Theory of phase locking in lasers with "all-to-all" optical coupling

S. Yu. Kurchatov, V. V. Likhanskiĭ, and A. P. Napartovich

Troitsk Institute for Innovative and Thermonuclear Research, 142092 Troitsk, Moscow Region, Russia
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We investigate emission from an array of lasers with a random spread of frequencies when all lasers are coupled with all others. We show that as the eigenfrequency detuning increases, either emission is quenched or the coherent steady-state emission regime is replaced by a nonstationary regime. Stable phase-locking regions are determined analytically and numerically. We show that the time lag in the gain results in cooperative phase locking at a maximum detuning close to the relaxation frequency. The peak brightness achievable is then greater than that of a system of identical lasers with a common pump.

1. INTRODUCTION

Many nonlinear systems[1,2] (a collection of Josephson junctions, gasdynamic vortices, neural networks, evolutionary models, and economic models) are dynamical systems with global coupling, i.e., feedback proportional to a mean taken over an array of interacting elements. One relatively simple example of this class of systems is a set of lasers that are all optically coupled to one another. To a good approximation, this sort of coupling can be implemented experimentally by placing an iris diaphragm at the system's common focus.[3] A study of this system can additionally shed light on the dynamically complex behavior of an array with global coupling. Moreover, such studies are of practical interest from the standpoint of obtaining high-power radiation with low angular divergence.

To address this problem, it is necessary to have a coordinated system of N lasers emitting a coherent field with minimal phase excursions at the output aperture. This can be achieved if the characteristics of all the lasers—in particular, the resonator eigenfrequencies—are approximately the same. In actuality, there will always be differences among the eigenfrequencies, which can be either static or randomly varying in time; thus far, static deviations among the eigenfrequencies have received the most attention. For example,[4] when the resonator eigenfrequency mismatches are bounded, the field at the output aperture exhibits a domain structure in which the individual domains tend to differ in phase by a value close to π. The mean domain size is governed by the relationship between the optical coupling coefficient and the magnitude of the eigenfrequency mismatch. This correlation length places a lower limit on the divergence of the total emission that does not depend on the size of the array as a whole. Likewise,[4] the advent of global feedback in a laser array with nearest-neighbor optical coupling significantly broadens the parameter range over which all the laser fields remain phase-locked.

Random variation of the eigenfrequencies with time introduces an element of self-averaging into the interactions among the lasers of the array. As the maximum frequency deviation increases, the resulting mean field at the output aperture, which determines the brightness of the emitter, behaves like the magnetic moment of a ferromagnet that is being heated, i.e., it undergoes a phase transition.[5] Furthermore, it was noted in Ref. 5 that the loss of coherence (as characterized by the order parameter) can take place via the emergence of topological solitons engendered by the eigenfrequency fluctuations.

The dynamics of an array of lasers with global feedback and a delayless optical medium has recently been studied numerically.[6] The eigenfrequency deviations were static, with a distribution that was approximately Lorentzian. Four different regimes were detected numerically: phase-locked, partially phase-locked (an analog of the domain regime in Ref. 4), independent emission by each laser, and oscillation between an order and disordered state. Given that the authors detected an abrupt increase in fluctuations of the order parameter (the mean field at the system output) when the order parameter itself underwent an abrupt decrease, we are dealing here with the analog of a thermodynamic

In the present paper, along with a numerical analysis, we conduct an analysis of feasible dynamical regimes for the fields of an array of lasers with global feedback, which enables us to find an explicit criterion for the quenching of emission or transition to a nonstationary regime. It has been shown elsewhere,[7] based on the behavior of two optically coupled lasers, that time delays in the medium can engender dynamically complex signal emission. We show below that time-delayed gain in an array of lasers with global feedback gives rise to a new effect, which we call cooperative field locking.

