Capillary pinning of foam in porous media

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A theoretical model is developed which explains the mechanism for the propagation of pressure through a porous sample filled with foam. On the basis of our considerations we propose a one-dimensional model of foam: a chain of bubbles in a channel with corrugated walls. We use a mean field theory to analyze two possible mechanisms for the flow of the foam: by transport of the bubbles and by slipping of individual layers. In the latter case the mathematical model of the effect is the same as the Frenkel–Kontorova model for the flow of dislocations in crystals. We show that in stationary regimes the two mechanisms lead to the same flow curves.

1. INTRODUCTION

The main feature of the flow of gas in porous media in the presence of foam is the fact that its mobility is lowered by several orders of magnitude.[1] This is caused both by the blocking of the paths for the gas by individual layers of foam (i.e., by thin films intersecting the pore channel) and by the friction of the layers or bubbles on the wall of the effective channel.

In Ref. 2 we constructed a phenomenological theory of the motion of the foam based upon the concept of the flow as the motion of an ensemble of bubbles bound together in a chain: strings of bubbles. We suggested treating the simultaneous flow of the chains along a system of self-organized active channels as the flow of the foam. In the present paper we consider a microscopic theory of such a flow. In Sec. 2 we are mainly concerned with an explanation of the mechanism by which a pressure gradient penetrates a sample fully saturated by foam. Observations show that even when there is no flow the pressure gradient in a sample that has practically no free gas channels is nonvanishing (see the survey of Ref. 3). If we neglect the compressibility of the gas, to change the pressure by an amount Δp in a sample with a macroscale L we need to overcome a huge capillary barrier $\Delta p \sim \sigma L/r^2$, where σ is the surface tension coefficient and r is the size of the pore. The experimental values are significantly lower than the estimate we have just given. It is found that when we take the compressibility into account we are able to resolve this problem. The mathematical problem is close to the problem of determining the critical field for vortex formation in Josephson and similar junctions.[4–8]

The explanation for the appearance of a mean field in a sample makes it possible to study possible mechanisms for the flow of the foam from the standpoint of an effective field theory. For instance, in Sec. 3 we consider the flow of the foam in which the bubbles are forced through the pore channels. We propose a new soliton mechanism for the flow in Sec. 4; this is reminiscent of the Frenkel–Kontorova model[7] of the flow of dislocations in crystals. We show that one can derive the phenomenological equations of Ref. 2 from the equations of the original microscopic process.

2. STRING OF BUBBLES IN A BLOCKED CHANNEL WITH RIGID WALLS; CAPILLARY EFFECTS

To construct a microscopic theory for the flow of foam we must specify the picture of the flow at the microlevel. We first dwell on the description of a model of an active channel which was introduced earlier. Let us recall the basic experimental facts.

It has been shown[9–11] by direct observations that the active channels along which the layers of foam primarily flow are effectively smooth. These channels may be either strictly pore channels or channels with walls which are partially formed by jammed layers of foam. Moreover, the cross-sectional area of an active channel is practically constant. Reaching individual pore channels, which have a cross-section with a large oscillation amplitude, a layer either disappears due to purely mechanical (stability) reasons, or it sticks in the entrance. Another part of the pore channels with cross-sections which have a large oscillation amplitude forms the active channels by filling the large pores with foam with a characteristic cell size of order the diameter of the entrance of the narrow part of the channel.[11] The layers move along such a channel practically uninterruptedly.

Using these facts we shall model an active channel by a capillary with a variable cross section of radius

$$r=r_0+\delta\cos\left(\frac{2\pi x}{\lambda}\right),\quad \frac{\delta}{r_0}\ll 1,\quad \frac{\delta}{\lambda}\ll 1, \tag{1}$$

where δ is the amplitude of the oscillations and λ is the period of the channel. The condition for the blocking of the channel means that the leading layer of the string is in the channel with the smallest radius. When we apply a load the capillary forces try to contain the layers in the entrances of the channel, whereas the elastic forces caused by the compression of the gas in the bubbles force the layers to move to a new equilibrium position. The competition between these forces leads to a new nontrivial equilibrium state of the bubble chain in a blocked channel which, in turn, significantly affects the hydrodynamics of the flow of the foam. For clarity we consider possible equilibrium structures for a string of bubbles confined to a channel with rigid walls. Previously[2] we studied the action of the elastic forces, so

