Surface magnetic oscillations in a uniaxial antiferromagnet

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Low-frequency excitations in a uniaxial two-sublattice easy-axis antiferromagnet in a constant magnetic field directed along a selected axis are studied. It is shown that the equations of macroscopic electrodynamics can be used to analyze the magnetic and electromagnetic excitations in bounded magnetic materials. Attention is focused on the propagation of surface waves along the boundary of a sample occupying the half-space $y>0$ or a plate of thickness L.

1. INTRODUCTION: FORMULATION OF THE PROBLEM

The spectrum of low-frequency long-wavelength electromagnetic oscillations of a magnetically ordered dielectric can be investigated by analyzing the macroscopic Maxwell equations

$$\text{curl } \mathbf{h} = \frac{1}{c}\frac{\delta \mathbf{d}}{\delta t}, \quad \text{curl } \mathbf{e} = -\frac{1}{c}\frac{\delta \mathbf{b}}{\delta t}, \tag{1}$$

together with the constitutive equations which relate the induction vectors $\mathbf{d}$ and $\mathbf{b}$ to the electric and magnetic field intensity vectors $\mathbf{e}$ and $\mathbf{h}$ (lower case letters denote variable field intensities and inductions).

We shall be interested in the oscillations caused by the rotation of the magnetization vectors of the magnetic sublattices of an antiferromagnet. The corresponding frequencies are much lower, as a rule, than the atomic frequencies which determine the frequency dispersion of the permittivity. We can therefore assume

$$\mathbf{d} = \hat{\varepsilon}\mathbf{e}, \tag{2}$$

where $\hat{\varepsilon}$ is the static permittivity tensor of the medium. The magnetic susceptibility tensor μ_{ik} describes the dynamics of the magnetic moments of the sublattices:

$$b_i = \mu_{ij}(\omega, \mathbf{k}) h_j. \tag{3}$$

Here and below, ω is the frequency and $\mathbf{k}$ is the wave vector of the wave. The spatial dispersion of the components of the tensor μ_{ij} is caused by the existence of spin waves (magnons) which transport magnetic excitations through the sample.[1]

In the present paper, we shall study low-frequency excitations in a uniaxial two-sublattice easy-axis (EA) antiferromagnet in a uniform time-independent magnetic field $\mathbf{H}$ directed along a selected axis. The magnetic field alters the magnetic structure of the antiferromagnet.

In easy-axis antiferromagnets in a weak field H (where $H<H_{SF}$),$H_{HF} = M\sqrt{2\delta(\beta-\beta')}, M—$ the magnetic moments of the sublattices are antiparallel and directed along the axis (just as in the case $H=0$), and a first-order reorientational (spin-flop) phase transition occurs at $H=H_{SF}$: The magnetic moments of the sublattices flip over and occupy a symmetric position making an angle θ with the axis, where $\cos\theta = H/H_E, H_E = 2\delta M$. For fields $H=H_E \gg H_{SF}$ the magnetic sublattices "collapse," i.e., a second-order (spin-flip) phase transition occurs.

We have simplified somewhat the picture of the reorientational transition in antiferromagnets. In particular, taking into account the anisotropy constants of the next higher order can cause a first-order phase transition to split into two second-order phase transitions, or the first-order transition can acquire a hysteresis loop (a more detailed discussion is given in Ref. 2).

In antiferromagnets in which the exchange energy is much higher than the anisotropy energy, $H_E \gg H_{SF}$, the ratio H_{SF}/H_E is a small parameter (see discussion below). Figure 1 displays the equilibrium configurations of the magnetic moments of easy-axis antiferromagnets.

In bounded magnets the position of the magnetic moments relative to the plane of the sample is important. We shall assume that the magnetic moments are always parallel to the surface of the sample.

To complete the description of the geometry of the problem, we note that the wave propagates in the direction perpendicular to the symmetry axis (an exception is the picture described in Sec. 2). The coordinate axes are chosen as follows: The x-axis is oriented along the direction of propagation of the wave, the y-axis is oriented perpendicular to the surface, and the z-axis is oriented along the symmetry axis. We recall that $H_z=H$ and $H_x=H_y=0$ (see Fig. 2).

The magnetic susceptibility tensor of the antiferromagnet depends strongly on the magnetic configuration.[2] In all cases of interest to us we can take

$$\hat{\mu} = \begin{pmatrix} \mu_1 & i\mu' & 0 \\ -i\mu' & \mu_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

in addition, $\mu_1=\mu_2$ holds outside the interval (H_{SF}, H_E) and there is no gyrotropy ($\mu'=0$). The expressions for μ_1, μ_2, and μ' are presented in Table I; spatial dispersion and dissipation are neglected. Most of the notation is standard. We shall discuss in the concluding section the range of applicability of the results obtained by means of the expression (4).

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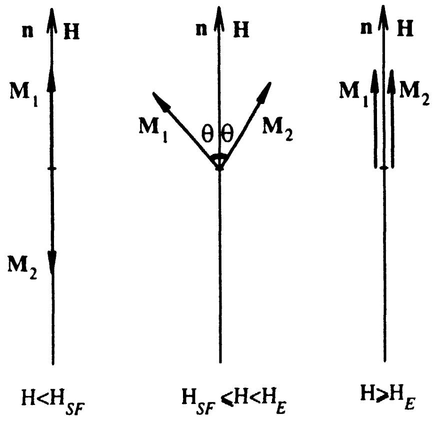

FIG. 1. Equilibrium states of an easy-axis antiferromagnet.