2. BOUNDARIES OF THE COHERENT EMISSION REGIME

We make a number of assumptions in studying emission regimes of a fully optically coupled laser array: with no coupling, the field in each laser can be characterized by one longitudinal and one transverse mode; the laser gains are proportional to the population differences between two resonant levels in the active medium (two-level approximation); and the principal difference between lasers is in the frequency of their eigenmodes. A system consisting of N

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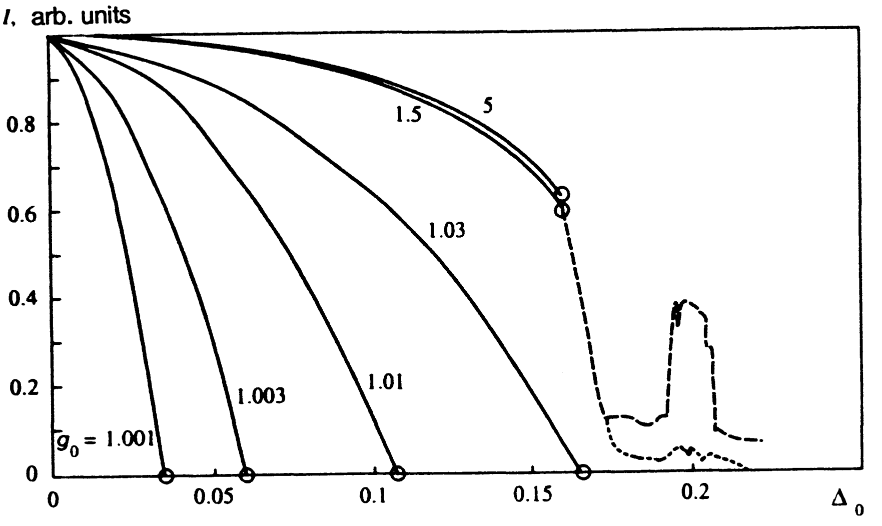

FIG. 1. Time-averaged brightness of a system of lasers as a function of the width of the eigenfrequency distribution for various values of g_0: steady-state emission (solid curve), dynamical regime with $\tau=50$ (dashed curve), dynamical regime with delayless medium (dotted curve).

coupled lasers possesses at least $3N$ degrees of freedom: N complex amplitudes E_k and N active-medium gain coefficients g_k.

The dynamical equations for the fields in a fully coupled laser array are[3]

$$\dot{E}_k=(g_k-1-M)E_k+i\Delta_k E_k+\frac{M}{N}\sum_{m=1}^{N} E_m, \quad (1)$$

$$\tau\dot{g}_k=g_0-g_k-|E_k|^2 g_k. \quad (2)$$

Delays due to the propagation of light in the coupling channels is negligible, assuming that the coupling channel is short compared with the length of the resonant cavity. The following dimensionless quantities appear in the system of equations (1) and (2) (N is the total number of lasers):

$$|E_k|^2=\frac{|\tilde{E}_k|^2 g_0}{E_s^2 g_n},\quad t=\frac{\tilde{t}g_n l}{\tau_p},\quad \Delta_k=\frac{\tilde{\Delta}_k\tau_p}{g_n l},\quad M=\frac{\tilde{M}}{g_n l},$$

$$g_0=\frac{\tilde{g}_0}{g_n},\quad \tau=\frac{\tilde{\tau}g_n l}{\tau_p},$$

where $\tilde{E}_k$ is the complex field amplitude in the kth laser, E_s is the saturation field, $\tau_p=2L/c$ is the round-trip travel time for light in each resonator, L is the resonator length, $g=\tilde{g}/g_n$, $\tilde{g}$ is the active-medium gain, g_n is the gain threshold, $\tilde{g}_0$ is the small-signal gain, l is the length of the active medium, $\tilde{\Delta}_k$ is the eigenfrequency offset of the kth resonator from the average across all lasers, $\tilde{M}$ is the coupling coefficient, which is the same for all lasers in the system, and $\tilde{\tau}$ is the relaxation time in the active medium.

The first term on the right-hand side of Eq. (1) accounts for gain and loss in traversing the resonator, including the effects of light coupled to other lasers, the second term is the change in the phase of the field (we assume that $|\Delta_k\tau_p|\ll 1$), and the third accounts for injection of the mean field into each laser.

In Eq. (2), g_0/τ accounts for pumping of the active medium, $-g/\tau$ for relaxation of the gain, and the last term for the decrease in population difference of the resonant levels as the lasers radiate.

