Nearly ideal two-dimensional Fermi gas in a weak magnetic field

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For a two-dimensional nearly ideal Fermi gas in weak magnetic fields, for which the chemical potential corresponds to a large Landau-level index, the question of the correctness of the Fermi-liquid description of the system is investigated by a diagram-technique method. The interaction between the particles is assumed to be short-range. Because the spectrum of two-dimensional particles in a magnetic field is discrete, the quasiparticles are undamped and the bare Landau levels are split. For the self-energy part of the one-particle Green's function the class of diagrams that describes the resulting splitting of the Landau levels is identified. The results are compared with the classic Landau Fermi-liquid theory.

1. INTRODUCTION

The investigation of the thermodynamic properties of a system in a magnetic field H (in particular, the oscillations of the magnetic susceptibility[1]) is based on certain ideas about the energy spectrum of the electrons in a magnetic field. Here, the question of the character of this spectrum in the presence of interaction between the particles is of fundamental significance. A pertinent investigation of this problem in the model of an isotropic Fermi liquid was carried out in Refs. 2 and 3 for a three-dimensional system. The results obtained in these papers show that in a sufficiently weak magnetic field the spectrum of the quasiparticles near the Fermi surface coincides with the spectrum of free electrons with a mass renormalized on account of the interaction.

In view of the great interest in the investigation of the properties of a system of interacting two-dimensional electrons in a magnetic field, the question arises as to whether the results obtained in Refs. 2 and 3 are valid when applied to this system. The fundamental difference between the self-energy part in a system of two-dimensional electrons and in the three-dimensional case was first noted in Ref. 2. In this paper it was shown that in the general case the dependence of the self-energy part Σ of the one-particle Green's function on the magnetic field depends not only on the operator $\hat{\mathbf{p}}-(e/c)\mathbf{A}$, but also explicitly on the magnetic-field intensity H. In weak magnetic fields, when the cyclotron energy is small compared with the chemical potential, for a three-dimensional system the explicit dependence on the magnetic-field intensity can be neglected. This is what permitted the energy spectrum of a three-dimensional system in a weak magnetic field to be investigated in Ref. 3 using the general Landau Fermi-liquid theory.

The aim of the present paper is to investigate the character of the energy spectrum of a system of two-dimensional particles in a magnetic field in the model of a nearly ideal Fermi gas. It is well known that for a three-dimensional system this model corresponds completely to Landau Fermi-liquid theory, and makes it possible to find the characteristic quantities of this theory in explicit form.[4,5] In particular, in Ref. 5 it was established which class of diagrams describes the Fermi-liquid properties in the model of a nearly ideal gas. At the same time, in Ref. 6, this model was used to study the properties of a three-dimensional system in the presence of a weak magnetic field and it was shown that the results correspond completely to ideas of the system as a Fermi liquid.

In a previous paper,[7] we obtained the first results characterizing the behavior of a two-dimensional system in a weak magnetic field in the model of an almost ideal gas. We investigated the character of the self-energy part of the one-particle Green's function to second order of perturbation theory. In the present paper we perform a systematic analysis of the energy spectrum of the system for the case when, in the absence of interaction, all Landau levels up to the Fermi energy are completely filled. The latter circumstance makes it possible to investigate vigorously the character of the splitting of the Landau levels and to find the particle occupation numbers. The results obtained show that the behavior of a nearly ideal two-dimensional Fermi gas in a weak magnetic field corresponds, to a certain extent, to Landau Fermi-liquid theory.

2. ENERGY SPECTRUM

In the approximation to be considered it is assumed that the magnetic field is weak in the sense that the condition

$$\hbar\omega_c/\mu \ll 1, \tag{1}$$

is fulfilled, where the cyclotron frequency is $\omega_c = eH/mc$ and μ is the chemical potential of the system.

The interaction between the particles is short-range:

$$ak_F \ll 1, \tag{2}$$

where a is the characteristic range of the interaction and k_F is the wave vector of the particles on the Fermi surface. The interaction is repulsive in character, and for the scattering of two particles by each other we can confine ourselves to the Born approximation, the condition for which is the inequality

$$\frac{mU_0}{\hbar^2}\ln\frac{1}{ak_F} \ll 1, \quad U_0 = \int U(r)d^2x, \tag{3}$$

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where m is the unrenormalized particle mass and $U(r)$ is the short-range pair-interaction potential.

