Stimulated scattering of high-intensity radiation in a plasma

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(Submitted 27 October 1994)
Zh. Éksp. Teor. Fiz. **108**, 182–192 (July 1995)

Stimulated Mandel'shtam–Brillouin scattering of a powerful pump wave ($\gamma_0 > \kappa v_s$, where γ_0 is the instability growth rate, κ is the wave vector of the low-frequency perturbation, and v_s is the ion-sound velocity) is studied under conditions in which dissipation and extraction of the energy of the modified low-frequency wave from the interaction region have no time to take effect. Expressions are derived for the amplitudes of the waves in the linear scattering stage, both in the presence of an initial frequency offset $\Delta\omega$ between the high-frequency waves and in the absence of such an offset. It is shown that the solution at $\Delta\omega=0$ is self-similar for times shorter than $1/\kappa v_s$, before the wave properties of internal motion in in density perturbations of the medium come into play, and the rate at which instability develops is at its highest. Experimental data are interpreted on the basis of the theoretical ideas developed.

The development of more and more powerful sources of electromagnetic radiation of different frequency ranges imposes new demands on the theory of interaction of such radiation with matter. One of the most important effects accompanying the propagation of a powerful wave in a medium is stimulated scattering and, in particular, stimulated Mandel'shtam–Brillouin scattering (SMBS). The effects are widely discussed in the literature devoted to high-temperature laser plasma, in which various high-intensity radiations often affect one another, producing a complex scattering pattern.[1–3] On the other hand, presently available high-power sources of microwave radiation—free-electron lasers—make it possible to cleanly initiate the process fairly in a large-scale laboratory plasma of a given concentration.[4,5]

The modified decay of a high-frequency wave into a similar wave and sound in infinite plasma has been thoroughly studied.[6] In a layer, such a backscattering process has been studied mainly numerically.[7,8] Below we examine it in an approximation with a given field of the pump wave, and make some estimates for the nonlinear stage.

We begin with the following system of equations for the complex-valued amplitude of a quasimonochromatic electric field of high-frequency electromagnetic waves, $\mathbf{E}(t,x)\exp^{-i\omega_0 t}$, and the perturbation of the concentration n of a homogeneous plasma (both quantities are assumed to vary slowly with time):

$$2i\,\frac{\omega_0}{c^2}\frac{\partial \mathbf{E}}{\partial t}+\frac{\partial^2\mathbf{E}}{\partial x^2}+\frac{\omega_0^2}{c^2}\,\varepsilon\mathbf{E}=0,$$
$$\frac{\partial^2 n}{\partial t^2}-v_s^2\,\frac{\partial^2 n}{\partial x^2}=\frac{N_0}{16\pi N_{\rm cr}M}\,\frac{\partial^2}{\partial x^2}|\mathbf{E}(t,x)|^2, \qquad (1)$$

with ω_0 the pumping frequency, $\varepsilon=1-(N_0+n)/N_{\rm cr}$ the plasma's dielectric constant, N_0 and $N_{\rm cr}=m\omega_0^2/4\pi e^2$ the unperturbed and critical concentrations, and m and M the electron and ion masses. The right-hand side of the equation for n describes the effect of the averaged ponderomotive force of high-frequency radiation pressure, and dissipation is ignored.[1)]

Substituting into Eqs. (1) the solution for $\mathbf{E}$ in the form of the sum of the impinging wave $\mathbf{E}_0,\mathbf{k}_0$ and the reflected wave $\mathbf{E}_1,\mathbf{k}_1$ and the solution for the perturbation n in spatial synchronism with these waves,

$$E(t,x)=E_0e^{-ik_0x}+E_1(t,x)e^{ik_1x},$$
$$n(t,x)=\tilde{n}(t,x)e^{-i\kappa x}+\tilde{n}^*(t,x)e^{i\kappa x}, \qquad (2)$$
$$\boldsymbol{\kappa}=\mathbf{k}_0-\mathbf{k}_1,\quad \mathbf{k}_0\simeq-\mathbf{k}_1,\quad \mathbf{E}_0\|\mathbf{E}_1$$

($\mathbf{k}_{0,1}$, and $\boldsymbol{\kappa}$ are the corresponding wave vectors), we arrive at the following equations for the amplitudes E_1 and $\tilde{n}$, assuming that these quantities change little on the scale $2\pi/k_0$:

$$\frac{\partial E_1}{\partial x}=i\alpha E_0\tilde{n}^*,\quad \frac{\partial^2\tilde{n}}{\partial t^2}+\omega_s^2\tilde{n}=\beta E_0E_1^*, \qquad (3)$$

where

$$\alpha=-\frac{\omega_0^2}{c^22k_1N_{\rm cr}},\quad \beta=-\frac{\kappa^2N_0}{16\pi MN_{\rm cr}}$$

are the coefficients of nonlinear interaction, $\omega_s=\kappa v_s$, and $E_0=\text{const}$. In the first equation in (3) we have ignored the term with $\partial E_1/\partial t$, since we assumed that the time that it takes the scattered fast wave to travel through the layer is much shorter than the characteristic time that it takes for an instability to develop.

