Quantum limit on classical force measurements using an active interferometric displacement sensor

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It is shown to be feasible, using an active interferometric displacement sensor, to continuously measure a classical force at an accuracy beyond the standard quantum limit. Rather than detecting the phase, one measures a specially chosen quadrature component of the wave generated by the sensor; that component is squeezed, due to a ponderomotive nonlinearity. Two types of active sensors are considered: those with gain provided by three-level atoms, and parametric amplifiers. Allowance is made for noise due to spontaneous emission and errors due to mirror losses and radiative drag.

1. INTRODUCTION

With the accuracy of experimental measurements of classical forces constantly on the rise, achieving the standard quantum limit no longer appears to be an unrealistic goal. This has piqued interest in methods of achieving even higher accuracy.

One of the most promising force detectors is the optical interferometric displacement sensor consisting of a Fabry–Perot etalon with one mirror that can undergo translational oscillatory motion (Fig. 1). Information about the signal to be measured is encoded by the phase of the reflected wave E_2. At present, such sensors have been proposed for use as gravitational detectors (for example, in the LIGO project) and in a number of other fundamental experiments.

If the phase of E_2 is tracked continuously, the measurement accuracy is bounded by the standard quantum limit, which is reached at the optimum power level of the pump E_1.[1–4] This bound results from the back influence of the sensor on the light wave. If the signal is given by the force

$$\begin{cases} F_s = F\sin\omega_F t, & -\frac{t_F}{2} \leq t \leq \frac{t_F}{2}, \\ F_s = 0, & t < -\frac{t_F}{2}, \quad t > \frac{t_F}{2} \end{cases}$$

($t_F = 2\pi n/\omega_F$, n is an integer), which acts on a free mass m, the standard quantum limit can be written in the form $F_{SQL} = \sqrt{m\hbar\omega_F^2/t_F}$. For definiteness, we will assume below that the force being measured has the same form as F_s.

There are several ways to mitigate the back influence. One can make use of a measuring instrument with noise correlated in a special manner,[4] or prepare the pump wave E_1 in a supercoherent (squeezed) state,[5] rather than a coherent state. The actual implementation of such correlation or squeezing, however, is difficult.

It has been shown[6–8] that the displacement sensor itself is a generator of squeezed radiation. Let the wave incident upon the stationary mirror be in a coherent state; the light pressure is proportional to the square of the wave amplitude. The greater the amplitude, the higher the pressure (and thus the greater the displacement of the mirror), thereby increasing the phase shift of the reflected wave. This means that amplitude and phase fluctuations in the reflected wave will be interdependent. A ponderomotive nonlinearity results in squeezing of the reflected wave.

The squeezing of light due to a ponderomotive nonlinearity has been proposed as a means of detecting forces beyond the standard quantum limit[7,8]; the assumption was that the pump wave E_1 is in a coherent state, with no preparatory squeezing. All that is necessary to circumvent the standard quantum limit is to detect a specially determined quadrature component of the phase of E_2, rather than the phase itself, using a modified homodyne setup.[9]

A similar means of improving measurement sensitivity was apparently first suggested in Ref. 10. That method is only quasioptimal, however, and enables one to achieve accuracy beyond the standard quantum limit only for signals in a narrow frequency range inversely proportional to $|\langle E_1\rangle|$.

An active displacement sensor can be used instead of a passive one. Kulagin and Rudenko[11] show that the insertion of a parametric amplifier near the lower oscillation threshold between the interferometer plates reduces the measurement error below that achieved with a conventional interferometric sensor (the back influence of fluctuations is not taken into account). They go on to show[12] how one can use parametric amplifiers to specially induce a terminal cascade reaction, which can be used to squeeze the backward wave. Panov and Rudenko[13] study a displacement sensor in oscillatory mode from a classical standpoint, based on a lumped-element arrangement.

In the present paper, we analyze two types of active interferometric displacement sensors using a quantum theoretic approach: these are based on a three-level atomic amplifier and a parametric amplifier (Fig. 2) (employing a medium with $\chi^{(2)}$ susceptibility). We study the influence of fluctuations in spontaneous emission (for the atoms) and in the pump noise (for the parametric amplifier) on experimental accuracy. Losses in the interferometer mirrors are taken into consideration, as is radiative drag.