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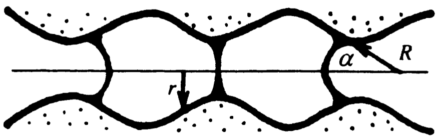

FIG. 1. Sketch illustrating the action of capillary forces on the various sections of a corrugated channel.

here we confine ourselves to the action of the capillary effects. One usually assumes[1] that the drop in pressure Δp across a layer is completely determined by Laplace's law (Fig. 1)

$$\Delta p=\frac{4\sigma}{R},$$

where σ is the surface tension coefficient of the lamina. One also assumes that the layer intersects the wetted walls at a right angle. Using Eq. (1), Laplace's law, and the formula for the elastic force from Ref. 2 we get the equation for the equilibrium of a string of bubbles in a blocked channel

$$\left(1+\frac{d\rho}{ds}\right)^{-2}\frac{d^2\rho}{ds^2}=\mu \sin \rho, \quad \mu=\frac{4\sigma\delta}{p_g r_0 \lambda}. \tag{2}$$

The left-hand side is determined here by the nonlinear equation of state of the bubble chain, and the right-hand side up to terms of order $O(\delta^2/\lambda^2)$ is an expression for the capillary force acting on the layer when it is displaced over a distance ρ from its equilibrium position. The Lagrangian coordinate s' and displacement ρ' are normalized using the period λ of the channel, as follows,

$$\rho=2\pi\frac{\rho'}{\lambda}, \quad s=2\pi\frac{s'}{\lambda}, \quad \mu=\frac{4\sigma\delta}{p_g r_0 \lambda}, \tag{3}$$

where p_g is the pressure of the gas in an individual bubble before it is deformed. Unless otherwise specified, primes indicate dimensional variables in what follows.

The appearance of nontrivial equilibrium shapes of the bubble string is clearly demonstrated in the language of dynamical systems. We rewrite Eq. (2) in the form of the set

$$\dot{\rho}=\frac{1}{u}-1, \tag{4}$$

$$-\dot{u}=\mu \sin \rho. \tag{5}$$

Equations (4) and (5) have a first integral

$$\ln u-u=E+\mu \cos \rho, \tag{6}$$

where E is a constant which is to be determined. From an analysis of Eq. (6) we can, depending on the value of the total energy E, elucidate the features of the internal structure of a blocked string (Fig. 2). Independent of whether the string spreads or contracts, the layer tries to leave the broad parts of the channel (the singular points $\dot{\rho}=0$, $\rho=2\pi n$, $n=0,\pm 1,\pm 2$ on the phase portrait). At the same time there are narrow parts of the channel, $\dot{\rho}=0$, $\rho=\pi(2n+1)$, which are stable equilibrium states for the layers of the string. Let us consider the case of compression, i.e., let us imagine that at a point infinitely far to the right there is a layer blocking the string, while one applies a pressure drop on the left. If in that case the energy lies in the range

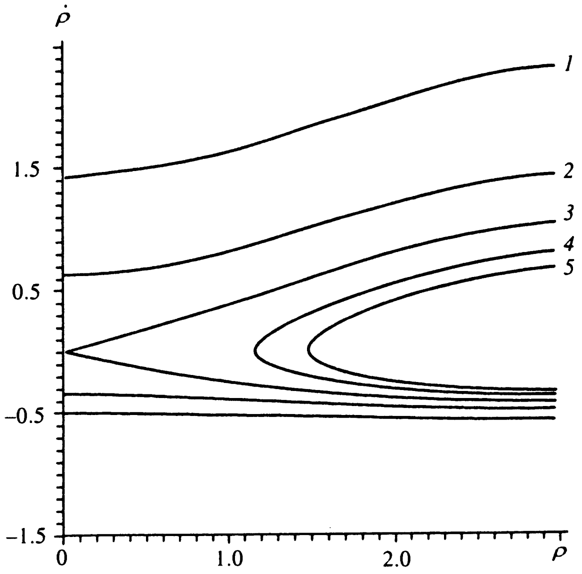

FIG. 2. Characteristic shape of the phase portrait of the set (4), (5) (μ=0.1) and various values of E:*1*—$E=-1.4$; *2*—$E=-1.2$; *3*—$E-1.1$; *4*—$E=-1.04$; *5*—$E=-1.01$.