The boundary and initial conditions imposed on the system of equations (3) are assumed to be

$$E_1(t,0)=\mathscr{E}e^{i\Delta\omega t},\quad \tilde{n}(0,x)=0,\quad \tilde{n}_t'(0,x)=0 \qquad (4)$$

($\Delta\omega=\omega_0-\omega_1$ is the initial frequency offset between the interacting high-frequency waves), bearing in mind that in our case (high fields), in contrast to ordinary SMBS, instability can also develop from initial fields unshifted in frequency in relation to the pump field, and in particular , when this field is partially reflected in a linear way from regions beyond the layer (e.g., a region with critical concentration, the chamber walls, etc.).[2)]

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The system of second-order partial differential equations (3) can be analyzed by a method similar to the one used in analyzing a similar (first-order) system for ordinary SMBS,[10,11]—using the Laplace transform, for example. But we start with a simpler case where we neglect the term $\omega_s^2\tilde{n}$ (short times) in (3) and put $\Delta\omega=0$ in (4). The equations then resemble those describing time-dependent stimulated temperature scattering (and SMBS, apart from the factor i preceding the coefficient α), the only difference being that the new equations contain a higher-order derivative of $\tilde{n}$. Hence, in $t(3)$ we might expect self-similar solutions analogous to the existing solutions of the processes cited above (see, e.g., Ref. 12).[3)] Indeed, it can easily be verified that in this case the self-similar substitution $b=\tilde{n}/t^2$ and $\xi=xt^2$ transforms (3), in accordance with (4), to the following equation:

$$4\xi^2\frac{d^3E_1}{d\xi^3}+10\xi\frac{d^2E_1}{d\xi^2}+2\frac{dE_1}{d\xi}-iAE_1=0, \tag{5}$$

where $A=\alpha\beta|E_0|^2$. All necessary initial conditions for this equation can be obtained from (5) and from the fact that $E_1(\xi=0)=\mathscr{E}$ [see (4)], which in fact is the only condition needed in what follows. As in Ref. 12, we seek an asymptotics solution of Eq. (5) in the form

$$E_{1\infty}=\mu(\xi)\exp(\sqrt[3]{iqA\xi}),\quad \sqrt[3]{qA\xi}\gg 1, \tag{6}$$

where $\mu(\xi)$ is a weakly varying factor, and q is a constant. Substituting (6) into Eq. (5) and keeping only the first derivative $d\mu/d\xi$, we arrive by successive approximation at the following:

$$\mu(\xi)=\frac{\alpha}{\sqrt[6]{qA\xi}},\quad q=\frac{27}{4}. \tag{7}$$

Here the constant α can obviously be determined only from the exact solution, which for this type of equations can be found by expanding E_1 in the neighborhood of the regular singular point ξ_1 (for Eq. (5), $\xi_1=0$) (see Ref. 14):

$$E_1=\sum_{\nu=0}^{\infty}\alpha_\nu(\xi-\xi_1)^\nu.$$

The coefficients α_ν can be determined successively via recursion relations (α_0 is determined by the initial value of $\mathscr{E}$) obtained by substituting this series into Eq. (5). As a result we get

$$E_1=\mathscr{E}\sum_{\nu=0}^{\infty}\frac{(iA\xi)^\nu}{\nu!(2\nu!)}, \tag{8}$$

which after matching (8) with (6) for large ξ yields

$$\alpha\approx\mathscr{E}(0.26-i\cdot 0.07). \tag{9}$$

Now we examine a more general case, the complete system of equations (3), but with $\Delta\omega=0$ in (4). Normalizing the terms in the second equation in (3) to ω_s^2 and then applying the Laplace transformation to the functions of the new time variable $\tilde{t}=\omega_s t$, from the resulting system of equations combined with (4) we determine the image function F of the amplitude of the scattered wave, which we write here as

$$F(p,x)=\frac{\mathscr{E}}{p}\exp\left[\frac{iAx}{\omega_s^2}\left(1-\frac{1}{1+1/p^2}\right)\right]. \tag{10}$$