In doing the calculations, we have assumed that a) E_1 is a vacuum fluctuation field in the coherent state; b) both techniques work near the threshold of oscillation; c) there are no

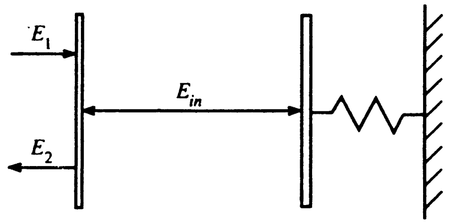

FIG. 1. Passive interferometric displacement sensor. The force to be detected affects the movable mirror. It is possible to surpass the sensitivity given by the standard quantum limit in measuring the quadrature component (rather than the phase) of the reflected wave E_2.

intrinsic losses in the mechanical system or the pump resonator (used in the parametric amplifier), and losses in the measurement interferometer are small ($1 \gg q \gg R$, where q is the transmission coefficient of the interferometer input mirror, and R is the loss factor in both mirrors); d) perturbation theory is applicable, i.e., the mean value of any observable in the system is much greater than its rms deviation during the measurement interval; e) the sensor is operating in the mass-controlled domain ($\omega_F \gg \omega_M$, where ω_M is the natural frequency of the mechanical oscillator); f) the relaxation time of the resonator is greater than the time over which the force acts, i.e., $q \ll L_0\omega_F/c$ (L_0 is the separation between the walls of the measurement resonator and c is the speed of light), a condition that can hold, for example, in the LIGO project.

In the first section of the present paper, we consider an active parametric displacement sensor. Near the sensor's threshold of oscillation, it is possible to suppress pump fluctuations and surpass the level of accuracy imposed by the standard quantum limit. The influence of these fluctuations will decrease as the resonator pump power W_p increases. If the mirrors are lossless, then

$$F_{\min 1} \simeq \nu_1 F_{SQL} \left(\frac{W_{SQL}}{W_p} \right)^{1/6}. \tag{1}$$

Here

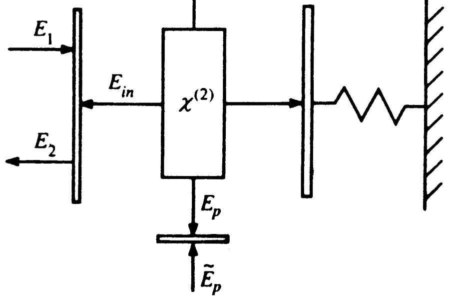

FIG. 2. Active interferometric displacement sensor based on a parametric amplifier. It is possible to surpass the sensitivity given by the standard quantum limit in measuring the quadrature component. The ultimate measurement accuracy, which depends on radiative drag, can be higher than that of a passive sensor. Increasing the power associated with the field $\tilde{E}_p$ can lead to unlimited suppression of pump noise.

$$W_{SQL} = \frac{mc^2\omega_F}{2q} \frac{\omega_F}{\omega_0} \left(\frac{L_0\omega_F}{c} \right)^2$$

is the power required within the passive interferometric displacement sensor to achieve the standard quantum limit, and from this point onward, ν represents a factor of order unity. If the transmission coefficient of the front mirror of the pump resonator is 10^{-6}, $\omega_F = 300\ \mathrm{sec}^{-1}$, $\omega_0 = 3 \cdot 10^{15}\ \mathrm{sec}^{-1}$, $m = 10^3 g$, $L = 4 \cdot 10^5$ cm, and the pump power $\tilde{E}_p$ in the active medium is of order $10^3 W$, then we have $F_{\min 1} = 0.1 F_{SQL}$.

The second section concerns the sensor with a three-level atomic active medium. We show that if the mirror are lossless ($R = 0$), it is possible to achieve sensitivity better than the standard quantum limit outside the gain bandwidth Γ of the atoms. The minimum detectable force is then

$$F_{\min 1} \simeq \nu_2 F_{SQL} \left(\frac{\Gamma^2}{\omega_F^2 + \Gamma^2} \right)^{1/4}. \tag{2}$$

The present state of the art in atomic force sensors is such that their accuracy is no better than the standard quantum limit, due to noise in the active medium. There is some hope, however, that atomic or molecular transitions will be identified that are suitable for making an oscillator with a narrow enough bandwidth that the sensor sensitivity can be improved.

In either case, mirror losses limit the minimum detectable force. Assuming that these losses dominate other mechanisms that bound the measurement sensitivity, we have

$$F_{\min 2} \simeq \nu_3 F_{SQL} \left(\frac{R}{q} \right)^{1/4}. \tag{3}$$

For $R = 10^{-5}$ and $q = 3 \cdot 10^{-3}$, we have $F_{\min 2} = 0.38\ F_{SQL}$.