$$E<E^{**}=-1-\mu,$$

the gradient of the shift will always be nonvanishing, which causes an infinite displacement of the layers of the foam: "laminar condensation." This effect is clearly demonstrated by the example of a string of finite length L. In the situation we have described one can neglect the right-hand side of Eq. (2). Using the equation of state

$$p=p_g\left(1+\frac{d\rho}{ds}\right)^{-1},$$

we arrive at the conclusion that the solution of the problem corresponds to a string with a uniform distribution of the pressure p, equal to the load pressure. The layers in such a string are distributed according to a linear law

$$\rho=\frac{p-p_g}{p}(s-L).$$

It is clear from this equation that for $p\gg p_g$ the layer with a Lagrangian coordinate $s=0$ tends to occupy the same pore as the blocking layer, i.e., it tends to shift over a distance of the order L. All intermediate layers also shift into this pore. It is just in that sense that one can speak of a laminar condensation.

When the energy lies in the range

$$E<E^{**}$$

the shift of the layers is limited by the capillary forces and, generally speaking, is not uniform along the length of the channel: the set (4) and (5) allows solutions with a period $2T$, of the form

$$\rho(s+2T)=\rho(s)+2\pi.$$

In other words, in this case a periodic chain of domains appears along the string with a period which is different from the period of the channel. If, on the other hand, we have

$$E=E^{**},$$

then the shift of the layers is described by the separatrix and the period of the structure which then occurs is infinite. Although the problem (4), (5) can be solved in the parametric form

$$\ln u(t)-u(t)=E+\mu\cos\rho,$$

$$s=\int\frac{dt}{(1+t)^2\sqrt{\mu^2-(\ln u(t)-u(t)-E)^2}},$$

$$u(t)=(1+t)^{-1}, \tag{7}$$

it is difficult to use Eq. (7) to analyze the physical situation. However, when the parameter μ is much smaller than unity one can carry out a rather complete asymptotic study of the problem. It is just this case which is the most interesting one for the majority of applications. For instance, for foams which are stabilized by a surface-active substance[12] we have $p_g\approx 1$ bar, $r_0\approx 10^{-3}$ cm, $\sigma\approx 10$ dyne/cm and hence $\mu=10^{-2}\delta/\lambda\ll 1$. To determine the threshold for the formation of a modulated structure along the string up to terms of order $O(\sqrt{\mu})$ we can therefore use the approximate equation

$$\frac{d^2\rho}{dx^2}=\sin\rho,\quad x=\sqrt{\mu}s. \tag{8}$$

In this formulation the problem becomes equivalent to the problem of determining the critical field for vortex formation in Josephson contacts,[4,5] the problem of the pinning of a charge density wave,[6] or the problem of the phase transition in elastic chains described by the Frenkel–Kontorova model.[7,8] Without giving the detailed calculations we merely present the solution of the problem corresponding to a single domain wall

$$\rho=4\ \mathrm{arctg}\ \exp[-(x-x_0)]. \tag{9}$$

Here we have $\rho=2\pi$ as $x\rightarrow-\infty$, i.e., under the action of the applied pressure drop all layers behind the wall shift by a period, whereas in front of the wall we have $\rho=0$ as $x\rightarrow\infty$, i.e., the layers remain in the unperturbed state. To determine the critical pressure drop we must equate the change in the internal energy of the string to the work done by the external forces. To form a chain of domains with period T requires an amount of work

$$\Delta H=(p-p_g)\Delta l, \tag{10}$$

to be performed, where Δl is the average change in the length of the string

$$\Delta l=\frac{1}{2T}\int_{-T}^{T}\frac{d\rho}{ds}\,ds. \tag{11}$$

The change in the internal energy of the domain can be written in the obvious form

$$\Delta E=\frac{1}{2T}\int_{-T}^{T}\left(-p_c\cos\rho+\frac{p_g}{2}\left(\frac{d\rho}{ds}\right)^2\right)ds, \tag{12}$$

where $p_c=p_g\mu$. Letting the period T tend to infinity and using Eqs. (9)–(12) we find for the critical pressure drop the value

$$\Delta p^*=p-p_g=\frac{4}{\pi}\sqrt{p_gp_c}. \tag{13}$$

For pressure drops below the critical one the laminas therefore remain in the unperturbed state and the field does not penetrate into the channel. In that case one can talk about an effective screening of the applied pressure drop by the foam. Above the critical pressure drop the bubble string splits up into a set of regions bounded by domain walls. Further increase in the pressure leads to an increase in the number of domain walls, which ultimately leads to laminar condensation when they overlap.