We express the exponential function by a Taylor series in powers of $z\equiv p^{-2}$ in the neighborhood of the point $z=0$. As a result, Eq. (10) acquires the following form

$$\begin{aligned}F(p,x)&=\frac{\mathscr{E}}{p}\left[1+\sum_{\nu=1}^{\infty}\frac{B_\nu}{\nu!\,p^{2\nu}}\right]=\frac{\mathscr{E}}{p}\left\{1+\frac{iAx}{\omega_s^2}\frac{1}{p^2}\right.\\&+\frac{1}{2}\left[\left(\frac{iAx}{\omega_s^2}\right)^2-2\frac{iAx}{\omega_s^2}\right]\frac{1}{p^4}+\frac{1}{3!}\left[\left(\frac{iAx}{\omega_s^2}\right)^3\right.\\&\left.-6\left(\frac{iAx}{\omega_s^2}\right)^2+6\frac{iAx}{\omega_s^2}\right]\frac{1}{p^6}+\frac{1}{4!}\left[\left(\frac{iAx}{\omega_s^2}\right)^4-12\left(\frac{iAx}{\omega_s^2}\right)^3\right.\\&\left.\left.+36\left(\frac{iAx}{\omega_s^2}\right)^2-24\frac{iAx}{\omega_s^2}\right]\frac{1}{p^8}+\cdots\right\},\end{aligned} \tag{11}$$

$$B_\nu=\sum_{j=1}^{\nu}\left(\frac{iAx}{\omega_s^2}\right)^j c_{j,\nu},\quad c_{j,j}=1,\quad c_{j-1,j}=-j(j-1),$$

$$c_{1,j}=(-2)(-3)\cdots(-j),$$

$$c_{j-h,j}=-(2j-h-1)c_{j-h,j-1}+c_{j-h-1,j-1},\quad h\geqslant 1,$$

$$c_{j,h}=0\quad\text{if } j<1 \text{ or } j>h.$$

Now we can use the expansion theorem[15] to go from $F(p,x)$ to the original function $E_1(t,x)$.

$$\begin{aligned}E_1(t,x)&=\mathscr{E}\left\{1+\sum_{\nu=1}^{\infty}\frac{B_\nu(\omega_s t)^{2\nu}}{\nu!(2\nu!)}\right\}=\mathscr{E}\left\{1+\frac{1}{2}\frac{iAx}{\omega_s^2}(\omega_s t)^2\right.\\&+\frac{1}{2\cdot(4!)}\left[\left(\frac{iAx}{\omega_s^2}\right)^2-2\frac{iAx}{\omega_s^2}\right](\omega_s t)^4\\&+\frac{1}{3!\cdot 6!}\left[\left(\frac{iAx}{\omega_s^2}\right)^3-6\left(\frac{iAx}{\omega_s^2}\right)^2+6\frac{iAx}{\omega_s^2}\right](\omega_s t)^6\\&+\frac{1}{4!\cdot 8!}\left[\left(\frac{iAx}{\omega_s^2}\right)^4-12\left(\frac{iAx}{\omega_s^2}\right)^3+36\left(\frac{iAx}{\omega_s^2}\right)^2\right.\\&\left.\left.-24\frac{iAx}{\omega_s^2}\right](\omega_s t)^8+\cdots\right\}.\end{aligned} \tag{12}$$

Clearly, if we allow only for the first terms within the square brackets, the series (12) degenerates into (8). This requirement in fact means that we are considering times for which $\omega_s t\lesssim 1$. Inclusion of the other terms in (12) leads to a slowing down in instability development, just as the term $\omega_s^2\tilde{n}$ in the equation for the density in (3) in the infinite-plasma model reduces the growth rate of the modified decay.[6] Moreover, note that this analogy makes it possible to simplify Eq. (12), for instance, in the case discussed below of powerful short radiation pulses in which $\omega_s t > 1$ is still not too large. Indeed, if in this case we write the solution in the form (6), we arrive at the following equation for the slowly varying pre-exponential factor $\mu(t,x)$ (which is no more self-similar):

$$\frac{2}{9}(iqA)^{2/3}t^{-2/3}x^{-1/3}\left[\mu+2t\frac{\partial\mu}{\partial t}+2x\frac{\partial\mu}{\partial x}\right]$$
$$+\omega_s^2\left[\frac{1}{3}(iqA)^{1/3}t^{2/3}x^{-2/3}\mu+\frac{\partial\mu}{\partial x}\right]=0. \qquad (13)$$

This equation has a solution in the form $\mu(t,x)=\mu_1(t,x)\mu_2(t,x)$, where $\mu_1\equiv\mu(\xi)$ of Eq. (7) is the solution of Eq. (13) as $\omega_s\to 0$, and μ_2 can be found by the following procedure. According to Eq. (6), the "instantaneous" increment of the given instability is

$$\Gamma(t,x)\sim\frac{\partial E_1}{\partial t}\Big/E_1\sim\frac{2}{3}\frac{(iqAx)^{1/3}}{t^{1/3}}\left[1-\frac{1}{2(iqAxt^2)^{1/3}}\right]. \qquad (14)$$