There is another mechanism besides the ones mentioned above that limits measurement accuracy, and that is important when F_{min1} and F_{min2} are sufficiently small—namely, radiative drag (when an electromagnetic wave is reflected from the moving mirror, the frequency of the reflected wave and the radiation pressure depend on the mirror velocity) resulting from a combination of the Doppler effect and the resonator effect (the mechanical system exhibits additional stiffness due to coupling to an optical mode). For an oscillator with an atomic-level amplifier, the sensitivity will be the same as that of a passive displacement sensor:

$$F_{\min 3} \simeq \nu_4 F_{SQL} \left(\frac{\omega_F}{\omega_0} \right)^{1/2}. \tag{4}$$

In a parametric amplifier, the accuracy will be higher:

$$F_{\min 3} \simeq \nu_5 F_{SQL} \left(\frac{\omega_F}{\omega_0} \frac{16\omega_F^2 L_0^2}{qc^2} \right)^{1/2}. \tag{5}$$

We can explain these results by noting that if the oscillator operates near its oscillation threshold, the relaxation time of the resonator will increase within the gain bandwidth of the active medium. When condition f above is satisfied, this will reduce the radiative drag.

The expressions for the minimum detectable force were obtained for the optimum value of the power W within the resonator used for the measurement. Rewriting (1)–(3) in the form $F_{\min}\simeq F_{SQL}\zeta$ ($\zeta<1$), we have

$$W\simeq W_{SQL}\zeta^{-2}. \tag{6}$$

To reach the limits (4)–(5), the required power is

$$W\simeq W_{SQL}q\frac{\omega_0}{\omega_F}\left(\frac{\omega_F L_0}{c}\right). \tag{7}$$

We now proceed to examine the proposed methods directly.

2. ACTIVE INTERFEROMETRIC DISPLACEMENT SENSORS WITH A PARAMETRIC AMPLIFIER

The temporal evolution of this sensor (Fig. 2) is given by a set of three equations:

$$\dot{E}_{in}=-\left(\frac{Q}{2}\frac{c}{2L_0}+i\omega_R\right)E_{in}+i\frac{\omega_0}{L_0}E_{in}\left(x+\frac{i}{\omega_0}\dot{x}\right)$$
$$-2igE_{in}^{+}E_p+\frac{c}{2L_0}(\sqrt{q}E_1+\sqrt{R}E_{fl}),$$
$$\dot{E}_p=-\left(\frac{q}{2}\frac{c}{2L_1}+2i\omega_0\right)E_p-igE_{in}^2+\frac{c}{2L_1}\sqrt{q}\tilde{E}_p,$$
$$\ddot{x}+2\delta_R\dot{x}+\omega_M^2x=\frac{S}{\pi m}E_{in}^{+}E_{in}+\frac{F_s}{m}. \tag{8}$$

Here g is the coupling constant between the between the pump resonator mode and the interferometric displacement sensor mode, L_1 is the distance between the pump resonator mirrors, and $\tilde{E}_p$ is the external electromagnetic field driving the pump mode oscillations. To streamline the calculations, we have assumed both the transmission coefficients of the input mirrors and the mirror areas in the two resonators to be equal. The carrier frequency of the wave $\tilde{E}_p$ and the eigenfrequency of the pump resonator are equal to twice the frequency of the measurement resonator (with radiation pressure taken into account). The requirement that the system operate near the oscillation threshold means that the field amplitude E_p in the pump resonator depends weakly on the field E_{in} in the measurement part of the system. The setup shown here is phase-sensitive. Let $\langle\tilde{E}_p\rangle=i|\langle\tilde{E}_p\rangle|e^{-2i\omega_0 t}$. If we then equate observables as in the previous section, we have from (8) that

$$\langle E_p\rangle=i\frac{Qc}{8L_0g},\quad \langle E_{in}\rangle=\sqrt{\frac{qc}{4L_1g}\left(\frac{2|\langle\tilde{E}_p\rangle|}{\sqrt{q}}-\frac{Qc}{8L_0g}\right)},$$
$$\omega_0=\frac{\omega_R}{1+\delta_R c/L_0\omega_M^2}. \tag{9}$$

In these equations, we assume for definiteness that $\langle\tilde{E}_p\rangle$ lags $\langle\tilde{E}_{in}\rangle$ in phase by $\pi/2$. (It can also lead by $\pi/2$; the actual phase of the generated wave can only be determined experimentally, as these two phases are equally likely.)

