Surface magnetic field generation by fast moving charged particles

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We calculate the surface magnetic field generated near a metal surface by a fast moving charged particle. Numerical results are given for the case where a proton moves parallel to an aluminum surface. © *1995 American Institute of Physics.*

1. INTRODUCTION

Surface electromagnetic waves generated by fast moving charged particles has been studied by many authors, starting with Ritchie.[1] It is believed (see, e.g., Ref. 2) that surface plasma waves with an electric field described by a scalar potential φ are generated on the surface of the metal. A more detailed analysis (see, e.g., Ref. 3 and the cited literature therein) shows that such a description is only approximate, since it ignores the retardation effect. Retardation considerably changes the nature of the surface waves as $\lambda \to \infty$, but if λ does not exceed the wavelength $\lambda_{\rm ph}$ of a plasma-frequency photon, allowing for retardation in describing surface electromagnetic oscillations yields only small corrections. Hence it is clear that the only phenomena that occur at distances large compared to $\lambda_{\rm ph}$ are sensitive to a time lag in electromagnetic signals. Specifically, the potential of the wake polarization field of a moving charged particle undergoes changes on distance greater than $\lambda_{\rm ph}$. Bohr[4] was the first to show that the contributions to the stopping power of matter from near and far collisions is approximately the same. Since a Coulomb field is long-range, distances greater than $\lambda_{\rm ph}$ contribute considerably to deceleration of particles, and one should expect certain modifications in the formulas for the contribution of surface modes to the stopping power. The most important corollary of retardation is the generation of a surface magnetic field, which can play an important role in the polarization of elementary particles, atoms, and molecules in their movement near the surface. In this paper we analyze in detail the magnetic field generated by a charged particle moving with a constant velocity parallel to a surface.

2. ELECTRIC AND MAGNETIC FIELDS INDUCED BY A CHARGED PARTICLE MOVING NEAR A SURFACE

The mean fields that appear near a surface thanks to generation of surface polaritons by a moving particle were calculated in Ref. 5. Let the z axis of the coordinate system be directed from the metal into a vacuum at right angles to the metal surface $z=0$. The general expression for the mean magnetic field at the point $\mathbf{x}=(\mathbf{r},z)$ has the form

$$\mathbf{H}(\mathbf{x},t)=\sum_{\mathbf{q}_\parallel}\mathbf{H}_0\cdot 2\,\mathrm{Re}\{Q_{\mathbf{q}_\parallel}(t)\exp[-i\omega_s t+i\mathbf{x}\mathbf{q}(z)]\}. \quad (1)$$

Here $\omega_s\equiv\omega_s(\mathbf{q}_\parallel)$ is the surface polariton energy (we use natural units here), $\mathbf{q}_\parallel$ is the polariton momentum, and $\mathbf{q}(z)=(\mathbf{q}_\parallel,i\tilde{\kappa}(z)\cdot\mathrm{sign}(z))$, with $\tilde{\kappa}(z)=\kappa_1\theta(z)+\kappa_2\theta(-z)$, where $\theta(z)$ is the Heaviside step function, and the electromagnetic field damping factors in the vacuum, κ_1, and in the metal, κ_2, are

$$\kappa_1=\sqrt{q_\parallel^2-\omega_s^2/c^2},\quad \kappa_2=\sqrt{q_\parallel^2-\varepsilon(\omega_s)\omega_s^2/c^2},$$

with $\varepsilon(\omega)$ the metal's dielectric constant on the assumption that spatial dispersion is negligible, and c the speed of light. The functions $Q_{\mathbf{q}_\parallel}(t)$ are determined by the following integrals of the electric current density $j(\mathbf{x},t)$ of the moving particle:

$$Q_{\mathbf{q}_\parallel}(t)=\frac{i}{c}\int_{-\infty}^{t}dt'\int d^3x'\,\mathbf{g}_{\rm tr}^*(\mathbf{q}_\parallel,z')\mathbf{j}(\mathbf{x}',t')\times\exp[i\omega_s t'-i\mathbf{q}_\parallel\mathbf{r}'-\tilde{\kappa}(z')|z'|]. \quad (2)$$

