# Polarization of radiation in a rotating universe

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(Submitted 19 June 1995)
Zh. Éksp. Teor. Fiz. **108**, 1889–1898 (December 1995)

A study is made of the effect of the gravitation field in a cosmological model with expansion and rotation on the polarization of radiation. It is shown that the effect of rotation of the plane of polarization is observable in terms of real astrophysical measurements. Moreover, calculations confirm the data of Birch on the dipole anisotropy of the distribution on the celestial sphere of the relative angles between the directions of polarization and the principal axis of the image of a radio source at cosmological distances. © *1995 American Institute of Physics.*

## 1. INTRODUCTION

Earlier[1] we considered a large class of cosmological models with rotation and analyzed possible observable physical effects. In particular, using the Newman–Penrose formalism we investigated the rotation of the polarization vector in the shear-free Gödel metric

$$ds^2=dt^2-2\sqrt{\sigma}R(t)e^{mx}dtdy-R^2(t)\times(dx^2+ke^{2mx}dy^2+dz^2) \tag{1.1}$$

with nontrivial expansion

$$\vartheta=3\frac{\dot{R}}{R} \tag{1.2}$$

and rotation

$$\Omega=\sqrt{\frac{1}{2}\,\Omega_{\mu\nu}\Omega^{\mu\nu}}=\frac{m}{2R}\sqrt{\frac{\sigma}{k+\sigma}}. \tag{1.3}$$

Similar rotation–polarization effects have been studied in noncosmological metrics of rotating compact objects.[2–7]

However, it is clear that the calculations in Ref. 1 do not give directly observable quantities for the angle of rotation of the polarization during motion along a light ray, since the rotation of the polarization vector $f^\mu$ was considered relative to local coordinate frames, and the choice of these is obviously arbitrary both along any ray as well as in the complete spacetime (1.1). At the same time, the gravitational field in which the radiation propagates influences the image of the source.[5–7]

In order to eliminate the arbitrariness in the coordinates and separate an observable physical effect, it is necessary to compare the direction of the polarization vector with some reference direction that is specified in advance at each point of the ray. The very formulation of the problem suggests a natural choice: In real astronomical observations, the angle between the principal axis of the source image and the polarization vector is directly measurable. This quantity does not depend on the choice of the coordinates, so that calculation of the difference between the directions of the principal axis of the image and of the polarization at an arbitrary point of the ray makes it possible to distinguish the observable effect of polarization rotation in a cosmological model with rotation.

In Ref. 8, an attempt was made to solve such a problem for the metric (1.1) by the "representative beam method." However, this method is based on the incorrect assumption that the properties of a beam of rays in a gravitational field can be specified arbitrarily without regard to the properties of the isotropic geodesics on the corresponding spacetime manifold. In particular, in the general case for the metric (1.1) (as also for more general models with rotation) it is not possible to require the vanishing of the optical scalar $\omega$ on an arbitrary isotropic geodesic.

In Refs. 1 and 9, we obtained exact solutions of the equations of isotropic geodesics in the cosmology (1.1), and this makes it possible to treat exhaustively the properties of all beams of rays propagating in a rotating universe.

## 2. ISOTROPIC CONGRUENCES AND OPTICAL SCALARS

In the geometrical-optics approximation, it is convenient to describe the evolution and deformation of the image of a source by means of the optical scalars introduced by Sachs.[10] Below, we shall calculate these quantities for an arbitrary ray, and this will enable us to find the position of the principal axis of the image at every point of an arbitrary isotropic geodesic. In a remarkable way, this immediately determines the angle in which we are interested, i.e., the angle between the principal axis of the image and the polarization vector, provided an accurate choice is made of the isotropic Newman–Penrose tetrad $(l,n,m,\bar{m})$. Namely, if we make the real vector $l^\mu$ coincide with the wave vector $k^\mu$ tangent to the given isotropic geodesic, then the polarization vector $f^\mu$ will lie in the spacelike plane spanned by the complex-conjugate vectors $(m^\mu,\bar{m}^\mu)$. Further, we can rigidly tie $f^\mu$ to one of these basis vectors (say $m^\mu$). (Technically, this is not difficult: Without affecting $l$, by means of local Lorentz rotations we can always change each of the three vectors $n,m,\bar{m}$ in such a way that they become covariantly constant along the geodesic congruence $l$.) Thus, the rotation of the principal axis of the source image determined by the "refracting" properties of the gravitational field relative to a given isotropic frame automatically gives the required observable effect.