Let the pump resonator relaxation time be less than the time over which the force acts (this is possible if $L_0>L_1$). We then have for the fluctuations in the output wave E_2

$$b_2=Nb_1+K(b_1+b_{1-}^{+})+\sqrt{\frac{R}{q}}[(1+N)d+K(d+d_{-}^{+})]$$
$$+\sqrt{\frac{R_1}{q}}[(1+N)d_2+K(d_2+d_{2-}^{+})]+UF_s(\Omega), \tag{10}$$

where

$$\frac{R_1}{q}\simeq 2\frac{W}{W_p},\quad N=\frac{i(4L_0/c)-2R-Q\xi}{-i\Omega(4L_0/c)+Q(2+\xi)},$$
$$K=\frac{4q}{[-i\Omega(4L_0/c)+Q\xi][-i\Omega(4L_0/c)+Q(2+\xi)]}\times\left(\frac{Q}{2}+i\frac{4W\omega_0}{mc^2Z_1(\Omega)}\right),$$
$$U=\frac{4i\sqrt{W_q\omega_0/\hbar}}{[-i\Omega(4L_0/c)+Q(2+\xi)]mcZ_1(\Omega)},\quad \xi=\frac{WL_1}{W_pL_0}.$$

The operators $d_2=d_2(2\omega_0+\Omega)$ and $d_{2-}^{+}=d_{2-}^{+}(2\omega_0-\Omega)$ characterize fluctuations induced by the pump wave $\tilde{E}_p$.

We can write the impedance of the mechanical system in the form

$$Z_1(\Omega)=\omega_M^2-\Omega^2-2i\delta_R\Omega\left(1+\frac{4}{-i\Omega(4L_0/c)+\xi Q}\right). \tag{11}$$

It can easily be seen that radiative drag decreases when condition f above holds.

3. INTERFEROMETRIC DISPLACEMENT SENSOR BASED ON A THREE-LEVEL ATOMIC OSCILLATOR

The temporal evolution of the sensor is given by

$$\dot{E}_{in}=-\left(\frac{Q}{2}\frac{c}{2L_0}+i\omega_R\right)E_{in}+i\frac{\omega_0}{L_0}E_{in}\left(x+\frac{i}{\omega_0}\dot{x}\right)$$
$$+\mu_1\sigma_-+\frac{c}{2L_0}(\sqrt{q}E_1+\sqrt{R}E_{fl}),$$
$$\ddot{x}+2\delta_R\dot{x}+\omega_M^2x=\frac{S}{\pi m}E_{in}^{+}E_{in}+\frac{F_s}{m},$$
$$\dot{\sigma}_-=-(i\omega_0+\Gamma)\sigma_-+\mu_2E_{in}(\sigma_{22}-\sigma_{11})+F_{12},$$
$$\dot{\sigma}_{11}=-\Gamma\sigma_{11}+\mu_2(E_{in}^{+}\sigma_-+E_{in}\sigma_+)+F_{11},$$
$$\dot{\sigma}_{22}=w_{20}(1-\sigma_{11}-\sigma_{22})-\Gamma\sigma_{22}$$
$$-\mu_2(E_{in}^{+}\sigma_-+E_{in}\sigma_+)+F_{22}. \tag{12}$$

In these equations, $Q=q+R$; ω_R is the natural frequency of the unloaded resonator; ω_0 is the oscillation frequency; σ_-, σ_+, σ_{11}, and σ_{22} are operators that describe the three-level atomic medium[14]; $\mu_1=N_1\mu E_0$ and $\mu_2=\mu/E_0$ are the atom-field coupling constants (N_1 is the number of atoms in the resonator, $E_0=(\hbar\omega_0/2V)^{1/2}$ is the electric field attributable to one photon in the resonator mode, V is the effective volume of the oscillation mode, and $\mu=\langle\bar{p}\bar{E}_0/\hbar\rangle$ is the mean coupling constant between the dipole moment $\bar{p}$ and the field $\bar{E}_0$ of one photon); x is the position of the mechanical oscillator; $\delta_R=2W/mc^2$ is the radiative drag coefficient; $W=(Sc/2\pi)|\langle E_{in}\rangle|^2$ is the radiative power inside the resonator; S is the area of each resonator mirror; m is the mass of the movable mirror; $(c/2L_0)\sqrt{R}E_{fl}$ is the Langevin force resulting from mirror losses; and F_{12}, F_{11}, and F_{22} are Langevin forces related to the atomic linewidth Γ and the atomic population inversion factor w_{20}.[14]