The Hamiltonian of the interaction has the form

$$\hat{H}_{\rm int}=\frac{U_0}{2}\int d^2x d^2x' \hat{\psi}^+_\sigma(\mathbf{r})\hat{\psi}^+_{\sigma'}(\mathbf{r}')\hat{\psi}_{\sigma'}(\mathbf{r}')\hat{\psi}_\sigma(\mathbf{r}), \quad (4)$$

where the indices σ,σ' give the spin projection.

Below, we shall investigate the properties of the self-energy part of the one-particle Green's function. We use standard notation, and therefore we refer the reader to the monograph Ref. 8 for all the details. We should draw attention, however, to one very important aspect. In view of the degeneracy of the Landau levels, it is necessary, generally speaking, to use the temperature diagram technique and make the transition to zero temperature (with the aim of determining the quasiparticle spectrum) by means of analytic continuation.[8] In this paper we confine ourselves to the situation when, in the absence of the magnetic field, all Landau levels are completely filled (up to the Fermi energy) or empty. This condition makes the ground state of the system nondegenerate, and, in principle, makes it possible to use the zero-temperature diagram technique from the outset.

The following remark pertains to the calculation of diagrams in the two-dimensional case in the presence of a weak magnetic field. The corresponding expressions are most simply obtained as follows. In the absence of the magnetic field each vertex appearing in the expression for the self-energy part of the Green's function is proportional to the quantity

$$U_0\int \frac{d^2p_1d^2p_2d^2p_3}{(2\pi\hbar)^4}\,\delta(\mathbf{p}+\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3)f(\epsilon_p,\epsilon_{p_1},\epsilon_{p_2},\epsilon_{p3}), \quad (5)$$

where f is a certain function that depends on the energies ϵ_{p_i} of the interacting particles. The transition to the discrete particle spectrum in a magnetic field reduces to the following. The δ-function appearing in the expression (5) should be represented in the form of the standard integral of a exponential, after which one must integrate over all angles. The integration over the variables p_i^2 is replaced by summation over the Landau-level indices by means of the relation $p_i^2=2m\hbar\omega_c(N_i+1/2)$, where N_i is the level index:

$$\int dp_i^2 \to \sum_{N_i} 2m\hbar\omega_c. \quad (6)$$

Finally, we obtain the following expression for the second-order correction to the self-energy part of the Green's function in a weak magnetic field:

$$\Sigma^{(2)}(\omega,N) =\left(\frac{mU_0}{\hbar^2}\right)^2\frac{m}{(2\pi)^2}\frac{(\hbar\omega_c)^3}{\hbar^2} \times\sum_{N_1,N_2,N_3} I(N,N_1,N_2,N_3) \times\frac{n(N_1)[1-n(N_2)][1-n(N_3)]+[1-n(N_1)]n(N_2)n(N_3)}{\omega+\mu+\epsilon(N_1)-\epsilon(N_2)-\epsilon(N_3)}, \quad (7)$$

where $n(N)$ is the Fermi distribution function. The quantity $I(N,N_1,N_2,N_3)$ is an integral of Bessel functions:

$$I(N,N_1,N_2,N_3)=\int_0^\infty xdxJ_0(x\alpha)J_0(x\alpha_1)J_0(x\alpha_2)J_0(\alpha_3), \quad (8)$$

where $\alpha_i=\sqrt{2(N_i+1/2)}/l_H$, and the magnetic length is $l_H=(c\hbar/eH)^{1/2}$. The appearance of Bessel functions in the expression (8) is not accidental, but reflects the fact that in a magnetic field the one-particle Green's function is determined by means of Laguerre polynomials (see Ref. 3), whose asymptotic forms for large values of the index are Bessel functions.

The integral (5) can be calculated (see Ref. 9), and is equal to

$$I=\frac{1}{\pi^2}\begin{cases}\frac{1}{b}\mathbf{K}\left(\frac{a}{b}\right), & a<b\\ \frac{1}{a}\mathbf{K}\left(\frac{b}{a}\right), & b<a\end{cases}, \quad (9)$$

where

$$a^2=\alpha\alpha_1\alpha_2\alpha_3,\quad 16b^2=[(\alpha+\alpha_1)^2-(\alpha_2-\alpha_3)^2] \times[(\alpha_2+\alpha_3)^2-(\alpha-\alpha_1)^2],$$

and $\mathbf{K}(x)$ is a complete elliptic integral of the second kind.