We describe the construction of the isotropic frame. In accordance with what we have said above, we can proceed from an arbitrary Newman–Penrose frame and by means of Lorentz rotations transform it to the required configuration. For definiteness, we take the isotropic frame used earlier in Ref. 1 [see (6.2) in Ref. 1]. Our main notation and terminology in the Newman–Penrose formalism are taken from Ref. 7, and therefore in what follows we restrict ourselves to just the minimum of explanations.

We submit the original tetrad [see (6.2) of Ref. 1] to the three following successive Lorentz transformations.

1) We first make a rotation of class III, which leaves the directions of $l$ and $n$ unchanged:

$$l \to A^{-1} l, \quad n \to A n, \quad m \to m, \quad \bar{m} \to \bar{m},$$

where

$$A = \frac{\sqrt{2}}{(1+\cos\theta)} \frac{R}{R_0}. \tag{2.1}$$

Here $R_0$ and $\theta$ are constant parameters of the transformation, the meaning of which will be clarified later.

2) We then make a Lorentz rotation of class II, which transforms the field of vectors $l$ into a geodesic congruence with nonaffine parametrization:

$$l \to l + b^* m + b\bar{m} + b^* n, \quad n \to n,$$

$$m \to m + bn, \quad \bar{m} \to \bar{m} + b^* n,$$

where $b$ and $b^*$ are complex-conjugate functions of the cosmological coordinate time,

$$b(t) = i \frac{\sin\theta}{\sqrt{2}} \frac{R_0}{R} e^{-i\Phi}. \tag{2.2}$$

Here the function $\Phi(t)$ is determined as a solution of the ordinary differential equation

$$\frac{d\Phi}{dt} = -\frac{m}{R} \frac{\sqrt{\frac{\sigma}{k+\sigma}} + \sin\theta \sin\Phi}{1 + \sqrt{\frac{\sigma}{k+\sigma}} + \sin\theta\sin\Phi}. \tag{2.3}$$

3) Finally, for the geodesic congruence $l$ we make the parametrization affine by means of a Lorentz rotation of class III:

$$l \to l, \quad n \to n, \quad m \to e^{i\Psi} m, \quad \bar{m} \to e^{-i\Psi} \bar{m},$$

where

$$\Psi(t,z) = z \frac{m}{2} \sqrt{\frac{\sigma}{k+\sigma}} + \Phi(t). \tag{2.4}$$

After this series of transformations, the isotropic tetrad takes the final form

$$l = \frac{R_0}{R} \left\{ \left[ 1 + \sqrt{\frac{\sigma}{k+\sigma}} \sin\theta \sin\Phi \right] \partial_t + \frac{1}{R} \sin\theta \cos\Phi \partial_x + \frac{e^{-mx}}{R\sqrt{k+\sigma}} \sin\theta \sin\Phi \partial_y + \frac{1}{R} \cos\theta \partial_z \right\}, \tag{2.5}$$

$$n = \frac{R}{R_0} \left( \frac{1}{1+\cos\theta} \right) \left[ \partial_t - \frac{1}{R} \partial_z \right], \tag{2.6}$$

$$m = \frac{e^{i\Psi}}{\sqrt{2}} \left\{ \left[ \sqrt{\frac{\sigma}{k+\sigma}} + i \frac{\sin\theta}{1+\cos\theta} e^{-i\Phi} \right] \partial_t + \frac{i}{R} \partial_x + \frac{e^{-mx}}{R\sqrt{k+\sigma}} \partial_y - \frac{i}{R} \frac{\sin\theta}{1+\cos\theta} e^{-i\Phi} \partial_z \right\}, \tag{2.7}$$

$$\bar{m} = \frac{e^{-i\Psi}}{\sqrt{2}} \left\{ \left[ \sqrt{\frac{\sigma}{k+\sigma}} - i \frac{\sin\theta}{1+\cos\theta} e^{i\Phi} \right] \partial_t - \frac{i}{R} \partial_x + \frac{e^{-mx}}{R\sqrt{k+\sigma}} \partial_y + \frac{i}{R} \frac{\sin\theta}{1+\cos\theta} e^{i\Phi} \partial_z \right\}. \tag{2.8}$$