We now express in terms of this increment the slowing down of the process, assuming the process to have the same form as in the time-dependent problem (infinite plasma):

$$\mu_2=\exp\left(-\sigma\frac{\omega_s^2}{\Gamma(t,x)}t\right)\approx\left[1-\frac{3}{4}\sigma\frac{\omega_s^2t^2}{(iqAxt^2)^{2/3}}\right]$$
$$\times\exp\left[-\frac{3}{2}\sigma\frac{\omega_s^2t^2}{(iqAxt^2)^{1/3}}\right], \qquad (15)$$

where σ is a constant, and we have assumed that

$$\frac{\omega_s^2t^2}{(qAxt^2)^{2/3}}\ll 1. \qquad (16)$$

Clearly, the product $\mu_1\mu_2$ at $\sigma=\frac{1}{2}$ satisfies Eq. (13) if (16) holds. However, the same product approximately satisfies Eq. (13) for other fairly arbitrary constants σ in front of the expression in the square brackets in (15). Comparison with the exact solution (the present case with (12)) makes it possible to select the appropriate constant. As a result we get

$$E_1(t,x)\approx\frac{\mathscr{E}(0.26-0.07i)}{(6.75Axt^2)^{1/6}}\left[1+\frac{2.5(0.87-0.5i)^2\omega_s^2t^2}{(6.75Axt^2)^{2/3}}\right]$$
$$\times\exp\left\{(6.75Axt^2)^{1/3}\left[0.87\left(1-\frac{0.75\omega_s^2t^2}{(6.75Axt^2)^{2/3}}\right)\right.\right.$$
$$\left.\left.+0.5i\left(1+\frac{0.75\omega_s^2t^2}{(6.75Axt^2)^{2/3}}\right)\right]\right\}, \qquad (17)$$

where we have set $q=6.75$ (see Eq. (7)). This function serves as a good approximation of the series (12) for times obeying condition (16). Note that the slowness of variation of the function $\mu(t,x)$ needed for the approximation (17) to be correct is consistent with (16), as can easily be seen from (16).

Finally, we examine the situation where $\Delta\omega\neq 0$ in (4). The approach is the same as in the previous case. The image function F differs from (10) in its pre-exponential factor, which is actually the image of the boundary condition (4) ($x=0$). The modification is

$$\frac{\mathscr{E}}{p}\to\frac{\mathscr{E}}{p-i\Delta\omega/\omega_s}=\frac{1}{1-i\Delta\omega/p\omega_s}\frac{\mathscr{E}}{p} \qquad (18)$$

(note that here we have also gone to the new time variable $\tilde{t}=\omega_st$). Expanding the first factor in (18) powers of $(i\Delta\omega/p\omega_s)^\nu$ in the neighborhood of $p=\infty$, and multiplying the latter with the series (11), we also arrive at a series representation of $F(p,x)$

$$F(p,x)=\frac{\mathscr{E}}{p}\left[1+\sum_{\nu=1}^{\infty}\frac{\mathscr{D}_\nu}{p^\nu}\right]=\frac{\mathscr{E}}{p}\left\{1+\frac{i\Delta\omega}{\omega_s}\frac{1}{p}+\left[\left(\frac{i\Delta\omega}{\omega_s}\right)^2\right.\right.$$
$$\left.+\frac{iAx}{\omega_s^2}\right]\frac{1}{p^2}+\left[\left(\frac{i\Delta\omega}{\omega_s}\right)^3+\frac{iAx}{\omega_s^2}\frac{i\Delta\omega}{\omega_s}\right]\frac{1}{p^3}$$
$$+\left[\left(\frac{i\Delta\omega}{\omega_s}\right)^4+\frac{iAx}{\omega_s^2}\left(\frac{i\Delta\omega}{\omega_s}\right)^2+\frac{1}{2}\left(-\frac{2iAx}{\omega_s^2}\right.\right.$$
$$\left.\left.+\left(\frac{iAx}{\omega_s^2}\right)^2\right)\right]\frac{1}{p^4}+\left[\left(\frac{i\Delta\omega}{\omega_s}\right)^5+\frac{iAx}{\omega_s^2}\left(\frac{i\Delta\omega}{\omega_s}\right)^3\right.$$
$$\left.\left.+\frac{1}{2}\left(-\frac{2iAx}{\omega_s^2}+\left(\frac{iAx}{\omega_s^2}\right)^2\right)\frac{i\Delta\omega}{\omega_s}\right]\frac{1}{p^5}+\dots\right\}, \qquad (19)$$

where

$$\mathscr{D}_\nu=\left(\frac{i\Delta\omega}{\omega_s}\right)^\nu+\sum_{h=1}^{[\nu/2]}\frac{B_h}{h!}\left(\frac{i\Delta\omega}{\omega_s}\right)^{\nu-2h},$$

with the coefficients B_h given in (11), and the square brackets in the upper summation limit denoting the integral part of the number within the brackets.