Here $\mathbf{g}_{\rm tr}(\mathbf{q}_\parallel,z)$ are the vector coupling constants of the external transport particles and the surface polaritons:

$$\mathbf{g}_{\rm tr}(\mathbf{q}_\parallel,z)=i\frac{c^2}{\omega_s^2\varepsilon(\omega_s,z)}[\mathbf{q}(z)\mathbf{H}_0],$$

where $\varepsilon(\omega_s,z)=\theta(z)+\varepsilon(\omega_s)\theta(-z)$. The real vector $\mathbf{H}_0$ is perpendicular to the surface and to the plasmon momentum in such a way that the triad $(\mathbf{H}_0,\mathbf{q}_\parallel,\mathbf{e}_z)$ forms a right-handed system of perpendicular vectors. The magnitude of $\mathbf{H}_0$ is

$$H_0=\left(\frac{8\pi\omega_s^3}{Sc^2}\right)^{1/2}\left\{\frac{2q_\parallel^2}{\kappa_2\varepsilon(\omega_s)}+\frac{2q_\parallel^2}{\kappa_1}+\frac{\omega_s}{\kappa_2\varepsilon_2^2(\omega_s)}\times\left[2q_\parallel^2-\frac{\omega_s^2}{c^2}\,\varepsilon_s(\omega_s)\right]\frac{d\varepsilon_2(\omega_s)}{d\omega_s}\right\}^{-1/2}, \quad (3)$$

where S is the normalization surface area of the interface. The dispersion dependence $\omega_s(q_\parallel)$ in Eqs. (1)–(3) can be found from the equation $\kappa_1=-\kappa_2/\varepsilon(\omega)$ and has the property that at a certain critical value $q_\parallel=q_c$ the curve representing the $q_\parallel$-dependence of ω_s cuts off. The value q_c is the maximum momentum of surface polaritons. In this paper we determine q_c from the point of intersection of the ω_s vs. $q_\parallel$ curve with the edge of the band of electron–hole excitations of the metal's electron gas, $\omega=q_\parallel(q_\parallel+2p_{\rm F})/2$, where $p_{\rm F}$ is the Fermi momentum. The critical surface-plasmon momentum was found in a similar way by Wirkborg and Inglesfield.[6]

In the special case in which a point charge Z_1 moves at right angles to the surface along a path $\mathbf{x}_0(t)=-\mathbf{e}_z vt$, the surface magnetic field generated by that charge has, in cylin-

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drical coordinates, only a φ-component. After replacing the sum in Eq. (1) by an integral we get for this case

$$H_\varphi=-\frac{Z_1 vcS}{\pi}\int_0^{q_c}dq_\parallel\frac{q_\parallel H_0^2}{\omega_s^2}J_1(q_\parallel r)$$
$$\times\exp[-\tilde{\kappa}(z)|z|]\int_{-\infty}^{t}dt'\frac{\sin(\omega_s(t-t'))}{\varepsilon(\omega_s-vt')}$$
$$\times\exp[-\tilde{\kappa}(-vt')v|t'|], \qquad (4)$$

where $J_1(\xi)$ is the first-order Bessel function. The field H_φ, as expected, is continuous at the boundary $z=0$ and is oriented according to the right-hand screw rule. After the particle has crossed the surface, surface field-strength vibrations are induced. Since $\kappa_2>\kappa_1$, the field in the medium decays faster than in the vacuum.