In the event of fast inversion relaxation of the active medium (with a delayless medium), Eq. (2) is replaced by

$$g(E_k)=\frac{g_0}{1+|E_k|^2}. \quad (3)$$

We assume that the eigenfrequency differences among the resonators are uniformly distributed over the interval $[-\Delta_0/2, \Delta_0/2]$, in contrast to the Lorentzian distribution assumed in Ref. 6. Here Δ_0 is the width of the distribution. In numerical models for the solution of Eqs. (1)–(2), the frequency mismatch Δ_k is set by a random number generator.

The degree of field coherence in the system of optically coupled lasers can be characterized in terms of the axial brightness of the total radiation in the far zone, which is proportional to the squared modulus of the total field. When eigenfrequency offsets among the resonators are small, the phase of each laser field will be constant in time and different from the phase of the mean field. As the spread in frequencies increases, so will the phase shifts among the fields. Various scenarios come into play for the breakdown of coherence and decrease in brightness as frequency mismatches become more significant; these depend upon the relationship between the magnitude of optical coupling and the gain of the active medium.

Figure 1 shows the dependence of the time-averaged brightness on Δ_0, normalized to the maximum value of the brightness (at $\Delta_0=0$), for various g_0-1 and M. The curves were calculated for a large number of lasers, $N>100$, and the results were independent of the specific random frequency distribution.

For low small-signal gain ($g_0-1<M$), as the width of the frequency distribution increases, the laser array continues to emit coherently (curves labeled $g_0=1.001$–1.03 in Fig. 1), but the total brightness drops to zero. This is a consequence of destructive interference among fields with different phases. For $g_0-1<M$, it is clear from Eq. (1) that an individual laser cannot emit without injection of the mean field. The mean field decreases with increasing Δ_0, and all lasers are quenched at some threshold value of Δ_0.

Another scenario for the breakdown of order is played out when the gain greatly exceeds the small-signal threshold value ($g_0-1>M$: curves for $g_0=1.5$ and $g_0=5$ in Fig. 1), and at some critical value the system moves from a stationary coherent emission regime into a nonstationary one. The nonstationarity is accompanied by a reduction in system brightness, due to time-averaging of the total field. At high pump levels in the active medium, the field amplitudes vary only slightly, and the brightness reduction is associated with time-averaging of the dynamical phase variations of the fields.

To determine the relationship between the critical values of Δ_0 and the system parameters, we consider the two cases in more detail.

a. $g_0-1<M$. When the small-signal gain is only slightly above threshold ($g_0-1<M$), the field amplitude in each laser decreases with increasing width of the frequency distribution, and is eventually completely quenched. It is easy to obtain a criterion for coherent emission when $g_0-1<M$, since the fields tend to zero at the quenching boundary ($g_k\equiv g_0$) and Eqs. (1) and (2) then constitute a linear system. The development of emission or quenching depends on the eigenvalues of the characteristic equation,

$$\begin{vmatrix} D_1-\lambda & M/N & \cdots & M/N \\ M/N & D_2-\lambda & \cdots & M/N \\ \cdots & \cdots & \cdots & \cdots \\ M/N & M/N & \cdots & D_N-\lambda \end{vmatrix}=0, \tag{4}$$

$$D_k=g_0-1-M+i\Delta_k+M/N. \tag{5}$$

Diagonalizing the matrix in Eq. (4), we obtain the equation for the eigenvalues λ:

$$\left(D_1-\lambda-\frac{M}{N}\right)\left(D_2-\lambda-\frac{M}{N}\right)\cdots\left(D_N-\lambda-\frac{M}{N}\right)$$
$$\times\left(\frac{N}{M}+\frac{1}{D_1-\lambda-M/N}+\frac{1}{D_2-\lambda-M/N}+\cdots+\frac{1}{D_N-\lambda-M/N}\right)=0. \tag{6}$$