In order that the integral (8) be nonzero, it is necessary that certain inequalities be fulfilled. In particular, the following conditions should be fulfilled:

$$\alpha+\alpha_1\geqslant|\alpha_2-\alpha_3|,\quad \alpha_2+\alpha_3\geqslant|\alpha-\alpha_1|.$$

These inequalities are related to the conditions imposed on the magnitudes of the momenta that appear in the δ-function in the expression (5).

It follows from the expressions (8)–(9) that, in the range of values of N_i that lies near the Landau-level index N_0 corresponding to the Fermi level, the quantity $I(N,N_1,N_2,N_3)$ is

$$I\approx\frac{l_H^2}{2N_0\pi^2}\ln\frac{4a}{\sqrt{|a^2-b^2|}},\quad \sqrt{|a^2-b^2|}\ll 1. \quad (10)$$

Taking this result into account, we can estimate as follows the value of the self-energy part of the Green's function in the second order of perturbation theory:

$$\Sigma^{(2)}\propto\left(\frac{U_0}{l_H^2}\right)^2\frac{\hbar\omega_c}{\mu}\ln N_0\sum_{\{N\}}\frac{A(\{N\})}{\omega+\mu-E(\{N\})}, \quad (11)$$

where the notation $\{N\}$ refers to the set of Landau levels that appears in the expression (7), and the quantities $A(\{N\})$ are of order unity.

The characteristic energy is

$$\frac{U_0}{l_H^2}=\frac{U_0 m}{\hbar^2}\hbar\omega_c\ll\hbar\omega_c. \quad (12)$$

An important feature of the expression (7) for the self-energy part of the Green's function is the summation (not integration) over the values of the intermediate energies appearing in it. This is connected with the discrete character of the energy spectrum of a two-dimensional system in a magnetic field. In the three-dimensional case, in the model of a weakly nonideal gas, the expression (7) describes effective-

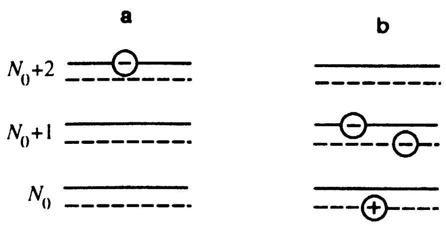

FIG. 1. Resonance states for a particle in the Landau level with index N_0+2. The state (a) corresponds to an initial state with one particle above the Fermi level. The state (b) contains an extra electron-hole pair. The levels depicted by the solid line and dashed line take into account the spin splitting in the magnetic field. The symbols − and + describe a particle and a hole, respectively.

mass renormalization and quasiparticle damping. This also applies to a two-dimensional system in the absence of a magnetic field. The situation changes radically for a two-dimensional system in a magnetic field. In the summation over the numbers N_i in the expression (7) there can be terms for which the condition

$$N_2+N_3=N+N_1$$

is fulfilled, i.e., $\Sigma^{(2)}$ can contain a resonance contribution of the form

$$\Sigma^{(2)}_{\text{res}}=\frac{B_N^{(2)}}{\omega+\mu-\epsilon(N)}. \tag{13}$$

The presence in $\Sigma^{(2)}$ of a term of the form (13) corresponds to resonance between different states of the system, while for a particle situated in a Landau level higher than the chemical potential the resonance state corresponds to two particles and one hole (see Fig. 1). An analogous situation also arises for a hole. Thus, in the case of a continuous spectrum the poles of the expression (7) describe the damping of the quasiparticles, while for a discrete spectrum they correspond to resonance between different states of the system.

In the general case, the expression for the self-energy part has, obviously, the following form:

$$\Sigma(N,\omega)=\Sigma_{\text{res}}(N,\omega)+\Sigma_{n\text{res}}(N,\omega). \tag{14}$$

In the model of a nearly ideal Fermi gas, for the nonresonance term $\Sigma_{n\text{res}}$ we can confine ourselves to the contribution in second order of perturbation theory.

We now discuss the resonance term Σ_{res} in more detail. It is easy to see that the situations for the cases when the Landau level with index N_0 corresponding to the Fermi energy is completely and incompletely filled differ sharply. We shall consider first the case in which the level with the Fermi energy is incompletely filled, when a situation is possible in which the resonating state corresponds to the condition

$$N_1=N_2=N_3=N=N_0.$$

Thus, an extra particle at the Fermi level decays into two particles and a hole with the same energies. An analogous situation is valid for a hole. It is entirely obvious that each of the particles and holes that arise can decay again, and the system passes, e.g., into a state with three particles and two holes, and so on. Thus, the number of resonating particles becomes infinite.