It is readily shown by direct verification that $l$ is a geodesic congruence with affine parametrization, $l^\nu \nabla_\nu l^\mu = 0$. We now clarify the meaning of the functions and constant parameters that occur in (2.5)–(2.8). For this, we recall the construction of the exact solutions of the equations of isotropic geodesics in the metric (1.1) (see the detailed but somewhat different derivation in Refs. 1 and 9).

The key property is the existence of three ordinary, $\xi_{(i)} = 1, 2, 3$, and one conformal, $\xi_{(0)}$, Killing vectors for the metric (1.1):

$$\xi_{(0)} = R\partial_t, \quad \xi_{(1)} = \frac{1}{m} \partial_x - y\partial_y, \quad \xi_{(2)} = \partial_y, \quad \xi_{(3)} = \partial_z. \tag{2.9}$$

We consider the geodesic equations $k^\nu \nabla_\nu k^\mu = 0$, where $k^\mu = dx^\mu/ds$, $k^\mu k_\mu = 0$, is the tangent vector to the curve $x^\mu(s)$ with affine parameter $s$. We specify the initial conditions as in Refs. 1 and 9; without loss of generality (the geometry is spatially homogeneous), we choose the position of the observer at the point $P = (t = t_0,\ x = 0,\ y = 0,\ z = 0)$ and characterize each geodesic that passes through $P$ by spherical angles $(\theta, \phi)$, which determine the initial direction of the ray in the local Lorentz basis of the observer at $P$:

$$k_P^a = (h_\mu^a k^\mu)_P = (1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta).$$

The Killing vectors (2.9) determine four first integrals:

$$q_0 = \xi_{(0)}^\mu k_\mu, \quad q_i = -\xi_{(i)}^\mu k_\mu, \quad i = 1, 2, 3. \tag{2.10}$$

As is clear from what was said above, their values on the ray passing through $P$ in the direction $(\theta, \phi)$ are

$$q_0 = R_0, \quad q_1 = \frac{R_0}{m} \sin\theta\cos\phi, \quad q_2 = R_0(\sqrt{\sigma} + \sqrt{k+\sigma} \sin\theta\sin\phi), \quad q_3 = R_0 \cos\theta. \tag{2.11}$$

Here we have written $R_0=R(t_0)$. From (2.10), we obtain the explicit form of the wave vector tangent to the given geodesic:

$$k=\frac{dx^\mu}{ds}\partial_\mu=\frac{1}{R^2}\left[\frac{R}{k+\sigma}(\sqrt{\sigma}q_2e^{-mx}+kq_0)\partial_t+m(q_1+yq_2)\partial_x+\frac{k}{k+\sigma}e^{-mx}(q_2e^{-mx}-\sqrt{\sigma}q_0)\partial_y+q_3\partial_z\right]. \tag{2.12}$$

The integration of the expression (2.12) described in Ref. 1 leads to the following exact solutions of the equations of isotropic geodesics:

$$e^{-mx}=\frac{\sqrt{\sigma}+\sqrt{k+\sigma}\sin\theta\sin\Phi}{\sqrt{\sigma}+\sqrt{k+\sigma}\sin\theta\sin\phi}, \tag{2.13}$$

$$y=\frac{\sin\theta(\cos\Phi-\cos\phi)}{m(\sqrt{\sigma}+\sqrt{k+\sigma}\sin\theta\sin\phi)}, \tag{2.14}$$

$$z=\left(\frac{k+\sigma}{k}\right)\cos\theta\left[\int_{t_0}^{t}\frac{dt'}{R(t')}+\sqrt{\frac{\sigma}{k+\sigma}}\frac{\Phi-\phi}{m}\right], \tag{2.15}$$

where the function $\Phi$ is given by Eq. (2.3) with the initial condition $\Phi(t_0)=\phi$.