2. VOLUME POLARITON AND MAGNETOSTATIC OSCILLATIONS

In accordance with the title of this paper, we are concerned mainly with the propagation of surface waves along the boundary of a sample occupying the half-space $y>0$ or a plate of thickness L. First, however, we shall derive the dispersion law for volume magnetic polaritons. As a simplification we assume (and we shall adhere to this assumption throughout this paper) that the permittivity is isotropic ($\varepsilon_{ik}=\varepsilon\delta_{ik}$, where $\varepsilon>1$). The extension to the anisotropic case does not present any difficulties.

According to Eqs. (1)–(4), waves with two polarizations can propagate perpendicular to the axis in an antiferromagnet. In waves with one type of polarization e_y and h_z are different from zero, the dispersion relation does not contain the components of the magnetic susceptibility tensor, and

$$\omega=\frac{kc}{\sqrt{\varepsilon}}. \tag{5}$$

This is a bulk electromagnetic wave which is not associated with oscillations (rotations) of the magnetic moments of the sublattices. In waves with the other type of polarization e_z, h_x, and h_y are different from zero, and the dispersion relation contains μ_1, μ_2, and μ' in the combination

$$\mu_{\text{eff}}=\frac{\mu_1\mu_2-(\mu')^2}{\mu_1}. \tag{6}$$

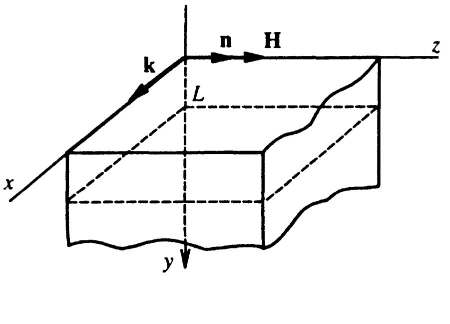

FIG. 2. Geometry of the problem.

The frequency ω as a function of the wave vector k is obtained by solving the dispersion relation

$$k^2=\frac{\omega^2}{c^2}\,\varepsilon\mu_{\text{eff}}(\omega). \tag{7}$$

This is an elementary excitation (quasiparticle), a volume magnetic polariton corresponding to an electromagnetic wave interacting with the oscillations (rotations) of the magnetic moments of the sublattices. It is obvious that volume magnetic polaritons exist at all frequencies for which $\mu_{\text{eff}}>0$.

The values of μ_{eff} for different cases are presented in the last column of Table I, and curves for a magnetic polariton which correspond to these cases are displayed in Fig. 3. The function $\omega(k)$ specific to a magnetic polariton is related to the resonance properties of μ_{eff}. In the limits $\omega\to 0$ and $\omega\to\infty$ the magnetic polariton "converts" into a photon:

$$\omega\simeq\frac{ck}{\sqrt{\varepsilon\mu_2}},\quad \omega\to 0;\quad \omega\simeq\frac{ck}{\sqrt{\varepsilon}},\quad \omega\to\infty. \tag{8}$$

Since $\mu_2(0)>1$ holds, the phase velocity satisfies $(\omega/k)_{\omega\to 0}<(\omega/k)_{\omega\to\infty}$. The effective magnetic susceptibility μ_{eff} is an even function of the frequency with a positive derivative $d\mu_{\text{eff}}/d\omega>0$.

In the limit $\omega\to 0$ we can write

$$\mu_{\text{eff}}|_{\omega\to 0}\simeq\mu(0)\left(1+\frac{\omega^2}{\omega_0^2}\right).$$

Hence there follows an expression for the phase velocity

$$c(\omega)|_{\omega\to 0}\simeq\frac{c}{\sqrt{\varepsilon\mu(0)}}\left(1-\frac{\omega^2}{2\omega_0^2}\right).$$

In the limit $\omega\to\infty$ we have

$$\mu_{\text{eff}}|_{\omega\to\infty}\simeq 1-\frac{\omega_\infty^2}{\omega^2},\quad c(\omega)|_{\omega\to\infty}\simeq\frac{c}{\sqrt{\varepsilon}}\left(1+\frac{\omega_\infty^2}{2\omega^2}\right).$$

Expressions for the frequencies ω_0 and ω_∞ and for $\mu(0)$ can be easily written out for all fields (see Table II).

The notation "$\omega\to\infty$" is conventional: The frequency ω must be much lower than the atomic frequencies (see above). Moreover, the magnetic permeability is of limited use: It is inapplicable at optical frequencies.[3] In the case at hand, however, there is no limitation problem: $\mu_{\text{eff}}\to 1$ as the frequency increases. Note that this is a very general property: The magnetic permeabilities employed in the theory of magnetically ordered media automatically "switch off" as the frequency ω increases.

The problem mentioned in the last paragraph may be important when the model of "rigid" magnetic sublattices, which is adopted here, is applied to magnets in which the magnetic atoms have anomalously low excited atomic states, the excitation corresponding to a rearrangement of the atomic shell such that the magnitude of the spin (magnetic moment) of the magnetic atom changes.

