($|\tau|\ll\Delta_0$, $\tau\neq 0$) when $\mathbf{H}=0$. As follows from (61), when $p>p_c$ ($\tau>0$), in this case four ranges of magnetic fields can be demarcated in the dependence of ψ_i on $\mathbf{H}$:

$\beta\ll 1$:

$$\psi_1\simeq\frac{1}{2}(\sigma_2/\sigma_1)^{1/2}, \quad \psi_2\simeq\frac{1}{2}(\sigma_1/\sigma_2)^{1/2},$$

$$\sigma_{xe}\simeq(\sigma_1\sigma_2)^{1/2}, \tag{63a}$$

$1\ll\beta\ll h^{1/2}\tau^{-t}$:

$$\psi_1\simeq\frac{\beta}{2}(\sigma_2/\sigma_1)^{1/2}, \quad \psi_2\simeq\frac{\beta}{2}(\sigma_1/\sigma_2)^{1/2},$$

$$\sigma_{xe}\simeq\beta^{-1}(\sigma_2\sigma_1)^{1/2}, \tag{63b}$$

$h^{1/2}\tau^{-t}\ll\beta\ll\tau^{-t}$:

$$\psi_1\sim\beta^2\tau^{t}, \quad \psi_2\sim\tau^{-t}, \quad \sigma_{xe}\sim\sigma_1\tau^{t}, \tag{63c}$$

$\beta\gg\tau^{-t}$:

$$\psi_1\sim\tau^{-t}, \quad \psi_2\sim\tau^{-t}(\beta\tau^{t})^2, \quad \sigma_{xe}\sim\sigma_1\beta^{-2}\tau^{-1}. \tag{63d}$$

Here $h=\sigma_2/\sigma_1$, and the corresponding values of σ_{xe} are presented along with ψ_1 and ψ_2 (compare with Ref. 6). The numerical multipliers of order unity were omitted in (63c) and (63d).

Expressions (63a) and (63b) correspond to the case in which the system is found within the smearing-out region ($\tau\ll\Delta_H$), so that $f(p,\lambda)\simeq\lambda^{1/2}$ and the expressions (60) and (61) take the following form when $\beta\ll h^{1/2}\tau^{-t}$

$$\psi_1\simeq\frac{1}{2}h^{1/2}(1+\beta^2)^{1/2}, \quad \psi_2\simeq\frac{1}{2}h^{-1/2}(1+\beta^2)^{1/2},$$

$$\sigma_{xe}\simeq\left(\frac{\sigma_1\sigma_2}{1+\beta^2}\right)^{1/2}. \tag{64}$$

In the range of magnetic fields $1\ll\beta\ll h^{1/2}\tau^{-t}$ the anomalous conductivity appears, and ψ_1 and ψ_2 increase linearly ($\propto H$) with increasing $\mathbf{H}$. Equations (63c) and (63d) correspond to the situation in which $\tau\gg\Delta_H$, the system has already "left" the smearing-out region, and expression (57a) with the replacement $h\rightarrow\lambda$ is valid for $f(p,\lambda)$. In this case, omitting the numerical multipliers of order unity, for $\beta\gg h^{1/2}\tau^{-t}$ from (60) and (61) we obtain

$$\psi_1\sim\frac{\beta^2\tau^2}{1+(\beta\tau^{t})^2}, \quad \psi_2\sim\frac{1}{\tau^{t}}\frac{1}{1+(\beta\tau^{t})^2},$$

$$\sigma_{xe}\sim\frac{\sigma_1\tau^{t}}{1+(\beta\tau^{t})^2}. \tag{65}$$

In this range of magnetic fields the anomalous conductivity and the effects associated with it are absent. The case of $\tau<0$ can be treated in a similar manner.

According to (63)–(65) $\psi_1\ll\psi_2$ for all $\beta\ll\tau^{-t}$, and, conversely, $\psi_1\gg\psi_2$ in the range $\beta\gg\tau^{-t}$, so that the values of ψ_1 and ψ_2 are comparable when $\beta\sim\tau^{-t}$. We note that the inequality $\psi_1\ll\psi_2$ is perfectly natural for the system under consideration ($h=\sigma_2/\sigma_1\ll 1$) near p_c, since the electric field is displaced from the high-conductivity (first) component (see, for example, Ref. 11, as well as Sec. 4 in this communication). However, as was noted in Ref. 6, in the ultrastrong magnetic fields $\beta\gg\tau^{-t}$ insulating inclusions exhibit some properties of ideal conductors, so that the electric field is concentrated mainly in the first component even in this case: $\psi_1\gg\psi_2$.

When $\beta\ll h^{1/2}\tau^{-t}$, for q_1 and q_2 from (44) and (64) we obtain

$$q_1\simeq q_2\simeq 1/2. \tag{66}$$

In the range $\beta\ll h^{1/2}r^{-t}$ the system is found within the smearing-out region ($\tau\ll\Delta_H$), so that, as was noted above, $Q_1\simeq Q_2\simeq Q/2$, whence follows (66). Accordingly, in the range $\beta\gg h^{1/2}\tau^{-t}$ from (44) and (65) we find

$$q_1\simeq 1, \quad q_2\sim h/(\beta\tau^{t})^2\ll 1. \tag{67}$$

In this range of magnetic fields the system is found outside the smearing-out region ($\tau\gg\Delta_H$), and Joule heat is evolved mainly in the first component.

Thus, along with σ_{xe} and σ_{xe} the study of ψ_i[as well as φ, see (53)] as functions of p and $\mathbf{H}$ is of unquestionable interest. Such a complex investigation (for example, by numerical methods) of galvanomagnetic phenomena in two-component media should provide diverse information on the properties of such systems. In the two-dimensional case there is considerable interest, in particular, in the study of the effects predicted in Ref. 1, viz., the anomalous conductivity, the saturation of the effective Hall parameter, and the linear ($\propto H$) increase in the fluctuations of the electric field strength and the current density. It should, however, be noted that, according to the results in Ref. 6 and the present article, all these effects take place in a finite (when $p \neq p_c$) range of magnetic fields and in a narrow vicinity of the critical point, and they exhibit asymptotic behavior (as $H \rightarrow \infty$) at the isolated point $p = p_c = 1/2$.

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