It is easy to show that substitution of (2.13)–(2.15) in (2.12) gives (2.5). Thus, we have elucidated the meaning of the constant parameters $\theta$ and $R_0$ and the function $\Phi$ in (2.5)–(2.8): They specify the direction of the geodesic congruence (2.5) in the local frame of reference of the observer positioned at the point $P$. The congruence (2.5) contains the distinguished ray (which can be called the central ray of the given congruence) seen by the observer $P$ in the direction $(\theta,\phi)$ on his celestial sphere. Varying the angle parameters, we obtain the complete family of geodesic congruences that describe radiation fluxes in an arbitrary direction.

Direct calculations of the spin coefficients (see the definitions in Ref. 7) lead to the results

$$\varepsilon=0,\quad \kappa=0, \tag{2.16}$$

$$\lambda=0,\quad \nu=0, \tag{2.17}$$

$$\rho=-\frac{R_0}{R^2}\left\{\dot{R}\left[1+\sqrt{\frac{\sigma}{k+\sigma}}\sin\theta\sin\Phi\right]+\frac{m}{2}\frac{k}{k+\sigma}\frac{\sin\theta\cos\Phi}{1+\sqrt{\frac{\sigma}{k+\sigma}}\sin\theta\sin\Phi}\right\}+i\frac{R_0}{R^2}\frac{m}{2}\cos\theta\frac{\sqrt{\frac{\sigma}{+k+\sigma}}+\sin\theta\sin\Phi}{1\sqrt{\frac{\sigma}{k+\sigma}}\sin\theta\sin\Phi}, \tag{2.18}$$

$$\sigma=\frac{R_0}{R^2}\frac{m}{2}\frac{k}{k+\sigma}\frac{e^{2i\Psi}\sin\theta}{1+\sqrt{\frac{\sigma}{k+\sigma}}\sin\theta\sin\Phi}\{-\cos\Phi[2\cos\theta-1-2\cos^2\Phi(\cos\theta-1)]+i\sin\Phi[\cos\theta-2\cos^2\Phi(\cos\theta-1)]\}, \tag{2.19}$$

$$\tau=\frac{e^{i\Psi}}{\sqrt{2}}\left\{-\sqrt{\frac{\sigma}{k+\sigma}}\frac{\dot{R}}{R}+\frac{\sin\theta}{1+\cos\theta}e^{-i\Phi}\left[\frac{m}{2R}\sqrt{\frac{\sigma}{k+\sigma}}-i\frac{\dot{R}}{R}+\frac{m}{R}\frac{k}{k+\sigma}\frac{\sin\theta\sin\Phi}{1+\sqrt{\frac{\sigma}{k+\sigma}}\sin\theta\sin\Phi}\right]\right\}, \tag{2.20}$$

$$\pi=\frac{e^{-i\Psi}}{\sqrt{2}}\left\{\sqrt{\frac{\sigma}{k+\sigma}}\frac{\dot{R}}{R}+\frac{\sin\theta}{1+\cos\theta}e^{i\Phi}\left[\frac{m}{2R}\sqrt{\frac{\sigma}{k+\sigma}}-i\frac{\dot{R}}{R}\right]\right\}, \tag{2.21}$$

$$\mu=\frac{1}{R_0(1+\cos\theta)}\left(\dot{R}+i\frac{m}{2}\sqrt{\frac{\sigma}{k+\sigma}}\right), \tag{2.22}$$

$$\alpha=\frac{e^{-i\Psi}}{\sqrt{2}}\left\{i\frac{m}{2R}+\left(-\sqrt{\frac{\sigma}{k+\sigma}}+i\frac{\sin\theta}{1+\cos\theta}e^{i\Phi}\right)\times\left[\frac{\dot{R}}{R}+i\frac{m}{2R}\sqrt{\frac{\sigma}{k+\sigma}}+i\frac{m}{2R}\frac{k}{k+\sigma}\frac{\sin\theta\sin\Phi}{1+\sqrt{\frac{\sigma}{k+\sigma}}\sin\theta\sin\Phi}\right]\right\}, \tag{2.23}$$