An analogous process is also possible for an arbitrary particle or hole. This implies that it is necessary to take into account the contribution to the self-energy part from resonance terms arising from all orders of perturbation theory. Our analysis has shown that in the case of an incompletely filled Fermi level it is necessary to take a very wide class of diagrams into account, and we have not yet succeeded in obtaining a closed expression for the self-energy part, since the vertex part that appears in the expression requires the inclusion of diagrams that lie outside the framework even of the "parquet" approximation.

The situation changes radically in the case of a completely filled Landau level. To see this, we shall consider the expression under the summation in Eq. (7), and rewrite it in the following form:

$$\frac{n(N_1)[1-n(N_2)][1-n(N_3)]}{\omega+\mu+\epsilon(N_1)-\epsilon(N_2)-\epsilon(N_3)} + \frac{[1-n(N_1)]n(N_2)n(N_3)}{\omega+\mu+\epsilon(N_1)-\epsilon(N_2)-\epsilon(N_3)}. \tag{15}$$

Here, the first term describes the possible decay of a particle above the Landau level, while the second describes the same for a hole. It follows from the expression (15) that in the case when all the occupation numbers $n(N_i)$ are equal to zero or unity the decay of a particle is possible only when the particle energy corresponds to a Landau level whose index is greater than the index of the Fermi level by at least two. For a hole, decay is possible beginning from the first Landau level lying below the Fermi energy.

The simplest resonance for a particle is shown in Fig. 1. For a particle with Landau-level index $N=N_0+3$ (the index N_0 corresponds to the Fermi energy) the number of resonating states is equal to 4. The corresponding states are shown in Fig. 2. The number of resonating states increases very rapidly with increase of the quantity k, which is defined by the relation

$$N=N_0+k,$$

where N is the index of the Landau level occupied by the particle. The number of resonating states determines the number of sublevels into which the bare level with index N will split. Of great importance is the fact that, in the case of a completely filled Fermi surface $[n(N)=1, N\leqslant N_0]$, the number of resonating states is finite.

Because the resonating states include some in which there are more than two particles (e.g., as in the situation represented in Fig. 2), in the determination of the self-energy part Σ it is not possible to confine oneself to second order of perturbation theory. The simplest insertions in Σ are depicted in Fig. 3. An insertion of type (a) (whether it is of the zero-sound or Cooper type) describes the interaction of particles and holes in states resonating with the initial states. Insertions of type (b) involve replacing the bare one-particle

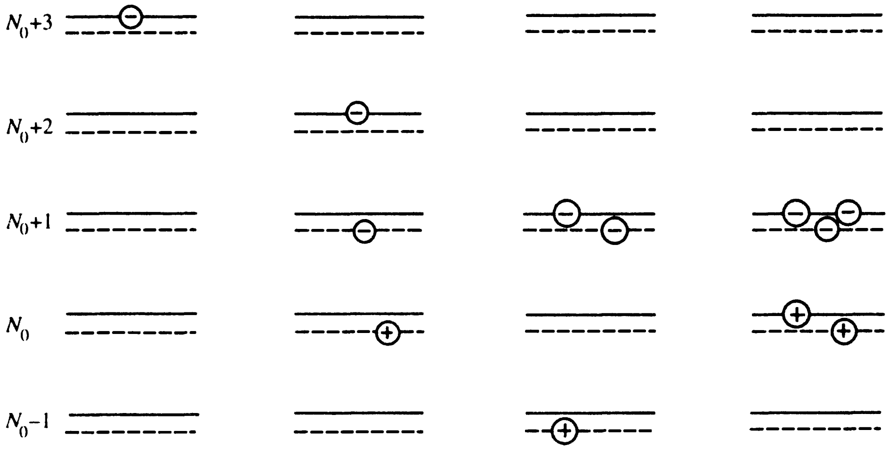

FIG. 2. Resonance states for the initial state with a particle in the Landau level with index $N=N_0+3$. The notation corresponds to that in Fig. 1.

Green's function by the exact Green's function. An important point is that these insertions have entirely different orders of magnitude.