Concluding this section, we note that the results we have obtained are not restricted to channels with the shape (1). The phase portrait has the same qualitative structure for any capillary with a periodically changing cross-sectional area. The domain formation effect must therefore also occur. Only the estimate (13) can change. However, it is extremely important for further applications that the domain formation effect allows the field to penetrate into the channel for significantly lower pressure drops than one expects at first sight, $\Delta p^*\ll\Delta p\sim\sigma L/r_0^2$. Turning to a porous medium, one may conclude that above the threshold Δp^* each string feels the action of the mean field, independent of whether it moves or is pinned. The critical pressure drop necessary for the formation of domain walls is much smaller than the one observed for a foam stabilized by a surface-active substance ($4\sigma/p_gr_0\sim 10^{-2}$; see Ref. 12) and we can neglect it. However, for a foam appearing as the result of the boiling of a liquid we have $\sigma/p_gr_0\sim 1$ and the capillary threshold may be quite appreciable. In what follows we consider only the first case and carry out our analysis using mean field theory. The introduction of a mean field enables us to distinguish hydrodynamic mechanisms for the flow of foam which differ in their character, depending on whether the bubbles or the layers move. In the first case, when all bubbles in the string move simultaneously, elastic effects are insignificant. This appears to be so since it is as if the whole string is forced like a single bubble through the porous channel under the action of the applied pressure drop.[13] In the opposite case, when the layers move, collective effects play a basic role and the displacement of the string takes place in the form of travelling waves. The next section is devoted to a study of the macroeffects following from the mechanism of the motion of foam through the transport of bubbles.

3. FOAM HYDRODYNAMICS: BUBBLE TRANSPORT

We consider a porous medium, completely filled with foam, consisting of a system of active channels—capillaries with a variable cross-section of the form (1) with some distribution law for their radii. If randomly connected they are able to form a connected system of channels for the transport of foam: an infinite cluster. One does not observe, as a rule, the flow of a free gas for small pressure gradients. We shall therefore assume that in this limit the gas is transferred only through the motion of the bubbles. Moreover, we shall assume that all bubbles move along the channels with a constant velocity. The ability of a separate "infinite" channel connecting opposite boundaries of the macrosample to conduct is then determined by the element with the smallest radius, more precisely, by the connecting section from a wide to a narrow capillary in which the action of the capillary forces is a maximum. If the walls of the infinite channel were rigid the flow rate of the foam through it would be determined by the difference in pressure at the entrance and the exit of the capillary. However, in reality the walls of an active channel consist partially of pinned layers of the foam, i.e, they are able to transfer the load. Each layer in the channel is thus subject to the mean field of the surroundings.

We can estimate the critical pressure gradient required for starting the flow of the foam. Neglecting hysteresis effects[14] we shall assume that the smallest pressure gradient necessary for sustaining a stationary flow is the same as the critical one. To obtain the required estimate it is sufficient to study the behavior of the system near the percolation threshold.

Near the percolation threshold an infinite cluster can be viewed as a grid of channels with an average distance between the nodes equal to the correlation radius ξ of the conducting chains

$$\xi \approx r_0(f-f_c)^{-\nu}. \tag{14}$$

Here f is the fraction of the conducting channels, f_c is its threshold value, and $\nu \approx 0.9$ is[15] the critical index. Hence we can estimate the effective force exerted on a bubble by its surroundings to be

$$\tau \approx -A\xi \mathbf{u} \nabla p, \tag{15}$$

where ∇p is the pressure gradient applied to the sample, $\mathbf{u}$ is a unit vector directed along the element, and A is the cross-sectional area of the channel.