$$\beta=\frac{e^{i\Psi}}{\sqrt{2}}\frac{k}{k+\sigma}\frac{m}{2R}\frac{1}{1+\sqrt{\frac{\sigma}{k+\sigma}}\sin\theta\sin\Phi}\left(i+\frac{\sin\theta}{1+\cos\theta}e^{-i\Phi}\sin\theta\sin\Phi\right), \tag{2.24}$$

$$\gamma=-\frac{1}{R_0(1+\cos\theta)}\left(\dot{R}+i\frac{m}{2}\sqrt{\frac{\sigma}{k+\sigma}}+i\frac{m}{2}\frac{k}{k+\sigma}+\frac{\sin\theta\sin\Phi}{1+\sqrt{\frac{\sigma}{k+\sigma}}\sin\theta\sin\Phi}\right). \tag{2.25}$$

Here and in what follows, the dot denotes a derivative with respect to $t$. We hope that the use of $\sigma$ for the constant in (1.1) and for the spin coefficient will not cause confusion.

As can be seen from (2.18) and (2.19), Sach's optical scalars $\Theta=-\mathrm{Re}\,\rho$, $\omega=\mathrm{Im}\,\rho$, and $\sigma$ contain a nontrivial dependence on the rotation parameters of the metric (1.1). Note that in view of (2.16) it is not possible by any Lorentz transformations of the frame (2.5)–(2.8) that do not affect the geodesic congruence $l$ to make any of the optical scalars vanish. At the same time, by a class I rotation,

$$l \to l, \quad n \to n + a^* m + a\bar{m} + a^* a l, m \to m + a l, \quad \bar{m} \to \bar{m} + a^* l, \qquad (2.26)$$

it is possible to annihilate the spin coefficient $\pi$. The transformation (2.26) is remarkable in that it leaves invariant $\kappa=0$, $\varepsilon=0$ and also does not change the optical scalars: $\rho \to \rho$, $\sigma \to \sigma$. We shall not calculate the function $a$ explicitly; it can be determined as solution of the differential equation $l^\mu \partial_\mu a + \pi^* = 0$. We shall denote the frame obtained from (2.5)–(2.8) after the transformation (2.6) by $(l,n,m,\bar{m})$, as before.

## 3. ROTATION OF THE POLARIZATION

Thus, we have as a result constructed a field of isotropic frames $(l,n,m,m)$ with the following properties: 1) the spin coefficients satisfy $\kappa=\varepsilon=\pi=0$; 2) (therefore) the congruence $l$ is geodesic with affine parametrization; 3) [also by virtue of 1)] the vectors $n,m,\bar{m}$ are covariantly constant along the isotropic geodesics $l$. This last gives the key to the solution of our main problem. Namely, let the observer at $P$ detect radiation in the direction $(\theta,\phi)$ on his local celestial sphere. For analysis of the observations, we then choose the congruence (2.5) with central ray $l=k$ connecting the point $P$ with the center of the radiation source $S$. At an arbitrary point, for example, at $P$, we make the polarization vector $f^\mu$ coincide with $m^\mu$; this ensures that $f^\mu = m^\mu$ holds everywhere on the ray $l=k$. Thus, the change in the orientation of the image of the source will, by the construction of the frame $(l,n,m,\bar{m})$, be described by the angle between the polarization vector and the direction of the principal axis of the image.

It is well known that the characteristics of the image of an object (orientation, shape, size) are determined by the optical scalars. We suppose that we cut out from the congruence (2.5) a thin beam of rays with central geodesic $l=k$. Suppose that at the point $x^\mu(s_1)$ with value of the affine parameter $s=s_1$ on the central ray $x^\mu(s)$ the cross section of the beam is the two-dimensional area with boundary

$$\zeta^\mu(s_1,\varphi) = \zeta(\varphi)\bar{m}^\mu + \zeta^*(\varphi) m^\mu,$$

with parameter range $0 \leq \varphi \leq 2\pi$. More concretely, we shall be interested in the case of an elliptical image. Without loss of generality, we can assume that at $x^\mu(s_1)$ the ellipse with principal semiaxes $a$ and $b$ is oriented in such a way that the directions of the semiaxes are given by the real vectors $e_1 = (m+m)/\sqrt{2}$ and $e_2 = (m-m)/i\sqrt{2}$. Then

$$\zeta(\varphi) = \frac{1}{\sqrt{2}} \left( \frac{a+b}{2} e^{i\varphi} + \frac{a-b}{2} e^{-i\varphi} \right). \qquad (3.1)$$