Indeed, it is not difficult to convince oneself that processes describing the interaction of particles and holes [insertions of type (a)] make a contribution to the self-energy part of order

$$\left(\frac{U_0}{l_H^2}\right)^3\left(\frac{\ln N_0}{N_0}\right)^2\frac{1}{[\omega+\mu-\epsilon(N)]^2}. \tag{16}$$

This implies that, as a result of the interaction, the energy of a three-particle resonating state acquires a correction of order

$$\frac{U_0}{l_H^2}\frac{1}{N_0}\ln N_0. \tag{17}$$

This is much smaller than the splitting of the bare level, which, as follows from the expression (7), is of order

$$\frac{U_0}{l_H^2}\left(\frac{1}{N_0}\ln N_0\right)^{1/2}. \tag{18}$$

The contribution of the insertion describing the renormalization of the one-particle Green's function has the form

$$\left(\frac{U_0}{l_H^2}\right)^4\left(\frac{1}{N_0}\ln N_0\right)^2\frac{1}{[\omega+\mu-\epsilon_N]^3}. \tag{19}$$

The splitting of the levels is such that

$$\omega+\mu-\epsilon_N\sim\frac{U_0}{l_H^2}\left(\frac{1}{N_0}\ln N_0\right)^{1/2},$$

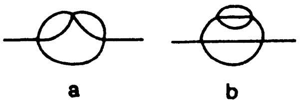

FIG. 3. The simplest insertions into the self-energy part. Diagram (a) takes into account the interaction between particles in a state resonating with the initial state, and diagram (b) corresponds to renormalization of a bare one-electron Green's function.

so corrections of the form (19) must be taken into account, since their contribution is comparable with the magnitude of the splitting of the Landau level on account of the resonances (18).

Thus, the analysis shows that in the case of a very weak magnetic field it is possible to disregard the interaction of particles and holes in states resonating with a bare one-particle (one-hole) state. On the other hand, in the calculation of the self-energy part it is possible to confine oneself to the "skeleton" diagram of second order of perturbation theory, having inserted the exact one-particle Green's functions into the internal lines. A very important point here is that, in resonance conditions, the Green's functions that appear in the internal lines are those of particles with energy smaller than the energy of the initial state. Thus, a fairly simple scheme arises for calculating the splitting of the Landau levels. The level corresponding to the Fermi energy, and the first level above this level, remain unsplit. The levels with indices N_0+2 and N_0-1 are split into two sublevels; the order of magnitude of the splitting energy is described by the expression (18), and its exact expression is described by the self-energy part calculated in second order of perturbation theory with the use of the bare Green's functions. In the calculation of the (fourfold) splitting of the Landau levels with index N_0+3, etc., for the Green's functions corresponding to the lower-lying levels it is necessary to use the Green's functions calculated in the previous stage.

Here, the order of magnitude of the spacing between the sublevels (near the Fermi level) corresponds to the expression (18) with a certain numerical coefficient. We must draw attention to the following extremely important fact. If we are considering the Landau level with index N_0+k, the number of states containing two particles and one hole and resonating with the initial state (so that the condition

$$\epsilon(N)+\epsilon(N_1)=\epsilon(N_2)+\epsilon(N_3)$$

is valid) increases with k as $k(k-1)/2$. Thus, the expression (18) must be multiplied by this number. In the approach under consideration, we should confine ourselves to those values of $k\gg1$ for which the inequality

$$k(k-1)\frac{U_0}{l_H^2}\left(\frac{1}{N_0}\ln N_0\right)^{1/2}\ll\hbar\omega_c \tag{20}$$

is fulfilled, i.e., for which the splitting of the levels is much smaller than the spacing between the bare Landau levels. In classic Landau Fermi-liquid theory this circumstance corresponds to increase of the damping of the quasiparticles with increase of the distance from the Fermi level. This implies that as the number k increases both the multiplicity of the splitting and its absolute value increase. At the present stage we have not succeeded in obtaining a general expression describing the dependence of the multiplicity of the splitting on the index k.