According to Eqs. (6) and (7), for $\mu'\neq 0$ the quasistatic limit ($kc/\omega\to\infty$) coincides with $\mu_1=0$. If we take into account the spatial dispersion of the magnetic permeability, the

TABLE I.

$H=0$	$\mu_1=\mu_2=\mu$ $\mu'\equiv 0$	$\mu=1+\frac{4\pi}{\delta}\frac{\Omega_{SF}^2}{\Omega_{SF}^2-\omega^2}$
$0<H<H_{SF}$	$\mu_1=\mu_2=\mu$ $\mu'\neq 0$	$\mu=1+\frac{2\pi}{\delta}\Omega_{SF}^2\frac{1}{\Omega_{SF}^2-(\Omega-\omega)^2}+\frac{1}{\Omega_{SF}^2-(\Omega+\omega)^2}$ $\mu'=\frac{2\pi}{\delta}\Omega_{SF}^2\frac{1}{\Omega_{SF}^2-(\Omega-\omega)^2}-\frac{1}{\Omega_{SF}^2-(\Omega+\omega)^2}$
$H_{SF}\leq H<H_E$	$\mu_1\neq\mu_2$ $\mu'\neq 0$	$\mu_1=1+\frac{\delta}{4\pi}\frac{\Omega_M^2\cos^2\theta}{\Omega_\theta^2-\omega^2}$ $\mu_2=1+\frac{4\pi}{\delta}\frac{\Omega_\theta^2}{\Omega_\theta^2-\omega^2}$ $\mu'=\frac{\Omega_M\omega\cos\theta}{\Omega_\theta^2-\omega^2}$
$H\geq H_E$	$\mu_1=\mu_2=\mu$ $\mu'\neq 0$	$\mu=1+\frac{\Omega_M\Omega}{\Omega^2-\omega^2}$ $\mu'=\frac{\Omega_M\omega}{\Omega^2-\omega^2}$
$H=0$	μ_{eff}	$1+\frac{4\pi}{\delta}\frac{\Omega_{SF}^2}{\Omega_{SF}^2-\omega^2}$
$0<H<H_{SF}$	μ_{eff}	$\frac{\left(\left(1+\frac{4\pi}{\delta}\right)\Omega_{SF}^2-\Omega^2-\omega^2\right)^2-4\Omega^2\omega^2}{(\Omega_{SF}^2-\Omega^2-\omega^2)\left(\Omega_{SF}\left(1+\frac{4\pi}{\delta}\right)-\Omega^2-\omega^2\right)-4\omega^2\Omega^2}$
$H_{SF}\leq H<H_E$	μ_{eff}	$\frac{\Omega_\theta^2\left(1+\frac{4\pi}{\delta}\right)+\Omega_M^2\cos^2\theta\left(1+\frac{\delta}{4\pi}\right)-\omega^2}{\Omega_\theta^2+\frac{\delta}{4\pi}\Omega_M^2\cos^2\theta-\omega^2}$
$H\geq H_E$	μ_{eff}	$\frac{(\Omega+\Omega_M)^2-\omega^2}{\Omega^2+\Omega\Omega_M-\omega^2}$

Note. $H_{SF}=\sqrt{2\delta(\beta-\beta')}$, $H_E=2\delta M$, $\Omega_{SF}=gH_{SF}$, $\Omega=gH$, $\Omega_M=8\pi gM$, $\Omega_\theta^2=(gM)^2[4\delta^2\cos^2\theta-2\delta(\beta-\beta')\sin^2\theta]$, $\cos\theta=H/H_E$, $\mu_0=1+\frac{4\pi}{\delta}$, $\delta\gg\beta$, $\beta'\sim 1$, $\beta-\beta'>0$.

frequency $\omega=\omega_s(k)$ of a spin wave propagating perpendicular to the magnetization (of course, for $ak\gg 1$, where a is the lattice constant) can be determined as a function of the wave vector k from the equation

$$\mu_1(\omega,k)=0. \tag{9}$$

Given the magnetic susceptibility and taking the spatial dispersion into account, we can of course determine the dispersion relation for spin waves propagating in an arbitrary direction from the equations of magnetostatics

$$\text{curl}\,\mathbf{h}=0,\quad \text{div}\,\mathbf{b}=0,\quad b_i=\mu_{ik}(\omega,\mathbf{k})h_k, \tag{10}$$

whence we immediately obtain

$$k_i\mu_{ik}(\omega,\mathbf{k})k_k=0, \tag{11}$$

and Eq. (9) in the particular case $\mathbf{k}=(k,0,0)$.

In the interval (H_{SF},H_E) the susceptibility components written out above (third row in Table I) do not pick up a branch of the oscillations, the rotations of the magnetic moments around the symmetry axis. For $k=0$ the characteristic frequency of such oscillations is zero.