For motion along the central ray, the shape, size, and orientation of the cross section of the beam (i.e., the image of the object) change. One can show[11] that at the neighboring point with affine parameter $s_2 = s_1 + \delta s$ the boundary is given by the expression

$$\zeta^\mu(s_2) = \zeta^\mu(s_1) + \nabla_\nu \; l^\mu \delta s \zeta^\nu(s_1),$$

from which we find

$$\zeta \to \zeta' = \zeta + \delta\zeta, \quad \delta\zeta = -(\rho\zeta + \sigma\zeta^*)\delta s. \qquad (3.2)$$

Note that the transformation (2.26) does not change the spin coefficients $\rho$ and $\sigma$, so that in the isotropic frame we have constructed these quantities are given by the expressions (2.18) and (2.19), respectively.

It is convenient to write the ellipse obtained at $s_2$ from (3.1) in the form

$$\zeta'(\varphi) = \frac{1}{\sqrt{2}} (M_1 e^{i\varphi} + M_2 e^{-i\varphi}), \qquad (3.3)$$

where the complex quantities $M_{1,2} = |M_{1,2}| e^{i\chi_{1,2}}$ characterize the deformation and rotation of the image.

Comparing (3.1), (3.3), and (3.2), we find

$$|M_1| = \frac{a+b}{2} \left( 1 + \Theta\delta s - \frac{a-b}{a+b} \,\mathrm{Re}\, \sigma\delta s \right),$$

$$\chi_1 = -\left( \omega + \frac{a-b}{a+b} \,\mathrm{Im}\, \sigma \right) \delta s, \qquad (3.4)$$

$$|M_2| = \frac{a-b}{2} \left( 1 + \Theta\delta s - \frac{a+b}{a-b} \,\mathrm{Re}\, \sigma\delta s \right),$$

$$\chi_2 = -\left( \omega + \frac{a+b}{a-b} \,\mathrm{Im}\, \sigma \right) \delta s. \qquad (3.5)$$

As can be seen from (3.4) and (3.5), the deformation and change in the orientation of the image are related. Similarly, the optical scalars $\omega = \mathrm{Im}\,\rho$ and $\sigma$ contribute to both effects. We are interested mainly in the rotational effects.

We give the expression for the rotation angle $\eta$ of the principal axis of the image relative to the polarization vector:

$$\delta\eta = \frac{1}{2}(\chi_1 + \chi_2) = -\omega\delta s - \frac{a^2+b^2}{a^2-b^2} \,\mathrm{Im}\, \sigma\delta s. \qquad (3.6)$$

We find the total value by integrating along the central ray from the source $S$ to the observer $P$. However, for the subsequent physical analysis it is sufficient to consider approximate estimates of the observable effects. We use the Kristian–Sachs expansion methods,[5] in which all geometrical quantities are represented in the form of series in powers of the affine parameter $s$. We recall, in particular, that the distance $r$ between $P$ and $S$, which is evidently determined by the size of the source, is related to the affine parameter $s$ by the expansion

$$s = \frac{r}{(k^\mu u_\mu)_P} \left( 1 + \frac{1}{12} \frac{R_{\mu\nu} \; k^\mu k^\nu}{(k^\mu u_\mu)^2} r^2 + \ldots \right)_P, \qquad (3.7)$$

where $u^\mu$ is the 4-velocity of the observer, and $B_P$ describes the value of an arbitrary quantity $B$ at the point of observation $P$. For functions of the cosmological time calculated at

$P$, i.e., at the time of observation $t=t_0$, we shall also use a notation with index 0, for example, scale factor $R_0=R(t_0)$, Hubble factor $H_0=(\dot{R}/R)_P$, etc.

It follows from (2.18) and (2.19) that in the direction of the rotation axis ($\theta=0$) the optical scalars are

$$\omega_P=\Omega_0, \quad \Theta_P=H_0, \quad \sigma\equiv 0, \tag{3.8}$$

and, thus, the effect of the rotation of the plane of polarization becomes "pure" and maximal.