Interestingly enough, the proof that band gaps exist in the electron energy spectrum of a superconductor in BCS theory[8] reduces to the solution of a similar eigenvalue problem. Given the distribution $f(\Delta)$ of eigenfrequency offsets normalized to the total number of lasers, $\int f(\Delta)\,d\Delta=N$, with $N\gg1$, we have that when $\mathrm{Re}(M)>0$, the eigenvalue with the largest real part $\lambda_{\max}$, which is the analog of the BCS ground-state energy, can be obtained from the integral equation

$$\int\frac{f(\Delta)d\Delta}{\lambda_{\max}-i\Delta-(g_0-1-M+M/N)}=\frac{N}{M}. \tag{7}$$

Equation (7) follows directly from the vanishing of the last factor in the dispersion relation. For a uniform random distribution of eigenfrequencies over the interval $[-\Delta_0/2, (\Delta_0/2)]$,

$$\lambda_{\max}=g_0-1-M+\frac{\Delta_0}{2}\cot\frac{\Delta_0}{2M}. \tag{8}$$

It can easily be shown that the requirement for emission to develop reduces to $\mathrm{Re}\lambda_{\max}>0$. Inserting (8), this requirement takes the form $\mathrm{Re}\lambda<0$ for other collective modes)

$$g_0-1>\mathrm{Re}\left[|M|e^{i\varphi}-\frac{\Delta_0}{2}\cot\frac{\Delta_0}{2|M|e^{i\varphi}}\right]. \tag{9}$$

Here $|M|$ and ϕ are the modulus and phase of the coupling coefficient.

b. $g_0-1>M$. In this second case, there is no field quenching with increasing Δ_0, since the gain in each laser is high enough to support emission in the absence of an additional optically coupled signal. As noted above, there is an attendant abrupt brightness reduction with increasing Δ_0 resulting from the development of nonstationary emission and variations in the relative phases of the fields. The transition from a coherent, steady-state regime to a nonstationary one takes place at some critical value of Δ_0.

We can estimate that critical value using perturbation theory, noting that when $g_0-1\gg M$, the field amplitudes of the various lasers are only slightly different. We assume steady-state fields of the form

$$E_k=A_k\exp i\Psi_k. \tag{10}$$

For a real coupling coefficient, we have the following equations for the amplitudes A_k and phases Ψ_k:

$$\dot{A}(\Delta_k)=[g(\Delta_k)-1-M]A(\Delta_k)+MCA(0)\cos\Psi(\Delta_k),$$
$$\dot{\Psi}(\Delta_k)=\Delta_k-MC\frac{A(0)}{A(\Delta_k)}\sin\Psi(\Delta_k), \tag{11}$$

where C is a constant common to all of the lasers, given by

$$CNA(0)=\sum_k E(\Delta_k). \tag{12}$$

The steady-state solution implied by (11) is

$$\sin\Psi(\Delta_k)+\frac{\Delta_kA(\Delta_k)}{MCA(0)}, \tag{13}$$

$$\cos\Psi(\Delta_k)=\frac{[1+M-g(\Delta_k)]A(\Delta_k)}{MCA(0)}. \tag{14}$$

The phase of the field differs from the phase of the total field by a constant; the larger the offset of the resonator eigenfrequency from the mean, the larger the constant. A steady-state regime is feasible as long as the maximum phase remains below $\pi/2$, so the stability limit for a steady-state solution is given by

$$\Psi(\Delta_m)=\frac{\pi}{2},\quad \Delta_m=\frac{\Delta_0}{2}. \tag{15}$$

To determine C we make use of Eq. (12), which in continuous variables takes the form

$$CA(0)=\frac{1}{2\Delta_m}\int_{-\Delta_m}^{\Delta_m}A(\Delta)\sqrt{1-\left[\frac{A(\Delta)\Delta}{A(0)MC}\right]^2}\,d\Delta. \tag{16}$$