In writing Eqs. (12), we have taken the working atomic transition frequency to be equal to the oscillation frequency ω_0, which for simplicity we have assumed to be equal to the frequency of the measurement resonator (with allowance for radiation pressure). The oscillation condition near threshold ($\mu_2|\langle E_{in}\rangle|/\Gamma\ll 1$) implies that the Langevin forces F_{11} and F_{22} make a much smaller contribution to the system than does F_{12}. For convenience in subsequent calculations, we can express the latter in terms of a fluctuating electric field analogous to E_{fl}:

$$F_{12}=\frac{c}{2L_0}\sqrt{\frac{Q\Gamma^2}{\mu_1^2}}E_{fl1}^{+}. \tag{13}$$

Solving for E_{in} in (12), we obtain for the output wave

$$E_2=\sqrt{q}E_{in}-E_1. \tag{14}$$

Putting the observables of the system in the form

$$E=\langle E_{in}\rangle+e^{-i\omega_0 t}\int_{-\infty}^{\infty}d\Omega\sqrt{\frac{\hbar(\omega_0+\Omega)}{Sc}}b(\omega_0+\Omega)e^{-i\Omega t},$$

$$x=\langle x\rangle+\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\Omega x(\Omega)e^{-i\Omega t},$$

$$\sigma_-=\langle\sigma_-\rangle+e^{-i\omega_0 t}\int_{-\infty}^{\infty}d\Omega\sigma_-(\omega_0+\Omega)e^{-i\Omega t},$$

$$\sigma_{ii}=\langle\sigma_{ii}\rangle+\int_{-\infty}^{\infty}d\Omega\sigma_{ii}(\Omega)e^{-i\Omega t},$$

we obtain for the oscillation frequency and amplitude

$$\omega_0=\frac{\omega_R}{1+\delta_R c/L_0\omega_M^2},$$

$$\frac{\mu_1\mu_2}{\Gamma}\frac{w_{20}/(w_{20}+\Gamma)}{1+4\mu_2^2|\langle E_{in}\rangle|^2/\Gamma^2}=\frac{Qc}{4L_0}. \tag{15}$$

We then have for the fluctuations in the emerging wave

$$b_2=Nb_1+K(b_1+b_{1-}^{+})+\sqrt{\frac{R}{q}}[(1+N)d+K(d+d_-^{+})]+\sqrt{\frac{Q}{q}\frac{\Gamma^2}{\Gamma^2+\Omega^2}}[(1+N)d_{1-}^{+}+K(d_1+d_{1-}^{+})]+UF_s, \tag{16}$$

where

$$N=\frac{2q+i\Omega(4L_0/c)-Q[1-\Gamma/(\Gamma-i\Omega)]}{-i\Omega(4L_0/c)+Q[1-\Gamma/(\Gamma-i\Omega)]},$$

$$K=i\frac{16Wq\omega_0}{mc^2Z(\Omega)}\{-i\Omega(4L_0/c)+Q[1-\Gamma/(\Gamma-i\Omega)]\}^{-1}\times\{-i\Omega(4L_0/c)+Q[1-(\Gamma/(\Gamma-i\Omega))\times[1-4\mu_2^2|\langle E_{in}\rangle|^2(2-i\Omega/\Gamma)/(\Gamma-i\Omega)^2]]\}^{-1},$$

$$U=\frac{4i\sqrt{Wq\omega_0/\hbar}}{\{-i\Omega(4L_0/c)+Q[1-\Gamma/(\Gamma-i\Omega)]\}mcZ(\Omega)},$$

$$Z(\Omega)=\omega_M^2-\Omega^2-2i\delta_R\Omega\times\left\{1+4\left\{-i\Omega\frac{4L_0}{c}+Q\left[1-\frac{\Gamma}{\Gamma-i\Omega}\times\left[1-\frac{4\mu_2^2|\langle E_{in}\rangle|^2(2-i\Omega/\Gamma)}{(\Gamma-i\Omega)^2}\right]\right]\right\}^{-1}\right\}.$$

In these four equations, $Z(\Omega)$ is the effective impedance of the mechanical oscillator; $b_1\equiv b_1(\omega_0+\Omega)$ and $b_{1-}^{+}\equiv b_{1-}^{+}(\omega_0-\Omega)$ are the spectral amplitudes of the annihilation and creation operators in the damped wave; d and d_-^{+} are the mirror-loss fluctuation operators; and d_1 and d_{1-}^{+} are the spontaneous-emission noise operators in the active medium.