From (19), by the expansion theorem yields

$$E_1(t,x)=\mathscr{E}\left[1+\sum_{\nu=1}^{\infty}\frac{\mathscr{D}_\nu(\omega_st)^\nu}{\nu!}\right]=\mathscr{E}\left\{1+\frac{i\Delta\omega}{\omega_s}\omega_st\right.$$
$$+\left[\left(\frac{i\Delta\omega}{\omega_s}\right)^2+\frac{iAx}{\omega_s^2}\right]\frac{(\omega_st)^2}{2}+\left[\left(\frac{i\Delta\omega}{\omega_s}\right)^3\right.$$
$$\left.+\frac{iAx}{\omega_s^2}\frac{i\Delta\omega}{\omega_s}\right]\frac{(\omega_st)^3}{3!}+\left[\left(\frac{i\Delta\omega}{\omega_s}\right)^4\right.$$
$$+\frac{iAx}{\omega_s^2}\left(\frac{i\Delta\omega}{\omega_s}\right)^2+\frac{1}{2}\left(-\frac{2iAx}{\omega_s^2}\right.$$
$$\left.\left.+\left(\frac{iAx}{\omega_s^2}\right)^2\right)\right]\frac{(\omega_st)^4}{4!}+\left[\left(\frac{i\Delta\omega}{\omega_s}\right)^5\right.$$
$$+\frac{iAx}{\omega_s^2}\left(\frac{i\Delta\omega}{\omega_s}\right)^3+\frac{1}{2}\left(-\frac{2iAx}{\omega_s^2}\right.$$
$$\left.\left.\left.+\left(\frac{iAx}{\omega_s^2}\right)^2\right)\frac{i\Delta\omega}{\omega_s}\right]\frac{(\omega_st)^5}{5!}+\dots\right\}. \qquad (20)$$

This expression, like (12), can be simplified in the limit similar to (16). Here the equation similar to (13) has the form[4)]

$$\frac{2}{9}\frac{(iqA)^{2/3}}{t^{2/3}x^{1/3}}\left[\mu(1+2i\Delta\omega t)+2t\frac{\partial\mu}{\partial t}\left(1+\frac{3}{2}\frac{i\Delta\omega t}{(iqAxt^2)^{1/3}}\right)\right.$$
$$\left.+2x\frac{\partial\mu}{\partial x}\left(1+3\frac{i\Delta\omega t}{(iqAxt^2)^{1/3}}\right)\right]+(\omega_s^2-(\Delta\omega)^2)$$

$$\times\left[\frac{1}{3}(iqA)^{1/3}t^{2/3}x^{-2/3}\mu+\frac{\partial\mu}{\partial x}\right]=0. \quad (21)$$

If

$$\frac{\Delta\omega t}{(qAxt^2)^{1/3}}\ll 1 \quad (22)$$

and (16) holds, the equation has the approximate solution

$$\mu(t,x)\approx\mu_1(t,x)\mu_2(t,x)\mu_3(t,x),$$

where $\mu_1\mu_2$ is the solution of Eq. (13) found earlier, and

$$\mu_3=e^{-i\Delta\omega t}\left[1+2.5\ (0.87-0.5i)\frac{i\Delta\omega t}{(6.75Axt^2)^{1/3}}\right]$$
$$\times\exp\left[-0.75(0.87-0.5i)\frac{(\Delta\omega t)^2}{(6.75Axt^2)^{1/3}}\right], \quad (23)$$

[cf. (15) and (17)]. The factor 2.5 here has also been found through comparison with the exact solution (20). Thus, the expression for the field $E_1(t,x)$ is represented in this case by the right-hand side of (17) multiplied by $\mu_3\exp(i\Delta\omega t)$.

Thus, as Eq. (23) shows, in contrast to processes of modified decay in infinite media, where energy is pumped "up" and "down" the spectrum more or less symmetrically, with "upward" pumping (modulation instability), perhaps prevailing, the situation here is quite different: there is strong "downward" scattering ($\Delta\omega>0$). To estimate the linewidth of this scattering, we can use the same formula as in ordinary decay processes (see Eq. (14)):

$$|\Gamma|\sim|\Delta\omega_{\rm lim}|, \text{ or } \sqrt[3]{6.75Axt^{22}}\sim|\Delta\omega_{\rm lim}|t,$$

which follows from Eq. (17) and (23). The exact solution (20) shows that the line is somewhat narrower in the anti-Stokes direction and somewhat wider in the Stokes direction, as stated earlier.