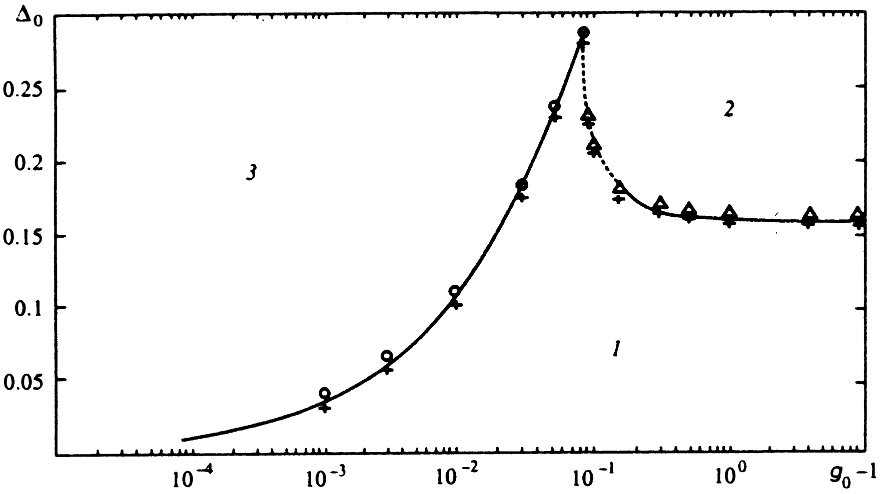

FIG. 2. Parameter domains in g_0 and Δ_0 for $M=0.1$ in which various emission regimes occur: 1) steady-state emission; 2) dynamical regimes; 3) quenching. Solid curves represent the analytic criteria of (9) and (20)–(23). Numerical results are indicated by $+$, $\triangle$, and $\bigcirc$ for the three cases above.

It is possible to obtain a solution of the foregoing equations in the limit $g_0-1\gg M$, for which the various lasers emit at approximately the same intensity. In zeroth order we can take $A(\Delta)\equiv A(0)$, and the condition for the onset of oscillations can be written

$$\Delta_m>\frac{\pi}{4}M. \tag{17}$$

To first order, the field intensity in a laser with frequency offset Δ can be approximated by the quadratic function

$$I(\Delta)=I(0)\left[1-\beta\left(\frac{\Delta}{\Delta_m}\right)^2\right]. \tag{18}$$

We have from (13) and (14) that

$$I(\Delta)\{\Delta^2+[1+M-g(\Delta)]^2\}=(MC)^2I(0), \tag{19}$$

and the value of $I(0)$ can be obtained by combining (3) and (14) at $\Delta=0$:

$$I(0)=\frac{g_0}{1+M(1-C)}-1. \tag{20}$$

Substituting (18) and (20) into (19), expanding in powers of $\beta(\Delta/\Delta_m)^2$, and equating first-order terms in $(\Delta/\Delta_m)^2$ to zero, we obtain

$$\beta=\left(\frac{\Delta_m}{MC}\right)^2\left\{1+\frac{2g_0I(0)}{MC[1+I(0)]^2}\right\}^{-1}. \tag{21}$$

Substituting the value of $I(\Delta)$ into (16), we have to second order in β

$$C=\frac{\pi}{4}\left(1-\frac{\beta}{4}\right). \tag{22}$$

Furthermore, (13) and (15) yield

$$\Delta_m=MC\sqrt{1-\beta}. \tag{23}$$

For $g_0-1\gg M$, the nonlinear equations (20)–(23) have a unique solution in the neighborhood of $\beta=0$, and can be solved iteratively.

The analytic criteria thus derived and numerical modeling of the emission from a large laser array with global feedback make it possible to map the various operating regimes. The solid curves in Fig. 2 are the analytic boundaries separating the various emission regimes (*1* corresponds to steady-state coherent emission, *2* to nonstationary emission, and *3* to quenching). The $+$, $\triangle$, and $\bigcirc$ markers correspond to emission with parameters g_0 and Δ_0 obtained by solving Eqs. (1) and (2) numerically.

3. NONSTATIONARY EMISSION BY AN ARRAY OF LASERS WITH GLOBAL OPTICAL FEEDBACK

The temporal behavior of the fields in an array of optically coupled lasers can be conveniently illustrated with an Argand phase diagram. As in Ref. 6, at any given time, the field of each laser—which is characterized by an amplitude and phase—can plotted as a point in the complex plane. Figure 3 shows phase diagrams of laser emission starting at the time of complete phase-locking for three sets of parameter values, corresponding to coherent emission (*1*, $M=0.1$, $g_0=1.5$, $\Delta_0=0.15$), temporally nonstationary field behavior (*2*, $M=0.1$, $g_0=1.5$, $\Delta_0=0.20$), and quenched emission (*3*, $M=0.1$, $g_0=1.001$, $\Delta_0=0.20$). In addition to the dynamical field of each laser in the Argand diagram, Fig. 3 shows the amplitude and phase of the mean field.