We underscore the fact that Eqs. (9)–(11) and, consequently, their solutions are approximate. Because of gyrot-

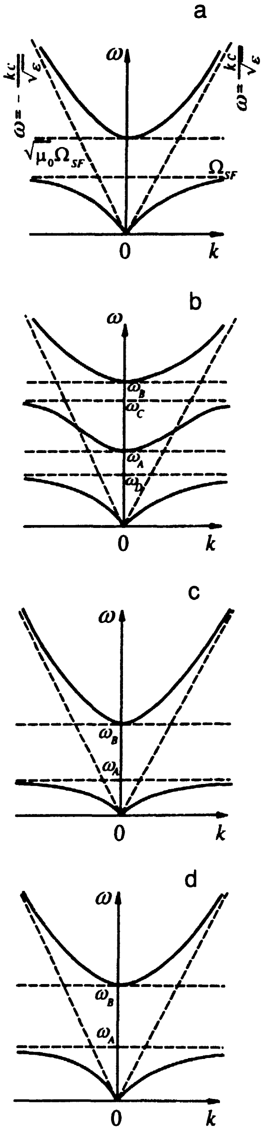

FIG. 3. Schematic plot of $\omega=\omega(k)$ for a volume magnetic polariton: a—$H=0$; b—$0<H<H_{SF}$, $\omega_A=\Omega_{SF}(1+2\pi/\delta)-\Omega$, $\omega_B=\Omega_{SF}(1+2\pi/\delta)+\Omega$, $\omega_C=\Omega_{SF}\times(1+\pi/\delta)+\Omega$, $\omega_D=\Omega_{SF}(1+\pi/\delta)-\Omega$; c—$H_{SF}\leq H<H_E$; $\omega_A=\sqrt{\Omega_\theta+(\delta/4\pi)\Omega_M^2\cos^2\theta}$, d—$H_E\leq H$, $\omega_A+\sqrt{\Omega^2+\Omega\Omega_M}$, $\omega_B=\Omega+\Omega_M$.

ropy, for $h_x\neq 0$ and $\mu_1=0$, the component $b_y\neq 0$ and the magnetic oscillations "engage" the electric field $\mathbf{e}$, which ultimately changes the dispersion law of the oscillations. The change introduced in the dispersion law for spin waves by the finiteness of the speed of light can be written out for $kc\gg\omega$. Near the frequency where $\mu_1(\omega,k)$ vanishes

$$\mu_{\text{eff}}\simeq\frac{\Omega_s^2}{\omega_s^2(k)-\omega^2}$$

and the "corrected" spin-wave frequency is

$$\omega_s(k)\simeq\omega_{s0}(k)\left\{1-\frac{\Omega_s^2}{2c^2k^2}\,\varepsilon\right\}. \tag{12}$$

Here, $\omega_{so}(k)$ is a root of Eq. (9), and Ω_s can be written out for all cases considered. The last formula is especially important, if ε has an appreciable imaginary part ε'', since it describes an additional, nonmagnetic mechanism of damping of spin waves. In the case of a semiconductor or metal we have $\varepsilon''=4\pi c\sigma/\omega$, where σ is the conductivity.

When there is no gyrotropy ($H=0$), we have $\mu_1=\mu_2=\mu$, $\mu'=0$, and $\mu_{\text{eff}}\equiv\mu$. The equation

$$\frac{1}{\mu(\omega,k)}=0 \tag{13}$$

corresponds to the quasistatic limit, and

$$\mu(\omega,k)=0 \tag{14}$$

follows from the equations of magnetostatics, the latter equation being exact, since for $h_x\neq 0$ and $b_x=\mu(\omega)h_x=0$ the electric field $\mathbf{e}\equiv 0$. We note that in Eqs. (7) and Eq. (13), which follows from them, $h_y\neq 0$. The longitudinal branch of the oscillations exists, strictly speaking, only for $H=0$ and, as we shall now show, only when the wave propagates in a direction perpendicular to the symmetry axis of the crystal [we recall that $\mu_3\equiv 1$; see Eq. (4)]. Indeed, if the wave propagates at an angle α with respect to the symmetry axis, there exist two magnetic polaritons with different polarization and the dispersion relations

$$k^2=\frac{\omega^2}{c^2}\,\varepsilon\mu,\quad k^2=\frac{\omega^2}{c^2}\,\varepsilon\frac{\mu}{\mu\sin^2\alpha+\cos^2\alpha}, \tag{15}$$

while the equations of magnetostatics give

$$\mu\sin^2\alpha+\cos^2\alpha=0. \tag{16}$$

Comparing Eqs. (15) and (16), we can see that for $\alpha\neq\pi/2$ the dispersion relation for a magnetostatic wave is identical to the limiting dispersion relation ($kc\to\infty,\omega\not\to\infty$) for one of the magnetic polaritons, but for $\alpha=\pi/2$ only one magnetic polariton exists [the first of Eqs. (15)] and the magnetostatic wave [Eq. (16)] "splits off" from the photon and Eq. (16) becomes exact:

$$\mu=0.$$

According to the first row of Table I, the frequency of a longitudinal magnetostatic wave is

$$\omega^2=\mu_0\Omega_{SF}^2,\quad \omega=\sqrt{\mu_0}\Omega_{SF}\,. \tag{17}$$

The frequency of the quasistatic wave (as the limit of a magnetic polariton) is $\omega_{ks}=\Omega_{SF}$, close to the frequency of the longitudinal wave. It should be remembered that the polarizations of the waves are substantially different: the wave (17) satisfies $h_x\neq 0$ and $h_y=h_z=0$, while a quasistatic wave satisfies $h_y\neq 0$ and $h_x=h_z=0$.

Figure 3 displays schematically the dispersion laws for a volume magnetic polariton for different values of the magnetic field. Table II gives the frequencies ω_{ks} in the magnetostatic limits.