Near the percolation threshold the main contribution to the flow rate of the foam comes only from the chains which are oriented along the applied pressure gradient.[16] In this limit one can also neglect their topological transformations. The flow rate Q of the foam averaged over all elements of the sample is then determined by the bubble velocity averaged over the elements of those capillary chains which are parallel to the applied pressure gradient

$$Q=\frac{A}{\pi \xi^2}\left\langle \frac{\partial p'}{\partial t'} \right\rangle. \tag{16}$$

The average bubble velocity in an infinite channel is completely determined by those elements for which the resistance to the motion of the layers is the largest. Since the capillary force acting on a layer depends significantly on the shape of the capillary one can hardly claim universality for the capillaries used in the model. Moreover, the law for the resistance to the motion of a layer along a channel with a different geometry is also not universal.[17] We must therefore restrict ourselves merely to a qualitative study of the problem. In the limiting case of an absolutely rigid bubble the law for the resistance for a layer is linear for slow flows and the friction constant γ can be estimated to be $\gamma \approx 2\pi \nu_\omega r$, where ν_w is the viscosity of the film wetting the pore and r is the channel radius.[2] Using as a model for the connecting section a conical capillary we can write down the equation for the motion of a layer in the form

$$2\nu_w \frac{\partial \rho'}{\partial t'} = -4\sigma \sin \alpha + (r_w - \rho' \tan \alpha)\tau. \tag{17}$$

Here r_w, is the radius of the wide connected channel and α is the angle of the cone. Integrating Eq. (17) we get the average velocity of the layer along a conical capillary of length L_r:

$$\overline{\frac{\partial \rho'}{\partial t'}} = \frac{L_r}{T \cos \alpha} = \frac{L_r \tau'}{2\nu_w \cos \alpha}\left[\ln \frac{\tau' - 4\sigma \sin \alpha / r_w}{\tau'(1-r_n/r_w) - 4\sigma \sin \alpha / r_w}\right]^{-1},$$

$$\tau' = -\xi \mathbf{u} \nabla p, \tag{18}$$

where r_n is the radius of the narrow channel.

If the resistance law is nonlinear, $F=\gamma(\partial \rho'/\partial, t')^\kappa$, the time spent by a layer in squeezing through the narrowing section is determined by the integral

$$T=\frac{r_w \gamma}{(\pi r_w \tau')^{1/\kappa} \tan \alpha} \int_0^{r_n/r_w} \frac{d\rho}{[(1-\rho)(b-\rho)]^{1/\kappa}},$$

$$b = 1 - \frac{4\sigma \sin \alpha}{r_w \tau'}. \tag{19}$$

Equations (18) and (19) together with Eq. (16) give us the equation of motion for the foam near the percolation threshold. It is clear from these solutions that the flow of the foam occurs only when the pressure gradient exceeds the threshold value

$$|\nabla p^*| = \frac{4\sigma \sin \alpha}{(r_w - r_n)\xi}. \tag{20}$$

Since the correlation radius ξ can be several orders of magnitude larger than the pore radius [see Eq. (14)] the estimate (20) looks very reasonable. The flow of the foam in a porous medium can thus be blocked by an insignificant number of small pores of size $r_w - r_n$ distributed with a frequency $1/\xi$. This idea has been expressed several times before[3,18] and a relation similar to (20) has been suggested[3] without any explanation of the appearance of a mean field in a sample saturated by foam.

We now turn to an analysis of the law for the flow of foam near the percolation threshold. We first consider the linear case. It follows from Eq. (18) that near the critical pressure gradient the velocity increases logarithmically, i.e., its derivative is infinite near the threshold. Such a behavior of the foam has so far not been observed, which speaks in favor of a nonlinear friction law with index $\kappa < 1/2$. Under this condition the derivative of the velocity tends to zero, which is characteristic for flows known up to the present.[1] The reason for the appearance of small indices $\kappa < 1/2$, may be that in the flow both the film wetting the surface of the pore and the layers involved.[13] In this case it makes sense to speak of the forcing of the bubble as a whole through the porous channel rather than the slipping of separate layers of the foam. The interaction between surface and viscous forces then leads to a nonlinear friction law with the necessary index.[13,17] This mechanism looks rather plausible far from the threshold for the capillary coupling to the liquid phase. However, near the threshold the wetting liquid is retained rather efficiently by the capillary forces, so that the foam is most probably transferred because the layers slip. Just this situation was considered earlier phenomenologically.[2] In the next section we analyze the mechanism for such a flow using a specific microscopic model.