3. THE FIELD GENERATED BY A PARTICLE MOVING PARALLEL TO THE SURFACE

Suppose that a point-like charged particle moves with a constant velocity v in a vacuum at a fixed distance a from the surface of the metal. We point the y axis along the direction of v. Then

$$\mathbf{H}=-Z_1\frac{v}{c}\int\frac{dq_x\,dq_y}{(2\pi)^2q_\parallel}\theta(q_c-q_\parallel)K(q_\parallel,z)q_y[\mathbf{q}_\parallel\mathbf{e}_z]$$
$$\times\left\{\frac{\exp(iq_x x+iq_y y')}{\omega_s-q_y v-i0}+\frac{\exp(-iq_x x-iq_y y')}{\omega_s-q_y v+i0}\right\}, \qquad (5)$$

where $y'=y-vt$, we have put $S=1$, and $K(q_\parallel,z)=\exp\{-\kappa_1 a-\tilde{\kappa}(z)|z|\}c^2\kappa_1H_0^2/\omega_s q_\parallel$.

At low velocities $v<v_c$, where $v_c=\omega_s(q_c)/q_c$, Eqs. (5) become

$$H_x=-\frac{2Z_1 v}{\pi^2 c}\int_0^{q_c}dq_\parallel\omega_s K(q_\parallel,z)\int_0^{q_\parallel}$$
$$\times dq_x\sqrt{q_\parallel^2-q_x^2}\frac{\cos(xq_x)\cos(y\sqrt{q_\parallel^2-q_x^2})}{\omega_s^2-v^2(q_\parallel^2-q_x^2)}, \qquad (6)$$

$$H_y=-\frac{2Z_1 v}{\pi^2 c}\int_0^{q_c}dq_\parallel\omega_s K(q_\parallel,z)\int_0^{q_\parallel}dq_y q_y$$
$$\times\frac{\sin(y'q_y)\sin(x\sqrt{q_\parallel^2-q_y^2})}{\omega_s^2-q_y^2v^2}. \qquad (7)$$

Above the critical velocity one must allow for the pole, which yields the following expression for both components:

$$H_\alpha=\theta(y')\Phi_\alpha(x,y';v)+\theta(-y')[\Phi_\alpha(x,-y';-v)$$
$$+\theta(v-v_c)\frac{2Z_1}{\pi v^2 c}\int_0^{\eta_c}\frac{\omega_s^2K(q_\parallel,z)d\eta}{q_\parallel-(\omega_s/v^2)(d\omega_s/dq_\parallel)}$$
$$\times\Psi_\alpha(x,y')], \qquad (8)$$

where

$$\Psi_x(x,y)=\cos(x\eta)\sin\left(\frac{y\omega_s}{v}\right),$$

$$\Psi_y(x,y)=-\frac{v\eta}{\omega_s}\sin(x\eta)\cos\left(\frac{y\omega_s}{v}\right).$$

In Eqs. (8) with $y'>0$,

$$\Phi_\alpha(x,y';v)=-\frac{2Z_1 v}{\pi^2 c}\int_0^{q_c}dq_\parallel K(q_\parallel,z)\mathrm{Re}\left\{i\int_0^\infty d\xi\right.$$
$$\times\exp(-y'\xi^2)[X_\alpha(x,y',q_\parallel)-X_\alpha(x,y',$$
$$\left.-q_\parallel)]\right\},$$

where

$$X_x(x,y,q_\parallel)=\frac{(q_\parallel-i\xi^2)^2}{\sqrt{\xi^2+2iq_\parallel}}\frac{\cos(x\xi\sqrt{\xi^2+2iq_\parallel})}{\omega_s+q_\parallel v-iv\xi^2}$$
$$\times\exp(-iq_\parallel y),$$

$$X_y(x,y,q_\parallel)=\xi(\xi^2+iq_\parallel)\frac{\sin(x\xi\sqrt{\xi^2+2iq_\parallel})}{\omega_s+q_\parallel v-iv\xi^2}$$
$$\times\exp(-iq_\parallel y).$$