Stimulated Mandel'shtam–Brillouin backscattering of powerful short pulses has been studied quite thoroughly in experiments in the microwave range.[4,5] The source of intense radiation, a relativistic carcinotron, generated single pulses whose length was $\tau\sim0.4\ \mu$s, and power I_0 was several tens of megawatts; the wavelength was 3.2 cm. The width of the spectrum being investigated was varied in steps , $\delta f\sim2$ MHz, 4 MHz, or 15 MHz, depending on the which of three oscillator designs was chosen. A quasioptical line with a transmission factor of approximately 50% was used to focus the microwave radiation inside a large vacuum chamber 360 cm long and 62 cm in diameter containing a previously prepared nonisothermal helium plasma ($T_e\sim20$ eV and $T_i\leq1$ eV) with maximum concentration $N_0\sim6\times10^{11}\ \text{cm}^{-3}\sim N_{\rm cr/2}$. The beam's cross-sectional distribution was Gaussian. Inside the active scattering zone (in the beam caustic), roughly 23 cm in diameter and about 100 cm long, the oscillator electron velocity ($v_\sim\sim2\times10^8\ \text{cm s}^{-1}$) was of the order of the thermal velocity, and hence the frequently employed weak-coupling approximation ($\gamma_0<\omega_s$) is certain to be violated.[5)]

The experimentally observed temporal variation of the reflection coefficient ($I_0\sim20$ MW and the spectrum bandwidth $\delta f\sim4$ MHz), the dependence of the peak value of the reflection coefficient on the incident power for various spectral characteristics of the radiation from the microwave source, and the angular scattering diagram are shown in Figs. 1–3

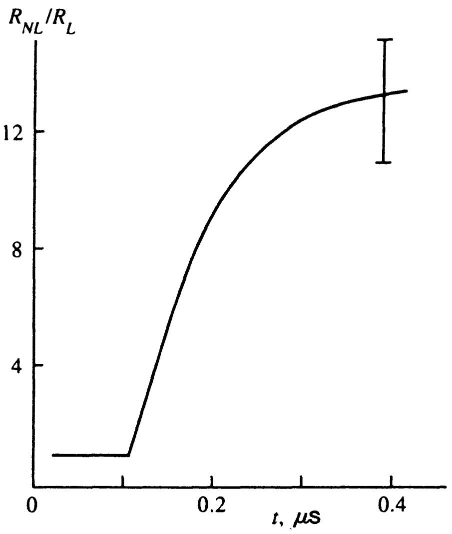

FIG. 1. Time dependence of the reflection coefficient for an incident 20-MW beam of microwave radiation.

Figure 1 depicts the time dependence of the reflection coefficient for a 20-MW beam (nonlinear mode) normalized to the reflection coefficient corresponding to the low-power case, $R_{\rm NL}(t)/R_{\rm L}$ ($R_{\rm L}\approx2\%$). In the first 100 ns, the SMBS process is masked by spurious reflections from absorbers of microwave radiation and the structural elements of the vacuum chamber, since the initial level of the Stokes wave (reflection almost strictly backwards) constitutes only a small fraction ($\sim10^{-2}$) of the overall reflection, which has a fairly isotropic angular spectrum. A nonlinear reflection emerges, $t_{\rm th}\sim0.1\ \mu$s, the instantaneous increment, which according to (14) (the linear stage in stimulated scattering) is defined as $\text{Re}\Gamma\sim2\times10^7\ \text{s}^{-1}$ ($\omega_s t_{\rm th}\sim0.63$ and $E_0\sim7\ \text{kV cm}^{-1}$), is close to the value observed in experiments, $\sim(2-3)\times10^7\ \text{s}^{-1}$. By this time, as Eq. (17) implies, the gain in the intensity of the scattered Stokes wave reaches

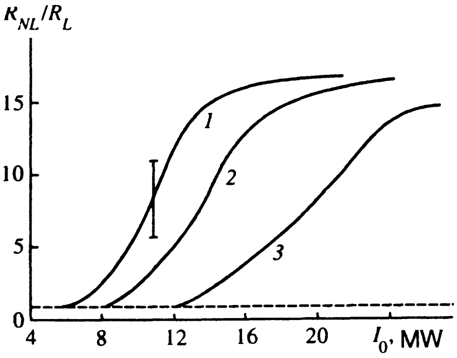

FIG. 2. Reflection coefficient as a function of the power of a microwave radiation beam with a frequency spectrum of the following width δf: 2 MHz (curve *1*), 4 MHz (curve *2*), and 15 MHz (curve *3*).