As we showed in Ref. 12, cosmological rotation is manifested predominantly in the direction $\theta=0$; however, in the models (1.1) with parameters

$$\frac{k}{\sigma}\ll 1 \tag{3.9}$$

rotational effects are important for all directions on the celestial sphere. Therefore, in what follows we shall consider the case (3.9).

Taking into account (3.7) and (3.9), we finally obtain from (3.6), (2.18), and (2.19) for the angle of rotation of the plane of polarization relative to the principal axis of the image of the source the expression

$$\eta=\Omega_0 r \cos\theta+O\left(\frac{k}{\sigma}\right). \tag{3.10}$$

where, as in Ref. 1, we have omitted the higher powers of the Kristian–Sachs expansion in $r$. We note remarkable agreement with the previous results [cf. (7.1) of Ref. 1]. The disagreement with the conclusions of Ref. 8 is explained, as we have already noted, by the incorrect assumption of that the properties of the so-called representative beam that forms the image of the source can be specified arbitrarily. Without considering this question in detail, we note, however, the following. An isotropic frame $(l',n',m',\bar{m}')$ with the desired properties can be obtained by means of a local Lorentz rotation, for example, from $(l,n,m,\bar{m})$ (2.5)–(2.8). The requirement of geodicity of the new congruence $l'$ and covariant constancy of the triplet $n',m',\bar{m}'$ is equivalent to the conditions that the spin coefficients be trivial: $\kappa'=0$, $\varepsilon'=0$, $\pi'=0$. These six equations (three complex equations) in general exhaust the freedom of the Lorentz transformations, which depend on six functions. Thus, in the general case it is possible to make an additional "adjustment" of the frame $(l',n',m',\bar{m}')$ to achieve, for example, vanishing of the optical scalar $\omega'=0$. In fact, in Ref. 8 there is no explicit construction of either the isotropic tetrad or the representative beam with the declared properties.

## 4. CONCLUSIONS

Thus, the main result of our paper is the confirmation of the previous conclusion of the existence in the cosmological models (1.1) of the effect of rotation of the polarization vector of the radiation of a (radio) source with respect to the principal axis of its image. The resulting dipole anisotropy (3.10) in the distribution of the rotation angle confirms the observational data of Birch.[13,14]

It is important to note that the condition (3.9) is not necessary. It is simply that in this case the effect has a purely dipole nature and can be conveniently compared with Birch's observations. It is clear from (2.18) and (2.19) that the dipole component is also dominant in the effect if (3.9) is not satisfied, but certain distortions are superimposed on this component and they ultimately lead to a more complicated picture of the observed angular distribution. Thus, our result should not be regarded as an argument "for" or "against" the observational data of Refs. 13 and 14. In contrast, the results of Birch and new astrophysical observations will ultimately help to choose a correct cosmological model with rotation and estimate its parameters $(k,\sigma,m)$.

We make some concluding remarks.

If the source originally has a circular profile, then obviously (3.6) becomes meaningless. In this case, (3.1) becomes

$$\zeta=ae^{i\varphi}, \tag{4.1}$$

but at $s+\delta s$ the observer will still see the ellipse (3.3). Instead of (3.4) and (3.5), we find

$$|M_1|=1+a\Theta\,\delta s, \quad \chi_1=-\omega\delta s, \tag{4.2}$$

$$|M_2|=a|\sigma|, \quad \chi_2=\pi+\arg\sigma, \tag{4.3}$$

and, therefore, the angle of rotation of the polarization vector with respect to the observed principal axis of the source is

$$\delta\eta=-\frac{1}{2}\omega\delta s+\frac{\pi}{2}+\frac{\arg\sigma}{2}. \tag{4.4}$$

As we see, for spherical sources the picture of the effect seen by the observer at $P$ does not differ qualitatively from the one described above.

The distortion of the image is also interesting in rotating cosmological models. However, in the metric (1.1), and *a fortiori* in the case (3.9), it is not significant, as an additional analysis of the expression (2.19) shows.


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Translated by Julian B. Barbour