The integrals in Eqs. (8) are the pole contributions, with the variable η related to the variable $q_\parallel$ through the relationship $\eta(q_\parallel)=\sqrt{q_\parallel^2-\omega_s^2/v^2}$, and $\eta_c=\eta(q_c)$. Accordingly, all functions of $q_\parallel$ must be expressed as functions of $q_\parallel(\eta)$. In calculating the integrals the values of square roots are taken according to the convention adopted in the FORTRAN programming language, i.e., the real part of the root is assumed

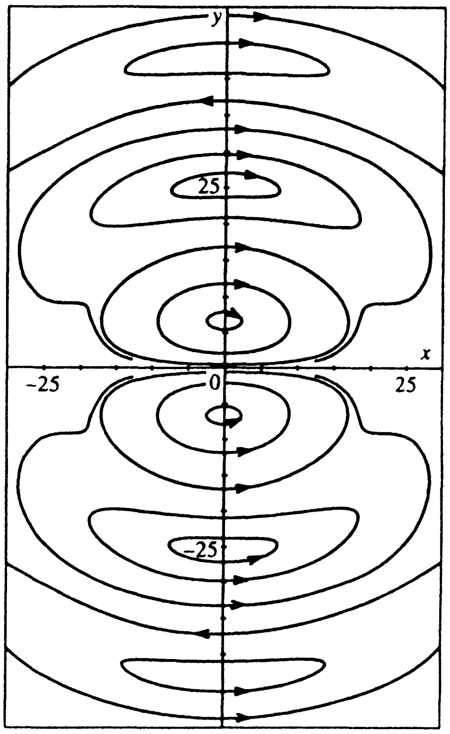

FIG. 1. Magnetic lines of force on the surface of aluminum that are generated as a result of a proton moving with a velocity $v=1$ nat.un. ($v<v_c$) at a distance of 1 nat.un. from the surface. The proton is moving in a vacuum along the y axis above the origin of coordinates.

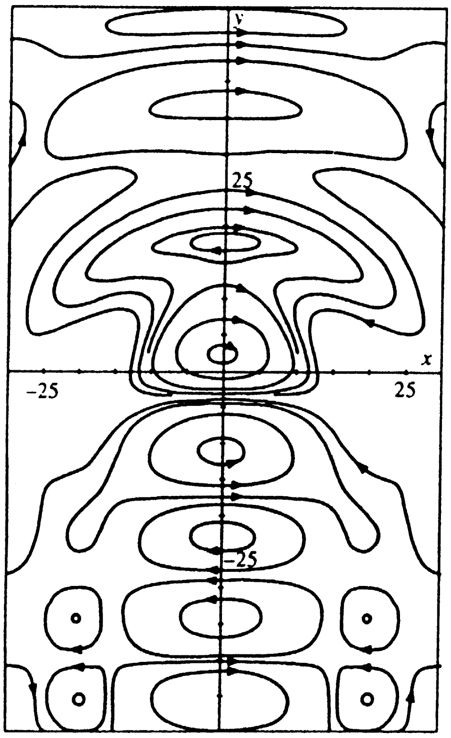

FIG. 2. The same as in Fig. 1 but at $v = 1.5$ nat.un. ($v > v_c$).

positive. What is also taken into account is that for $y' < 0$ we must go, in the pole contribution, to the lower edge of the cut, where the root $(q_\parallel^2 - q_y^2)^{1/2}$ has a negative value. Numerical calculations carried out independently using Eqs. (6)–(8) yielded results in the $v < v_c$ range that coincided to within the given computer accuracy. By applying the Maxwell equations to the expressions obtained for the magnetic field one can directly obtain equations for calculating the electric field components and components of the forces acting on the moving particle.