4. FOAM HYDRODYNAMICS: LAMINAR TRANSPORT

We remind ourselves that under normal conditions separate bubbles appear as the structural elements of the foam. In a porous medium one can use a coarse description of the hydrodynamics and a string of bubbles can be used for such an element. We verified earlier that it is just the internal properties of the bubble strings which are responsible for the penetration of the field into a sample. However, the successful explanation by Hirasaki and Lawson[13] of the law for the flow of foam when a gas and a liquid are simultaneously present was responsible for the broad acceptance of the string model. In Hirasaki and Lawson's treatment all bubbles in the string moved simultaneously, and it follows from the results of Sec. 3 that the structural organization of the foam is unimportant: the string flow as a single bubble in the mean field.

Nonetheless, another mechanism for the flow of a string is also possible which takes into account its internal structure. In its general features this mechanism reminds one of the Frenkel-Kontorova model for the flow of dislocations in crystals.[7] In this approach the layers move along an active channel one after the other so that each layer "jumps" over a period of the channel from one equilibrium position to another and lies dormant until a new displacement wave catches up with it. In such a motion practically no energy is spent, so that the capillary forces are directed in opposite directions in the expansion and the contraction of the channel (see Fig. 1) while their action averaged over a period is compensated by the corresponding change in the elastic forces. Hydrodynamically the situation looks as if the shift of each layer involves only the surface layer of the wetting film. In fact, the formation of a new surface behind the moving layer requires exactly as many molecules of the wetting liquid as are freed when the surface in front of the layer is compressed. The molecules of the surface-active substance remain during the flow pinned behind a definite bubble. The layer thus slips along the film which is wetting the pore surface, and the depth to which the hydrodynamic perturbation penetrates is comparable to the thickness of the layer, i.e., much larger than molecular dimensions. In this case we can therefore use a macroscopic description for the mass exchange processes between the surfaces, and to a first approximation the resistance force can be estimated to be $F=2\pi\nu_w r\partial\rho'/\partial t'$.[2 1)]

When writing down the equation of motion for the foam layers we take into account that in the range of energies $E\simeq E^{**}$, i.e., up to the onset of the laminar condensation, the shifts of the layers are limited by the action of the capillary forces. In that case we can neglect the nonlinear elasticity. In the situation considered here it is just that range of energies which if of most interest, so we can write down the equation of motion in the form

$$\frac{\partial\rho}{\partial t}=\frac{\partial^2\rho}{\partial s^2}-\mu\sin\rho+\Upsilon. \tag{21}$$

Here Υ is the dimensionless external force, the dimensionless variables correspond to the set (3), and we have also

$$t=\frac{t'}{t_0},\quad t_0=\frac{\nu_w\lambda}{\pi r_0 p_g}.$$

In accordance with the proposed mechanism the displacement wave caused by the external force Υ propagates along the section under consideration and shifts the layers by a period. We shall assume in what follows that the section of the active channel which is oriented along the direction $\mathbf{u}(s)$ contains at most one domain wall, which separates the bubbles shifted by a period from the elements of the string that are in the unperturbed state. Assuming that the characteristic length a of a separate element of the active channel lies in the range $r_0\ll a\ll\xi$ (ξ is the length of the string) we can describe the profile of the laminar displacement by Eqs. (8) and (9) with $x=\sqrt{\mu}(s-vt)$.

In the limits as $\Upsilon\rightarrow 0$ the propagation speed v of the displacement wave can be found from the condition that Eq. (21) has a solution

$$-v\int_{-\infty}^{\infty}\left(\frac{d\rho}{dx}\right)^2dx=\Upsilon\int_{-\infty}^{\infty}\frac{d\rho}{dx}\,dx. \tag{22}$$