For steady-state coherent emission with a real coupling coefficient M, the Argand diagrams indicate the dynamics of emergence into a regime with constant brightness over the entire array from an initial state in which all the laser fields are phase-locked. The slight precession of the total field vector at constant angular velocity is due to the nonzero mean frequency mismatch in the random sample Δ_k. As the number of lasers N increases, the mean frequency of the field will be proportional to $N^{-1/2}$, and the precession frequency will decrease. If the coupling coefficient is complex $(Me^{i\phi})$, the angular velocity of the mean field vector in the steady-state coherent regime will be approximately $M\sin\phi$, and will be a weak function of the number of lasers.

When the active medium has a short relaxation time $(\tau\ll 2(g_0-1-M)/M^2C^2)$, the dynamical behavior of an array of lasers with global feedback, and with an average uniform distribution of relative phases of the laser fields on the unit circle in the Argand phase diagram, turns out to be statistically stable. The temporal average of the brightness of

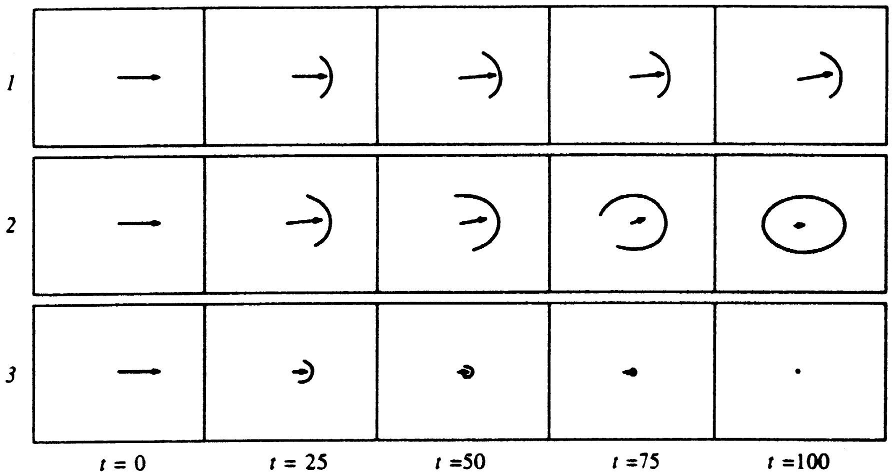

FIG. 3. Transition of a system out of a completely phase-locked state: 1) to synchronized emission ($M=0.1$, $g_0=1.5$, $\Delta_0=0.15$); 2) to a regime with time-dependent field behavior ($M=0.1$, $g_0=1.5$, $\Delta_0=0.20$); 3) quenching ($M=0.1$, $g_0=1.001$, $\Delta_0=0.20$).

the N lasers is then proportional to $1/N$ of the brightness of a perfectly phase-locked system, which is consistent with previous results.[6]

Two frequencies characterize the problem for parameter domain 2. One is dictated by the loss of stability of phase-locked steady-state emission, and by phase variations in some of the lasers relative to the mean field. Under these circumstances, the mean field is modulated, and that modulation is periodic in the vicinity of the boundary between parameter domains 1 and 2 (Fig. 2). The typical modulation period of the mean field has a form similar to that for two coupled lasers,[7]

$$T_{\mathrm{in}}=\frac{2\pi}{\sqrt{\Delta_0^2-M^2C^2(1-\beta)}}, \tag{24}$$

where C and β are obtained by solving (20)–(23).