4. NUMERICAL RESULTS AND DISCUSSION

In the vicinity of the metal–vacuum interface the lines of magnetic force are closed contours with planes parallel to the surface. In carrying out specific calculations we assumed that $\varepsilon(\omega) = 1 - \omega_0^2/\omega^2$, where ω_0 was taken equal to the plasma frequency in the aluminum electron gas. At low velocities $v < v_c$, the lines of force turned out to be symmetric under reflection in the plane $y' = 0$. The reason is that surface polaritons are not excited in this velocity range. But if $v > v_c$, the reflection symmetry in the $y' = 0$ plane is broken, and wake oscillations of the system of lines of magnetic force develop. Figure 1 depicts the lines of force for $v < v_c$ in the case of a proton traveling at a distance of 1 (in natural units) from an aluminum flat surface with a velocity of 1.5. Figure 2 depicts similar results for $v = 1.5$ (the case of $v > v_c$). Note the breakdown of the symmetry under velocity inversion for $v > v_c$ and the formation and breaking away of additional cycles in the wake of the particle. For $v < v_c$ the Lorentz force proves to be directed out of the metal and into the vacuum, pushing the particle away from the surface. This

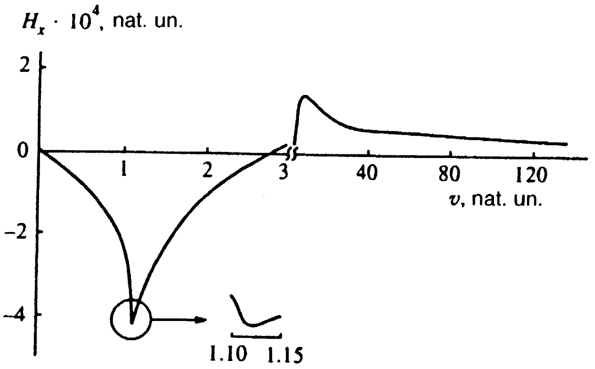

FIG. 3. The polarization magnetic field strength at a proton as a function of the proton's velocity. The proton is moving above an aluminum surface at $a = 1$ nat.un..

fact becomes clear if it is considered a result of the interaction of two oppositely directed currents, the moving change and its image.

Building the pattern of the lines of magnetic force prompted the development of an algorithm that took into account the features of the problem: the large cost of computer time required to calculate the magnetic-field components H_x and H_y, and the need of building lines of forces with a high degree of accuracy. The efficient Runge–Kutta–Merson method, with automatic choice of step size, was used to numerically integrate the differential equation $dy/dx = f(x,y) = H_y/H_x$. The initial conditions were chosen in the form $y(0) = 0$ and $y'(0) = 0$, which was achieved as a result of translation and rotation of the system of coordinates before each step along a line of force.

Figure 3 depicts the behavior of the magnetic field at the proton location as a function of proton velocity. Note the characteristic dip in the vicinity of the critical velocity. In the region of still higher velocities the Lorentz force changes sign. Here the component of the polarization-field energy flux parallel to the surface opposes the particle's motion, having a stopping effect on the particle. The change in sign of the Lorentz force at high particle velocities can be explained by the high inertia of the electron system of a solid, in view of which the image charge turns out to lag the real charge. The electron velocities in the surface region directly under the particle prove to be directed in opposition to **v**. The induced polarization current that forms the image flows parallel to the proton current and is the cause of Lorentz attraction to the surface.

[1]R. H. Ritchie, Phys. Rev. **106**, 874 (1957).
[2]P. M. Platzman and P. A. Wolff, *Waves and Interactions in Solid State Plasma*, Academic Press, New York (1973).
[3]V. M. Agranovich and D. L. Mills (eds.), *Surface Polaritons*, North-Holland, Amsterdam (1982).
[4]N. Bohr, "The penetration of atomic particles through matter," Danske Vid. Selsk. Mat.-Fyz. Medd. **18**, No. 8 (1948).
[5]G. M. Filippov, Izv. Vyssh. Uchebn. Zaved., Fiz. No. 1, 94 (1995).
[6]E. Wirkborg and J. E. Inglesfield, Phys. Scripta **15**, 37 (1977).

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