We underscore the fact that in the present case the limit $H=0$ is nontrivial. Of course, in the case $H=0$ μ_{eff} from the second row of Table I is identical to μ_{eff} from the first row. We note, however, that in this case the factors $\mu_0\Omega_{SF}^2-\Omega^2-\omega^2$ in the numerator and denominator cancel. As a result of this, the number of solutions (the number of values of the magnetostatic limit) decreases.

TABLE II.

$H=0$	$\omega_0^2=\Omega_{SF}^2\dfrac{\mu_0}{\mu_0-1}$	$\omega_\infty^2=\Omega_{SF}^2(\mu_0-1)$
$0\leq H<H_{SF}$	$\omega_0^2=\dfrac{(\Omega_{SF}^2-\Omega^2)(\mu_0\Omega_{SF}^2-\Omega^2)}{\Omega_{SF}^2(1+\mu_0)+2\Omega^2}$	$\omega_\infty^2=2\Omega^2+\Omega_{SF}^2(1+\mu_0)$
$H_{SF}\leq H<H_E$	$\omega_0^2=\Omega_\theta^2+\dfrac{\delta}{4\pi}\Omega_M^2\cos^2\theta$	$\omega_\infty^2=\Omega_\theta^2+\dfrac{\delta}{4\pi}\Omega_M^2\cos^2\theta$
$H\geq H_E$	$\omega_0^2=\Omega^2+\Omega\Omega_M$	$\omega_\infty^2=\Omega^2+\Omega\Omega_M$
$H=0$	$\mu(0)=\mu_0$	$\omega_{ks}^2=\Omega_{SF}^2$
$0<H\leq H_{SF}$	$\mu(0)=\dfrac{\mu_0\Omega_{SF}^2-\Omega^2}{\Omega_{SF}^2-\Omega^2}$	$\omega_{ks}^2=\Omega^2+\Omega_{SF}^2\dfrac{1+\mu_0}{2}\times \times\left(1\pm\dfrac{4\Omega^2+(\mu_0-1)^2\Omega_{SF}^2}{\Omega_{SF}^2(\mu_0^2-1)}\right)$
$H_{SF}\leq H<H_E$	$\mu(0)=\dfrac{\mu_0\Omega_\theta^2+\dfrac{\delta}{4\pi}\Omega_M^2\cos^2\theta}{\Omega_\theta+\dfrac{\delta}{4\pi}\Omega_M^2\cos^2\theta}$	$\omega_{ks}^2=\Omega_\theta+\dfrac{\delta}{4\pi}\Omega_M^2\cos^2\theta$
$H\geq H_E$	$\mu(0)=\dfrac{(\Omega+\Omega_M)^2}{\Omega^2+\Omega\Omega_M}$	$\omega_{ks}^2=\Omega^2+\Omega\Omega_M$

3. SURFACE MAGNETOSTATIC OSCILLATIONS

In any type of magnetic material the surface oscillations in the half-space $y>0$ can be described by the homogeneous magnetostatic equations

$$\mathrm{curl}\,\mathbf{h}=0,\quad \mathrm{div}\,\mathbf{h}=0,\quad y<0, \tag{18}$$

$$\mathrm{curl}\,\mathbf{h}=0,\quad \mathrm{div}\,\mathbf{b}=0,\quad b_i=\mu_{ik}(\omega)h_k,\quad y>0,$$

together with the boundary conditions

$$\mathbf{h}\Big|_{|y|\to\infty}=0,\quad \mathbf{h}_\tau\Big|_{y=-0}=\mathbf{h}_\tau\Big|_{y=+0},$$

$$b_y\Big|_{y=-0}=b_y\Big|_{y=+0}, \tag{19}$$

where $\mathbf{h}_\tau$ is a two-dimensional vector with the components h_x and h_z. Here, a fundamental point is that the spatial dispersion of the components of the magnetic susceptibility tensor must be neglected, because otherwise additional boundary conditions would have to be imposed (see, for example, Ref. 4).

We now introduce the magnetic potential φ according to the formula $\mathbf{h}=-\nabla\varphi$. To satisfy Eqs. (18) and all boundary conditions besides the last one, we must have

$$\varphi=\begin{cases}\varphi_0e^{|k|y+ikx}, & y<0,\\ \varphi_0e^{-\gamma y+ikx}, & y>0,\end{cases} \tag{20}$$

$$\gamma=|k|\sqrt{\frac{\mu_1(\omega)}{\mu_2(\omega)}}>0.$$

The requirement that the normal component of the induction vector be continuous gives the following dispersion relation for determining the frequency of a surface magnetostatic wave:

$$1+\mathrm{sign}\,k\cdot\mu'(\omega)+\sqrt{\mu_1(\omega)\mu_2(\omega)}=0. \tag{21}$$