The second characteristic time in this system is the period of damped relaxation oscillations. If the relaxation time of the active medium is much greater than the light transit time in the resonator, then $\tau\gg 1$ in the notation introduced above, and the frequency and decay constant of relaxation oscillations, w_c and γ, take the form

$$w_c=\sqrt{2(g_0-1-M)/\tau}, \tag{25}$$

$$\gamma=-g_0/2(1+M)\tau. \tag{26}$$

The former is usually called the Toda frequency, which refers to the effective potential to which the nonlinear oscillation problem reduces in a laser system.[9,10] For the parameter domain in which nonstationary emission takes place, the ratio of the two characteristic times can take on a range of values.

When $w_c T_{in}\gg 1$, we can assume the active medium to be delayless, and can therefore use Eqs. (1) and (3) to describe the dynamics. In this limiting case, the field dynamics can be reduced to a vector mapping model with global feedback,[1] taking the discrete time step to be equal to the modulation period of the mean field.

The most interesting emission regimes, however, are those in which the period of relaxation oscillations is close to the modulation period of the mean field. Since the global-feedback laser array under consideration is quite similar to nonlinear laser systems with an injected external signal[11] and Q-switched lasers,[12] we should expect it to display the same nonlinear and nonstationary modes of behavior inherent in those systems. In particular, period-doubling bifurcations and various paths to chaos, which are both typical of nonlinear dynamical systems, have been detected.

As an example, Fig. 4 shows emission with a modulated mean field, corresponding to the period-doubling case. The buildup of oscillations results in the main emitted pulse being produced in a time much shorter than the period. This situation is equivalent to laser emission with an injected external signal modulated at a multiple of the period of relaxation oscillations. Since the active media and resonator Q factors are assumed to be the same for all lasers in the array, the Toda frequencies will be the same as well. All of the laser fields will therefore become cooperatively phase-locked, regardless of significant differences among the eigenfrequencies. In this mode of operation, the peak brightness can be appreciably higher than the brightness of a completely phase-locked system (by a factor of 1.6 for the conditions illustrated in Fig. 4).

Cooperative phase-locking is attributable to a number of factors. The capture bandwidth of the laser fields rises with increasing external injection signal. The signal amplitude also increases as more and more fields are phase-locked. Another factor that enables brief pulses to reach high energies is the pumping of relaxation oscillations in the combined radiation/active medium system of the laser. Pumping of these oscillations can give rise to pulses whose peak intensity is much higher that in the steady state.

Cooperative phase-locking of fields consists in the following. If the width of the eigenfrequency mismatch distribution function is of the order of the Toda frequency and the active medium has a long relaxation time, instability can develop, leading to transition from an incoherent regime of laser operation (with vanishing order parameter) to phase-locked emission for a brief interval during pulsed periodic emission from each laser. The scenario for the development of instability is as follows. Modulation of the mean laser field at frequencies close to the Toda frequency leads to pumping of field fluctuations in each laser. This leads in turn to an increase in the number of in-phase lasers during pulse