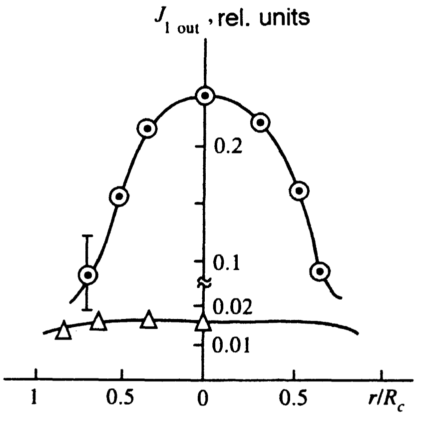

FIG. 3. The scattering diagram for the reflected radiation at high $I_0 \sim 20$ MW (⊙) and low $I_0 = 10$ kW (Δ, the linear case) power (R_c is the radius of the chamber).

$$\mathscr{G}(t_{\rm th}) \equiv \mathscr{G}_{\rm th} = \frac{|E_1(L)|^2}{\mathscr{E}^2} \sim 20, \tag{24}$$

which corresponds to the threshold of SMBS observation.

It was found that reducing the power of the incident beam reduces the reflection coefficient $R_{\rm NL}$ (see Fig. 2). Here the onset time of nonlinear scattering was found to be shifted closer to the end of the microwave pulse. At $I_0 = I_{0\,\rm th}$ the reflection coefficient $R_{\rm NL}$ at $t \sim \tau$ be the spurious linear reflection coefficient $R_{\rm L}$, and it becomes impossible to detect SMBX. Clearly, in this case the increase in intensity of the Stokes wave up to the observation threshold of stimulated scattering must correspond to the condition (24), on the basis of which an estimate of the threshold power can be made.

In the experiment with $\delta f \sim 4$ MHz, we see that the threshold power was $I_{0\,\rm th} \sim 9$ MW. To estimate it we must turn to Eq. (20) or Eq. (23), each of which takes into account the initial frequency offset between the interacting waves, since at large times $t \sim \tau$ they become significant. Indeed, the "average" distance between the modes of the amplitude spectra of the pump wave and the wave reflected from the chamber walls, which is actually the "average" distance between the modes of the pump wave itself, is $\Delta\omega \sim 2\pi\delta f$, and $2\pi\delta f \cdot \tau \sim 10$. As a result, the simplified expression (23) [together with (17)] yields an underestimated value of $|E_1(L)|^2$, since the condition (22) does not hold. However, a calculation using the exact solution (20) yields agreement with the threshold value $\mathscr{G}_{\rm th}$ introduced earlier.

At the same time, for beams with a lower δf (~ 2 MHz; curve *1* in Fig. 2) the results of calculations by (17) and (20) at $t \sim \tau$ are still comparable and yield a value $I_{0\,\rm th} \sim 5$ MW, and also agree with the experimental data.[6)] But if the pump wave spectrum is broad (curve *3* in Fig. 2), the wave's modes interact differently with the waves of the wideband reflected signal produced in this case, with only those modes effectively scattered that wind up within the linewidth specified above.

To understand the nonlinear stage in stimulated scattering, which follows the linear stage (Fig. 1), the following factors must be taken into account. Power measurements of the incident and reflected waves were done outside the chamber, roughly one meter from the active zone. The measured radial distribution of intensity in the reflected beam, $J_1(r)$, depicted in Fig. 3 proved to be about a factor 1.5 wider than that of the incident beam, $J_0(r)$. At the same time, owing to the exponential nature of the scattering process (6), the effective diameter of the gain region is smaller than the active zone width by a factor of approximately two, as probe measurements have revealed.[4,5] Note in this connection that the quantity directly measured in the plasma was the decrease, along the radius r measured from the chamber's axis, of the amplitude of the low-frequency wave $2\tilde{n}$ (the concentration perturbations). However, and this can easily be understood, the amplitude of the Stokes wave behaves in a similar way. The quantity $\tilde{n}$ was found to decrease e-fold over a distance of approximately 7 cm from the axis, and the scale of decrease, $|\tilde{n}|^2$, is obviously even smaller, roughly 5 cm (for comparison, from estimates based on (17) and assuming that $A \sim \exp\{-r^2/(12)^2\}$, we obtaint 9.5 cm and 7 cm, respectively).

Thus, the scattered beam, following its path in the plasma after amplification and then leaving the plasma, experiences significant diffractive and refractive scattering. As a result the quantity $|E_1|^2$, depicted in Fig. 1,[7)] proves to be smaller than its value at the exit from the active zone by a factor of at least three to four (the energy flux in the Stokes wave is conserved), and the SMBS instability saturation observed in the experiments is probably related primarily to the depletion of the pump wave ("corrosion" of the central part of the beam), and to the nonmonochromatic nature (δf) of the radiation and the frequency shift due to the term $\omega_s^2 \tilde{n}$ in Eqs. (3). To estimate the first factor (the others have already been mentioned) we can use Eq. (17), with $|E_0|t$ replaced by $\int_0^t \sqrt{|E_0|^2 - |E_1(t',L)|^2}\, dt'$ (here the radicand is a constant of the motion in x for the system of equations (3) augmented by a similar equation for the pump wave).[8)]

In all probability, there should be no low-frequency nonlineariteis for the short pulses considered here. Indeed, the experimentally observed relative perturbations of the plasma concentration, which in fact correspond to the Mach numbers $\mathscr{M}$ in them, amount to $n/N_0 \lesssim 0.05$. Hence the characteristic times of triggering such mechanisms (low-frequency hydrodynamic instability) are $\tau_{\rm LF} \sim (\mathscr{M}\omega_s)^{-1} \gg 1\ \mu$s.