The expression for the logarithmic damping coefficient γ shows that a surface wave can exist in the frequency range where $\mu_1(\omega)/\mu_2(\omega)>0$ holds. In the isotropic case (for $\mu_1=\mu_2$; see Table I) there is no such restriction, since $\gamma=|k|$ and Eq. (21) simplifies:

$$1+\mathrm{sign}k\cdot\mu'(\omega)+\mu(\omega)=0. \tag{22}$$

For $\mu'\neq0$ (gyroptropy) the wave is nonreciprocal: $\omega(k)\neq\omega(-k)$. This property, which is well known for ferromagnets,[5] arises in antiferromagnets only if $H\neq0$. For $H=0$ there is no gyrotropy ($\mu'\equiv0$), and irrespective of the direction of propagation of the wave (sign of k) the frequency of the wave (which we denote by ω_s) is

$$\omega_s=\sqrt{\frac{1+\mu_0}{2}}\Omega_{SF}. \tag{23}$$

Nonreciprocity appears for $0<H\leqslant H_{SF}$:

$$\omega_s=\sqrt{\frac{1+\mu_0}{2}}\Omega_{SF}\mp\Omega\,\mathrm{sign}\,k. \tag{24}$$

The anisotropy of the magnetic susceptibility complicates the problem, and for this reason for $H_{SF}<H\leqslant H_E$ Eq. (21) gives

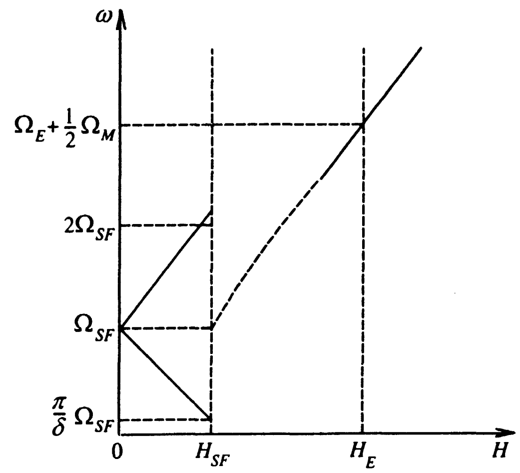

FIG. 4. Schematic plot of the frequency of a surface magnetostatic wave versus the magnetic field.

a cubic equation for ω. We shall not analyze this equation. We merely note that for $H_{SF} \ll H < H_E$ the anisotropy can be neglected and

$$\mu_1 \simeq \mu_2 \simeq \frac{\Omega^2\mu_0 - \omega^2}{\Omega^2 - \omega^2}, \quad \mu' = (\mu_0 - 1)\frac{\Omega\omega}{\omega^2 - \Omega^2}. \tag{25}$$

Hence, according to Eq. (22), $\omega_s = \Omega$ and it would seem that the wave exists for both $k > 0$ and $k < 0$. A more accurate analysis shows that for $k < 0$ the exact equation (21) has no solution. Moreover, it can be shown that the frequency ω_s is continuous at the point $H = H_E$.

Finally, for $H \geqslant H_E$ a surface magnetostatic wave exists only for $k > 0$ and

$$\omega_s = \tfrac{1}{2}\Omega_M + \Omega. \tag{26}$$

This is the standard Damon—Eshbach wave[5] in a ferromagnet with a magnetic moment $2M$ per unit volume. The function $\omega_s = \omega_s(H)$ is displayed schematically in Fig. 4.

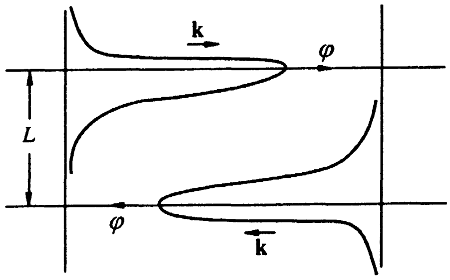

FIG. 5. Quasistatic oscillations in a plate.

4. QUASISTATIC OSCILLATIONS IN A PLATE

The spectrum of oscillations in an antiferromagnetic plate is very complicated. Here, we shall not make a complete and detailed investigation of the spectrum. We confine our attention to determining how a surface wave concentrated near one boundary ($y = 0$) is modified by the presence of a second boundary ($y = L$).

Interference of the waves reflected from the boundaries of the plate causes the frequency of the wave to depend on the wave vector, and as a result, the group velocity of the wave is different from zero ($v_s = \partial\omega_s/\partial k \neq 0$).

We recall that if the spatial dispersion and retardation are neglected, the frequency of quasistatic oscillations does not depend on the wave vector and hence such oscillations do not carry energy ($v_s = 0$). So as not to complicate the analysis, we confine our attention to the isotropic case, i.e., we exclude the interval of fields (H_{SF}, H_E). As shown in the preceding section, for $H_{SF} \ll H < H_E$ the anisotropy is small and the expressions obtained here describe (albeit, only approximately) this interval also.

If we can assume $\mu_1 = \mu_2$, then the magnetic potential φ in the plate is a superposition of functions of the form

$$e^{ikx - ky}, \quad e^{ikx + ky}.$$

TABLE III.