Substituting expression (9) into Eq. (22) we find

$$v=\frac{\Upsilon}{2\sqrt{\mu}}. \tag{23}$$

The motion of the domain walls can thus be treated as the slipping of effective layers along the active channel. It is just this kind of model of the string with the period of the chain being somewhat larger than the pore radius which was phenomenologically considered earlier.[2] In the proposed approach the period a of the chain is determined by the history of the loading and it depends, generally speaking, on the applied pressure drop.[5] It is sufficient for a justification of the phenomenological approach to check the formula for the foam flow postulated in Ref. 2. In the external force Υ we split off the term connected with the applied pressure gradient and the oscillating part caused both by the inhomogeneity of the porous space and by the contribution from the capillary forces. We then get the following formula for the force density

$$\frac{\delta\Upsilon}{\delta s}=-\frac{\beta\mathbf{u}(\mathbf{u}\nabla p)}{p_g}+\frac{\Phi}{p_g}, \tag{24}$$

where

$$\langle \Phi(s,t) \rangle = 0,$$

here β is a constant mean field and the averaging is carried out over the realizations of the forces Φ. Using (24) and changing to dimensional variables we find the foam flux in the sample,

$$J_k = -c \frac{\beta r_0}{4\sqrt{\mu \nu_w}} \int_0^{\xi} ds \langle u_k(s,t) u_i(s,t) \rangle \frac{\partial p}{\partial x_i}, \tag{25}$$

where c is the concentration of the bubble strings. Apart from a constant factor, expression (25) is the same as the corresponding phenomenological definition.[2] Introducing as before[2] a tensor order parameter

$$S_{ki} = \langle u_k(s,t) u_i(s,t) - \tfrac{1}{3} \delta_{ki} \rangle,$$

we can write the flux in the form

$$\mathbf{J} = \mathbf{J}_0 - c \frac{\beta r_0 \xi}{12\sqrt{\mu \nu_w}} \int_0^{\xi} ds S * \nabla p, \tag{26}$$

$$\mathbf{J}_0 = -c \frac{\beta r_0 \xi}{12\sqrt{\mu \nu_w}} \nabla p.$$

The first term in Eq. (26) can be related to the corresponding term in Darcy's law, while the second term is specific for the flow of gas in the presence of foam: the flux of the foam depends significantly on the orientation of the active channels. A change in the ordering occurs, firstly, as the result of the deformation of the channels in the flow and, secondly, as the result of diffusive motion of the strings—reptations. Both mechanisms have been analyzed before[2] using a model of the string in which the characteristic distance between the layers is substantially larger than the size of a pore. Since in this context one can treat a domain wall as an effective layer all results can automatically be taken over in the case considered here. Note that in this mechanism, in contrast to the previous one, the specific features of the porous medium manifest themselves through the hydrodynamics: the formation of active channels in the framework of the proposed flow mechanism occurs under the action of the hydrodynamic field.[2] The order parameter S in Eq. (26) is thus a functional of the velocity and of the pressure gradient. For small pressure gradients the number of elements of the channel oriented along the field is small, $S \to 0$, so that the foam flow rate also tends to zero; when we exceed a certain pressure gradient practically all elements of the channel are oriented along the field, and the order parameter S tends to 2/3, so that the foam flow rate is significantly increased. As a result the dependence of the filtration rate of the gas on the applied pressure gradient becomes nonlinear and reminds one of the law for the flow of a pseudoplastic.[2]

5. CONCLUSION

The analysis of the behavior of a one-dimensional chain of bubbles placed in a channel with a periodically changing cross-sectional area thus enables us to elucidate the mechanism for overcoming a capillary barrier by applying pressure. Thanks to the compressibility of the gas the effective screening by the foam of the applied pressure drop Δp is possible only under the restricted conditions $\Delta p < \Delta p^*$ [see Eq. (13)]. Above the threshold Δp^* the chain is split into a set of macroregions such that the shift of the layers in neighboring domains differs by the period of the channel. In the connecting sections—domain walls—the shift of the laminas is nonuniform, so that the pressure gradient is also nonvanishing. This is just in this way, i.e., through overlap of the domain walls, that the field penetrates into the channel. A similar situation arises also, in all probability, in a porous medium filled with foam; otherwise it is impossible to explain the existence of small pressure gradients $\Delta p / L \ll \sigma / r^2$ in macrosamples of size L. At the same time the critical pressure drops which have been observed in practice, above which foam flow starts, are appreciably higher than the threshold Δp^*. In this paper we have given an analysis of two possible hydrodynamic mechanisms explaining such a behavior.