On the other hand, in the transition from (1) to (3) we also neglected the spatial derivatives in the equation for n related to nonlinearity. This is likely to be permissible at least up to times

$$t_{SBS} \sim \frac{L_{\rm cr}}{v_s} = \pi \frac{k_1 c}{\omega_0} \sqrt{\frac{\omega_0}{\gamma_0}}\, \gamma_0^{-1} \simeq 1\ \mu\text{s},$$

where the estimate has been based on the ordinary SMBS, in which for $L_{\rm cr}$ we have taken the smallest nonlinear spatial scale possible in the problem. We also believe that at extremely high power, the role of the well-known double SMBS effect[17] diminishes because in scattering from a region of critical concentration, the wave winds up in resonance with the pump wave, a situation considered earlier in this paper.

1)Clearly, this one-dimensional model does not incorporate low-frequency ion-acoustic nonlinearities (see, e.g., Ref. 9) and, hence, can be used to describe the SMBS of powerful and fairly short pulses (see, e.g., the corresponding estimate at the end of the paper).

2)These conditions correspond to a situation in which there is a high-frequency wave at the entrance to the layer ($x=0$) propagating in the direction opposite to the pump wave; the amplitude of this high-frequency wave is much greater than the level of spontaneous scattering on thermal fluctuations of the medium's density. The condition imposed on $\tilde{n}_t'$ in (4) follows from the continuity equation for ions, which in turn leads to the equation for n in (1) ($n=n_i \simeq n_e$), with allowance for the fact that $v_i(0,x)=0$, where v_i is the hydrodynamic ion velocity. The boundary value $\tilde{n}(t,L)$ can be neglected at least up to times $t \ll L/v_s$, when even if the growth rate (see below) drops to values characterizing the transition of the process into ordinary SMBS, the transfer of the energy of the acoustic energy wave has still no effect on the solution over almost the entire layer, except a small region near the layer's right boundary ($x=L$, where the pump wave enters the medium), where the solution is modified by the value $\tilde{n}(t,L)$. This is the well-known condition for time-dependent SMBS.

3)Note that Kim *et al.*[13] pointed out the similarity, in the given approximation and in the nonlinear scattering stage, both of the solutions for the processes of stimulated temperature scattering and modified decay in an infinite plasma and of the methods by which these solutions were obtained.

4)This equation can easily be obtained from Eqs. (3) with the factors $\exp(-i\Delta\omega t)$ on the right-hand sides (this actually means that the offset factor has been shifted from the boundary condition (4) to the equations). Obviously, to match the solution of (21) and (6) with (20) the first must be multiplied by $\exp(i\Delta\omega t)$. This difference emerges automatically in the given transition and is described in (21) by the term $2i\Delta\omega t$ in the first set of parentheses.

5)Note that on the basis of the analysis done by Mounaix *et al.*[16] it is natural to expect that the initial equations (1) can be applied to a broader range of fields than those for which $v_\sim^2/v_{Te}^2 \ll 1$.

6)Here we have allowed for the fact that in addition to the increase in the intensity of the Stokes wave in the boundary value problem considered throughout this paper, the intensity increases somewhat initially in the infinite plasma model with growth rate γ_0 ($\gamma_0 \approx 6\times 10^7\ \mathrm{s}^{-1}$ for $I_0 \approx 20$ MW). This continues up to times $t_* \gtrsim (L/c)\omega_0/k_1c$ (allowing for retardation of the fast wave in the plasma). Hence $\mathcal{G}$th is several times the value given by (24). At the same time, we note that we cannot constrain t_* and estimate it on the basis of the $\gamma_0 \to \Gamma(t,x)$ transformation, where $\Gamma(t,x)$ is the growth rate calculated by (6), since the asymptotics of (8) does not yet apply at such short times.

7)actually, the reflection coefficient is defined as the ratio of the squares of the absolute values of the fields (reflected and incident) at the beams' axis.

8)Here it must be borne in mind that as the power of the wave travelling through the plasma decreases, so does the power of the wave reflected from the back wall of the chamber, i.e., the quantity $\mathcal{E}$ virtually decreases.

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Translated by Eugene Yankovsky