3. PARTICLE OCCUPATION NUMBERS

By virtue of the existence of resonances in a two-dimensional system in a magnetic field, the classic derivation of the expression for the difference between the particle-number and quasiparticle-number distributions[10] becomes invalid. In this paper we shall give a derivation of the corresponding expression, starting from the one-particle Green's function. In the presence of resonances it has the following form:

$$G(N,\omega)=\frac{1}{\omega+\mu-\epsilon(N)-\Sigma_{\text{res}}(N,\omega)-\Sigma_{n\text{res}}(N,\omega)}. \tag{21}$$

In the model under consideration, of a nearly ideal Fermi gas, in the calculation of the nonresonance contribution $\Sigma_{n\text{res}}$ we can confine ourselves to second order of perturbation theory. The quasiparticle spectrum ϵ is determined from the equation

$$\epsilon=\epsilon(N)+\Sigma_{\text{res}}(N,\epsilon-\mu)+\Sigma^{(2)}_{n\text{res}}(N,\epsilon(N)-\mu). \tag{22}$$

The Green's function (21) can be represented in the following approximate form:

$$G(N,\omega)\approx\frac{1}{\omega+\mu-\epsilon(N)-\Sigma_{\text{res}}(N,\omega)-\Sigma^{(2)}_{n\text{res}}(N,\xi_N)}+\frac{\Sigma_{n\text{res}}(N,\omega)-\Sigma_{n\text{res}}(N,\xi_N)}{[\omega+\mu-\epsilon(N)-\Sigma_{\text{res}}(N,\omega)-\Sigma_{n\text{res}}(N,\xi_N)]^2}, \tag{23}$$

where $\xi_N=\epsilon(N)-\mu$.

The second term in the expression (23) describes the difference of the particle and quasiparticle distributions (after integration over the frequency ω). It follows from this formula that the amounts by which the distribution differs from unity (below and up to the Fermi level) and from zero (above the Fermi level) are of order $(U_0 m/\hbar^2)^2\ll 1$. It should be emphasized, especially, that the discontinuity of the distribution function at the Fermi level in this case is conventional in character, by virtue of the discreteness of the spectrum of a two-dimensional system in a magnetic field. The distribution function now has the following form:

$$n_i(N)=\begin{Bmatrix}1-f_i^{(1)}(N), & N\leqslant N_0\\ f_i^{(2)}(N), & N>N_0\end{Bmatrix}, \tag{24}$$

where the subscript i corresponds to the index of a sublevel of the given Landau level with index N. Unfortunately, even in the absence of a magnetic field, because of the nontrivial nature of the integrals that arise it has not been possible to obtain an analytic expression for the f-functions.

The discontinuity in the particle distribution could have been represented as

$$Z=1-f^{(1)}(N_0)-f^{(2)}(N_0+1). \tag{25}$$

In Eq. (25) the sublevel index has been omitted, since for the model under consideration, of a nearly ideal Fermi gas, the function $f^{(i)}$ varies appreciably only over a distance of the order of the Fermi energy.

4. CONCLUSION

In this paper we have investigated the one-particle spectrum of a two-dimensional nearly ideal Fermi gas in the case of very weak magnetic fields. The principal feature of the system under consideration is the appearance of resonances corresponding to the formation of electron-hole pairs. This effect is due to the discrete character of the spectrum of two-dimensional particles in a magnetic field. In the case of a continuous spectrum the particle-decay process considered describes the damping of quasiparticles. In the case of an incompletely filled Fermi level we are concerned with an infinite number of resonating states, and come up against the problem of taking account of a very wide class of diagrams describing the self-energy part of the one-particle Green's function. The situation is substantially simpler for a completely filled Fermi level, i.e., with an integer filling factor. In this case it is necessary to take into account only a finite number of resonances. The calculations are further simplified because we are considering a weak magnetic field. This allows us to disregard the interaction between the electron-hole pairs that arise. Because of the presence of resonances, a bare Landau level splits into a finite number of sublevels, which depends on the energy difference between the level under consideration and the Fermi energy. The number of sublevels increases rapidly with distance from the Fermi level.

It is natural to ask how the energy spectrum changes as the filling factor moves away from an integer value. There is every reason to assert that, when the filling factor differs very little from an integer, there is no radical change in the spectrum. The point is that the probability of creation of electron-hole pairs at the Fermi level is proportional to the product $\nu_F(1-\nu_F)$, where ν_F is the filling factor of this level. It follows from this that when the condition $1-\nu_F\ll 1$ is fulfilled the corresponding processes are of low probability, and the principal contribution to the splitting of the Landau levels will be given by processes arising from the appearance of additional particles in empty Landau levels. This idea can also be used to treat the change in the particle distribution over the levels caused by the interaction, so that the filling factor ceases to be zero or unity in each level. As a result, the transition amplitudes between different resonance states is renormalized, but the corresponding corrections are small by virtue of the smallness of the parameter $(U_0 m/\hbar^2)^2$ that determines the deviation of the filling factors from the bare filling factors.

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