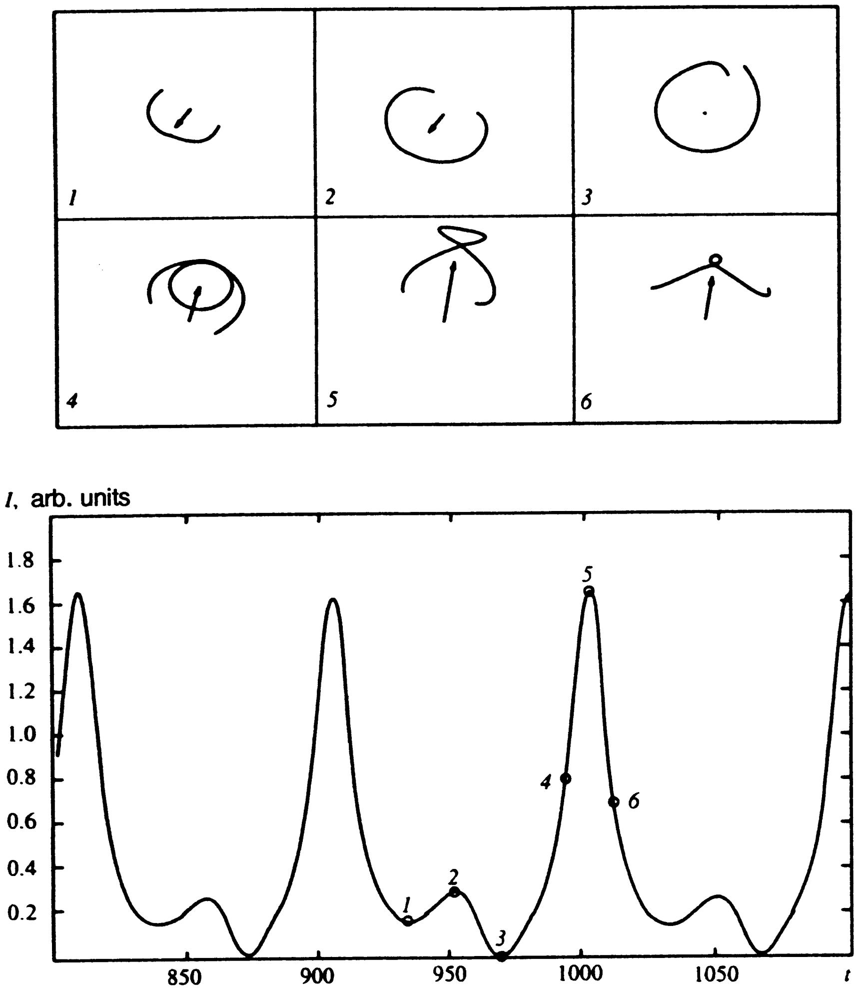

FIG. 4. Dynamics of total system brightness and laser fields for parameters corresponding to the peak in Fig. 1 ($g_0=1.5$, $M=0.1$, $\Delta_0=0.21, \tau=50$).

formation and, as a consequence, an increase in the depth of modulation of the total field, and so forth.

Figure 1 shows the dependence of the time-averaged brightness in the dynamical regime on the width of the frequency-mismatch distribution in the dynamical regime with $N=100$ lasers for $\tau=50$ (dashed curve) and for a delayless medium (dotted curve). With these parameters, the mean brightness attained when relaxation oscillations are pumped reaches approximately 40% of the brightness of a completely phase-locked system. This value is a relatively weak function of the number of lasers in the system, and is governed solely by the dimensionless parameters M, Δ_0, and g_0.

4. SUMMARY

In this paper, we have analytically and numerically investigated emission modes of a system consisting of a large number of optically coupled lasers with differing eigenfrequencies. We have considered a model in which each laser is optically coupled to every other. It has been shown that as the mismatch level throughout the system rises, it undergoes a transition from high-intensity coherent emission involving all of the lasers to emission with a significantly lower order parameter (the brightness of the total field divided by the maximum possible value). This transition can take place according to two different scenarios.

The first applies when the small-signal gain slightly exceeds a threshold value ($g_0-1\ll M$). In this case, increasing the mismatch level to the critical value leads to quenching of the emission.

The second scenario applies to the case $g_0-1\gg M$, in which an increase in the width of the mismatch distribution causes the breakdown of the coherent steady-state emission regime and a transition to dynamical regimes.

In an array of lasers with global optical coupling, a change in the parameters can result in bifurcations and a transition to chaos in the intensity of the total field. The dynamical behavior depends on time delays in the active medium. When they are large compared with the time of flight τ in the resonator, we encounter dynamical regimes in which the total field has high peak brightness due to resonant pumping of fluctuations at a multiple of the period of Toda relaxation oscillations. In this cooperative phase-locking regime, the mean and peak values of the order parameter depend weakly on the number of optically coupled lasers.

The breakdown of the coherent regime at high brightness for lasers with a spread in eigenfrequencies is analogous to the destruction of long-range order in a variety of physical problems: a superconducting state can be destroyed when the density of scatterers is high enough at zero temperature; sufficiently large irregularities in an external magnetic field in two-dimensional spin systems can lead to the disappearance of the total magnetic moment, etc. In contrast to these systems, however, the breakdown of order in an array of lasers can result from the complex dynamics of the laser fields.

The situation in which the laser eigenfrequencies fluctuate in time bears a striking resemblance to a thermodynamic phase transition induced by a temperature change. Here one can introduce a noise parameter analogous to the temperature; when it rises, a phase transition results.

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