H	$\mu'/\mu_1 = 0$
$H = 0$	0
$0 < H \le H_{SF}$	$-\frac{8\pi}{\delta}\Omega\Omega_{SF}^2\omega \times \times \left[\omega^2 - \Omega^2 + \Omega_{SF}^2\frac{1+\mu_0}{2}\left(1 - \frac{\mu_0 - 1}{\mu_0 + 1} + \frac{1}{\mu_0 - 1}\frac{\Omega^2}{\Omega_{SF}^2}\right)\right]^{-1} \times \times \left[\omega^2 - \Omega^2 + \Omega_{SF}^2\frac{1+\mu_0}{2}\left(1 + \frac{\mu_0 - 1}{\mu_0 + 1} + \frac{1}{\mu_0 - 1}\frac{\Omega^2}{\Omega_{SF}^2}\right)\right]^{-1}$
$H_{SF} \le H \le H_E$	$\dfrac{\Omega_M\omega\cos\theta}{\Omega_\theta^2 + \frac{\delta}{4\pi}\Omega_M^2\cos^2\theta - \omega^2}$
$H \ge H_E$	$\dfrac{\Omega_M\omega}{\Omega(\Omega_M + \Omega) - \omega^2}$

The potential outside the plates decays exponentially with damping rate $|k|$. If all the required boundary conditions are satisfied, we can easily derive the following dispersion relation:

$$\frac{(1+\mu)^2-(\mu')^2}{(1-\mu)^2+(\mu')^2}=e^{-2|k|L}, \tag{27}$$

where L is the thickness of the plate. It is obvious that Eq. (27) describes an invertible wave. In the limit $|k|L\to\infty$ the equation decomposes into two equations [compare with Eq. (22)]:

$$1+\mu+\mu'=0, \quad 1+\mu-\mu'=0. \tag{28}$$

Each equation describes a surface wave on one side of the plate. A wave propagating in the positive direction along the x-axis is concentrated near $y=0$ and a wave propagating in the negative direction is concentrated near $y=L$ (see Fig. 5). The direction of propagation is determined by the vector $[\mathbf{kn}]$, where the vector $\mathbf{n}$ is normal to the surface of the sample.

For $|k|L\ll 1$ the potential φ and hence also $\mathbf{h}$ and $\mathbf{m}$ are virtually uniform over the thickness of the plate. Such a wave can still be regarded as a surface wave, since it decays exponentially away from the surface of the plate (along the y-axis). The formulas for $H=0$ are especially simple. The equation determining $\omega=\omega(k)$ becomes

$$\mu(\omega)=-\sinh(|k|L/2), \tag{29}$$

whence, according to Table I,

$$\omega^2=\Omega_{SF}^2\left(1+\frac{4\pi}{\delta}\,\frac{1}{1+\sinh(|k|L/2)}\right). \tag{29'}$$

One can see that for any value of $|k|L$ the frequency of the wave is near the frequency of a uniform oscillation of the magnetic moment. For $0<H<H_{SF}$ there exist two frequencies of uniform oscillations and, correspondingly, two surface waves in the plate. It is easy to calculate $v=\partial\omega/\partial k$ from Eq. (29′) or similar formulas for other field intervals. In order of magnitude $v\simeq\Omega_{SF}L/\delta$ for $|k|L\simeq 1$.

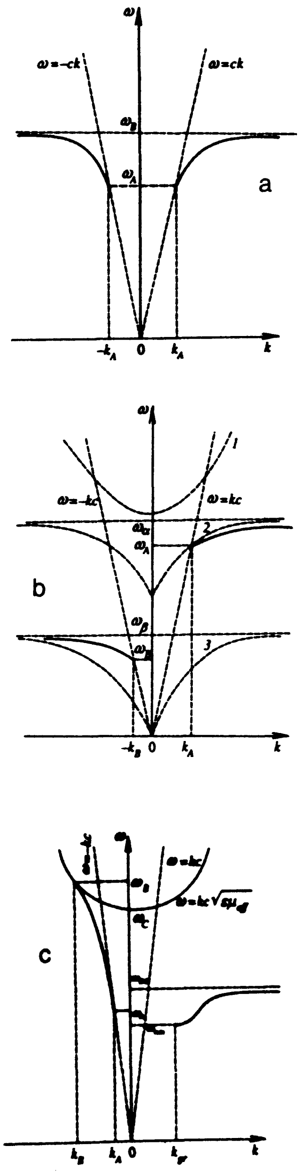

FIG. 6. Schematic plot of for a surface magnetic polariton: a—$H=0$, $\omega_A=\Omega_{SF}\sqrt{(\mu_0-1)/(\varepsilon-1)}$, $k_A=\Omega_A/c$, $\omega_B=\Omega\sqrt{(\mu_0+1)/2}$; b—$0<H_0\leqslant H_{SF}$, *1,2,3*—$\gamma=0$, $\omega_{\alpha,\beta}=\Omega_{SF}(1+\pi/\delta)\pm\ \Omega$, $\omega_{A,B}=\Omega_{SF}(1-(2\pi/\delta)\varepsilon/(\varepsilon-1)\pm\Omega$, $k_A=\Omega_A/c$, $k_B=\Omega_B/c$; c—$H_E\leqslant H$, $\omega_A^2=\Omega(\Omega+\Omega_M)+(\Omega\Omega_M)/(\varepsilon-1)$, $k_A=-\Omega_A/c, \omega_B^2\approx\varepsilon/(\varepsilon-1)(\Omega^2+\Omega\Omega_M)-1/2(\varepsilon-1)(\Omega+\varepsilon_M)(\Omega+2\Omega_M)+\ \Omega_M/2(\varepsilon-1)\ \sqrt{4\varepsilon^2\Omega(\Omega+2\Omega_M)-2\varepsilon\Omega(\Omega+\Omega_M)+(\Omega+\Omega_M)^2}$, $k_B=-\Omega_B/c$, $k_{gr}=(\Omega+\Omega_M)/c$, $\sqrt{\Omega/\Omega_M}$, $\omega_C=\Omega+\Omega_M$, $\omega_{DE}=\Omega+\Omega_M/2$, $\omega_{\min}=\sqrt{\Omega(\Omega+\Omega_M)}$.