The first one treats the foam flow as the forcing of individual bubbles or of bubbles bound together in a chain through a porous channel. Although just this mechanism served as the impetus for studying the bubble string model[13] the nature of the structural organization of the foam is unimportant in this case. The introduction of a mean field makes it possible to show that the law for the foam flow is completely determined by the friction law for an individual bubble at the wall of the channel. The minimum critical pressure gradient required to sustain the flow is determined by merely an insignificant number of small pores in which the action of the capillary forces is maximal. The number of such "dangerous" pores is determined solely by the structure of the conducting cluster near the percolation threshold and is independent of the hydrodynamics.

According to the second mechanism the foam flow can be treated as the motion of domain walls along a set of bubble strings. The particular features of the porous medium in this case show up both at the structural and at the hydrodynamical level: moving with the flux the string cannot unimpededly surmount the porous space. Its contour becomes entangled as the number of "collisions" between the domain walls and the porous matrix increases and also due to their random motions in response to pulsations of the effective force.[2] As a result the flow rate of the gas (of the bubbles) decreases. Although in this case the resulting flow curve is smooth, one can distinguish on it a section with an almost zero velocity while the pressure drop is nonzero. The apparent critical pressure drop in actual fact divides the diffusive regime of the motion of the domain walls from the convective one.

In both cases the shape of the flow curves is close to the one observed experimentally, so that to explain the actual flow mechanism one must analyze nonstationary experiments in which the mechanisms discussed here show up differently.

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[1)]A. N. Rozhkov obtained a similar estimate for the resistance force of a layer on a section of the active channel consisting of pinned foam layers, where the film was subject to stretching forces.

[1]A. R. Kovscek and C. J. Radke, in *Foams, Fundamentals and Applications in Petroleum Industry* (Ed. L. L. Schramm) Adv. in Chem., Ser. 242 (1992), p. 113.
[2]K. G. Kornev and V. N. Kurdyumov, Zh. Éksp. Teor. Fiz. **106**, 457 (1994) [JETP **79**, 251 (1994)].
[3]W. R. Rossen and P. A. Gauglitz, AIChE J. **36**(8), 1176 (1990).
[4]B. D. Josephson, Adv. Phys. **14**, 419 (1965).
[5]I. O. Kulik, Zh. Éksp. Teor. Fiz. **51**, 1952 (1966) [JETP **24**, 1307 (1967)].
[6]A. I. Larkin, Zh. Éksp. Teor. Fiz. **105**, 1793 (1994) [JETP **78**, 971 (1994)].
[7]Ya. I. Frenkel' and T. A. Kontorova, Zh. Éksp. Teor. Fiz. **8**, 1340 (1938).
[8]P. Bak, Rep. Prog. Phys. **45**, 587 (1982).
[9]K. T. Chambers and C. J. Radke, in *Interfacial Phenomena in Petroleum Recovery* (Ed. N. R. Morrow), Marcel Dekker Inc., New York (1990), Ch. 6, p. 191.
[10]K. T. Chambers, MS Thesis, Univ. of California, Berkeley (1990).
[11]A. Basilevsky, K. Kornev, and A. Rozhkov, in *Physical and Mathematical Modelling of Foams Flow through Porous Media* (Eds. V. M. Entov and V. I. Zemskih) IKU Sintef Group, Trondheim (1994), Ch. 5, p. 65.
[12]J. E. Hansen, Ph. D. Thesis, Roganald Univ. Center, Stavanger (1993).
[13]G. J. Hirasaki and J. B. Lawson, Soc. Petr. Eng. J. **25**(2), 176 (1985).
[14]A. H. Falls, J. J. Musters, and J. Ratulowski, Soc. Petr. Eng. Reservoir Eng. **4**, 155 (1989).
[15]J. W. Essam, Rep. Prog. Phys. **43**, 53 (1980).
[16]A. S. Skal and B. I. Shklovskiĭ, Fiz. Tekh. Poluprovodn. **8**, 1586 (1974) [Sov. Phys. Semicond. **8**, 1029 (1975)].
[17]J. Ratulowski and H. C. Chang, Phys. Fluids A1, 1642 (1989).
[18]V. M. Entov, in *Physical and Mathematical Modelling of Foams Flow through Porous Media* (Eds. V. M. Entov and V. I. Zemskih) IKU Sintef Group, Trondheim (1994), Ch. 1, p. 10.

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