4. OPTIMAL SIGNAL FILTERING

We apply the optimal signal detection method developed in Ref. 9 to the devices described above. This method employs a modified homodyne detector for noise like that given by Eq. (10).

A homodyne detector (Fig. 3), as is well known, can be used to measure the quadrature components of an electromagnetic wave. The difference photocurrent J is proportional to $E_{LO}(t)E_2^{+}(t)+\text{H. c.}$ ($E_{LO}(t)$ and $E_2(t)$ are the complex amplitudes of the local oscillator and signal). In order for fluctuations in $E_{LO}(t)$ not to make any appreciable contribution to the measured results, the power associated with E_{LO} must be high: $|E_{LO}|\gg k|E_2|$ (k is the squeezing factor of the signal field E_2). It is assumed that E_{LO} is in a coherent state. In that event, we can assume that the photocurrent $J(t)$ is proportional to the quadrature component $B(\theta,t)$,

$$B(\theta,t)=b_2(t)\exp(-i\theta)+b_2^{+}(t)\exp(i\theta), \tag{17}$$

where the angle θ is given by the phase of E_{LO}. Due to the fact that the measurement time is finite, we detect not the quadrature component but some mean value:

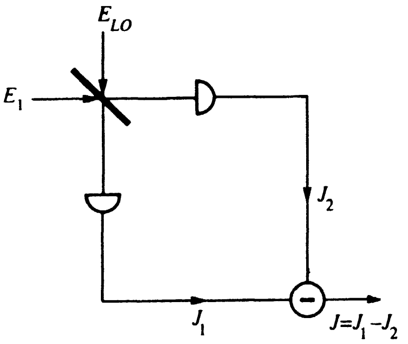

FIG. 3. Homodyne detector layout. If the power in the local oscillator E_{LO} is much greater than the power in E_2, the photocurrent difference $J = J_1 - J_2$ will be proportional to the quadrature component $B(\theta)$ of the signal wave, and the phase θ will be determined by the phase of the local oscillator.

$$B_T=\int_0^T \Phi(t)B(\theta,t)dt. \tag{18}$$

The modified homodyne method proposed in Ref. 9 entails phase-modulating the local oscillator. We can show that the averaged function $\Phi(t)$ and the phase modulation $\theta(t)$ can be chosen such that B_T contains no information on noise due to the back influence of fluctuations.

To simplify the mathematics, we assume that sensitivity is limited solely by radiative drag effects. if

$$g_S(\Omega)=\int_0^T \Phi(t)\sin\theta(t)e^{i\Omega t}\,dt,$$

$$g_C(\Omega)=\int_0^T \Phi(t)\cos\theta(t)e^{i\Omega t}\,dt,$$

then when

$$(N+2K_1)g_C(-\Omega)-2iK_2g_S(-\Omega)=0 \tag{19}$$

B_T can be put in the form

$$B_T=\int_{-\infty}^{\infty} -ig_S(-\Omega)(N(b_1-b_{1-}^+)+2UF_s(\Omega))d\Omega. \tag{20}$$

Here N, U, $K=K_1+K_2$, $K_1^*(-\Omega)=K_1(\Omega)$, and $K_2^*(-\Omega)=-K_2(\Omega)$ are parameters that describe the wave E_2.

We are now in a position to write the signal-to-noise ratio for the measured quantity[2]:

$$\kappa=\int_{-\infty}^{\infty}\frac{4|UF_s(\Omega)|^2d\Omega}{\pi|N|^2}. \tag{21}$$

In measuring a force of the form described above, the integral (21) can be evaluated over the closed interval $\Omega\in[\omega_F-\omega_F/2,\omega_F+\omega_F/2]$. Optimizing the signal-to-noise ratio by varying the pump power and calculating the corresponding integrals leads to Eqs. (4)–(5). We note in closing that mirror losses and fluctuations due to spontaneous emission do not alter the foregoing signal-detection strategy.

One fundamental difficulty encountered in experimentally implementing the proposed measurement technique is the need for a stable pump for the parametric amplifier-based sensor (this is not a problem for the three-level atomic amplifier), as well as a stable local oscillator. Whereas for the parametric oscillator we are faced with the strictly technical problem of designing a stable, high-power signal source to be simultaneously used as a pump and local oscillator, the problem is somewhat more complicated for the atomic oscillator due to the fact that the radiation is not phase-locked to the pump. If the atomic oscillator is not stable enough, it can be stabilized using a frequency standard. The measurement accuracy remains unchanged.

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