5. SURFACE MAGNETIC POLARITON

The solutions of the complete system of Maxwell equations (1) together with the constitutive equations (2) and (3) for a half-space include solutions which decay exponentially in both directions from the boundary (along the y-axis) and represent a traveling wave ($\propto e^{ikx}$) along the x-axis. According to Eq. (1) the penetration depth in a magnet is

$$\gamma=\sqrt{k^2-\frac{\omega^2}{c^2}\,\varepsilon\mu_{\text{eff}}}, \tag{30}$$

and in vacuum

$$\gamma_0=\sqrt{k^2-\frac{\omega^2}{c^2}}. \tag{31}$$

The continuity of the tangential components of the vectors $\mathbf{e}$ and $\mathbf{h}$ makes it possible to derive a dispersion relation which establishes a relation between ω and k:

$$\gamma-\frac{\mu'}{\mu_1}\,k+\mu_{\text{eff}}\gamma_0=0. \tag{32}$$

All quantities appearing in the expressions (30)–(32) are defined in Table I.

The formulas (31) and (32) show that surface magnetic polaritons exist only if

$$k^2\geqslant\frac{\omega^2}{c^2}\,\varepsilon\mu_{\text{eff}}(\omega), \quad k^2\geqslant\frac{\omega^2}{c^2}. \tag{33}$$

Analysis shows, however, that the limiting points at which a magnetic polariton exists do not always lie on the curves $\gamma_0(\omega,k)=0$ and $\gamma(\omega,k)=0$.

In Eq. (32) the coefficient of the term linear in k includes the ratio μ'/μ_1, which is different from zero when $H\neq 0$. In a magnetic surface polariton, just as in a magnetostatic surface wave, nonreciprocity occurs only when $H\neq 0$. The values of μ'/μ_1 for different values of the magnetic field are presented in Table III.

Figures 6a–c display the results of an analysis of Eq. (32). The schematic dependence of the frequency of a surface magnetic polariton on the wave vector is the same in the field intervals $H_{SF}<H\leqslant H_E$ and $H\geqslant H_E$. We have confined our attention to presenting this dependence and the values of the characteristic frequencies and wave vectors for the interval $H\geqslant H_E$, since in the interval $H_{SF}<H\leqslant H_E$ the expressions for the characteristic frequencies and wave vectors are often very complicated.

Surface magnetic polariton were first studied, we believe, in Ref. 4. In those cases when a magnetic-field-induced static magnetic moment $(H>H_{SF})$ is present in the antiferromagnet, the dispersion relation for a magnetic polariton in an antiferromagnet is very similar to the that for a magnetic polariton in a ferromagnet.

Two circumstances require additional comments.

1) Some expressions for the characteristic points contain $\varepsilon-1$ in the denominator (see, for example, the values of $\omega_{A,B}$ in the caption to Fig. 6b and c) and they do not admit the limit $\varepsilon\rightarrow 1$. Moreover, in the analysis we assumed $\varepsilon-1\gg 1/\delta$. The case $\varepsilon-1\ll 1/\delta$ requires a special analysis (compare Refs. 4 and 6).

2) For $k=k_{gr}$ (see Fig. 6 c) the penetration depth γ^{-1} vanishes and the macroscopic analysis is formally inapplicable. As shown in Ref. 4, taking into account the spatial dispersion of the magnetic permeability changes the numerical values of the parameters very little, so that the macroscopic approach is justified.

CONCLUSIONS

The main objective of the present analysis was to show that the equations of macroscopic electrodynamics can be used to analyze the magnetic and electromagnetic excitations in bounded magnets. If the structure of the magnetic material is more complicated (if several sublattices are present, and so on) the frequency dependence of the components of the tensor μ_{ik} will, of course, be more complicated. As a result, numerical analysis, which is made feasible by the existence of computers, may be required.

For $H\gg H_{SF}$ the properties of antiferromagnets are close to those of ferromagnets. This also came out in our analysis (see above). For $H\simeq H_{SF}$, when the distinguishing properties of antiferromagnets are especially clearly manifested, all characteristic frequencies are close to one another (their differences are of order $(4\pi/\delta)\ \Omega_{SF}\ll\Omega_{SF}$), and different types of excitations can be identified not so much according to the values of the characteristic frequencies as according to the properties of the characteristic modes (polarization, nonreciprocity, and so on).

Neglecting dissipative processes, of course, imposes stringent requirements on the quality of a sample and on the temperature: The differences $\Delta\omega$ of the characteristic frequencies must be greater than the inverse lifetime of the oscillations: $\Delta\omega>1/\tau$. If $\delta\gg 1$ holds, the stronger inequality $\omega\tau\gg\delta$ probably must be satisfied. However, we did not specially analyze the role of dissipative processes.

One would like to think that the results obtained in this paper will be used in experimetal investigations of antiferromagnets. It seems to us that surface waves can be observed by investigating scattering from the surface of a magnet (see, for comparison, Ref. 7) and the nonreciprocity of the waves for $H\neq 0$ $[\omega(k)\neq\omega(-k)]$ provides a reliable method of identifiation.

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